Expressions Using Letter-Numbers

i. Definition and Explanation

In mathematics, we often need to work with numbers that we don't know yet, or numbers that can change. Instead of using a blank space or a question mark, we use letters. These "letter-numbers" are a fundamental part of algebra.

What is an Algebraic Expression?

An algebraic expression (or an "expression using letter-numbers") is a mathematical phrase that combines numbers, letter-numbers (called variables), and operation signs (+, −, ×, ÷).

- **Think of it like a recipe:** It tells you the steps to combine ingredients (numbers and variables), but it doesn't give you the final result until you know the value of your variables.

- **Key Idea:** An expression does not have an equals sign (=). It's a phrase, not a complete sentence.

- x + 5 is an expression.

- x + 5 = 12 is an equation.

Example: Imagine you earn money for chores. You get a fixed amount of 10, plus10, plus3 for every hour you work. We can represent this as: 10 + 3h Here, h is the letter-number (variable) that stands for the number of hours you work.

ii. Key Points and Important Terms

To understand expressions, you need to know the language of algebra.

**Variable:** A letter that represents an unknown number or a number that can change.

Examples: x, y, a, n, t

Constant: A number that stands on its own and does not change. Its value is fixed.

Examples: In x + 7, the constant is 7.

Coefficient: The number that is multiplied by a variable. It's written right in front of the letter.

Examples: In 5y, the coefficient is 5. In $\frac{k}{3}$ (which is the same as $(\frac{1}{3})$ k), the coefficient is $\frac{1}{3}$. If you just see x, the coefficient is 1 (because 1x = x).
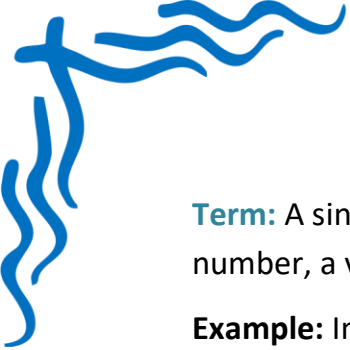
Term: A single part of an expression, separated by + or − signs. A term can be a number, a variable, or a product of numbers and variables.

Example: In the expression 4x - y + 8, there are three terms: 4x, y, and 8.

Operator: The symbol that indicates the operation to be performed (+, −, ×, ÷).

iii. Detailed Examples with Solutions

There are two main skills you need: writing expressions and evaluating them.

A. Writing Expressions from Words

Example 1: "The sum of a number a and 15."

> **Solution:** "Sum" means addition. So, the expression is a + 15.

Example 2: "7 less than a number p."

> **Solution:** "Less than" means you subtract from the number. So, the expression is p - 7.

Example 3: "Twice a number x, increased by 5."

> **Solution:** This has two steps.
>
> "Twice a number x" means 2x.
>
> "increased by 5" means add 5.
>
> The final expression is 2x + 5.

B. Evaluating Expressions

To evaluate an expression, you replace the variable with a given number and then calculate the result. Always remember the order of operations (PEMDAS/BODMAS)!
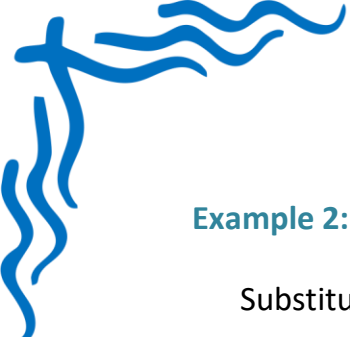
- Parentheses / Brackets
- Exponents / Orders
- Multiplication and Division (from left to right)
- Addition and Subtraction (from left to right)

Example 1: Evaluate 3y + 4 if y = 5.

> Substitute. Replace y with 5. 3(5) + 4
>
> Calculate. Follow PEMDAS (Multiplication first). 15 + 4 19
>
> **Solution:** The value of the expression is 19.

Example 2: Evaluate $\frac{m}{3} - 2$ if m = 21.

Substitute. Replace m with 21. $\frac{21}{3} - 2$

Calculate. Follow PEMDAS (Division first). 7 - 2 5

Solution: The value of the expression is 5.

iv. Summary of Main Concepts

- Algebraic Expressions use letters (variables) to represent unknown or changing numbers.

- They are made of terms, which include variables, coefficients, and constants.

- Expressions are like recipes; they don't have an equals sign.

- **Writing Expressions:** Translate keywords for addition, subtraction, multiplication, and division into mathematical symbols.

- **Evaluating Expressions:** Substitute the given number for the variable and use the Order of Operations (PEMDAS/BODMAS) to find the final value.

- Be careful with tricky phrases like "less than" and always multiply a coefficient and variable before adding or subtracting.