Comparing Expressions

i. Definition and Explanation

What are we doing when we "compare expressions"? Comparing algebraic expressions means determining the relationship between them for a specific value of the variable. We want to find out if one expression is greater than (>), less than (<), or equal to (=) the other.

An algebraic expression, like 2x + 5, doesn't have a single value on its own. Its value depends on the value of the variable x. Therefore, we can't say that 2x + 5 is always bigger than 3x - 1. The relationship might change depending on what number we substitute for x.

The core process is:

- **Substitute:** Replace the variable in each expression with the given number.
- **Simplify:** Calculate the value of each expression.
- **Compare:** Use <, >, or = to show the relationship between the two final values.

ii. Key Points and Important Terms

Algebraic Expression: A mathematical phrase that can contain numbers, variables (like x or y), and operation signs (+, -, ×, ÷).

- **Example:** 5y - 10

Variable: A letter used to represent a number that can change or is unknown.

- **Example:** In 4a + 7, the variable is a.

Constant: A number that does not change.

- **Example:** In 4a + 7, the constant is 7.

Evaluate: To find the numerical value of an expression by substituting a number for the variable.

Order of Operations (PEMDAS/BODMAS): The rules you must follow to simplify an expression correctly.

- Parentheses (or Brackets)
- Exponents (or Orders)
- Multiplication and Division (from left to right)
- Addition and Subtraction (from left to right)

Comparison Symbols:

- > : Greater Than
- < : Less Than
- = : Equal To

iii. Detailed Examples with Solutions

Example 1: Basic Comparison

Compare the expressions 3x + 4 and 5x - 2 when x = 3.

Step 1: Evaluate the first expression.

Expression A: 3x + 4

Substitute x = 3: 3(3) + 4

Simplify: 9 + 4 = 13

Step 2: Evaluate the second expression.

Expression B: 5x - 2

Substitute x = 3: 5(3) - 2

Simplify: 15 - 2 = 13

Step 3: Compare the results.

The value of Expression A is 13.

The value of Expression B is 13.

Conclusion: 13 = 13, so when x = 3, 3x + 4 = 5x - 2.

Example 2: Comparison with a Negative Value

Compare the same expressions, 3x + 4 and 5x - 2, but this time when x = -2.

Step 1: Evaluate the first expression.

Expression A: 3x + 4

Substitute x = -2: 3(-2) + 4

Simplify: -6 + 4 = -2

Step 2: Evaluate the second expression.

Expression B: 5x - 2

Substitute x = -2: 5(-2) - 2

Simplify: -10 - 2 = -12

Step 3: Compare the results.

The value of Expression A is -2.

The value of Expression B is -12.

On a number line, -2 is to the right of -12, so it is greater.

Conclusion: -2 > -12, so when x = -2, 3x + 4 > 5x - 2.

Notice how the relationship changed from = to > when we used a different value for x!

Example 3: Comparison with Exponents

Compare the expressions $a^2 + 5$ and 6a when a = 4.

Step 1: Evaluate the first expression.

Expression A: $a^2 + 5$

Substitute a = 4: $(4)^2 + 5$

Simplify: 16 + 5 = 21

Step 2: Evaluate the second expression.

Expression B: 6a

Substitute a = 4: 6(4)

Simplify: 24

Step 3: Compare the results.

The value of Expression A is 21.

The value of Expression B is 24.

Conclusion: 21 < 24, so when a = 4, $a^2 + 5 < 6a$.

iv. Summary of Main Concepts

- You cannot compare algebraic expressions without knowing the value of the variable.
- The process is to substitute the given value for the variable into both expressions.
- Always use the Order of Operations (PEMDAS) to evaluate each expression down to a single number.

- Compare the two resulting numbers using the symbols >, <, or =.
- The relationship between two expressions (which is greater, less, or equal) can change when the value of the variable changes.