Reading and Evaluating Complex Expressions

i. Definition and Explanation

What is a Numerical Expression? A numerical expression is a mathematical phrase that combines numbers and one or more operation symbols (+, -, ×, ÷).

- Simple Expression Example: 5 + 3

- Complex Expression Example: $15 + [(9 - 3) \times 2^2] \div 4$

What makes an expression "complex"? An expression becomes complex when it involves:

- Multiple operations.

- Grouping symbols like parentheses ( ), brackets [ ], or braces { }.

- Exponents (powers).

- Fractions and decimals.

What does it mean to "evaluate" an expression? To evaluate an expression means to find the single numerical value it represents. To do this correctly, we must perform the operations in a specific, universally agreed-upon order. This is called the Order of Operations.

ii. Key Points and Important Terms

The key to evaluating complex expressions is the Order of Operations. We use a mnemonic to remember this order: PEMDAS.

PEMDAS stands for:

P - Parentheses (and other Grouping Symbols)

- Always solve what's inside grouping symbols first.

- This includes parentheses ( ), brackets [ ], and braces { }.

- If there are nested groups (one inside another), work from the innermost group outward.

E - Exponents

- After handling groups, evaluate any exponents (powers).

- Example: $3^2 = 3 \times 3 = 9$

MD - Multiplication and Division

- Perform all multiplication and division from LEFT to RIGHT, in the order they appear.

- These two operations have equal priority. Don't do all multiplication before all division.

AS - Addition and Subtraction

- Finally, perform all addition and subtraction from LEFT to RIGHT, in the order they appear.

- These two operations also have equal priority.

Important Terminology:

- **Expression:** A mathematical phrase without an equals sign.

- **Evaluate:** To find the value of, or "solve," an expression.

- **Operation:** An action like addition, subtraction, multiplication, or division.

- **Grouping Symbols:** ( ), [ ], { } used to control the order of operations.

- **Exponent:** A number indicating how many times to multiply the base by itself.

iii. Detailed Examples with Solutions

Example 1: Basic PEMDAS

Evaluate: $20 - (4 + 2) \times 3^2$

P (Parentheses): Solve the operation inside the parentheses. $20 - (6) \times 3^2$

E (Exponents): Evaluate the exponent. $3^2 = 9$. $20 - 6 \times 9$

M (Multiplication): Perform the multiplication. $6 \times 9 = 54$. $20 - 54$

S (Subtraction): Perform the final subtraction. -34

Final Answer: -34

Example 2: Nested Groups and Division

Evaluate: $100 \div [(8 - 3) \times 2] + 7$

P (Parentheses - Innermost): Start with the innermost group $(8 - 3)$. $100 \div [(5) \times 2] + 7$

P (Brackets): Now solve the operation inside the brackets $[5 \times 2]$. $100 \div 10 + 7$

D (Division - Left to Right): Perform the division. $100 \div 10 = 10$. $10 + 7$

A (Addition): Perform the final addition. 17

Final Answer: 17

iv. Summary of Main Concepts

- To evaluate complex expressions, you must follow the Order of Operations.
- The acronym PEMDAS helps you remember the correct order:
  - ✓ Parentheses (and other groups, from inside out)
  - ✓ Exponents
  - ✓ Multiplication and Division (from left to right)
  - ✓ Addition and Subtraction (from left to right)
- Multiplication/Division and Addition/Subtraction are equal priority pairs. Their order is determined by which appears first from left to right.
- Show your work step-by-step to avoid errors and make it easy to find mistakes. The "funnel method" is a great way to stay organized.
- Practice is the key to mastery! The more you practice, the more natural the order of operations will become.