Removing Brackets (Simplifying Expressions)

i. Definition and Explanation

What is Simplifying an Expression? Simplifying an algebraic expression means rewriting it in the most compact and efficient way, without changing its value. This usually involves removing any brackets (parentheses) and combining "like terms".

What are Brackets? Brackets, ( ), are used in mathematics to group terms together. Think of them as a package. To work with the individual items inside the package, you first need to "unwrap" or "remove" the brackets.

The core mathematical rule we use to remove brackets is called the Distributive Property.

The Distributive Property This property states that multiplying a term by a group of terms in a bracket is the same as multiplying it by each term inside the bracket individually.

- In symbols: a(b + c) = ab + ac

ii. Key Points and Important Terms

Term: A single number, a variable, or numbers and variables multiplied together. Terms are separated by + or - signs.

Example: In 5x + 3y - 8, the terms are 5x, 3y, and -8.

Coefficient: The number that is multiplied by a variable.

Example: In the term 5x, the coefficient is 5.

Like Terms: Terms that have the exact same variable part (the same letters raised to the same power).

Examples: 7x and 2x are like terms. 5y and -3y are like terms.

Non-Examples: 7x and 7y are not like terms. 4x and $4x^2$ are not like terms.

The "Invisible 1": When you see a - sign in front of a bracket, like -(x + 5), it's helpful to think of it as -1. So, -(x + 5) is the same as -1(x + 5). This makes the multiplication rule clearer.

iii. Detailed Examples with Solutions (The Rules of Removing Brackets)

We can break down removing brackets into four main scenarios.

Rule 1: A Plus Sign + Before the Bracket

If there is a + sign (or no sign) before a bracket, you can remove the brackets without changing the signs of the terms inside.

Rule: + (a + b) = a + b and + (a - b) = a - b

Example: Simplify 7 + (x - 3)

The sign before the bracket is +, so we remove the brackets and the signs inside stay the same. 7 + x - 3

Combine like terms (7 and -3). x + 7 - 3

Solution: x + 4

Rule 2: A Minus Sign - Before the Bracket

If there is a - sign before a bracket, you must change the sign of every term inside the bracket when you remove it.

Rule: -(a + b) = -a - b and -(a - b) = -a + b

Example: Simplify 10 - (2y + 4)

The sign before the bracket is -. We change the sign of +2y to -2y and +4 to -4. 10 - 2y - 4

Combine like terms (10 and -4). -2y + 10 - 4

Solution: -2y + 6

Rule 3: A Number or Variable Before the Bracket (Distribution)

Multiply the term outside the bracket by every term inside the bracket.

Rule: k(a + b) = ka + kb

Example: Simplify 5(3x + 2)

Distribute the 5 to the 3x. 5 × 3x = 15x

Distribute the 5 to the +2. 5 × 2 = 10

Combine the results. 15x + 10

Solution: 15x + 10 (Cannot be simplified further as they are not like terms).

Rule 4: A Negative Number Before the Bracket

Distribute the negative number to every term inside, paying close attention to the sign rules (negative × positive = negative, negative × negative = positive).

Rule: -k(a - b) = -ka + kb

Example: Simplify -4(2m - 3)

Distribute the -4 to the 2m. -4 × 2m = -8m

Distribute the -4 to the -3. -4 × -3 = +12 (A negative times a negative is a positive!)

Combine the results. -8m + 12

Solution: -8m + 12

iv. Summary of Main Concepts

- **Goal:** To simplify expressions by removing brackets and combining like terms.
- **The Golden Rule:** The term outside the bracket must be applied to every term inside the bracket.
- **Key Rules Checklist:**
    1. + (…): Keep signs the same. Remove brackets.
    2. - (…): Flip/change all signs inside. Remove brackets.
    3. k(…): Multiply k by everything inside.
- **Final Step:** After removing all brackets, always scan the expression for any like terms that can be added or subtracted to make it even simpler.