DETERMINANTS

2.1 Basic Concepts

The eliminant of the variables from the equations

(i) $a_1x + b_1y = 0,\ a_2x + b_2y = 0$ (ii) $a_1x + b_1y + c_1z = 0,\ a_2x + b_2y + c_2z = 0$ is

is $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1 = \Delta$ $a_3x + b_3y + c_3z = 0,\ \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \Delta$

Expansion w.r.t. first row.

$$a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1\begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1\begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix} = a_1(b_2c_3 - b_3c_2) - b_1(a_2c_3 - a_3c_2) + c_1(a_2b_3 - a_3b_2)$$

Expansion w.r.t. first column.

$$a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - a_2\begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} + a_3\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} = a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - b_2c_1)$$

If you compare term by term you will observe that the two expansions are same.

2.2 Properties Of Determinant

1. The value of determinant is not altered by changing rows into columns and columns into rows.

2. If any two adjacent rows or two adjacent columns of a determinate are interchanged, the determinant retains its absolute value but changes its sign.

3. If any line of a determinant Δ be passed over p parallel lines the resultant determinate is $(-1)^p\ \Delta$.

4. If any two rows or two columns of a determinant are identical, then the determinant vanishes.

5. If each constituent in any row or in any column be multiplied by the same factor then the determinant is multiplied by that factor.

6. If each constituent in any row or column consists of r terms then the determinant can be expressed as the sum of r determinants.

7. If to or from each constituent of a row (or column) of a determinant are added or subtracted the equi-multiples of the corresponding constituent of any other row (or column) the determinant remains unaltered.

(a) The value of a determinant is obtained by multiplying the elements of any row (or column) with the correspond ing cofactors and adding the resulting products.

(b) If we multiply the elements of any row (or column) with the corresponding cofactors of any other row (or column) and add them, then the result is zero.

(c) If Δ' is the determinant formed by replacing the elements of a determinant Δ by their corresponding cofactors, then $\Delta' = \Delta^2$.

2.3 Special Determinants :

(1) Symmetric determinant. $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = abc + 2fgh - af^2 - bg^2 - ch^2$

In this the elements $a_{ij} = a_{ji}$ or the elements situated at equal distance from the diagonal are equal both in magnitude and sign.

(2) Skew symmetric determinant of odd order. $\begin{vmatrix} 0 & b & -c \\ -b & 0 & a \\ c & -a & 0 \end{vmatrix} = 0.$

All the diagonal elements are zero and $a_{ij} = -a_{ji}$ or the elements situated at equal distance from the diagonal are equal in magnitude but opposite in sign. Its value is zero.

(3) Circulant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -(a^3 + b^3 + c^3 - 3abc)$

The three rows or columns are the cyclic arrangement of the letters a, b, c, i.e. a, b, c ; b, c, a and c, a, b respectively.

(4) Factors of three important determinants :

(i) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

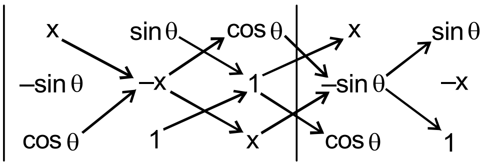
(ii) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

(iii) $\begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$

SOLVED PROBLEMS

Ex.1 ***Prove that the determinant*** $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ ***is independent of*** θ.

Sol. We shall expand this determinant, using Sarrus Method.



Thus, $\Delta = (-x^3 + \sin\theta\cos\theta - \sin\theta\cos\theta) - (-x\cos^2\theta + x - x\sin^2\theta) = -x^3$

Hence, the determinant is independent of θ.

Ex.2 ***Evaluate :***

(i) $\begin{vmatrix} 1 & x & y \\ 1 & x+y & y \\ 1 & x & x+y \end{vmatrix}$ ***(ii)*** $\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$

Sol. (i) Let $\Delta = \begin{vmatrix} 1 & x & y \\ 1 & x+y & y \\ 1 & x & x+y \end{vmatrix}$

Expanding it along the first column, we have

$$\Delta = 1\begin{vmatrix} x+y & y \\ x & x+y \end{vmatrix} - 1\begin{vmatrix} x & y \\ x & x+y \end{vmatrix} + 1\begin{vmatrix} x & y \\ x+y & y \end{vmatrix}$$

$$= [(x+y)^2 - xy] - [x(x+y) - xy] + [xy - xy - y^2]$$

$$= x^2 + y^2 + xy - x^2 - y^2 = xy$$

(ii) Let $\Delta = \begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$

$$= x\begin{vmatrix} x+y & x \\ x & y \end{vmatrix} - y\begin{vmatrix} y & x \\ x+y & y \end{vmatrix} + (x+y)\begin{vmatrix} y & x+y \\ x+y & x \end{vmatrix}$$

$$= x(xy + y^2 - x^2) - y(y^2 - x^2 - xy) + (x+y)(xy - x^2 - y^2 - 2xy)$$

$$= x^2y + xy^2 - x^3 - y^3 + x^2y + xy^2 + x^2y - x^3 - xy^2 - 2x^2y + xy^2 - x^2y - y^3 - 2xy^2$$

$$= -2x^3 - 2y^3 = -2(x^3 + y^3)$$

Ex.3 ***Solve the equation*** $\begin{vmatrix} x+\alpha & x & x \\ x & x+\alpha & x \\ x & x & x+\alpha \end{vmatrix}$ ***= 0, (a ≠ 0).***

Sol. Let

$$\Delta = \begin{vmatrix} x+\alpha & x & x \\ x & x+\alpha & x \\ x & x & x+\alpha \end{vmatrix}$$

$$= (x + a)\begin{vmatrix} x+a & x \\ x & x+a \end{vmatrix} - x\begin{vmatrix} x & x \\ x & x+\alpha \end{vmatrix} + x\begin{vmatrix} x & x+\alpha \\ x & x \end{vmatrix}$$

$$= (x + a)\,[(x + a)^2 - x^2] - x\,[x\,(x + a) - x^2] + x\,[x^2 - x\,(x + a)]$$

$$= (x + a)^3 - x^2\,(x + a) - ax^2 - ax^2$$

$$= (x + a)^3 - x^3 - 3ax^2$$

$$= x^3 + 3ax^2 + 3a^2x + a^3 - x^3 - 3ax^2$$

$$= 3a^2x + a^3$$

Equating it to zero, we have

$$3a^2x + a^3 = 0 \quad \Rightarrow \quad x = -\frac{a}{3}$$

Ex.4 ***If a, b and c are real numbers, and***

$$\Delta = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$$

Show that either a + b + c = 0 or a = b = c

Sol. Here,

$$\Delta = \begin{vmatrix} 2(a+b+c) & c+a & a+b \\ 2(a+b+c) & a+b & b+c \\ 2(a+b+c) & b+c & c+a \end{vmatrix} \qquad (C_1 \to C_1 + C_2 + C_3)$$

$$= 2\,(a + b + c)\begin{vmatrix} 1 & c+a & a+b \\ 1 & a+b & b+c \\ 1 & b+c & c+a \end{vmatrix}$$

$$= 2\,(a + b + c)\begin{vmatrix} 1 & c+a & a+b \\ 0 & b-c & c-a \\ 0 & b-a & c-b \end{vmatrix} \qquad (R_2 \to R_2 - R_1;\ R_3 \to R_3 - R_1)$$

$$= 2\,(a + b + c) \times 1 \begin{vmatrix} b-c & c-a \\ b-a & c-b \end{vmatrix}$$

$$= 2\,(a + b + c)\,[(b - c)\,(c - b) - (b - a)\,(c - a)]$$

$$= 2\,(a + b + c)\,[bc - c^2 - b^2 + bc - bc + ac + ab - a^2]$$

$$= 2\,(a + b + c)\,(ab + bc + ca - a^2 - b^2 - c^2)$$

Equating Δ to zero, we have

$$2\,(a + b + c)\,(ab + bc + ca - a^2 - b^2 - c^2) = 0$$

⇒ Either $a + b + c = 0$ or $ab + bc + ca = a^2 + b^2 + c^2$

⇒ Either $a + b + c = 0$ or $a = b = c$

Ex.5 ***Show that*** $\begin{vmatrix} \alpha & \alpha^2 & \beta+\gamma \\ \beta & \beta^2 & \gamma+\alpha \\ \gamma & \gamma^2 & \alpha+\beta \end{vmatrix} = (\beta-\gamma)(\gamma-\alpha)(\alpha-\beta)(\alpha+\beta+\gamma)$

Sol. Let

$$\Delta = \begin{vmatrix} \alpha & \alpha^2 & \beta+\gamma \\ \beta & \beta^2 & \gamma+\alpha \\ \gamma & \gamma^2 & \alpha+\beta \end{vmatrix} = \begin{vmatrix} \alpha-\beta & \alpha^2-\beta^2 & \beta-\alpha \\ \beta-\gamma & \beta^2-\gamma^2 & \gamma-\beta \\ \gamma & \gamma^2 & \alpha+\beta \end{vmatrix} \quad (R_1 \to R_1 - R_2;\ R_2 \to R_2 - R_3)$$

$$= (\alpha-\beta)(\beta-\gamma)\begin{vmatrix} 1 & \alpha+\beta & -1 \\ 1 & \beta+\gamma & -1 \\ \gamma & \gamma^2 & \alpha+\beta \end{vmatrix} = (\alpha-\beta)(\beta-\gamma)\begin{vmatrix} 0 & \alpha+\beta & -1 \\ 0 & \beta+\gamma & -1 \\ \alpha+\beta+\gamma & \gamma^2 & \alpha+\beta \end{vmatrix} \quad (C_1 \to C_1 + C_3)$$

$$= (\alpha-\beta)(\beta-\gamma)(\alpha+\beta+\gamma)\begin{vmatrix} \alpha+\beta & -1 \\ \beta+\gamma & -1 \end{vmatrix}$$

$$= (\alpha-\beta)(\beta-\gamma)(\alpha+\beta+\gamma)(-\alpha-\beta+\beta+\gamma) = (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha+\beta+\gamma)$$

Ex.6 ***Show that*** $\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca)$

Sol. Using $C_1 \to C_1 + C_2 + C_3$, we have

$$\Delta = \begin{vmatrix} a+b+c & -a+b & -a+c \\ a+b+c & 3b & -b+c \\ a+b+c & -c+b & 3c \end{vmatrix} = (a+b+c)\begin{vmatrix} 1 & -a+b & -a+c \\ 1 & 3b & -b+c \\ 1 & -c+b & 3c \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & -a+b & -a+c \\ 0 & a+2b & a-b \\ 0 & a-c & a+2c \end{vmatrix} \quad (R_2 \to R_2 - R_1;\ R_3 \to R_3 - R_1)$$

$$= (a+b+c) \times 1.\begin{vmatrix} a+2b & a-b \\ a-c & a+2c \end{vmatrix} = (a+b+c)[(a+2b)(a+2c)-(a-b)(a-c)]$$

$$= (a+b+c)(3ab+3bc+3ca) = 3(a+b+c)(ab+bc+ca)$$

Ex.7 ***Prove that*** $\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 4+3p+2q \\ 3 & 6+3p & 10+6p+3q \end{vmatrix} = 1$

Sol. Let

$$\Delta = \begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 4+3p+2q \\ 3 & 6+3p & 10+6p+3q \end{vmatrix} = \begin{vmatrix} 1 & 1+p & 1+p+q \\ 0 & 1 & 2+p \\ 0 & 3 & 7+3p \end{vmatrix} \quad (R_2 \to R_2 - R_1;\ R_3 \to R_3 - R_1)$$

$$= \begin{vmatrix} 1 & 2+p \\ 3 & 7+3p \end{vmatrix} = (7+3p) - 3(2+P) = 1$$

Ex.8 ***If a, b, c are in AP, prove that :*** $\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix} = 0$

Sol. Let $\Delta = \begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$

$$= \begin{vmatrix} x+2 & x+3 & x+2a \\ 1 & 1 & 2(b-a) \\ 2 & 2 & 2(c-a) \end{vmatrix} \qquad (R_2 \to R_2 - R_1;\ R_3 \to R_3 - R_1)$$

$$= \begin{vmatrix} x+2 & x+3 & x+2a \\ 1 & 1 & 2(b-a) \\ 0 & 0 & 2(c-a)-4(b-a) \end{vmatrix} \qquad (R_3 \to R_3 - 2r_2)$$

$$= [2\,(c-a) - 4\,(b-a)] \begin{vmatrix} x+2 & x+3 \\ 1 & 1 \end{vmatrix}$$

$= [2c - 2a - 4b + 4a]\,(x + 2 - x - 3)$
$= [2c + 2a - 4b]\,(-1) = (4b - 2a - 2c)$ (....1)

Since, a, b, c are in AP,
$2b = a + c$
$\therefore$ $4b = 2a + 2c$
Hence by (1), $\Delta = 0$

Ex.9 ***Prove that*** $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc\,(a+b+c)^3$

Sol. Replacing C_2 by $C_2 - C_1$ and C_3 by $C_3 - C_1$, we have

$$\Delta = \begin{vmatrix} (b+c)^2 & a^2-(b+c)^2 & a^2-(b+c)^2 \\ b^2 & (c+a)^2-b^2 & 0 \\ c^2 & 0 & (a+b)^2-c^2 \end{vmatrix} = (a+b+c)^2 \begin{vmatrix} b^2+c^2+2bc & a-b-c & a-b-c \\ b^2 & c+a-b & 0 \\ c^2 & 0 & a+b-c \end{vmatrix}$$

Replacing R_1 by $R_1 - R_2 - R_3$, we have

$$\Delta = (a+b+c)^2 \begin{vmatrix} 2bc & -2c & -2b \\ b^2 & c+a-b & 0 \\ c^2 & 0 & a+b-c \end{vmatrix}$$

Replacing C_2 by $C_3 + \frac{1}{b}C_1$ and C_3 by $C_3 + \frac{1}{c}C_1$, we have

$$\Delta = (a+b+c)^2 \begin{vmatrix} 2bc & 0 & 0 \\ b^2 & c+a & \frac{b^2}{c} \\ c^2 & \frac{c^2}{b} & a+b \end{vmatrix}$$

$= 2bc\,(a+b+c)^2\,[(a+c)\,(a+b) - bc]$
$= 2bc\,(a+b+c)^2\,(a^2 + ab + bc - bc)$
$= 2abc\,(a+b+c)^3$

Ex.10 Find the values of h if area of a triangle is 4 sq. units and vertices are (i) (k, 0) (4, 0) (0, 2)

Sol. (i) The area of the triangle = $\frac{1}{2}\begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix}$

$= \frac{1}{2}$ [k (0 – 2) – 0(4 – 0) + 1(8 – 0)]

$= \frac{1}{2}$ (–2k + 8) = (4 – k) sq units

According to the question,

4 – k = 4 or 4 – k = –4

$\Rightarrow$ k = 0 k = 8

Hence, the possible values of k are 0 and 8.

(ii) Area of the triangle = $\frac{1}{2}\begin{vmatrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & k & 1 \end{vmatrix}$

$= \frac{1}{2}$ [–2 (4 – k) – 0 (0 – 0) + 1 (0 – 0)]

= (k – 4) sq units

According to the question,

k – 4 = 4 or k – 4 = –4

$\Rightarrow$ k = 8 k = 0

Hence, the possible values of k are 8 and 0.

Ex.11 Using determinants, find the equation of the line AB, joining

(i) A (1, 2) and B (3, 6) (ii) A (3, 1) and B (9, 3)

Sol. Let P (x, y) be any point on AB. Then, area of the triangle ABP is zero, since the points A, B and P are collinear. So,

(i) $\frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 3 & 6 & 1 \\ x & y & 1 \end{vmatrix} = 0$

$\Rightarrow$ $\frac{1}{2}$ [1 (6 – y) – 2(3 – x) + 1(3y – 6x)] = 0

$\Rightarrow$ 6 – y – 6 + 2x + 3y – 6x = 0

$\Rightarrow$ y = 2x

Hence, the equation of the line AB is y = 2x.

(ii) $\frac{1}{2}\begin{vmatrix} 3 & 1 & 1 \\ 9 & 3 & 1 \\ x & y & 1 \end{vmatrix} = 0$

$\Rightarrow$ $\frac{1}{2}$ [3 (3 – y) – 1(9 – x) + 1(9y – 3x)] = 0

$\Rightarrow$ 9 – 3y – 9 + x + 9y – 3x = 0

$\Rightarrow$ 2x = 6y

$\Rightarrow$ x = 3y

Hence, the equation of the line AB is x = 3y.

EXERCISE – I

UNSOLVED PROBLEMS

Q.1 If A = $\begin{bmatrix} 2 & 5 \\ 2 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 4 & -3 \\ 2 & 5 \end{bmatrix}$, verify that |AB| = |A| |B|

Q.2 Without expanding evaluate the determinant $\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix}$

Q.3 Without expanding evaluate the determinant $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ac \\ 1 & c & c^2-ab \end{vmatrix}$

Q.4 Without expanding evaluate the determinant $\begin{vmatrix} (a^x-a^{-x})^2 & (a^x-a^{-x})^2 & 1 \\ (a^y+a^{-y})^2 & (a^y-a^{-y})^2 & 1 \\ (a^2+a^{-z})^2 & (a^z-a^{-z})^2 & 1 \end{vmatrix}$

Q.5 Without expanding evaluate the determinant $\begin{vmatrix} \sin\alpha & \cos\alpha & \sin(\alpha+\delta) \\ \sin\beta & \cos\beta & \sin(\beta+\delta) \\ \sin\gamma & \cos\gamma & \sin(\gamma+\delta) \end{vmatrix}$

Q.6 Prove that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\ 1\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) = abc + ab + bc + ca$

Q.7 Prove that $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$

Q.8 Prove that $\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a^2+b^2+c^2)(a+b+c)(b-c)(c-a)(a-b)$

Q.9 Show that $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2ab \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^2$

Q.10 Solve : $\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$

Q.11 If $x \neq y \neq z$ and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$, then prove that $1 + xyz = 0$

Q.12 If a, b, c are all positive and are pth, qth and rth terms respectively of a G.P. then prove that $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0$

Q.13 Find the area of a triangle with vertices : (–3, 5), (3, –6), (7, 2)

Q.14 Find the value of λ so that the points given below are collinear $(\lambda, 2-2\lambda)$, $(-\lambda+1, 2\lambda)$ and $(-4, -\lambda, 6-2\lambda)$

Q.15 Using determinants, find the area of the triangle, whose vertices are (–2, 4), (2, –6) and (5, 4). Are the given points collinear ?

EXERCISE – II BOARD PROBLEMS

Q.1 Prove $\begin{vmatrix} 1 & x & x^3 \\ 1 & y & y^3 \\ 1 & z & z^3 \end{vmatrix}$ = (x – y) (y – z) (z – x) (x + y + z).

Q.2 Prove using properties of determinants $\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix} = 0$.

Q.3 Prove using properties of determinants

(a) $\begin{vmatrix} 1 & x+y & x^2+y^2 \\ 1 & y+z & y^2+z^2 \\ 1 & z+x & z^2+x^2 \end{vmatrix}$ = (x – y) (y – z) (z – x).

(b) $\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^3$.

Q.4 Using the properties of determinants, show that

(a) $\begin{vmatrix} 43 & 3 & 6 \\ 35 & 21 & 4 \\ 17 & 9 & 2 \end{vmatrix} = 0$.

(b) $\begin{vmatrix} 9 & 9 & 12 \\ 1 & 3 & -4 \\ 1 & 9 & 12 \end{vmatrix}$

Q.5 Using the properties of determinants, show that

(a) $\begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix} = a^2(a + x + y + z)$.

(b) $\begin{vmatrix} 0 & ab^2 & ac^2 \\ a^2b & 0 & bc^2 \\ a^2c & cb^2 & 0 \end{vmatrix}$.

Q.6 Using the properties of determinants, prove that $\begin{vmatrix} y+z & z & y \\ z & z+x & x \\ y & x & x+y \end{vmatrix} = 4xyz$

Q.7 Using the properties of determinants, solve for x $\begin{vmatrix} x+a & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$.

Q.8 Show that $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$ where a, b and c are in A.P.

Q.9 Using the properties of determinants, prove that

(a) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

(b) $\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ y+z & z+x & x+y \end{vmatrix}$ = (x – y) (y – z) (z – x) (x + y + z).

Q.10 If a, b and c are in A.P., show that $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$

Q.11 Using the properties of determinants, prove that

(a) $\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$ **(b)** $\begin{vmatrix} 3a & -a+b & -a+c \\ a-b & 3b & c-b \\ a-c & b-c & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca)$

Q.12 Using the properties of determinants, prove that

(a) $\begin{vmatrix} 1 & bc & bc(b+c) \\ 1 & ca & ca(c+a) \\ 1 & ab & ab(a+b) \end{vmatrix} = 0.$ **(b)** $\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$

Q.13 If x, y, z are different and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$, show that $xyz = -1$.

Q.14 If a, b and c are all positive and distinct, show that $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ has a negative value.

Q.15 Solve for x : $\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0.$

Q.16 Using the properties of determinants.

(a) $\begin{vmatrix} x & x^2 & 1+ax^3 \\ y & y^2 & 1+ay^3 \\ z & z^2 & 1+az^3 \end{vmatrix} = (1+axyz)(x-y)(y-z)(z-x)$

(b) $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$

(c) $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2$ **(d)** $\begin{vmatrix} 1 & a^2+bc & a^3 \\ 1 & b^2+ca & b^3 \\ 1 & c^2+ab & c^3 \end{vmatrix} = -(a-b)(b-c)(c-a)(a^2+b^2+c^2).$

Q.17 Using properties of determinants, prove the following :

(a) $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$ **(b)** $\begin{vmatrix} a^2+1 & ab & ac \\ ba & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$

(c) $\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix} = 1$

(d) $\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3 + b^3 + c^3 - 3abc$

Q.18 Using properties of determinants, prove the following :

(a) $\begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (a+c)^2 & bc \\ ac & bc & (a+b)^2 \end{vmatrix} = 2abc\,(a+b+c)^3$

(b) $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2\,(a+b+c)^3$

Q.19 Using properties of determinants, solve the following for x :

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$$

Q.20 Using properties of determinants, show that $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$

Q.21 Using properties of determinants prove the following :

$$\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix} = y^2(x+y)$$

Answers

EXERCISE – 1 (UNSOLVED PROBLEMS)

2. 0 **3.** 0 **4.** 0 **5.** 0

10. $x = \frac{11}{3}, \frac{11}{3}, \frac{2}{3}$

13. 46 sq. unit **14.** $\lambda = \frac{1}{2}, -1$ **15.** 35 sq. units, No

EXERCISE – 2 (BOARD PROBLEMS)

4. (b) 576 **5.** (b) $2a^3b^3c^3$ **7.** 0 or 3a **15.** $\frac{2}{3}$ or $\frac{11}{3}$