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# EMI & AC

# BASIC CONCEPTS AND FORMULAE

#### 1. MAGNETIC FLUX

Magnetic Flux through a surface of area A placed in a uniform magnetic field is

 $\phi_m = \vec{B}.\vec{A} = BA\cos\theta, \theta$  being angle between  $\vec{B}$  and  $\vec{A}$ . If magnetic field is not uniform, then

 $\phi_m = \int \vec{B} \cdot \vec{d} A$ . Where integral extends for whole area A.

The S.I. unit of magnetic flux is weber. Magnetic flux is a **scalar quantity**; because of being scalar product of two vectors  $\vec{B}$  and  $\vec{A}$ .

#### 2. ELECTROMAGNETIC INDUCTION

Whenever the magnetic flux linked with a coil changes, an emf is induced in the coil. The induced emf lasts so long as the change in magnetic flux lasts. This phenomenon is called *electromagnetic induction*.

#### 3. FARADAY'S LAWS

 Whenever there is a change in magnetic flux linked with of a coil, an emf is induced in the coil. The induced emf is proportional to the rate of change of magnetic flux linked with the coil.

i.e., 
$$\propto \frac{\Delta \alpha}{\Delta t}$$

(ii) emf induced in the coil opposes the change in flux, i.e.,

$$\mathbf{e} \propto \frac{\Delta \phi}{\Delta t} \Rightarrow \mathbf{e} = -\mathbf{k} \frac{\Delta \phi}{\Delta t}$$

where k is a constant of proportionality.

In S.I. system  $\phi$  is in weber, t in second, e in volt, then k = 1, so  $e = -\frac{\Delta \phi}{\Delta t}$ 

If the coil contains N-turns, then  $e = -N \frac{\Delta \phi}{t}$ 

#### 4. INDUCED CURRENT AND INDUCED CHARGE

If a coil is closed and has resistance R, then current induced in the coil,

 $i = \frac{e}{R} = -\frac{N}{R}\frac{\Delta\phi}{\Delta t}$  ampere

Electromagnetic induction and Alternating Current

Induced charge,  $q = I\Delta t = -\frac{N\Delta\phi}{R} = \frac{\text{Total flux linkage}}{\text{Resistance}}$ 

#### 5. LENZ'S LAW

It sates that the direction of induced emf is such that it tends to produce a current which opposes the change in magnetic flux producing it.

#### 6. EMF INDUCED IN A MOVING CONDUCTING ROD

EMF induced in a conducting rod of length I moving with velocity v in a magnetic field of induction B. such that B, I and v are mutually perpendicular, is given by

 $\mathbf{e} = \mathbf{B}\mathbf{v}\mathbf{I}.$ 

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Symbol of inductor

#### 7. SELF INDUCTANCE

When the current in a coil is changed, a back emf is induced in the same coil. Tis phenomenon is called self-induction. If L is self-inductance of coil, then

$$N\phi \propto I$$
 or  $N\phi = LI \Longrightarrow L = \frac{N\phi}{I}$   $e = -L\frac{\Delta I}{\Delta t}$ 

Also induced emf  $e = -L \frac{\Delta I}{\Delta t}$ 

The unit of self inductance is henry (H). The self inductance acts as intertia in electrical circuits; so it is also called **electrical inertia**.

The self inductance of a solenoid consisting core of relative permeability  $\mu_r$  is

 $L = \mu_r \mu_o n^2 A I$ 

where  $n = \frac{N}{L}$  is the number of turns per metre length.

#### 8. MUTUAL INDUCTION

When two coils are palced nearby and the current in one coil (often called primary coil) is changed, the magnetic flux linked with the neighbouring coil (often called secondary coil) changes; due to which an emf of two coils, then  $\phi_2 \propto I_1$  or  $\phi_2 = MI_1$ 

## Definition of mutual inductance : $M = \frac{\Psi^2}{1}$

The mutual infuctance of two coils in defined as the magnetic flux linked with the secondary coil when the current in primary coil is 1 ampere.

Also induced emf in secondary coil  $e_2 = -M \frac{\Delta l_1}{\Delta t} \Rightarrow M = \frac{e_2}{\Delta l_1/\Delta t}$ .

The mutual inductance of twl coils is defined as the emf induced in the secondary coil when the rate of change of current in the primary coil is 1 A/s.

The unit of mutual inductance is also henry (H). The mutual inductance of two coils does not depend on the fact which coil carries the current and in which coil emf is induced i.e.,  $M_{12} = M_{21} = M$ 

If  $L_1$  and  $L_2$  are self-inductances two coils with 100% fulx linkage between them, then

 $M = \sqrt{L_1L_2}$ , otherwise  $M = k\sqrt{L_1L_2}$ where k is coefficient of flux linkage between the coils.

Mutual inductance of solenoid-coil system

 $M = \frac{\mu_0 N_1 N_2 A}{I}$ 



where A is area of coil, I is length of solenoid.  $N_1$  is nuber of turns in solenoid and  $N_2$  is number of turns in coil.

#### 9. EDDY CURRENTS

When a conductor is placed in a varying magnetic field the magnetic flux linked with the conductor changes, so induced current are induced in the body of conductor,



which causes heating of conductor.

The current in induced in the comductor are called the eddy currents. In varying magnetic field, the free electrons of conductor experience Lorentz force and traverse closed paths; which are equivalent to small current loops. |These currents are the eddy currents; they cause heating effect and sometimes the conductor becomes red-hot.

**Eddy current losses** may be reduced by using laminated soft iron cores in galvanometers, tranformers. etc. and making holes in conductor.

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10.	<b>NEED FOR DISPLACEMEN</b> Ampere's circuital law for inconsistent. therefore may current. It is given by	TCURRENT conduction current during charging of a capacitor was found xwell modified Ampere's circuital law by introducing displacement
	$I_d = \epsilon_0 \frac{d\phi_E}{dt}$ modified Ampere flux.	e's circuital law is $\oint \vec{B}.d\vec{I} = \mu_o \left(I + \epsilon_0 \frac{d\phi_E}{dt}\right)$ where $\phi_E = \text{electric}$
11.	<b>ALTERNATING CURRENT</b> Alternating current is or magnitude and direction perivalue of current is called <b>current - amplitu</b>	ne which changes in iodically. The maximum <b>de or peak value of current</b> .
	If $f = \frac{\omega}{2\pi}$ is frequency of all	ternating current, then it is expressed as
	$I = I_{c}$ Similarly alternating voltag	յ sin ωt e (or emf) is
	v – v	
12.	MEAN AND RMS VALUE OF The mean value of alternat (I <sub>mean</sub> While for half cycle it is	<b>FALTERNATING CURRENTS</b> ting current over complete cycle is zero $f_{\text{full cycle}} = 0$
	An instrument read root me	$\int_{\text{half cycle}} = \frac{1}{\pi} = 0.030 I_0$ ean square values as $\sqrt{(1^2)} = -\frac{I_0}{\pi} = 0.707I_0$
	Flectromagnetic Induction	and Alternating Current
13.	<b>PHASE DIFFERENCE BETV</b> Area may be controlled by inductance and capacitance	<b>VEEN VOLTAGE AND CURRENT</b> resistance, inductance and capacitance. Due to the pressence of e, current is usually not in phase with applied voltage. In general
	V = V	$\frac{1}{0}\sin\omega t$
	where $\phi$ is the phase differ	$\sin(\omega t + \phi)$
14.	<b>IMPEDANCE AND REACTANCE</b> The hindrace offered by a circuit to flow of AC is called impedance. It is denote	
	$z = \frac{V}{I}$	$V = \frac{V_o}{I_o}$ ohm
	<b>Reactance</b> : The hindrance offered by inductance and capacitance in ac circuit is called reactance. It is demoted by X. The hindrance due to inductance alone is called the inductive reactance while that due to capacitance alone is called the capacitive reactance.	
	Inductive reactance, $X_L = $	ωL
	Capacitive reactance,	$X_{c} = \frac{I}{\omega C}$

#### **15. PURELY RESISTIVE CIRCUIT**

If a circuit contains pure resistance, then phase  $\phi$  =oi.e., current and voltage are always in the same phase. Impedance, Z = R

#### 16. PURELY INDUCTIVE CIRCUIT

by

If a circuit contains pure inductance, then  $\phi = -\frac{\pi}{2}$ , i.e. current lags behind the applied voltage

ωt

an angle 
$$\frac{\pi}{2}$$
. i.e.,  $V = V_0 \sin \theta$ 

$$\mathbf{I} = \mathbf{I}_0 \sin\left(\boldsymbol{\omega} \mathbf{t} - \frac{\pi}{2}\right)$$

In this case Z = inductive reactance,  $X_1 = \omega L$ The inductive reactance increases with the increase of frequency of AC linearly (fig.b.)

#### **17. PURELY CAPACITIVE CIRUIT**

If circuit contains pure capacitance, the  $\phi = \frac{\pi}{2}$ , i.e. current leads the applied voltage by angle  $\pi/2$ .

Vc

i.e. 
$$V = V_0 \sin \omega t$$
.  $I = I_0 \sin \left( \omega t + \frac{\pi}{2} \right)$ 

Impedace = capacitance reactance,  $X_{C} = \frac{1}{\omega C}$ Clearly capacitance reactance ( $X_{C}$ ) is inversely proportional to the frequency f (fig. b).

#### 18. LC OSCILLATION

A circuit containing inductance L and capacitance C is called a resonant ciruit. If capacitance is charged initially and ac source is removed, then electrostatic energy of capacitor  $(q_0^2/2C)$ )

is converted into magnetic energy of inductor  $\left(\frac{1}{2}LI^2\right)$  and vice versa perriodically; such

oscillation of energy are called LC oscillation. The frequency is given by  $\omega_0 = \frac{1}{\sqrt{LC}}$ 

# **19. LCR SERIES CIRCUIT**

If a circuit contains inductance L, capacitance C and R L C resistance R, fed by AC voltage V = V<sub>0</sub> sin  $\omega$ t then impedance R, fed by AC voltage V = V<sub>0</sub> sin  $\omega$ t then impedance

$$Z = \sqrt{R^2 + \left(X_C^{-X_L}\right)^2}$$

$$\phi = \tan^{-1} \frac{X_{C}^{-X}}{D}$$

$$V = V_0 \sin \omega t$$

and phase  $\phi = \tan^{-1}$ 

Net voltage 
$$V = \sqrt{V_{R}^{2} + (V_{C} - V_{I})^{2}}$$



 $X_{1} = \omega L$ the increase  $\pi/2$ Phase diagram (b)

(a)



(b) f or  $\omega$ 

## 20. RESONANT CIRCUITS

(i) Series LCR circuit : In series LCR circuit, when phase  $(\phi)$  between current and voltage is zero, the circuit is said to be resonant circuit. In resonant circuit

Electromagnetic induction and Alternating Current  $\omega = \frac{1}{\sqrt{1 C}}$ 

Resonant angular frequency  $\omega_r = \frac{I}{\sqrt{LC}} \Rightarrow (\text{linear})\text{frequency } f_r = \frac{\omega_r}{2\pi} = \frac{I}{2\pi\sqrt{LC}}$ 

At resonant frequency  $\phi = 0, V = V_R$ 

#### Quality factor (Q)

i.e.

The quality factor (Q) of a series LCR ciruit is given by the ratio of resonant frequency to frequency band width of the resonant 0.707  $\rm I_{\rm o}$  curve

$$Q = \frac{\omega_r}{\omega_2 - \omega_1} = \frac{\omega_r l}{R}$$

Clearly, smaller the value of R. larger is the qualit factor and sharper the resonance. Thus quality factor. determines the nature of sharpness of resonance.

(ii) Parallel Resonant circuit : A circuit containing inductance L and capacitance C in parallel and fed by ac voltage is called parallel resonant circuit. In parallel resonant circuit

√LC

$$i_{C} = i_{L} \Rightarrow X_{C} = X_{L}$$
, frequency  $\omega_{r} =$ 

#### 21. POWER DISSIPATION IN AC CIRCUIT IS

$$P = V_{rms} I_{rms} \cos \phi = -\frac{1}{2} V_0 I_0 \cos \phi$$
 (where  $\cos \phi = \frac{R}{2}$  is the power factor.)

#### 22. WATTLESS CURRENT

In purely inductive or purely capacitive circuit, power loss is zero. In such a circuit current flowing is called wattless current. In LCR circuit at resonace, the power loss is maximum.

Wattless component of current =  $I_{rms} \sin \phi$ 

Power compoent of current =  $I_{rms} \cos \phi$ 

#### 23. ACGENERATOR

It is a device to vonvert mechanical energy into electrical energy based on the phenomenon of electromagneti induction. If a coil of N turns, area A is rotated with frequency f in uniform magnetic field of induction B, then motional emf in coil (if initially it is perpendicular to field) is

 $e = NBA \omega \sin \omega t$  with  $\omega = 2\pi f$ 

Peak emf,  $e_o = NBA\omega$ .

#### 24. TRANSFORMER

A tranformer is a device which converts low ac voltage into high ac voltage and vice versa. It works on the principle of mutual induction. If  $N_p$  and  $N_s$  are the number of turns in primary and secondary coils,  $V_p$  and  $I_p$  are voltage and current in primary coil, then voltage ( $V_s$ ) and current ( $I_s$ )in secondary coil will be

$$V_{s} = \left(\frac{N_{S}}{N_{P}}\right)V_{P} \text{ and } I_{S} = \left(\frac{N_{P}}{N_{S}}\right)I_{P}$$

Step up transformer increases the voltage while step down transformer decreases the voltage.

In step down transformer  $V_S < V_P$  so  $N_S < N_P$ 



## SOLVED PROBLEMS

- 1. What is meant by impedance ? Give its unit. Using the phasor diagram or otherwise, derive an expression for the impedance of an ac circuit containing L, C and R in series. Find the expression for resonant frequency ?
- **Sol. Impedance :** The hindrance offered by a circuit to the flow of ac is called impedance. Mathematically it is the ratio of rms voltage applied and rms current produced in circuit i.e., Z

 $= \frac{V}{I}$  Its unit is ohm ( $\Omega$ ).

**Expression for Impedance in LCR series circuit :** Suppose resistance R, inductance L and capacitance C are connected in series and an alternating source of voltage  $V = V_0 \sin \omega t$  is applied across it (fig. a) On account of being in series, the current (*i*) flowing through all of them is the same.



Suppose the voltage across resistance R is  $V_R$ , voltage across inductance L is  $V_L$  and voltage across capacitance C is  $V_c$ . The voltage  $V_R$  and current i are in the same phase, the voltage  $V_L$  will lead the current by angle 90° while the voltage  $V_c$  will lag behind the current by angle 90° (fig. b) Clearly  $V_c$  and  $V_L$  are in opposite directions, therefore their resultant difference =  $V_c - V_L$  if ( $V_c > V_c$ )

Thus  $V_{R}$  and  $(V_{c} - V_{L})$  are mutually perpendicular and the phase difference between them is 90°. As applied voltage across the circuit is V, the resultant of  $V_{R}$  and  $(V_{c} - V_{L})$  will also be V. From fig.

(i)

But

where  $X_c = \frac{1}{\omega C}$  = capacitance reactance and  $X_L = \omega L$  = inductive reactance

$$V = \sqrt{(Ri)^{2} + (X_{c}i - X_{L}i)^{2}}$$
  
rcuit, 
$$Z = \frac{V}{i} = \sqrt{R^{2} + (X_{c} - X_{L})^{2}}$$

 $V^{2} = V_{R}^{2} + (V_{C} - V_{L})^{2} \rightarrow V = \sqrt{V_{R}^{2} + (V_{C} - V_{L})^{2}}$ 

 $V_{R} = R i, V_{C} = X_{C} i$  and  $V_{L} = X_{L} i$ 

i.e.,

: Impedance of ci

**Resonant frequency :** The phase difference ( $\phi$ ) between current and voltage  $\phi$  is given by

$$\tan \phi = \frac{X_{C} - X_{L}}{R}$$
For resonance  $\phi = 0$ , so  $X_{C} - X_{L} = 0$   
 $\Rightarrow \qquad \frac{1}{\omega C} = \omega L \Rightarrow \omega^{2} = \frac{1}{LC}$ 
Resonant frequency  $\omega_{r} = \frac{1}{\sqrt{LC}}$ 

 $Z = \sqrt{R^2 + (X_C - X_L)^2} = \sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}$ 

# 2. Describe briefly the principle, construction and working of a transformer. Why is its core laminated ?

**Sol. Transformer :** Transformer is a device by which an alternating voltage may be decreased or increased. This is based on the principle of mutual - induction.

**Construction :** It consists of laminated core of soft iron, on which two coils of insultated copper wire are separately wound. These coils are kept insulated from each other and from the iron-core, but are coupled through mutual induction. The number of turns in these coils are different. Out of these coils one coil is called primary coil and other is called the secondary coil. The terminals of primary co8ls are connected to Ac mains and the terminals of the secondary coll are connected to external circuit in which alternating current of desired voltage is required. Transformers are of two types :



**1. Step up Transformer :** It transforms the alternating low voltage to alternating high voltage and in this the number of turns in secondary coil is more than that in primary coil. (i.e.,  $N_s > N_p$ )

**2. Step down Transformer :** It transforms the alternating high voltage to alternating low voltage and in this the number of turns in secondary coil is less than that in primary coil (i.e.,  $N_{c} < N_{p}$ )

**Working :** When alternating current source is connected to the ends of primary coil, the current changes continuously in the primary coil; due to which the magnetic flux linked with the secondary coil changes continuously, therefore the alternating emf of same frequency is developed across the secondary.

Let  $N_p$  be the number of turns in primary coil,  $N_s$  the number of turns in secondary coil and  $\phi$  the magnetic flux linked with each turn. We assume that there is no leakage of flux so that the flux linked with each turn of primary coil and secondary coil is the same. According to Faraday's laws the emf induced in the primary coil

$$\epsilon_{p} = -N_{p} \frac{\Delta \phi}{\Delta t} \qquad \dots(1)$$
  
mf induced in the secondary coil  
$$\epsilon_{s} = -N_{s} \frac{\Delta \phi}{\Delta t} \qquad \dots(2)$$

From (1) and (2)

and e

If the resistance of primary coil is negligible, the emf ( $\varepsilon_p$ ) induced in the primary coil, will be equal to the applied potential difference ( $V_p$ ) across its ends. Similarly if the secondary circuit is open, then the potential difference  $V_s$  across its ends will be equal to the emf ( $\varepsilon_s$ ) induced in it; therefore

$$\frac{V_{s}}{V_{p}} = \frac{\varepsilon_{s}}{\varepsilon_{p}} = \frac{N_{s}}{N_{p}} = r(say) \qquad \dots (4)$$

where  $r = \frac{N_s}{N_p}$  is called the transformation ratio. If  $i_p$  and  $i_s$  are the instantaneous currents in

*.*..

So

So

primary and secondary coils and there is no loss of energy; then

Power in primary = Power in secondary

$$V_{p} i_{p} = V_{s} i_{s}$$
$$\frac{i_{s}}{i_{p}} = \frac{V_{p}}{V_{s}} = \frac{N_{p}}{N_{s}} = \frac{1}{r}$$

In step up transformer,

$$N_s > N_p \rightarrow r > 1$$
;

$$V_s > V_p$$
 and  $i_s < i_p$ 

i.e. step up transformer increases the voltage but decreases the current.

#### In step down transformer,

 $N_s > N_p \rightarrow r < 1$  ;  $V_s < V_p$  and  $i_s > i_p$ 

i.e., step down transformer decreases the voltage, but increases the current.

**Laminated core :** The core of transformer is lamined to reduce the enrgy losses due to eddy currents, so that its efficiency may remain nearly 100 %

...(5)

# 3. Show diagrammatically two different arrangements used for winding the primary and secondary coils in a tranformer. Assuming the transformer to be an ideal one, write the expression for the ratio of its

(i) output voltage to input voltage

#### (ii) output current to input current.

## Mention two reasons for energy losses in an actual tranformer.

**Sol.** Arrangements of winding of primary and secondary coil in a tranformer are shown in figure (a) and (b)



(i) Ratio of output voltage to input voltage

# $\frac{V_s}{V_p} = \frac{N_s}{N_p}$

(ii) Ratio of output current to input current

# $\frac{I_s}{I_p} = \frac{I_p}{N_s}$

#### Reasons for energy losses in a transformer

(i) Joule Heating : Energy is lost in resistance of primary and secondary winding as heat (I<sup>2</sup> Rr).

(ii) Flux Leakage : Energy is lost due to coupling of primary and secondary coils not being perfect.

i.e., whole of magnetic flux generated in primary coil is not linked with the secondary coil.

4. Explain with the help of a labelled diagram, the principle and working of an a.c. generator ? Write the expression for the emf generated in the coil in terms of speed of rotation. Can the current produced by an a.c. generator be measured with a moving coil galvanometer.

- **Sol. AC generator :** A dynamo or generator is a device which converts mechancial energy into electrical energy. It is based on the principle of electromagnetic induction. **Construction :** It consists of the four main parts :
- (i) **Field Magnet :** It produces the magnetic field. In the case of a low power dynamo, the magnetic field is generated by a permanent magnet, while in the case of large power dynamo, the magnetic field is produced by an electromagnet.
- (ii) Armature : It consists of a large number of turns of insultated wire in the soft iron drum or ring. It can revolve reound an axle between the two poles of the field magnet. The drum or ring serves the two purposes : (i) It serves as a support to coils and (ii) It increases the magnetic field due to air core being replaced by an iron core :
- (iii) **Slip Rings :** The slip rings  $R_1$  and  $R_2$  are the two metal rings to which the ends of armature coil are connected. These rings are fixed to the shaft which rotates the armature coil so that the rings also rotate along with the armature.
- (iv) **Brushes :** These are two flexible metal plates or carbon rods  $(B_1 \text{ and } B_2)$  which are are fixed and constantly touch the revolving rings. The ouput current in extyernal load  $R_L$  is taken through these brushes.

Working : When the armature coil is rotated in the strong magnetic field, the magnetic flux linked with the coil changes and the current is induced in the coil, its direction being given by Fleming's right hand rule. Considering the armature to be in vertical positoin and as it rotates in anticlockwise direction, the wire ab moves upward and cd downward, so that the direction of induced current is shown in fig. In the external circuit, the current flows along  $B_1$   $B_1$   $B_2$ . The direction of current remains unchanged during the first half turn of armature. During the second half revolution, the wire ab moves downward and cd upward, so the direction of current is reversed and in external circuit if flows along B, R, B,. Thus the direction of induced emf and current changes in the external circuit after each half revolution.



If N is the number of turns in coil, f the frequency of rotation. A area of coil and B the magnetic induction, then induced emf

$$e = -\frac{d\phi}{dt} = \frac{d}{dt}(NBA(\cos 2\pi ft))$$

Obviously, the emf produced is alternating and hence the current is also alternating Current produced by an ac generator cannot be measured by moving coil ammeter, because the average value of ac over fully cycle is zero.

# 5. A jet plane is travelling west at 450 ms<sup>-1</sup>. If the horizontal component of earth's magnetic field at that place is $4 \times 10^{-4}$ T and the angle of dip is 30°, find the emf induced between the ends of wings having a span of 30 m.

**Sol.** The wings of jets plane will cut the vertical component of earth's magnetic field, so emf is induced across the wings. The vertical component of eath's magnetic field

$$V = H \ tan \ \theta \label{eq:V}$$
 Given  $H = 4.0 \times 10^{-4} \ T\!\! . \ \theta = 30^{\circ}$ 

V = (4.0 × 10<sup>-4</sup> T) tan 30° = 4 × 10<sup>-4</sup> × 
$$\frac{1}{\sqrt{3}}$$
 =  $\frac{4}{\sqrt{3}}$  × 10<sup>-4</sup> T

Induced emf across the wing

$$\varepsilon = Vvl$$
  
Given v = 450 ms<sup>-1</sup>, l = 30 m  
$$\varepsilon = \left(\frac{4}{\sqrt{3}} \times 10^{-4}\right) \times (450) \times 30 \text{ V} = 3.12 \text{ V}$$

- 6. A bulb of resistance 10 Ω connected to an inductor of inductance L, is in series with an ac source marked 100 V, 50 Hz. If the phase angle between the voltage and current is
  - $\frac{\pi}{4}$  radian, calculate the value of L.

Sol. Given

 $R = 10 \Omega$  E = 100 V, f = 50 Hz,

In RL circuit Phase angle  $\phi$  is given by

$$tan\phi = \frac{X_{L}}{R} = \frac{\omega L}{R} = \frac{2\pi fL}{R}$$

 $L = \frac{R \tan \phi}{2\pi f} = \frac{10 \times \tan \frac{\pi}{4}}{2 \times 3.14 \times 50}$ 

 $\Rightarrow$ 

7. An indcutor of unknown value, a capacitor of 100  $\mu$ F and a resistor of 10  $\Omega$  are connected in series to a 200 V, 50 Hz a.c. source. It is found that the power factor of the circuit is unity. Calcualte the inductance of the inductor and current amplitude.

10×1

314

10

314

1

= 0.0318 A

**Sol.** For power factor unity, 
$$X_{L} = X_{C} \Rightarrow \omega L = \frac{1}{\omega C}$$
  
 $\Rightarrow L = \frac{1}{\omega^{2}C} = \frac{1}{(2\pi f)^{2}C} = \frac{1}{4\pi^{2}f^{2}C}$   
Given f = 50 Hz, C = 100  $\mu$ F = 100 × 10<sup>-6</sup> F = 10<sup>-4</sup> F  
 $\therefore L = \frac{1}{4 \times (3.14)^{2} \times (50)^{2} \times 10^{-4}}$ H = 0.10 H  
Current amplitude,  $I_{0} = \frac{V_{0}}{Z}$   
At resonance,  $I_{0} = \frac{V_{0}}{R} = \frac{200\sqrt{2}}{10} = 20\sqrt{2}A = 20 \times 1.414$  A = 28.3 A

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#### **EXERCISE - I**

# UNSOLVED PROBLEMS

- **1.** Sate Lenz's law. How is it useful to find the direction of current in a circuit ?
- **2.** Discuss oscillations in LC circuit.
- **3.** Show that the energy stored in an inductor L when a current  $I_0$  is established through it is  $U = \frac{1}{2}LI_0^2$
- **4.** Find an expression for phase difference between current and voltage in (i) purely inductive circuit and (ii) purely capacitive circuit.
- **5.** Explain construction and working of an ac generator.
- **6.** What are eddy currents ? How can they be minimised ? Write two advantage of these currents.
- 7. The rate of change current 2 A/s induces an emf 40 mV in a solenoid, what is the self inductance of this solenoid ?
- 8. A 40  $\Omega$  resistor, 3 mH inductor and 2 $\mu$ F capacitor are connected in seires to a 110 V, 50 Hz ac source. Calcualte the current in the circuit.
- **9.** The self-inductance of a coil is 10 H. If the induced emf in the coil be 120 volt, find the rate of change of current in the coil.
- **10.** Determine the impedance of a circuit if reactance of C and L and 340  $\Omega$  and 300  $\Omega$  respectively and R is 30  $\Omega$ .
- **11.** An a.c. generator consists of a coil of 100 turns and cross sectional area,  $3m^2$ , rotating at an angular frequency of 60 rad/second in a uniform magnetic field of 0.04 T. The resistance of the coil is 500 ohm. Calculate (i) the maximum current drawn from the generator and (ii) the maximum power dissipated in the coil.

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EXERCISE - II
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# **BOARD PROBLEMS**

**1.** Calculate the (i) impedance (ii) wattless current of the given ac circuit.



- 2. An inductor 200 mH, capacitor 500  $\mu$ F, resistor 10  $\Omega$  are connected in series with a 100 V, variable frequecy ac source. Calculate the
  - (i) frequency at which the power factor of the circuit is unity.
  - (ii) current amplitude at this frequency,
  - (iii) Q-factor.
- **3.** A circular copper disc 10 cm in radius rotates at 20  $\pi$  rad/s about an axis through its centre and perpendicualr to the disc. A uniform magnetic field of 0.2 T acts perpendicular to the disc.
  - (i) Calculate the potential difference developed between the axis of the disc and the rim.
  - (ii) What is the induced current in the circuit whose terminals are connected between centre of disc and point of rim and the resistance of the circuit is 2  $\Omega$ .

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- **4.** The instantaneous current from an ac source is  $I = 5 \sin (314 t)$  ampere. What are the average and rms value of an alternating current ?
- **5.** A resistor of  $200 \Omega$  and a capacitor of  $40 \mu$ F are connected in series to a 220 V ac source with angualr frequency w = 300 Hz. Calculate the rms voltages across the resistor and the capacitor. Why is the algebraic sum of these voltage more than the source voltage ? How do you resolve this paradox ?

[**Hint :** Due to phase diff. of  $\frac{\pi}{2}$  between V<sub>c</sub> and V<sub>R</sub>

Hence  $\sqrt{V_R^2 + V_C^2} = 220 V$  ]

- 6. Calculate the current drawn by the primary coil of a transformer which steps down 200 V to 20 V to operate a device of resistance 20  $\Omega$ . Assume the efficiency of the transformer to be 80%
- **7.** A rectangular coil of area A, number of turns N is rotated of f rev s<sup>-1</sup> in a uniform magnetic field B, the field being perpendicual to coil. Prove that the maximum emf induced in the coil is  $2\pi f$  NBA.
- **8.** A 0.5 m long metal rod PQ completes the circuit as shown in the figure. The area of the circuit is perpendicualr to the magnetic field of flux density 0.15 T. If the resistance of the total circuit is  $3\Omega$ , calculate the force needed to move the rod in the direction as indicated with a constant speed of 2 ms<sup>-1</sup>.
- **9.** When an inductor L and a resistor R in series are connected across a 12 V, 50 Hz supply, a current of 0.5 A flows in the circuit. The current differs in phase from applied voltage by  $\pi/3$  radian. Calculate the value of R.
- **10.** A small piece of metal wire is dragged across the gap between the pole piece of a magnet in 0.5 s. The magnetic flux between the pole pieces is known to be  $8 \times 10^{-4}$  Wb. Estimate the induced emf in the wire.
- **11.** The instantaneous voltage from an ac source is given by V = 300 sin 314 t. What is the rms value of the source ?
- **12.** An a.c. source of frequency 50 Hz is connected to a 50 mH inductor and a bulb. The bulb glows with some brightness. Calculate the capacitance of the capacitor to be connected in series with the circuit, so that the bulb glows with maximum brightness.
- **13.** A 200 V variable frequency ac source is connected to a series combination of L = 5 H,  $C = 80 \mu$ F and  $R = 40 \Omega$  Calculate (i) angular frequency of source to get the maximum current in the circuit (ii) current amplitude at resonance and (iii) power dissipation in the circuit.
- 14. An inductor L, a capacitor 20  $\mu$ F, a resistor 10  $\Omega$  are connected in series with an ac source of frequency 50 Hz. If the current is in phase with the voltage, calculate the inductance of the inductor.
- 15. How does the mutual inductance of a pair of coils change when(i) distance between the coils is increased and(ii) number of turns in the colis is increased ?
- **16.** A light metal disc on the top of an electromagnet is thrown up as the current is switched on. Why ? Give reason.
- **17.** (a) For a given a.c.,  $i = i_m \sin \omega t$ , show that the average power dissipated in a resistor R over a complete cycle is  $1/2 i_m^2 R$ .
  - (b) A light bulb is rated at 120 W for a 240 V a.c. supply. Calculate the resistance of the bulb.