1. ELASTICITY

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A body is said to be **rigid** if the relative positions of its constituent particles remain unchanged when external deforming forces are applied to it. The nearest approach to a rigid body is diamond or carborundum.

Actually no body is perfectly rigid and every object can be deformed to some extent or other by the application of suitable forces. All these deformed bodies, however, regain their original shape or size, when the deforming forces are removed.

The property of matter by virtue of which a body tends to regain its original shape and size after the removal of deforming forces is called **elasticity**.

1.1 Some Definitions

- **Deforming Force**

  External force which tends to bring about a change in the length, volume or shape of a body is called **deforming force.**

- **Elasticity**

  Elasticity is that property of a material of a body by virtue of which it opposes any change in its shape or size when deforming forces are applied on it, and recover its original state as soon as the deforming force are removed.

- **Perfectly Elastic Body**

  A body which perfectly regains its original form on removing the external deforming force, is defined as a **perfectly elastic body**. Example : quartz. - It is quite dose to a perfect elastic body.

- **Plastic Body**

  (a) A body which does not have the property of opposing the deforming forces, is known as a **plastic body**.

  (b) All bodies which remain in the deformed state even after the removed of the deforming forces are known as plastic bodies.

- **Restoring force**

  When an external force acts at any object then an internal resistance produced in the substance due to the intermolecular forces which is called **restoring force**.

  At equilibrium the numerical value of internal restoring force is equal to the external deforming force.

1.2 Stress

The restoring force acting per unit area of cross-section of the deformed body is called stress.

$$\text{Stress} = \frac{\text{Internal restoring force}}{\text{Area of cross section}} = \frac{F_{internal}}{A} = \frac{F_{external}}{A} \quad \text{(at equilibrium).}$$

SI UNIT : N/m^2 **Dimensions :** $[M^1L^{-1}T^{-2}]$

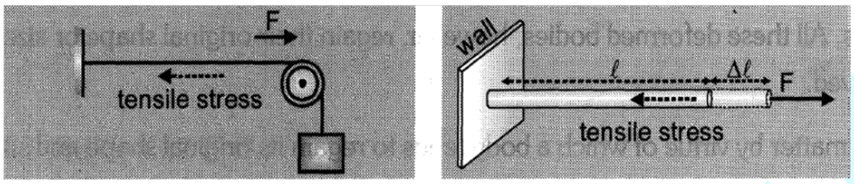
Type of stress –

(i) Longitudinal Stress

When the stress is normal to the surface of body, then it is known as **longitudinal stress.** There are two types of longitudinal stress :

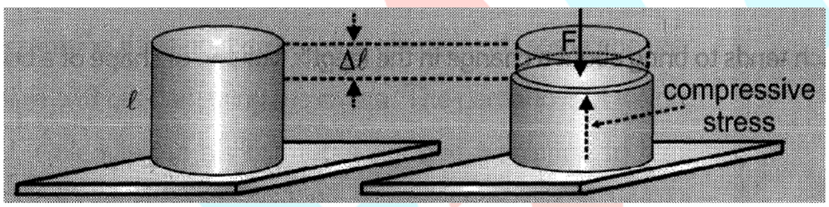
(a) Tensile Stress

The longitudinal stress, produced due to increase in the length of a body, is defined as **tensile stress.**



(b) Compressive Stress

The longitudinal stress, produced due to the decrease in the length of a body, is defined as **compressive stress.**



(ii) Volume Stress

If equal normal forces are applied over every unit surface of a body, then it undergoes a certain change in volume. The force opposing this change in volume per unit area is defined as **volume stress.**

(iii) Shear Stress

When the stress is tangential or parallel to the surface of a body then it is known as **shear stress.** Due to this stress, the shape of the body changes or it gets twisted but not its volume.

(iv) Breaking Stress

The stress required to cause the actual fracture of a material is called the **breaking stress** or ultimate strength.

Breaking stress = $\frac{F}{A}$; F = force required to break the body.

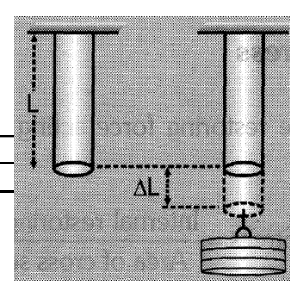
Dependence of breaking stress : (i) Nature of material (ii) Temperature (iii) Impurities.

Independence of breaking stress : (i) Cross sectional area or thickness (ii) Applied force.

Maximum load (force) which can applied on the wire depends on

(i) Cross sectional area or thickness (ii) Nature of material (iii) Temperature (iv) Impurities.

1.3 Strain

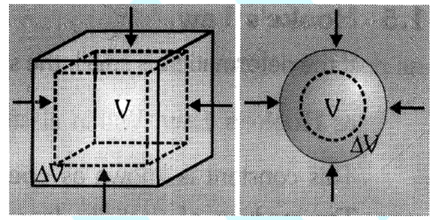


Strain = $\dfrac{\text{change in the dimension of the body}}{\text{original dimension of the body}}$

There are three types of strains :
Types of strains depend upon the directions of applied force.

(i) Longitudinal strain = $\dfrac{\text{change in length of the body}}{\text{initial length of the body}} = \dfrac{\Delta L}{L}$

Longitudinal strain is possible only in solids.

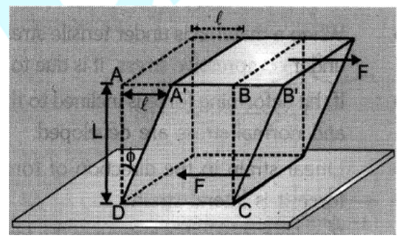


(ii) Volume strain = $\dfrac{\text{change in volume of the body}}{\text{original volume of the body}} = \dfrac{\Delta V}{V}$

(iii) Shear strain

When a deforming force is applied to a body parallel to its surface its shape (not size) changes. The strain produced in this manner is known as **shear strain.**

The stain produced due to a change in shape of the body is known as **shear strain.**

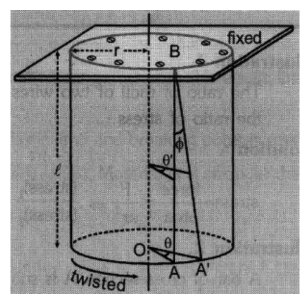


$\tan\phi = \dfrac{l}{L}$ (Here ϕ is very small)

Shear strain $\phi = \dfrac{l}{L}$

$\phi = \dfrac{\text{displacement of upper face relative to the lower face}}{\text{distance between two faces}}$

- **Relation Between angle of twist and Angle of shear**

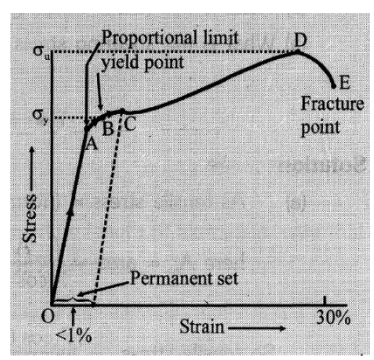


When a cylinder of length 'λ' and radius 'r' is fixed at one end and tangential force is applied at the other end, then the cylinder gets twisted. Figure shows the angle of shear ABA' and angle of twist AOA'.

Arc AA' = $r\theta$ and Arc AA' = $\lambda\phi$

so $r\theta = \lambda\phi \Rightarrow \phi = \dfrac{r\theta}{l}$

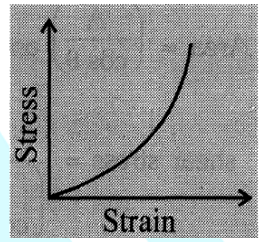
θ = angle of twist, ϕ = angle of shear.



1.4 Stress – Strain Graph

- In the region between O to A the curve is linear. Hooke's law is obeyed.
  In this region the solid behaves as an elastic body.
- In the region A to B stress and strain and not proportional but body regains its original shape and size when the load is removed.
- Point B is known as the elastic limit or yield point.
- If the load is increased further the stress developed than strains increases rapidly.

- In the region from B to D, when load is removed the body does not regain its original dimensions. Even when stress is zero the strain is not zero. The material is said to have a permanent set. The region beyond point B is known as the plastic region.
- Point D corresponds to the tensile strength; beyond this point additional strain is produced by even a reduced applied force and fracture occurs at point E.
- If plastic region is large then material will be ductile.
- If plastic region is small then material will be brittle.
- For some materials elastic region is very large and the material does not obey Hooke's law over most of the region. These are called elastomers e.g. Tissu of Aorta, rubber, etc.



1.5 Hooke's Law

If the deformation is small, the stress in a body is proportional to the corresponding strain; this fact is known , as **Hooke's Law**. Within elastic limit : stress $\propto$ strain $\Rightarrow \frac{\text{strees}}{\text{strain}}$ = constant.

This constant is known as **coefficient of elasticity or modulus of elasticity.**

The modulus of elasticity depends on the type of material temperature and impurity. It does not depend upon the values of stress and strain.

GOLDEN KEY POINTS

- When a material is under tensile stress restoring force is generated due to the intermolecular attraction while under compressive stress, it is due to the intermolecular repulsion.
- If the deforming force is inclined to the surface at an angular θ such that $\theta \neq 0$ and $\theta \neq 90°$ then both tangential and normal stress are developed.
- Linear strain in the direction of force is called longitudinal strain while in a direction perpendicular to the force it is lateral strain.
- Breaking stress· also measures the tensile strength.

Illustrations

Illustration 1.

The ratio of radii of two wires of same materials is 2 : 1. Find if they are stretched by the same force, the ratio of stress :

Solution:

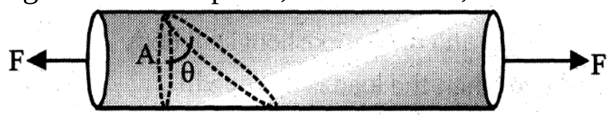
$$\text{Stress} = \frac{\text{force}}{\text{area}} = \frac{F}{\pi r^2} \Rightarrow \frac{(\text{stress})_1}{(\text{stress})_2} = \frac{F}{\pi r_1^2} \times \frac{\pi r_2^2}{F} = \left[\frac{r_2}{r_1}\right]^2 = \left[\frac{1}{2}\right]^2 = \frac{1}{4}.$$

Illustration 2.

A bar of cross-section A is subjected to equal and opposite tensile forces F at its ends. Consider a plane through the bar making an angle θ with a plane at right angles to the bar length.

(a) What is the tensile stress at this plane in terms of F, A and θ ?

(b) What is the shearing stress at this plane, in terms of F, A and θ ?



Solution:

(a) As tensile stress = (normal force / area)

here A_N = area = $\left(\frac{A}{\cos\theta}\right)$ and normal force $F_N = F\cos\theta$

So tensile stress = $\frac{F\cos\theta}{\left(\frac{A}{\cos\theta}\right)} = \frac{F\cos^2\theta}{A}$.

(b) As shear stress = (tangential force / area)

here Area = $\left(\frac{A}{\cos\theta}\right)$ and tangential force = F sinθ

So shear stress = $\frac{F\sin\theta}{\left(\frac{A}{\cos\theta}\right)} = \frac{F\sin\theta\cos\theta}{A} = \frac{F\sin 2\theta}{2A}$

Illustration 3.

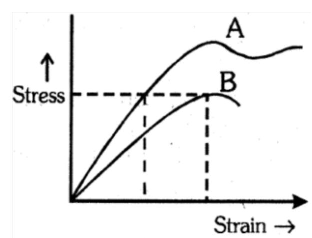
The upper end of a wire 1 meter long and 2 mm radius is clamped. The lower end is twisted through an angle of 45°. The angle of shear is .....

Solution:

$\phi = \frac{r}{l}\theta = \frac{2\times10^{-3}}{1}\times 45^\circ = 0.09^\circ$.

Illustration 4.

The stress versus strain graphs for two materials A and B are shown below. Explain the following



(a) Which material has greater Young's modulus ?
(b) Which material is more ductile ?
(c) Which material is more brittle ?
(d) Which of the two is more stronger material ?

Solution:

(a) Material A has greater value of Young's modulus, because slope of A is greater than that of B.
(b) Material A is more ductile because there is a large plastic deformation range between the elastic limit and the breaking point.
(c) Material B is more brittle because the plastic region between the elastic limit and breaking point is small.
(d) Strength of a material is determined by the stress required to cause fracture. Material A is stronger than material B.

Illustration 5.

A body of mass 10 kg is attached to a 30 cm long wire whose breaking stress is 4.8×10^7 N/m^2. The area of cross section of the wire is 10^{-6} m^2. What is the maximum angular velocity with which it can be rotated in a horizontal circle ?

Solution:

$$\frac{m\omega^2 l}{A} = \text{breaking stress (BS)} \Rightarrow \omega = \sqrt{\frac{(BS)A}{ml}} = \sqrt{\frac{4.8\times10^7\times10^{-6}}{10\times0.3}} = 4 \text{ rad/s}$$

Illustration 6.

The breaking stress of aluminium is 7.5×10^8 dyne/cm^2. Find the maximum length of aluminium wire that can hang vertically without getting broken. Density of aluminium is 2.7 g/cm^3.
Given : g = 980 cm/s^2,

Solution:

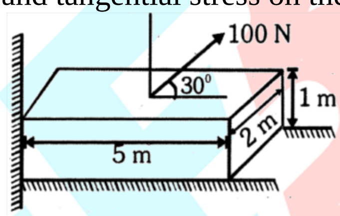
Let λ be the maximum length of the wire that can hang vertically without getting broken.
Mass of the wire, m = cross-sectional area (A) × length (λ) × density (ρ)
Weight of the wire = mg = $A\lambda\rho g$
This is equal to the maximum force that the wire can withstand.

$\therefore$ Breaking stress = $\frac{l A\rho g}{A} = \lambda\rho g$

or $7.5 \times 10^8 = \lambda \times 2.7 \times 980 \Rightarrow \lambda = \frac{7.5\times10^8}{2.7\times980} = 2.834 \times 10^5$ cm 2.83 km.

BEGINNER'S BOX - 1

1. Find out the longitudinal stress and tangential stress on the fixed block shown in figure.



2. A 2m long rod of radius 1 cm which is fixed at one end is given a twist of 0.8 radians at the other end. Find the shear strain developed.

3. The maximum stress that can be applied to the material of a wire employed to suspend an elevator is $\frac{3}{\pi} \times 10^8$ N/m^2. If the mass of the elevator is 900 kg and it moves up with an acceleration of 2.2 m/s^2 then calculate the minimum radius of the wire.

4. A human bone is subjected to a compressive force of 5.0×10^5N. The bone is 25 cm long and has an approximate cross sectional area of 4.0 cm^2. If the ultimate compressive strength of the bone is 1.70×10^8 N/m^2, will the bone be compressed or will it break under this force?

5. The breaking stress of steel is 7.9×10^8 N/m^2 and density is 7.9×10^3 kg/m^3. What should be the maximum length of a steel wire so that it may not break under its own weight?

6. A wire can bear a weight of 20 kg before it breaks. If the wire is divided into two equal parts, then each part will support a maximum weight .....

1.6 Types of Elasticity Coefficients

1. Young's Modulus of Elasticity 'Y'

Within elastic limit, the ratio of longitudinal stress to longitudinal strain is called Young's modulus of elasticity.

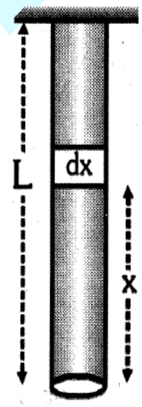
$$Y = \frac{\text{longitudinal stress}}{\text{longitudinal strain}} = \frac{F/A}{l/L} = \frac{FL}{lA}.$$

Within elastic limit, the normal force acting on a unit cross-sectional area of a wire due to which the length of the wire becomes double, is equivalent to the Young's modulus of elasticity of the material of the wire. If L is the original length of the wire, r is its radius and λ the increase in its length as a result of suspending a weight Mg at its lower end then Young's modulus of elasticity of the material of the wire is $Y = \frac{(Mg/\pi r^2)}{(l/L)} = \frac{MgL}{\pi r^2 l}$

Unit of Y : N/m^2 or pascal Dimensions of Y : $[M^1L^{-1}T^{-2}]$

- **Increment of length due to own weight**

Consider a rope of mass M and length L hanging vertically. As the tension at different points on the rope is different, stress as well as strain will be different at different points.



(i) maximum stress will be at the point of suspension

(ii) minimum stress will be at the lower end.

Consider an element of rope of length dx at x distance from the lower end,

then tension there $T = \left(\frac{M}{L}\right) x\ g$

So stress $= \frac{T}{A} = \left(\frac{M}{L}\right)\frac{xg}{A}$

Let increase in length of this element be dy then strain $= \frac{dy}{dx}$

So, Young modulus of elasticity $Y = \frac{\text{stress}}{\text{strain}} = \frac{\frac{M}{L}\frac{xg}{A}}{dy/dx} \Rightarrow \left(\frac{M}{L}\right)\frac{xg}{A}\,dx = Ydy$

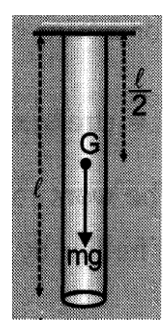
Summing up the expression for full length of the rope,

$$\frac{Mg}{LA}\int_0^L x dy = \int_0^{\Delta l} dy \Rightarrow \frac{Mg}{LA}\frac{L^2}{2} = Y\Delta\lambda \Rightarrow \Delta\lambda = \frac{MgL}{2AY}$$

[Since the stress is varying linearly we may apply the average method to evaluate strain.]

Alternate Method : Since the, weight acts at the centre of gravity, therefore

$\therefore$ the original length will be taken as $\frac{l}{2}$ $\quad \therefore Y = \frac{Mg \times \frac{l}{2}}{A \times \Delta l} \Rightarrow \Delta\lambda = \frac{Mgl}{2AY}$



But $M = (\lambda A)\rho$ $\quad \therefore \Delta\lambda = \frac{lA\rho gl}{2AY}$ or $\Delta\lambda = \frac{\rho gl^2}{2Y}$

2. Bulk's modulus of elasticity 'K' or 'B'

Within elastic limit, the ratio of the volume stress (i.e., change in pressure) to the volume strain is called bulk's modulus of elasticity.

$$K \text{ or } B = \frac{\text{volume stress}}{\text{volume strain}} = \frac{F/A}{\frac{-\Delta V}{V}} = \frac{\Delta P}{\frac{-\Delta V}{V}}$$

The minus sign indicates a decrease in volume with an increase in stress and vice-versa.

Unit of K : M/m^2 or pascal

Compressibility 'C'

The reciprocal of bulk's modulus of elasticity is defined as compressibility.

$C = \frac{1}{K}$; SI unit of C : m^2/N or pascal^{-1}

3. Modulus of Rigidity 'η'

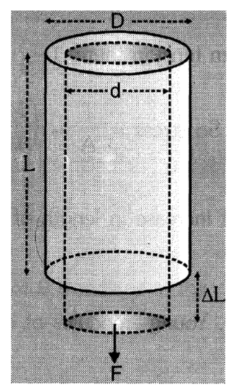
Within elastic limit, the ration of shearing stress to shearing strain is called modulus of rigidity of a material

$$\eta = \frac{\text{shearing stress}}{\text{shearing strain}} = \left(\frac{F_{tangential}}{\frac{A}{\phi}}\right) = \frac{F_{tangential}}{A\phi}$$

Note : Angle of shear 'ϕ' is always taken in radians

4. Poisson's Ratio (σ)

Within elastic limit, the ratio of lateral strain to the longitudinal strain is called Poisson's ratio.



$$\sigma = \frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{\beta}{\alpha}$$

$$\beta = \frac{-\Delta D}{D} = \frac{d - D}{d} \text{ and } \alpha = \frac{\Delta L}{L}$$

$-1 \le \sigma \le 0.5$ (theoretical limit)

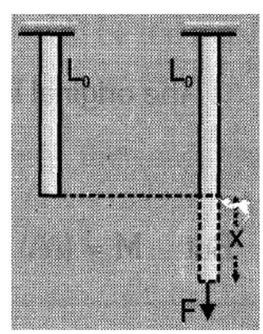
$\sigma \approx 0.2 - 0.4$ (experimental limit)

- **Relation between Y, K, η and σ :** (To be remembered)

$$Y = 3K(1 - 2\sigma), \qquad Y = 2\eta(1 + \sigma), \qquad \frac{9}{Y} = \frac{3}{\eta} + \frac{1}{K}.$$

1.7 Work done in stretching a wire (Potential energy of a stretched wire)

For a wire of length L_o stretched by a length x, the restoring elastic force is :



$$F = \text{stress} \times \text{area} = \left[\frac{x}{L_o}\right]A$$

The work required to be done against the elastic restoring forces to elongate it further by a length dx is,

$$dW = F.dx = \frac{YA}{L_o}x.dx$$

The total work done in stretching the wire from x = 0 to x = Δλ is,

$$W = \int_0^{\Delta l} \frac{YA}{L_o} x.dx = \frac{YA}{L_o}\left[\frac{x^2}{2}\right]_0^{\Delta l} \quad \text{or} \quad W = \frac{YA(\Delta l)^2}{2L_o} = \frac{1}{2} \times Y \times \left(\frac{\Delta l}{L_o}\right)^2 AL_0$$

$W = \frac{1}{2} \times Y \times (\text{strain})^2 \times \text{volume}$ $\quad$ $W = \frac{1}{2}$ (stress) (strain) (volume).

1.8 Factor Affecting Elasticity

- **Effect of Temperature**

  $T\uparrow \Rightarrow Y\downarrow$ Due to weakness of intermolecular force.

  When temperature is increased, the elastic properties in general decreases i.e. elastic constants decrease. Plasticity increases with temperature.

  For a special kind of steel, elastic constants do not vary appreciably with temperature. This steel is called INVAR steel.

- **Effect of Impurities**

  Y slightly increases with impurities. The inter molecular attraction strengthens impurities consequently, external deformation can be more effectively opposed.

- **Interatomic Force Constant :**

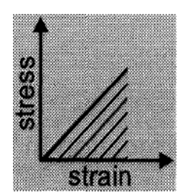
  k or $k_a = Y.\, r_0$

  Y = Young's modulus ; r_0 = interatomic distance under normal circumstances

GOLDEN KEY POINTS

- The value of K is maximum for solids and minimum for gases.
- Maxwell was the first to define bulk's modulus.
- For liquid and gases Young's modulus and modulus of rigidity are each equal to zero.
- For any ideal rigid body all the three elastic modulii are infinite .
- Modulus of rigidity (η) is the characteristic of solid material only as the fluids do not have a fixed shape.
- $W = \frac{1}{2}$ (Load) (extension) $= \frac{F}{2}\Delta\lambda$ [where $\Delta\lambda$ is the extension in length]
- This work done in elongating a wire is stored in the wire as elastic potential energy.

  Thus, the elastic potential energy density $u = \frac{W}{V} = \frac{1}{2}$ (stress) (strain)



- Potential energy density = area under the stress-strain curve.
- Young's modulus = Slope of the stress-strain curve.
- **Application of Elastic Behaviour of Materials :**

(1) Crane Cross-sectional area $A \geq \frac{W}{S_y} = \frac{mg}{S_y}$

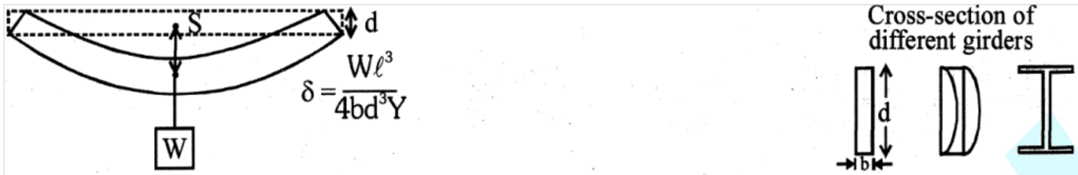
S_y = yield strength or breaking stress; W = weight of the object being lifted.

e.g. for 10 metric tone load

S_y for steel being 300×10^6 N/m^2

$A \geq 3.3 \times 10^{-4}\ m^2$

(2) Girder



$$\delta = \frac{W\ell^3}{4bd^3Y}$$

(3) Max height of mountain

$H\rho g$ = breaking stress / tensile strength for a rock = $30 \times 10^7\ N/m^2$

Illustrations

Illustration 7.

Two wires are made of the same metal. The length of the first wire is half that of the second wire and its diameter is double that of the second wire. If equal loads are applied on both the wires, find the ratio of increase in their lengths.

Solution:

$$Y = \frac{\frac{F}{A}}{\frac{\Delta l}{l}} \Rightarrow \Delta\lambda = \frac{Fl}{AY} = \frac{Fl}{\pi r^2 Y} \qquad \frac{\Delta l_1}{\Delta l_2} = \frac{4Fl_1}{\pi d_1^2 Y} \times \frac{\pi d_2^2 Y}{4Fl_2} = \frac{l_1}{l_2} = \frac{d_2^2}{d_1^2} = \frac{1}{2} \times \frac{1}{(2)^2} = \frac{1}{8}.$$

Illustration 8.

Two wires of diameter 0.25 cm, one made of steel and the other made of brass are loaded as shown in Fig. The unloaded length of steel wire is 1.5 m and that of brass wire is 1.0 m. Young's modulus of steel is 2.0×10^{11} Pa and that of brass is 0.91×10^{11} Pa. Calculate the elongations of the steel and brass wires. (1 Pa = 1 N/m^2)

Solution:

Θ The elongation in steel wire $\Delta\lambda_S$ $= \dfrac{Mgl_S}{\pi r^2 Y_S} = \dfrac{(4+6)\times 9.8 \times 1.5)}{3.14 \times (0.125 \times 10^{-2})^2 \times 2 \times 10^{11}} = 1.50 \times 10^{-4}$ m

The elongation in brass wire $\Delta\lambda_B$ $= \dfrac{6 \times 9.8 \times 1.0}{3.14 \times (0.125 \times 10^{-2})^2 \times 0.91 \times 10^{11}} = 1.32 \times 10^{-4}$ m

Illustration 9.

A copper wire of negligible mass, length 1 m and cross-sectional area $10^{-6}\ m^2$ is kept on a smooth horizontal table with one end fixed. A ball of mass 1kg is attached to the other end. The wire and the ball are rotating with an angular velocity of 20 rad/s. If the elongation in the wire is 10^{-3} m, obtain the Young's modulus of copper.

Solution:

Θ Centripetal force $F = m\omega^r r$ $\therefore$ Stress in the wire $= \dfrac{F}{A} = \dfrac{m\omega^2 r}{A}$ and Strain in the wire $= \dfrac{\Delta l}{l}$

Young's modulus $Y = \dfrac{\text{Stress}}{\text{Strain}} = \dfrac{m\omega^2 l.l}{A.\Delta l} = \dfrac{1 \times (20)^2 \times (1)^2}{10^{-6} \times 10^{-3}} = 4 \times 10^{11}\ N/m^2$.

Illustration 10.

By how much will a 3.0 m long copper wire elongate if a weight of 10 kg is suspended from the lower end with the upper end keeping fixed? The diameter of the wire is 0.4 mm. Given: Y for copper = $10^{11}\ N/m^2$ and g = 9.8 m/s^2.

Solution:

$$\Delta\lambda = \frac{F \times l}{\pi r^2 \times Y} \qquad \therefore \Delta\lambda = \frac{98 \times 3 \times 7}{22 \times (0.2 \times 10^{-3})^2 \times 10^{11}} = \frac{98 \times 21}{88 \times 10^3} = 2.34 \text{ cm.}$$

Illustration 11.

Calculate the force required to increase the length of a steel wire of cross-sectional area 10^{-6} m^2 by 0.5% given : $Y_{(\text{for steel})} = 2 \times 10^{11}$ N/m^2.

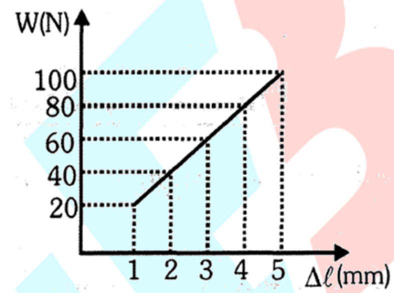
Solution:

$$\frac{l}{L} \times 100 = 0.5\% \qquad \Rightarrow \frac{l}{L} = 5 \times 10^{-3}$$

$$\text{so } F = YA\frac{l}{L} = 2 \times 10^{11} \times 10^{-6} \times 5 \times 10^{-3} = 10^3 \text{ N.}$$

Illustration 12.

The graph shows the extension of a wire of length l m suspended from a roof at one end and with a load W connected to the other end. If the cross sectional area of the wire is 1 mm^2, then the Young's modulus of the material of the wire is



Solution:

$$Y = \frac{F/A}{\Delta l / l} = \frac{Wl}{A\Delta l} \Rightarrow \frac{W}{\Delta l} = \frac{YA}{l} = \text{slope} \Rightarrow Y = \frac{l}{A}(\text{slope}) = \frac{1}{10^{-6}}\left(\frac{40-20}{(2-1)\times 10^{-3}}\right) = 2 \times 10^{10} \text{ N/m}^2.$$

Illustration 13.

A rubber rope of length 8 m is hung from the ceiling of a room. What is the increase in length of the rope due to its own weight? (Given: Young's modulus of elasticity of rubber = 5×10^6 N m^{-2} and density of rubber = 1.5×10^3 kg/m^3 and g = 10 m/s^2).

Solution:

$$\Delta\lambda = \frac{\rho g l^2}{2Y} = \frac{1.5 \times 10^3 \times 10 \times 8 \times 8}{2 \times 5 \times 10^6} = 9.6 \times 10^{-2} \text{ m} = 9.6 \times 10^{-2} \times 10^3 \text{ mm} = 96 \text{ mm.}$$

Illustration 14.

A sphere contracts in volume by 0.01% when taken to the bottom of sea 1 km deep. Find the bulk's modulus of the material of the sphere. Given: density of sea water is 1 g/cm^3, g = 980 cm/s^2.

Solution:

$$\frac{|\Delta V|}{V} = \frac{0.01}{100}, \; h = 1 \text{ km} = 10^5 \text{ cm}, \; \rho = 1 \text{g/cm}^3 \; ; \; \Delta P = 10^5 \times 1 \times 980 \text{ dyne/cm}^2, \; K = ?$$

$$K = \frac{\Delta P}{|\Delta V|} = \frac{\Delta P \times V}{|\Delta V|} = \frac{10^5 \times 980 \times 100}{0.01} \text{ dyne/cm}^2 = 9.8 \times 10^{11} \text{ dyne/cm}^2.$$

Illustration 15.

A rubber cord has a cross-sectional area 1 mm^2 and total unstretched length 10 cm. It is stretched to 12 cm and then released to project a mass of 5 g. Young's modulus for rubber is 5×10^8 N/m^2. Find the tension in the cord and velocity of the mass.

Solution:

Tension in the cord = $\frac{YA}{L}\Delta l = \frac{5\times10^8 \times 1\times10^{-6}\times 2\times10^{-2}}{10\times10^{-2}}$ = 100 N.

When the mass is released, elastic energy stored = Kinetic energy of mass $\Rightarrow \frac{1}{2}F \times \Delta\lambda = \frac{1}{2}mv^2$

$$v = \sqrt{\frac{F\times\Delta l}{m}} = \sqrt{\frac{100\times2\times10^{-2}}{5\times10^{-3}}} = 20 \text{ m/s.}$$

Illustration 16.

Young modulus of elasticity of steel is 2×10^{11} N/m^2. If interatomic distance for steel is 3.2 Aº, then find the interatomic force constant.

Solution:

$k = Y \times r_0 = 2 \times 10^{11} \times 3.2 \times 10^{-10} = 64$ N/m.

BEGINNER'S BOX - 2

1. A stress of 20×10^8 N/m^2 is developed when the length of a wire is doubled. Its Young's modulus of elasticity in N/m^2 will be ....

2. The ratio of lengths of two wires made up of the same material is 3 : 1 and the ratio of their radii is 1 : 3. The ratio of increments of lengths on account of suspending the same weight will be......

3. The following four wires are made of same material. Which one will have the largest elongation when subjected to the same tension ?

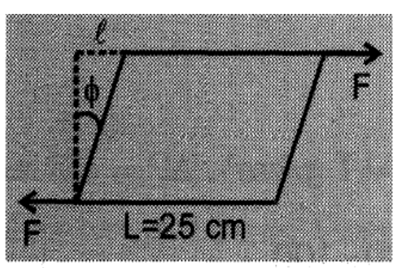
(1) Length 500 cm, diameter 0.05 mm. (2) Length 200 cm, diameter 0.02 mm.

(3) Length 300 cm, diameter 0.03 mm. (4) Length 400 cm, diameter 0. 01 mm.

4. The bulk's modulus of copper is 138×10^9 Pa. The additional pressure generated in an explosion chamber is 345×10^6 Pa. Then the percentage change in the volume of a piece of copper placed in this chamber will be ....

5. The compressibility of water per unit atmospheric pressure is 4×10^{-5}. Decrease in the volume of 100 cm^3 water of volume at 100 atmospheric pressure will be ....

6. Two parallel and opposite forces, each of magnitude 4000 N, are applied tangentially to the upper and lower faces of a cubical metal block of side 25 cm. If the shear modulus for the metal is 8×10^{10} Pa, then the displacement of the upper surface relative to the lower surface will be......



7. Young modulus of elasticity of brass is 10^{11} N/m^2. The increase in its energy on pressing a rod of length 0.1 m and cross-sectional area 1 cm^2 made of brass with a force of 10 kg along its length, will be ....

2. HYDRO-STATICS

Fluids are the substances that can flow or deforms. Therefore liquids and gases both are fluids.

Study of fluids at rest is called fluid statics or hydrostatics and the study of fluid in motion is called fluid dynamics or hydrodynamics. Fluid statics and fluid dynamics collectively known as fluid mechanics.

The intermolecular force in liquids are comparatively weaker than in solids. Therefore, their shapes can be changed easily. When external force (shear stress) are present, liquid can flow until it conforms to the boundaries of its container. Most liquids resist compression. Unlike a gas, a liquid does not disperse to fill every space of a container and it forms a free surface.

The intermolecular forces are weakest in gases, so their shapes and sizes can be changed much easily. Gases are highly compressible and occupy the entire space of the container quite rapidly. Unlike liquid, gases can't form free surface.

2.1 Density (ρ)

Mass per unit volume of a substance is defined as density. So density at a point of a fluid is expressed as

$$\rho = \underset{\Delta V \to 0}{\text{limit}} \frac{\Delta m}{\Delta V} = \frac{dm}{dV}$$

SI UNIT : kg/m^3 ; **CGS UNIT :** g/cc **Dimensions :** [ML^{-3}]

Density is a positive scalar quantity.

2.2 Specific Weight or Weight Density (w)

It is defined as the ration of the weight of the substance to its volume or the weight acting per unit volume of the fluid.

$$w = \frac{\text{weight (W)}}{\text{volume (V)}} \frac{mg}{V} = \left[\frac{m}{V}\right] g = \rho g.$$

SI UNIT : N/m^3 **Dimensions :** [ML^{-2}T^{-2}]

Specific weight of pure water is 9.81 kN/m^3 at 4ºC.

2.3 Relative Density

It is defined as the ratio of the density of the given fluid to the density of pure water at 4ºC.

$$\text{Relative density (R.D.)} = \frac{\text{density of given liquid}}{\text{density of pure water at } 4^{\circ}\,C} = \frac{\rho_l}{\rho_w}$$

Relative density is a unitless and dimensionless positive scalar quantity.

Being a dimensionless/unitless quantity R.D. of a substance is same in both SI and CGS system.

2.4 Specific Gravity

It is defined as the ratio of the specific weight of the given fluid to the specific weight of pure water at 4ºC.

$$\text{Specific gravity} = \frac{\text{specific weight of given liquid}}{\text{specific weight of pure water at } 4^\circ C\,(9.81\,kN/m^3)}$$

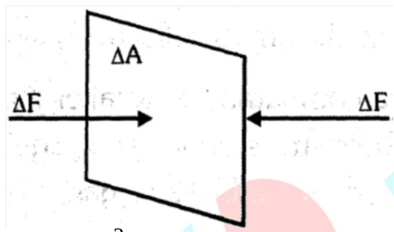
$$= \frac{\rho_l \times g}{\rho_w \times g} = \frac{\rho_l}{\rho_w} = \text{R.D. of liquid.}$$

Thus the specific gravity of a liquid is numerically equal to the relative density of that liquid and for calculation purposes they are used interchangeably.

2.5 Pressure

Pressure P is defined as the magnitude of the normal force acting per unit surface area.

$P = \dfrac{\Delta F}{\Delta A}$, here ΔF = normal force on a surface of area ΔA



SI UNIT : Pascal (Pa) ; 1 Pa = 1 N/m^2 **Dimensions :** $[ML^{-1}T^{-2}]$

Practical units : atmospheric pressure (atm), bar and torr.

1 atm = 1.01325×10^5 Pa = 1.01325 bar = 760 torr = 760 mm of Hg = 10.33 m of water

1 bar = 10^5 Pa; 1 torr = pressure exerted by 1 mm of mercury column= 133 Pa.

Pressure is a scalar quantity. This is because hydrostatic pressure is transmitted equally in all directions when force is applied, which shows that pressure is not associated with a definite direction.

Consequences of pressure

(i) Railway tracks are laid on wide wooden or iron sleepers. This is because the weight (force) of the train is spread over a large area of the sleeper. This reduces the pressure acting on the ground and hence prevents the yielding of ground under the weight of the train.

(ii) A sharp knife is more effective in cutting the objects than a blunt knife. The pressure exerted = Force / area. The sharp knife transmits force over a small area as compared to the blunt knife. Hence the pressure exerted in case of a sharp knife is more than that in case of a blunt knife.

(iii) A camel walks easily on sand but a horse cannot inspite of the fact that a camel is heavier than horse. This is because the area of camel's feet is large as compared to horse's feet. So the pressure exerted by camel on the sand is very small as compared to the pressure exerted by horse. Due to large pressure, sand under the feet of horse yields and hence it cannot walk easily on sand.

Type of Pressures

In our day to day activity we commonly encounter the following three types of pressures :

Pressure is of three types

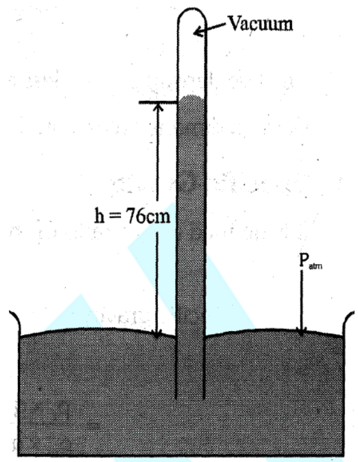
(i) Atmospheric pressure (P_o) (ii) Gauge pressure (P_{gauge}) (iii) Absolute pressure ($P_{abs.}$)

(1) Atmospheric pressure and Torricelli's experiment :- Force exerted by atmospheric column on unit cross-sectional area at mean sea level is called atmospheric pressure (P_o)

$P_o = 101.3\ kN/m^2$

$\therefore \quad P_o = 1.013 \times 10^5\ N/m^2$

At tube of length 1 m and uniform cross section is taken. It is filled with mercury and inverted into a mercury tray. The height of the mercury column in equilibrium inside the tube is 76 cm.



$\therefore$ atmospheric pressure $P_o = \rho gh$

$= 13.6 \times 10^3 \times 9.81 \times 76 \times 10^{-2}$

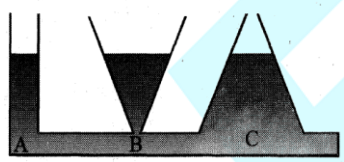
$= 1.013 \times 10^5\ N/m^2$

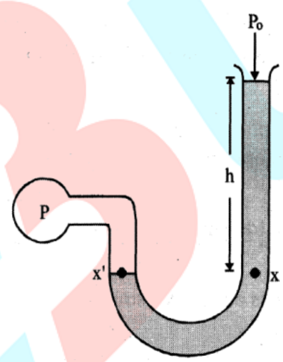
Note : The above apparatus is known as a barometer. Barometer is used to measure the atmospheric pressure.

(2) Gauge Pressure :- Excess Pressure over the atmospheric pressure ($P - P_{atm}$) measured with the help of pressure measuring instruments is called gauge pressure.

$$P_{gauge} = \frac{F}{A} = \frac{Mg}{A} = \frac{(volume \times density)g}{A} = \frac{(Ah)\rho g}{A}$$

$P_{gauge} = h\rho g$ or $P_{gauge} \propto h$





Pressure due to liquid $P_A = P_B = P_C$

$P_{x'} = P_x$

$P_{x'} = P_o + h\rho g$

Gauage pressure $= P_{x'} - P_o = h\rho g$

Note : Gauge pressure is always measured with the help of a **"manometer".**

(3) Absolute Pressure :- Sum of the atmospheric and gauge pressure is called absolute pressure.

$P_{abs} = P_{atm} + P_{gauge}$

$P_{abs} = P_o + h\rho g$

Pressure exerted by a liquid (Effect of gravity) :

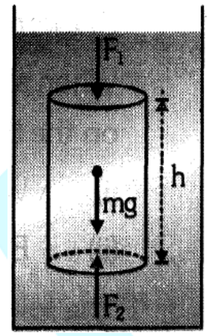
Consider a vessel containing liquid. As the liquid is in equilibrium, so every volume element of the fluid is also in equilibrium. Consider one such volume element in the form of a cylindrical column of liquid of height h and of area of cross section A. The various forces acting on the cylindrical column of liquid are :

(i) Force $F_1 = P_1$ A, acting vertically downward on the top face of the column. P_1 is the pressure of the liquid on the top face of the column.

(ii) Force $F_2 = P_2$ A, acting vertically upward at the bottom face of the cylindrical column. P_2 is the pressure of the liquid on the bottom face of the column.

(iii) W = mg, weight of the cylindrical column of the liquid acting vertically downward.

Since the cylindrical column of the liquid is in equilibrium, so the net force acting on the column is zero.



$F_1 + W - F_2 = 0 \Rightarrow P_1A + mg - P_2A = 0 \Rightarrow P_1A + mg = P_2A \therefore P_2 = P_1 + \dfrac{mg}{A}$ …(i)

Now, mass of the cylindrical column of the liquid is,

m = volume × density of the liquid = Area of cross section × height × density

$= Ah\rho$

$\therefore$ equation (i) becomes $P_2 = P_1 + \dfrac{Ah\rho g}{A}$, $P_2 = P_1 + h\rho g$ …..(ii)

P_2 is the absolute pressure at a depth h below the free surface of the liquid. Equation (ii), shows that the absolute pressure at a depth h is greater than the absolute pressure (P_1) by an amount equal to $h\rho g$.

Equation (ii) can also be written as $(P_2 - P_1) = h\rho g$ which is the difference of pressures between two points separated by a depth h.

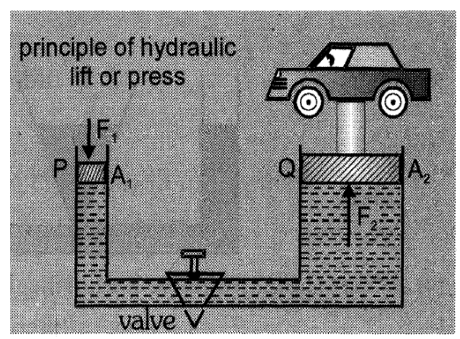
2.6 Pascal's Law

Pascal's law is stated in following ways-

- A liquid exerts equal pressures in all directions.
- If the pressure in an enclosed fluid is changed at a particular point, the change is transmitted to every point of the fluid and to the walls of the container without being diminished in magnitude.

Applications of pascal's law: hydraulic jacks, hydraulic lifts, hydraulic press, hydraulic brakes, etc

Hydraulic lift



$(A_2 > A_1)$

Pressure applied = $\dfrac{F_1}{A_1}$

$\therefore$ Pressure transmitted = $\dfrac{F_2}{A_2}$

$\because$ Pressure is equally transmitted $\therefore \dfrac{F_1}{A_1} = \dfrac{F_2}{A_2}$

$\therefore$ Upward force on A_2 is $F_2 = \dfrac{F_1}{A_1} \times A_2 = \dfrac{F_2}{A_1} \times F_1$

2.7 Buoyancy and Archimede's Principle

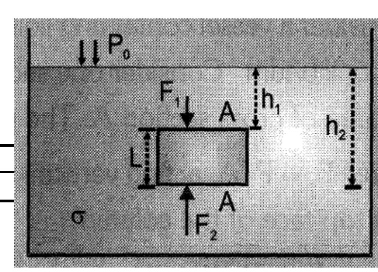
Buoyant Force

If a body is partially or fully immersed in a fluid, it experiences an upward force due to the fluid surrounding it. This phenomenon of force exerted by fluid on the body is called buoyancy and force is called buoyant force or force of up thrust.

Archimede's Principle

It states that the upward buoyant force on a body that is partially or totally immersed in a fluid is equal to the weight of the fluid displaced by it.

Consider a body immersed in a liquid of density σ.



Top surface of the body experiences a downward force

$F_1 = AP_1 = A[h_1.\sigma.g + P_0]$…….(i)

Lower face of the body will experience an upward force

$$F_2 = AP_2 = A[h_2.\sigma.g + P_0]\ldots\ldots.(ii)$$

As $h_2 > h_1$ so F_2 is greater than F_1

so net upward force $F = F_2 - F_1 = A\sigma g[h_2 - h_1]$

$\therefore$ $F = A.\sigma.g.L. = V_{in}.\sigma,g$ [Θ V_{in} = volume of the body submerged in the fluid = AL]

Principle of Floatation

When a body of density (ρ) and volume (V) is completely immersed in a liquid of density (σ), the forces acting on the body are :

(i) Weight of the body $W = Mg = V\rho g$ directed vertically downwards through the Centre of gravity of the body.

(ii) Buoyant force or Upthrust $Th = V\sigma g$ directed vertically upwards through Centre of buoyancy.

The following three cases are possible :

Case I : Density of the body is greater than that of liquid ($\rho > \sigma$)

In this case $W > Th$

So the body will sink to the bottom of the liquid.

$$W_{App} = W - Th = V\rho g - V\sigma g = V\rho g\,(1 - \sigma/\rho) = W\,(1 - \sigma/\rho).$$

Case II : Density of the body is equal to the density of liquid ($\rho = \sigma$)

In this case $W = Th$

So the body will float fully submerged in the liquid. It will be in neutral equilibrium.

$$W_{App} = W - Th = 0$$

Case III : Density of the body is lesser than that of liquid ($\rho < \sigma$)

In this case $W < Th$

So the body will float partially submerged in the liquid. In this case the volume of liquid displaced by the body (V_{in}) will be less than the volume of body (V). This ensures that Th equally to W

$$\therefore \quad W_{App} = W - Th = 0$$

The above three cases constitute the **laws of floatation** which states that a body will float in a liquid if weight of the liquid displaced by the immersed part of the body is at least equal to the weight of the body.

GOLDEN KEY POINTS

- For a solid body volume and density will be same as that of its constituent substance of equal mass i.e. if

  $M_{body} = M_{sub}$ then $V_{body} = V_{sub}$ and $\rho_{body} = P_{sub}$. But for a hollow body or body with air gaps or avities, $M_{body} = M_{sub}$ and $V_{body} > V_{sub}$ then $\rho_{body} < \rho_{sub}$

- If m_1 mass of liquid of density ρ_1 and m_2 mass of an immiscible liquid of density ρ_2 are mixed then

  $M_{mix} = m_1 + m_2$ and $V_{mix} = V_1 + V_2 = \dfrac{m_1}{\rho_1} + \dfrac{m_2}{\rho_2}$ $\quad\therefore\quad$ $\rho_{mix.} = \dfrac{M_{mix}}{V_{mix}} = \dfrac{m_1 + m_2}{\dfrac{m_1}{\rho_1} + \dfrac{m_2}{\rho_2}}$

  If liquids with same masses are mixed i.e. $m_1 = m_2 = m$ then $\rho_{mix.} = \dfrac{2\rho_1\rho_2}{\rho_1 + \rho_2}$ (Harmonic mean of individual densities)

- If V_1 volume of liquid of density ρ_1 and V_2 volume of liquid of density ρ_2 are mixed then

$$V_{mix.} = V_1 + V_2 \quad \text{and} \quad M_{mix.} = m_1 + m_2 = \rho_1 V_1 + \rho_2 V_2$$

$$\therefore \quad \rho_{mix.} = \frac{M_{mix}}{V_{mix}} = \frac{\rho_1 V_1 + \rho_2 V_2}{V_1 + V_2}$$

If liquids with same volumes are mixed i.e. $V_1 = V_2 = V$ then $\rho_{mix.} = \frac{\rho_1 + \rho_2}{2}$ (arithmetic mean of individual densities)

- Pressure due to liquid on a vertical wall is different at different depths, so average fluid pressure on side wall a of container = mean pressure = $\frac{h\rho g}{2}$ (h = height of wall)

- Buoyance force acts vertically upward through the centre of gravity (C.G.) of the displaced fluid. This point is called the centre of buoyancy (C.B.). Thus the centre of buoyancy is the point through which the force of buoyancy is supposed to act.

- Buoyant force or force of up thrust does not depend upon the characteristics of the body such as its mass, size, density, etc. However it depends upon the volume of the body inside the liquid.

  $$Th \propto V_{in}$$

- It depends upon the nature of the fluid as it is proportional to the density of the fluid. $\Rightarrow Th \propto \sigma$.
  This is the reason that force of upthrust on a fully submerged body is more in sea water than in pure water. (Θ $\sigma_{sea} > \sigma_{pure}$)

- The effective weight of a body decreases due to upthrust

  $W_{App} = W - Th$ (W is the true weight of the body)

  Decrease in weight = $W - W_{App}$ = Th = Weight of the fluid displaced

- Using Archimede's principle we can determine the relative density (R D) of a body as

  $$\text{R.D.} = \frac{\text{density of body}}{\text{density of pure water at } 4^\circ\text{C}} = \frac{\text{wt. of body}}{\text{wt. of equal volume of water}}$$

  $$= \frac{\text{wt. of body}}{\text{force of upthrust due to water}} = \frac{\text{wt. of body}}{\text{loss of wt. in water}} = \frac{\text{wt. of body in air}}{\text{wt. in air} - \text{wt. in water}} = \frac{W_A}{W_A - W_W}$$

- If a body is weighed in air (W_A), in water (W_A) and in a liquid (W_L), then

  $$\text{Specific gravity of liquid} = \frac{\text{loss of weight in liquid}}{\text{loss of weight in water}} = \frac{W_A - W_L}{W_A - W_W}$$

- In case of W = Th, the equilibrium of a floating body does not depend upon the variation in g though both thrust and weight depends upon g.

- The weight of a plastic bag full of airs is same as that of empty bag because the force of upthrust is equal to the weight of the air enclosed.

Illustrations

Illustration 17.

A hollow metallic sphere has inner and outer radii, as 5 cm and 10 cm respectively. If the mass of the sphere is 2.5 kg. Find the (a) density of the materia1, (b) relative density of the material of the sphere.

Solution:

Volume of the material of the sphere is

$$V = \left(\frac{4}{3}\right)\pi\left(r_2^3 - r_1^3\right) = \frac{4}{3} \times 3.14 \times \left[\left(\frac{10}{100}\right)^3 - \left(\frac{5}{100}\right)^3\right] = \frac{4}{3} \times 3.14 \times [0.001 - 0.000125]$$

$$= \frac{4}{3} \times 3.14 \times 0.000875 \text{ m}^3 = 0.00367 \text{ m}^3$$

(a) Therefore, density of the material of the sphere is $\rho = \frac{M}{V} = \frac{2.5}{0.00367}$ kg/m^3 = 681.2 kg/m^3

(b) Relative density of the material of the sphere $\rho_r = \frac{681.2}{1000} = 0.6812$

Illustration 18.

Two immiscible liquids of densities 2.5 g/cm^3 are taken in the ratio of their masses as 2.3 respectively. Find the average density of the liquid combination.

Solution:

Let masses be 2M & 3M then $V = V_1 + V_2 = \left(\frac{2M}{2.5} + \frac{3M}{0.8}\right)$ cm^3

Total mass = 2M + 3M = 5M

Therefore, the average density $\rho_{av} = \frac{5M}{V} = \frac{5M}{\frac{2M}{2.5} + \frac{3M}{0.8}} = \frac{2}{\frac{2}{2.5} + \frac{3}{0.8}} = \frac{10}{9.1}$ g/cm^3 = 1.09 g/cm^3

Illustration 19.

During a blood transfusion a needle is inserted in a vein where the gauge pressure is 2000 Pa. At what height must the blood container be placed so that blood may just enter the vein ? [Density of blood= 1.06 × 10^3 kg/m^3].

(1) 0.192 m (2) 0.182 m (3) 0.172 m (4) 0.162 m

Ans. (1)

Solution:

Pressure $P = h\rho g \quad \Rightarrow h = \frac{2000}{1.06 \times 10^3 \times 9.8} = 0.192$ m.

Illustration 20.

Calculate the depth of a well if the pressure at its bottom is 15 times that at a depth of 3 meters. Atmospheric pressure is 10 m column of water.

Solution:

Let the depth of the well be h then according to the question,

$P_{atm} + h\rho_w g = 15 (P_{atm} + 3\rho_w g)$

$h\rho_w g = 14 P_{atm} + 45 \rho_w g = 14 (10 \times \rho_w g) + 45 \rho_w g$

h = 185 m.

Illustration 21.

AU-tube contains water and methylated spirit separated by mercury. The mercury columns in the two arms are in level with 10.0 cm column of water in one arm and 12.5 cm column of spirit in the other. What is the specific gravity of spirit ?

Solution:

Since level of mercury in the two arms of U-tube is same and if P_a is the atmospheric pressure then

$$P_a + h_w\rho_w g = P_a + h_s\rho_s g \Rightarrow \frac{\rho_s}{\rho_w} = \frac{h_w}{h_s} = \frac{10}{12.5} = 0.8 \;\; \therefore \text{ Specific gravity of spirit} = 0.8$$

Illustration 22.

Two liquids that do not mix are poured into a U-shaped tube as shown in fig. Find the difference H in these heights of liquids in terms of ρ_1, ρ_2 h.

Solution:

Starting from the point B, we apply the manometric equation as –

$P_B + \rho_2 g\,(h + H) - \rho_1 gh = P_A$

Since $P_A = P_B = P_{atm}$ therefore

$$H = \left(\frac{\rho_1 - \rho_2}{\rho_2}\right)h$$

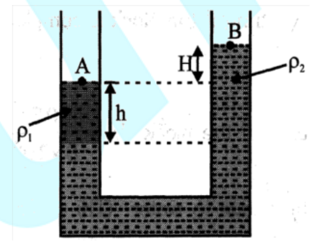
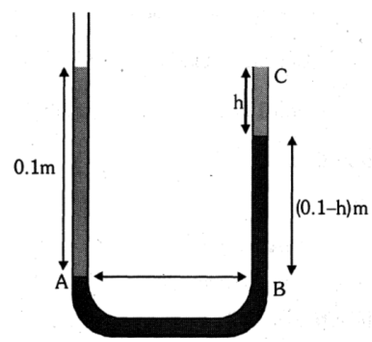


Illustration 23.

A vertical U-tube of uniform cross-section contains mercury in both arms. A glycerine (relative density = 1.3) column of length 10 cm is introduced into one of the arms. Oil of density 800 kg/m^3 is poured into the other arm until the upper surface of the oil and glycerine are at the same horizontal level. Find the length of the oil column.

Density of mercury is $13.6 \times 10^3\ kg/m^3$.



Solution:

Pressure at A and B must be same

Pressure at A= $P_0 + 0.1 \times (1.3 \times 1000) \times g$

Pressure at B = $P_0 + h \times 800 \times g + (0.1 - h) \times 13.6 \times 1000\ g$

$\Rightarrow 0.1 \times 1300 = 800\ h + (0.1 - h) \times 13600$

$\Rightarrow h = 0.096\ m = 9.6\ cm$

Illustration 24.

A hydraulic automobile lift is designed to lift cars with a maximum mass of 3000 kg. The area of cross-section of the piston carrying the load is 425 cm^2. What maximum pressure would the piston have to bear? (taking g = 10 m/s^2).

Solution:

According to Pascal's law $P_1 = P_2 = \frac{F_2}{F_1} = \frac{3000 \times 10}{425 \times 10^{-4}}$ Pa $= \frac{3000}{425} \times 10^5$ Pa $= 7.06 \times 10^5$ Pa.

Illustration 25.

A cubical box of wood of side 30 cm weighing 21.6 kg floats on water with two faces horizontal. Calculate the depth of immersion of wood.

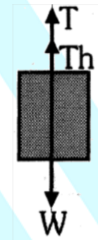
Solution:

For a floating body, the weight of displaced liquid should be equal to the weight of the block. Let x be the depth of immersion. Then $(x\times30\times30)\times1\times g = 21.6\times10^3 \times g \Rightarrow x = \frac{216\times10^3\times g}{30\times30\times g} = 24$ cm.

Illustration 26.

A block of brass of mass 0.5 kg and density 8×10^3 kg/m^3 is suspended from a string. What will be the tension in the string if the block is completely immersed in water? ($g = 10$ m/s^2)

Solution:



Volume of the block $V = \frac{0.5}{8\times10^3}$ m^3

upthrust due to water (Th) $= V\rho_w g = \frac{0.5}{8\times10^3} \times 10^3 \times 10 = \frac{5}{8} = 0.625$ N

The tension in the string $T = W - Th = mg - Th = 0.5 \times 10 - 0.625 = 4.375$ N.

Illustration 27.

A log of wood floats in water with $\frac{1}{5}$ of its volume above the surface. What is the density of wood?

Solution:

For a floating body, weight = force of upthrust $\Rightarrow V_B\rho_B.g = V_{in}.\rho g \quad \Rightarrow \quad V.\rho_B = \frac{4}{5}V.\rho_W$

$$\Rightarrow \rho_B = \frac{4}{5} \times 10^3 = 0.8 \times 10^3 \text{ kg/m}^3.$$

Illustration 28.

A body weighs 160 g in air, 130 g in water and 136 g in oil. What is the specific gravity of oil?

Solution:

Specific gravity of oil $= \frac{\text{loss of weight in oil}}{\text{loss of weight in water}} = \frac{160-136}{160-130} = \frac{24}{30} = \frac{8}{10} = 0.8.$

Illustration 29.

An iceberg is floating partially immersed in sea-water. The density of sea-water is 1.03 g/cm^3 and that of ice is 0.92 g/cm^3. What is the fraction of the total volume of the iceberg above the level of sea-water?

Solution:

In case of floatation weight = upthrust i.e.

$mg = V_{in}\sigma g$ or $V\rho g\ V_{in}\sigma g$

or $V_{in} = \frac{\rho}{\sigma}V$

so $V_{out} = V - V_{in} = V\left[1-\frac{\rho}{\sigma}\right]$

$\therefore \quad f_{out} = \frac{V_{out}}{V} = \left[1-\frac{\rho}{\sigma}\right] = \left[1-\frac{0.92}{1.03}\right] = \frac{0.11}{1.03} = 0.106$

Illustration 30.

A piece of ice floats in a liquid. What will happen to the level of liquid after the ice melts completely?

Solution:

Consider a liquid of density ρ_L with level A in a beaker. Let a piece of ice of mass m float in the liquid. The increase in level of the liquid is AB. Suppose V_D is the volume of liquid displaced.
Then, weight of the ice = weight of liquid displaced

$$mg = V_D \rho_L g \qquad \text{or} \qquad V_D = \frac{m}{\rho_L}$$

When the ice gets completely melted, let the level of (liquid + water) be at C. The difference levels A and C is due to the which got converted into water. Thus if volume of the molten ice (i.e., water) be V_0.

then, $\qquad V_0 = \dfrac{m}{\rho_w}$

Where ρ_w = density of water. Here we consider the following three cases –

(i) If $\rho_L > \rho_w$ then $V_0 > V_D$
i.e. the level of (liquid + water) will rise

(ii) If $\rho_L < \rho_w$ then $V_0 < V_D$ the level of (liquid + water) will come down

(iii) If $\rho_L = \rho_w$ then $V_0 = V_D$ then the level will remain unchanged.

Illustration 31.

A boat carrying a number of large stones is floating in water. What will happen to the water level if the stones are unloaded into the water ?

Solution:

Let M = mass of the boat, $\qquad$ m = mass of the stones
For floating condition
weight = upthrust
$(M + m)\, g = V_D\, \rho_w\, g$; where V_D = volume of water displaced

$$V_D = \frac{M}{\rho_w} + \frac{m}{\rho_w} \qquad \text{.....(1)}$$

After the stones are unloaded into the water

$$V_{D_1} = \frac{M}{\rho_w} \qquad (V_{D_1} = \text{volume of water displaced by boat})$$

$$V_{D_2} = \frac{m}{\rho_s} \qquad (V_{D_2} = \text{volume of water displaced by stones})$$

$\therefore$ total volume of water displaced $V_D = V_{D_1} + V_{D_2} = \dfrac{M}{\rho_w} + \dfrac{m}{\rho_s}$ ....(2)

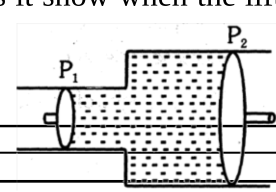
$\therefore \quad \dfrac{M}{\rho_w} > \dfrac{m}{\rho_s} \qquad \Rightarrow V_D > V_D$ $\quad$ So the water level will fall.

BEGINNER'S BOX - 3

1. When two metals with equal volumes are mixed together, the density of the mixture is 4 kg/m^3. When equal masses of the same two metals are mixed together, the mixture density is 3 kg/m^3. Calculate the densities of each metal.

2. A mercury barometer reads 75 cm in a stationary lift. What reading does it show when the lift is moving downwards, with an acceleration of 1 m/s^2 ?

3. The diameter of a piston P_2 is 50 cm and that of a piston P_1 is 10 cm.



What is the force exerted on P_2 when a force of 1 N is applied on P_1 ?

4. An open U-tube of uniform cross-section contains mercury. If 27.2 cm of water column is poured into one limb of the tube, how high does the mercury surface rise in the other limb from its initial level? [$\rho_w = 1$ g/cm^3 and $\rho_{Hg} = 13.6$ g/cm^3]

5. A certain block weighs 15 N in air, 12 N in water. When immersed in another liquid, it weighs 13 N. Calculate the relative density of (i) the block (ii) the other liquid.

6. A block of wood floats in water with two-third of its volume submerged. The block floats in oil with 0.90 of its volume submerged. Find the density of (i) wood and (ii) oil. Density of water is 10^3 kg/m^3.

7. A 700 g solid cube having an edge of length 10 cm floats in water. What volume of the cube is outside water?

8. If a block of iron of density 5 g/cm^3 and size 5 cm × 5 cm × 5 cm was weighed whilst completely submerged in water, what would be the apparent Weight in g – f (gram-force) ?

3. HYDRO-DYNAMICS

When a fluid moves in such a way that there are relative motions among the fluid particles, the fluid is said to be flowing.

3.1 Types of fluid flow

Fluid flow can be classified as :

- **Steady Flow**

  Steady flow is defined as that type of flow in which the fluid velocity at a point do not change with time. The fluid particle may have a different velocity at some other point.

  In steady flow all the particles passing through a given point follow the same path and hence a unique line, of flow. This line or path is called a streamline. Streamlines do not intersect each other, if they do so any particle at the point of intersection can move in either directions and consequently the flow cannot be steady.

- **Laminar and Turbulent Flow**

  Laminar flow is the flow in which fluid particles move along well-defined streamlines which are straight and parallel. In laminar flow the velocities at different points in the fluid may have different magnitudes, but their directions are parallel. Thus the particles move in laminae or layers sliding smoothly over the adjacent layer.

  Turbulent flow is an irregular flow in which the particles move in zig-zag way due to which eddy formation take place which are responsible for high energy losses.

- **Compressible and Incompressible Flow**

  In compressible flow the density of fluid varies from point to point i.e., the fluid density is not constant, whereas in an incompressible flow the density of the fluid remains uniform throughout. Liquids are practically incompressible while gases are highly compressible.

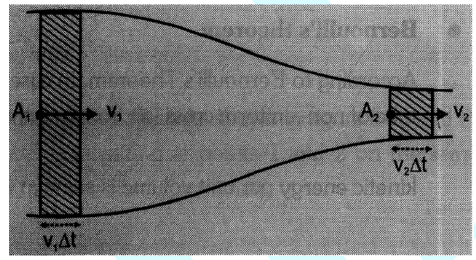
- **Rotational and Irrotational Flow**

Rotational flow is the flow in which the fluid particles while flowing along different path-lines also rotate about their own axes. In irrotational flow the particles do not rotate about their axes.

3.2 Equation of continuity

● The continuity equation is the mathematical expression of the law of conservation of mass in fluid dynamics.

● In the steady flow mass of the fluid entering into a tube of flow in a particular time interval is equal to the mass of fluid leaving the tube.



$\frac{m_1}{\Delta t} = \frac{m_2}{\Delta t}$ or $\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

for incompressible fluid $\rho_1 = \rho_2$

or $A_1 v_1 = A_2 v_2$ or Av = constant

Volume flux = Rate of flow = Volume of liquid flowing per second $Q = \frac{dV}{dt}$ Av

3.3 Bernoulli's Theorem

● Bernoulli's theorem is the mathematical expression of the law of mechanical energy conservation in fluid dynamics.

● Bernoullis theorem is applicable to ideal fluids. Characteristics of an ideal fluid are :

(i) The fluid is incompressible.
(ii) The fluid is non-viscous.
(iii) The fluid flow is steady.
(iv) The fluid flow is irrotational.

● Every volume at a point in an ideal fluid flow is associated with three kinds of energies :

(i) Kinetic Energy

If a liquid of mass (m) and volume (V) is flowing with velocity (v) then K.E. = $\frac{1}{2}mv^2$

and kinetic energy per unit volume = $\frac{K.E.}{volume} = \frac{1}{2}\frac{m}{V}v^2 = \frac{1}{2}\rho v^2$

(ii) Potential Energy

If a liquid of mass (m) and volume (V) is at a height (h) above the surface of the earth then its P.E. = mgh and potential energy per unit volume = $\frac{P.E.}{volume} = \frac{m}{V}gh = \rho gh$

(iii) Pressure Energy

If liquid moves through a distance (λ) due to pressure P on area A then

Pressure energy = Work done = force × displacement = pressure × area × displacement

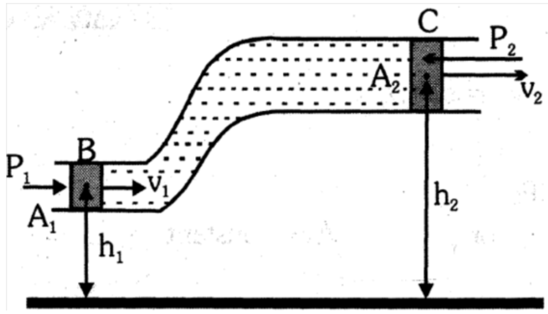
$= PA\lambda = PV$ $[\Theta \ A\lambda = \text{volume } V]$

Pressure energy per unit volume = $\frac{\text{pressure energy}}{\text{volume}} = P.$

● **Bernoulli's theorem**

According to Bernoulli's Theorem, in case of steady flow of incompressible and non-viscous fluid through a tube of non-uniform cross-section then the sum of the pressure, the potential energy per unit volume and the kinetic energy per unit volume is same at every point in the tube,

i.e., $P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$.



Consider a liquid flowing steadily through a tube of non- uniform cross-section as shown in figure. If P_1 and P_2 are the pressures at the two ends of the tube respectively, work done in pushing the volume ΔV of the incompressible liquid across sections B and C through the tube is

$$W = (P_1 - P_2)\Delta V \qquad \ldots\ldots(i)$$

This work is used by the liquid in two ways :

(i) In changing the potential energy of mass Δm (corresponding to the volume ΔV)

$$\Delta U = \Delta mg\,(h_2 - h_1) \qquad \ldots\ldots (ii)$$

(ii) In changing the kinetic energy $\Delta K = \frac{1}{2}\Delta m\,(v_2^2 - v_1^2)$

Now as the liquid is non-viscous, by work-energy theorem

$$W = \Delta U + \Delta K \qquad \text{i.e., } (P_1 - P_2)\,\Delta V = \Delta mg\,(h_2 - h_1) + \frac{1}{2}\Delta m\,(v_2^2 - v_1^2)$$

$$P_1 - P_2 = \rho g(h_2 - h_1) + \frac{1}{2}\rho\,(v_2^2 - v_1^2) \quad [\text{as } \rho = \Delta m / \Delta V]$$

$$P_1 + \rho gh_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho gh_2 + \frac{1}{2}\rho v_2^2 \Rightarrow P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$$

This equation is the **Bernoulli's equation** and expresses principle of conservation of mechanical energy in case of moving fluids.

The sum of pressure energy, kinetic energy and potential energy per unit volume remains constant along a streamline in an ideal fluid flow i.e.,

$$P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant} \qquad \text{(Energy per unit volume)}$$

or $$\frac{P}{\rho} + \frac{v^2}{2} + gh = \text{constant} \qquad \text{(Energy per unit mass)}$$

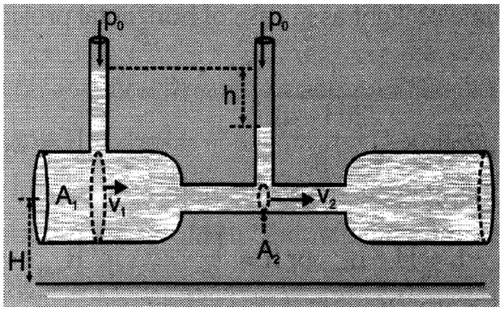
or $$\frac{P}{\rho g} + \frac{v^2}{2g} + h = \text{constant} \qquad \text{(Energy per unit weight)}$$

In the above equation $\frac{P}{\rho g}$ is called the pressure head, $\frac{v^2}{2g}$ is called the velocity head and h is called the gravitational / potential head.

3.4 APPLICATIONS OF BERNOULLI'S THEOREM

- **Venturimeter or Venturi Tube or Flowmeter**

Venturimeter is used to measure the flow velocities in an incompressible fluid. As shown in figure if P_1 and P_2 are the pressures and v_1 and v_2 are the velocities of the fluid of density ρ at points 1 and 2 on the same horizontal level and A_1 and A_2 be the respective areas, then from equation of continuity



$A_1v_1 = A_2v_2$ or $v_2 = \left[\frac{A_1}{A_2}\right]v_1$ ......(1)

From Bernoulli's equation for horizontal flow, $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

or $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho\left[\frac{A_1^2}{A_2^2}\right]v_1^2$ [from equation (i)]

or $P_1 - P_2 = \frac{1}{2}\rho v_1^2\left[\frac{A_1^2}{A_2^2} - 1\right]$

But $P_1 - P_2 = \rho gh$ [Θ difference in heights between the liquid surfaces in the two arms is h]

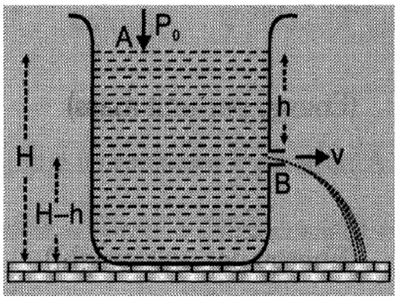
$\therefore$ $\rho gh = \frac{1}{2}\rho v_1^2\left[\frac{A_1^2}{A_2^2} - 1\right]$ $\therefore$ $v_1 = \sqrt{2gh\left[\frac{A_1^2}{A_2^2} - 1\right]} = A_2\sqrt{\frac{2gh}{A_1^2 - A_2^2}}$

If Q be the volume of liquid flowing per unit time $\Theta = A_1v_1 = A_2v_2 = A_1A_2\sqrt{\frac{2gh}{A_1^2 - A_2^2}}$

Thus at a point where the cross-sectional area is smaller velocity is greater and pressure is lower and vice versa.

- **Speed of efflux (Torricelli's Law)**

As shown in the figure the area of cross-section of the vessel A is very large as compared to that orifice B, therefore speed of liquid flow at A is zero i.e. $v_A \approx 0$. The fluid at sections A and B are at the same pressure P_0 (atmospheric pressure). Applying Bernoulli's theorem at A and B.



$P_0 + \rho gH + \frac{1}{2}\rho v_A^2 = P_0 + \rho g(H - h) + \frac{1}{2}\rho v_B^2$ or $\frac{1}{2}\rho v_B^2 = \rho gh$ or $v_B = \sqrt{2gh}$

This equation is same as that of the velocity acquired by a freely falling body after falling through h height and is known as **Torricelli's law.**

Writing the equation of uniformly accelerated motion in the vertical direction

$H - h = 0 + \frac{1}{2}gt^2$ (from $s_y = u_y t + \frac{1}{2} a_y t^2$)

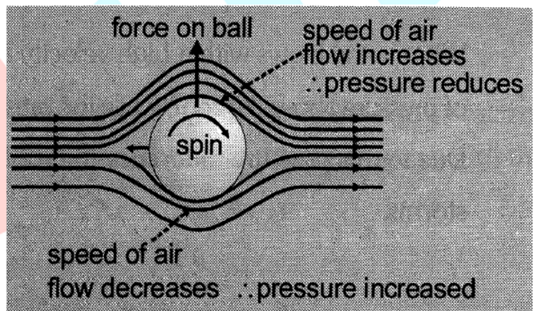
$\Rightarrow \quad t = \sqrt{\frac{2(H-h)}{g}}$, t = time of flight as in case of horizontal projection from the top of a tower.

Horizontal range $\quad R = v_x t = \sqrt{2gh} \times \sqrt{\frac{2(H-h)}{g}}$ or $R = 2\sqrt{h(H-h)}$

Range will be maximum when $\quad h = H - h$ or $h = \frac{H}{2}$ $\therefore$ $R_{max.} = 2\sqrt{\frac{H}{2}\left[H - \frac{H}{2}\right]} = H$

- **Magnus Effect (Observed in a Spinning Ball)**

Tennis and cricket players usually experience that when a ball is thrown spinning it moves along a curved path. This is called swing of the ball. This is due to the air which is being dragged round by the spinning ball. When the ball spins, the layer of the air around it also moves with the ball. So, as shown in figure the resultant velocity of air increases on the upper side and reduces on the lower side.

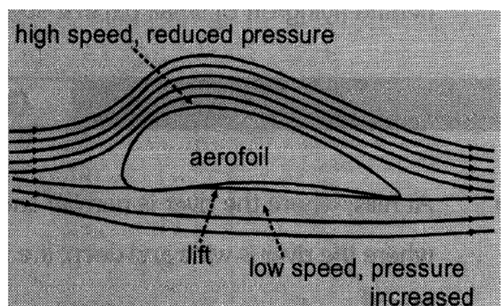


motion of a spin ball

Hence according to Bernoulli's theorem the pressure on the upper side becomes lower than that on the lower side. This pressure difference exerts a force on the ball due to which it moves along a curved path. This effect is known as Magnus-effect.

- **Aero foil**

This is a structure which is shaped in such a way so that its motion relative to a fluid produces a force perpendicular to the flow. As shown in the figure the shape of the aerofoil section causes the fluid to flow faster over the top surface than below the bottom i.e. the streamlines are closer above than below the aero foil. By Bernoulli's theorem the pressure at above reduced whereas that underneath it gets increased.



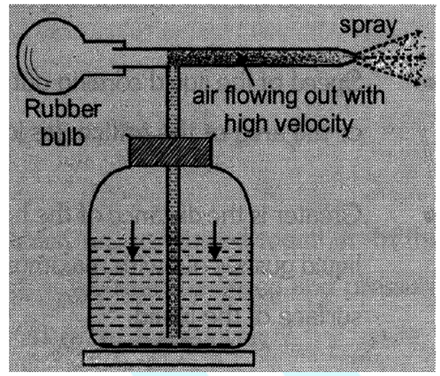
principle of an aerofoil

Thus a resultant upward thrust is generated normal to the flow and it is this force which provides most of the upward lift for an aeroplane.

Examples of aerofoils are aircraft wings, turbine blades and propellers.

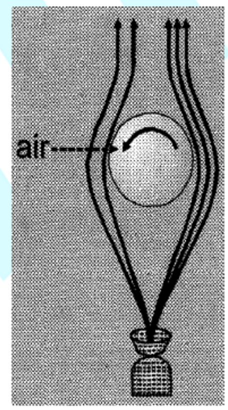
- **Sprayer or Atomizer**

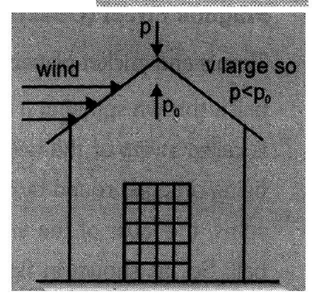
This is an instrument used to spray a liquid in the form of small droplets (fine spray). It consists of a vertical tube whose lower end is dipped in the liquid to be sprayed, filled in a vessel. The upper end opens in a horizontal tube. At one end of the horizontal tube there is a rubber bulb and the other end has is a fine bore (hole). When the rubber bulb is squeezed, air rushes out through the horizontal tube with very high velocity and thus the pressure reduces (according to Bernoulli's theorem). Consequently, the liquid in the vessel rises up and mixes with air in the form of small droplets which gets ejected in the form of a fine spray



Example : paint guns, perfume or deodaurant sprayer, etc.

- **Motion of the Ping-Pong Ball**
  When a ping-pong ball is placed on a vertical stream of water-fountain, it rises upto a certain height above the nozzle of the fountain and spins about its axis. The reason for this is that the streams of water rise up from the fountain with large velocity so that the air-pressure decreases. Therefore, whenever the ball tends to fall out from the stream, the outer air which is at atmospheric pressure pushes it back into the stream (in the region of low pressure). Thus the ball remains more or less stable in the fountain.





- **Blowing-off of Tin Roof Tops in Wind Storm l**
  When wind blows with a high velocity above a tin roof, it causes lowering of pressure above the roof, while the pressure below the roof is still atmospheric. Due to this pressure-difference the roof is lifted up and is blown away during storms.

- **Pull-in or Attraction Force by Fast Moving Train**
  If we are standing on a platform and a train passes through the platform with very high speed we are pulled towards the train. This is because as the train 'moves at high speed, the pressure close to the train decreases. Thus the air away from the train which is still at atmospheric pressure pushes us towards the train. The reason behind flying-off of small papers, straws and other light objects towards the train is also the same.

GOLDEN KEY POINTS

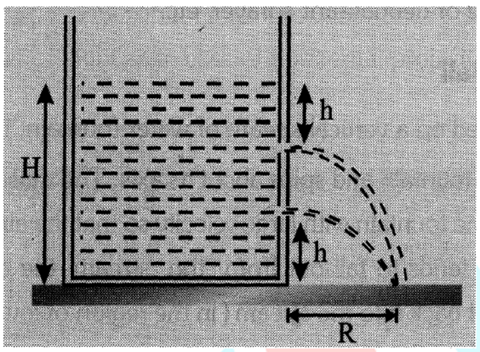
- At hills, where the river is narrow and shallow (i.e., small cross-section) the flow will be faster, while in planes where the river is wide and deep, (i.e., large cross section) the flow will be slower and so deep water appears to be still.

- Which water falls from a tap, the velocity of falling water under the action of gravity will increase with distance from the tap (i.e.,$v_2 > v_1$). So in accordance with continuity equation the cross-section of the water stream will decrease (i.e., $A_2 < A_1$), i.e., falling stream of water becomes narrower.
- Practically some energy of the fluid gets converted into heat energy and is lost. But Bernoulli's equation is derived without considering this loss of energy.

- Speed of the liquid coming out of the orifice is independent of the nature and quantity of liquid in the container or the area of the orifice. (as long as the orifice is small)

- Greater is the distance of the hole from the free surface of liquid greater will be the velocity of efflux. This is why liquid gushes out with maximum velocity from the orifice which is at maximum vertical distance below the free surface of the liquid.

- The horizontal range is same for liquid coming out of holes at equidistant from the liquid surface and the base .



Illustrations

Illustration 32.

The cylindrical tube of a spray pump has a, cross-section of 8 cm^2, one end of which has 40 fine holes each of area 10^{-8} m^2. If the liquid flows inside the tube with a speed of 0.15 m/min, then find the speed with which the liquid is ejected through the holes.

Solution:

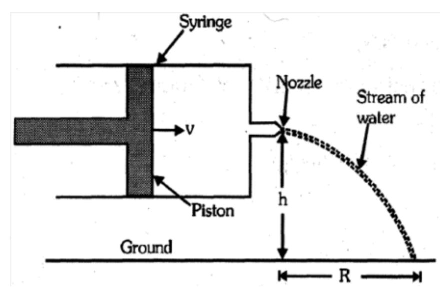
From equation of continuity

$A_1v_1 = A_2v_2$

$$(8 \times 10^{-4}) \times \left(\frac{0.15}{60}\right) = (40 \times 10^{-8}) \times v_2 \qquad \Rightarrow v_2 = 5 \text{ m/s.}$$

Illustration 33.

A syringe containing water is held horizontally with its nozzle at a height h above the ground as shown in fig. The cross-sectional areas of the piston and the nozzle are A and a respectively. The piston is pushed with a constant speed v. Find the horizontal range R of the stream of water on the ground.



Solution:

let v' be the horizontal speed of water when it emerges from the nozzle then from equation of continuity

$$Av = av' \Rightarrow v' = \frac{Av}{a}$$

Let t be the time taken by the stream of water to strike the ground then $h = \frac{1}{2}gt^2$

$$\Rightarrow \qquad t = \sqrt{\frac{2h}{g}} \Rightarrow \text{horizontal distance } R = v'\sqrt{\frac{2h}{g}} = \frac{Av}{a}\sqrt{\frac{2h}{g}}.$$

Illustration 34.

Water is flowing through two horizontal pipes of different diameters which are connected together. In the first pipe the speed of water is 4 m/s. and the pressure is 2×10^4 N/m^2. Calculate the speed and pressure of water in the second pipe. The diameters of the pipes are 3 cm and 6 cm respectively ?

Solution:

If A is the area of cross-section of a pipe at a point and v is the velocity of flow of water at that point, then by the principle of continuity Av = constant $\Rightarrow A_1v_1 = A_2v_2 \quad \Rightarrow \quad \pi r_1^2 v_1 = \pi r_2^2 v_2$

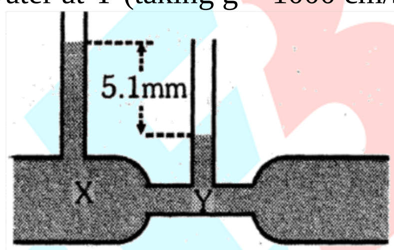
$$\Rightarrow v_2 = \left(\frac{r_1}{r_2}\right)^2 v_1 = \left(\frac{1.5\times10^{-2}}{3\times10^{-2}}\right)^2 \times 4 = 1 \text{ m/s.}$$

From Bernoulli's theorem : $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2 \Rightarrow P_2 = P_1 + \frac{1}{2}\rho(v_1^2 - v_2^2)$

$$\therefore \quad P_2 = 2 \times 10^4 + \frac{1}{2} \times (10^3) \times (16 - 1) = 2 \times 10^4 + 7.5 \times 10^3 = 2.75 \times 10^4 \text{ N/m}^2$$

Illustration 35.

The diagram (fig.) shows venturimeter through which water is flowing. The speed of water at X is 2 cm/s. Find the speed of water at Y (taking g = 1000 cm/s^2).



Solution:

By using Bernoulli's principle –

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2 \quad \Rightarrow P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) \quad \Rightarrow \rho gh = \frac{1}{2}\rho(v_2^2 - v_1^2)$$

putting the values in equation $1000 \times 0.51 = \frac{1}{2}(v_2^2 - 2^2) \Rightarrow v_2 = 32$ cm/s

Illustration 36.

In a test experiment on a model aeroplane in a wind tunnel, the flow speeds on the upper and lower surfaces of the wing are 70 m/s and 63 m/s respectively. What is the lift on the wing if its area is 2.5 m^2? The density of air is 1.3 kg/m^3.

Solution:

From Bernoulli's equation $P_2 - P_1 = \frac{1}{2}\rho(v_1^2 - v_2^2)$

Upward force on the wing, F = $(P_2 - P_1)$A, where A is area of the wing

$$F = \frac{1}{2}\rho(v_1^2 - v_2^2)\,A = \frac{1}{2} \times 10.3\,(70^2 - 63^2) \times 2.5 = 1.5 \times 10^3 \text{ N.}$$

Illustration 37.

A cylindrical tank 1 m in radius rests on a platform 5 m high. Initially, the tank is filled with water to a height of 5 m. A small plug whose area is 10^{-4} m^2 is removed from an orifice located on the side of the tank at the bottom. Calculate the :

(i) initial speed with which water flows out from the orifice

(ii) initial speed with which the water strikes the ground.

Solution:

(i) Applying Bernoulli's theorem between the water surface and the orifice,

$$P_0 + \frac{1}{2}\rho(0)^2 + \rho gh = P_0 + \frac{1}{2}\rho v^2 + \rho g(0)$$

$$\rho gh = \frac{1}{2}\rho v^2 \;;\; v = \sqrt{2gh} = \sqrt{2\times10\times5} = 10 \text{ m/s.}$$

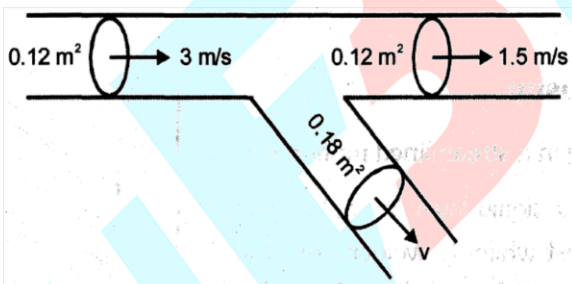
(ii) Let v' be the initial velocity with which the water strikes the ground

Then, applying Bernoulli's theorem between the top of the tank and the ground level, we get

$$v' = \sqrt{2g(H+h)} = \sqrt{2\times10\times10} = 10\sqrt{2} = 14.1 \text{ m/s}$$

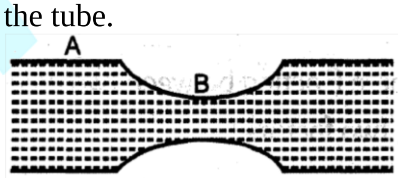
BEGINNER'S BOX - 4

1. An incompressible liquid flows as shown in the figure. calculate the speed v of the fluid in the lower branch.



2. A hole is made in a vessel containing water at a depth of 3.2 m below the free surface. What would be the velocity of efflux?

3. Water flows in a horizontal tube as shown in figure. The pressure of water changes by 600 N/m^2 between A and B where the area of cross-section are 30 cm^2 and 15 cm^2 respectively. Find the rate of flow of water through the tube.



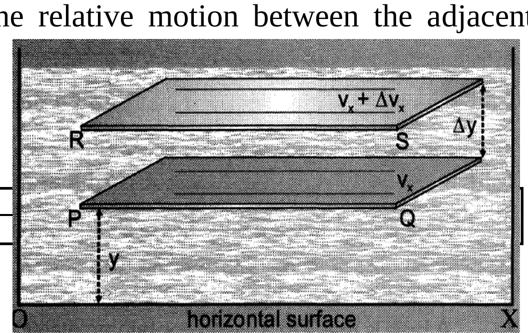
4. VILCOSITY

Viscosity is the property of a fluid (liquid or gas) by virtue of which it opposes the relative motion between its adjacent layers. It is the fluid friction or internal friction.

The internal tangential force which tends to retard the relative motion between the adjacent layers is called viscous force.

4.1 Newton's law of viscosity.

Suppose a liquid is flowing in a streamlined motion on a horizontal surface OX. The liquid layer in



contact with the surface is almost at rest while the velocity of other layers increases with increasing distance from the surface OX. The uppermost layer flows with maximum velocity.
Let us consider two parallel layers PQ and RS at distances y and y + Δy from OX. Let the change in velocity over a perpendicular distance Δy be Δv_x. The rate of change of velocity with distance perpendicular to the direction of flow i.e. $\frac{\Delta v_x}{\Delta y}$, is called velocity gradient.

According to Newton, the viscous force F acting between two adjacent layers of a liquid flowing in streamlined motion depends upon the following two factors :

(i) $F \propto$ contact area of the layers i.e. $F \propto A$

(ii) $F \propto$ velocity gradient between the layers i.e. $F \propto \frac{\Delta v_x}{\Delta y}$

Combining (i) and (ii) $F \mu A \frac{\Delta v_x}{\Delta y} \Rightarrow F = \eta A \frac{\Delta v_x}{\Delta y}$

where η is constant called **coefficient of viscosity** of the liquid.

Coefficient of viscosity $\eta = \frac{F/A}{v/l} = \frac{\text{Shear stress}}{\text{Strain rate}}$

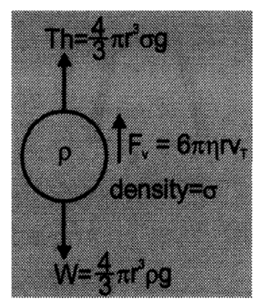
SI UNIT : N-sm^{-2} = Pa-s = poiseuille (PI) = deca poise
CGS UNITS : dyne-s/cm^2 = poise ; 1 decapoise = 10 poise.
Dimensions : $[M^1L^{-1}T^{-1}]$

4.2 Stoke's Law and Terminal velocity

- **Stoke's Law**

Stoke showed that if a small sphere of radius r is moving with a velocity v through a homogeneous stationary medium (liquid or gas), of viscosity η then the viscous force acting on the sphere is $\mathbf{F_v = 67\pi\eta r v}$.

- **Terminal Velocity**



When a solid sphere falls in a liquid, its accelerating velocity is controlled by the viscous force due to liquid and hence it attains a constant velocity which is known as the terminal velocity (v_T).
As shown in the figure when the body moves with constant velocity i.e. terminal velocity (with no acceleration) the net upward force (upthrust Th + viscous force F_v) balances the downward force (weight of the body W).

Therefore $Th + F_v = W \Rightarrow \frac{3}{4}\pi r^3 \sigma g + 6\pi\eta r v_T = \frac{3}{4}\pi r^3 \rho g \Rightarrow v_T = \frac{2}{9}\frac{r^2(\rho-\sigma)}{\eta}g$

where r = radius of body ρ = density of body
a = density of medium η = coefficient of viscosity.

Graph:
The variation of velocity with time (or distance) is shown in the adjacent graph.

- **Some applications of terminal velocity :**

(a) Actual velocity of rain drops is very small in comparison to the velocity which would have acquired by a body falling freely from the height of clouds.
(b) Descent of a parachute with moderate velocity.
(c) Determination of electronic charge in Milikan's oil drop experiment.

4.3 Reynold's Number (R_e)

The type of flow pattern (streamline, laminar or turbulent) is determined by a non-dimensional number called

Reynold's number (R_e). defined as

$$R_e = \frac{\text{Inertial force}}{\text{Viscous force}} = \frac{\rho v d}{\eta}$$

where ρ is the density of the fluid having viscosity η and flowing with a mean speed v. Here d denotes the lateral dimension of the obstacle or boundary of fluid flow.

Although there is no perfect demarcation for the value of R_e in case of laminar and turbulent flow but certain references take the value as :

R_e	< 1000	> 2000	Between 1000 to 2000
Type of flow	Streamline or laminar	turbulent	unsteady

Upon increasing the speed of flow gradually transition from laminar flow to turbulent flow takes place at certain speed. This speed is called **critical speed**. For fluids lower density and higher viscosity with laminar flow is more probable.

4.4 Dependency of viscosity

On Temperature of Fluid

(a) Since cohesive forces decrease with increase in temperature. Viscosity of liquids decreases with a rise in temperature.

(b) Viscosity of gases is the result of diffusion of gas molecule from one moving layer to other. With an increase in temperature, the rate of diffusion increases. Consequently the viscosity increases. Thus, the viscosity of gases increases with the rise in temperature.

On Pressure of Fluid

(a) Viscosity is normally independent of pressure. However liquids under extreme pressure often undergo an increase in viscosity.

(b) Viscosity of gases is practically independent of pressure.

4.5 Steady flow in capillary tube

Poiseuille's Formula

In case of steady flow of liquid of viscosity (η) in a capillary tube of length (L) and radius (r) under a pressure difference (P) across it, the volume of liquid flowing per second is given by :

$$Q = \frac{dV}{dt} = \frac{\pi P r^4}{8\eta L}$$

With the help of poiseuille's formula, coefficient of viscosity of a liquid can be determined.

GOLDEN KEY POINT

- Viscosity of fluid depends only on the nature of fluid and is independent of area considered or velocity gradient.
- Thin liquids like water alcohol are less viscous than thick liquids like blood, glycerin, honey.
- Viscosity of liquid is much greater (about 100 times more) than that of gases

Viscosity of water $\approx$ 0.01 poise, Visc6sity of air $\approx$ 200 μ poise

- As the temperature rises the atoms of the liquid become more mobile and the coefficient of viscosity η, falls. In a gas a temperature rise increases the random motion of atoms and as a consequence η increases.
- Due to small size of droplets forming the cloud, the terminal velocity is very small. So the clouds fall very slowly which appear to float in the sky.
- In turbulent flow the velocity of the fluid at any point varies rapidly and randomly with time oceanic currents and smoke rising from a burning stack of wood, oceanic currents are turbulent. Twinkling of stars is the result of atmospheric turbulence.
- Turbulence in a flow dissipates kinetic energy usually in the form of heat. This explains why racing cars and planes are designed to minimise turbulence.

Illustrations

Illustration 38.

There is a 1 mm thick layer of glycerine between a plate of area 100 cm^2 and a large plate. If the coefficient of viscosity of glycerine is 1.0 kg/ms, then what force is required to move the smaller plate with a velocity of 7 cm/s.

Solution:

$$\text{Required force } F = \eta A\frac{\Delta v}{\Delta x} = \frac{0.1\times100\times10^{-4}\times(7\times10^{-2})}{10^{-3}} = 0.7\text{ N}$$

Illustration 39.

The velocity of water in a river is 18 km/h at the surface. If the river is 5 m deep and the flow is streamlined, find the shearing stress between the horizontal layers of water assuming uniform velocity gradient. Viscosity of water is 10^{-3} poiseuille.

Solution:

As velocity at the bottom of the river will be zero,

$$\text{Velocity gradient } \frac{dv}{dy} = \frac{18\times10^3}{60\times60\times5} = 1\text{ s}^{-1}$$

$$\text{Shear stress} = \frac{F}{A} = \eta\frac{dv}{dy} = 10^{-3}\times1 = 1\times10^{-3}\text{ N/m}^2.$$

Illustration 40.

A drop of water of radius 0.0015 mm is falling in air. The co-efficient of viscosity of air is 1.8×10^{-5} kg/m-s. What will be the terminal velocity of the drop? Density of air can be neglected.

Solution:

$$v_T = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta} = \frac{2\times\left[\frac{15\times10^{-4}}{1000}\right]^2\times10^3\times9.8}{9\times1.8\times10^{-5}} = 2.72\times10^{-4}\text{ m/s}$$

Illustration 41.

The velocity of a small ball of mass M and density d_1, when dropped in a container filled with glycerine becomes constant after some time. If the density of glycerine is d_2, the viscous force acting on the ball will be :

Solution:

At equilibrium, the viscous force acting upward = Effective force downward

Effective force = $Vd_1g - Vd_2g$ $= V(d_1 - d_2)\,g = \frac{M}{d_1}(d_1 - d_2)g = Mg\left[1-\frac{d_2}{d_1}\right]$ $\quad [\because V = \frac{M}{d_1}]$

Illustration 42.

A spherical ball of radius 1×10^{-4} m and density 10^4 kg/m^3 falls freely under gravity through a distance h before entering a tank of water. If the velocity of the ball does not change, after entering the water find h. Viscosity of water is 9.8×10^{-6} N-s/m^2.

Solution:

After falling a height h velocity of the ball will become v = $\sqrt{2gh}$. After entering into the water as this velocity does not change, this velocity is equal to the terminal velocity,

$$\sqrt{2gh} = \frac{2}{9}r^2\left[\frac{\rho-\sigma}{\eta}\right]g$$

$$2gh = \left[\frac{2}{9}\times(10^{-4})^2\times\frac{(10^4-10^3)\times9.8}{9.8\times10^{-6}}\right]^2$$

$$\Rightarrow h = \frac{20\times20}{2\times9.8} = 20.41 \text{ m}$$

BEGINNER'S BOX - 5

1. A large wooden plate of area 10 m^2 floating on the surface of a river is made to move horizontally with a speed of 2 m/s by applying a tangential force. If the river is 1 m deep and the water in contact with the bed is stationary, find the tangential force needed to keep the plate moving. Coefficient of viscosity of water at the temperature of the river = 10^{-2} poise.

2. A square plate of 1m side moves parallel to a second plate with velocity 4 m/s. A thin layer of water exists between plate's. If the viscous force is 2 N and the coefficient of viscosity is 0.01 poise then find the distance between the plates in mm.

3. Find the terminal velocity of a rain drop of radius 0.01 mm. Coefficient of viscosity of air is 1.8×10^{-5} N-s/m^2 and its density is 1.2 kg/m^3. Density of water = 1000 kg/m^3. Take g = 10 m/s^2. (Force of buoyancy due to air is neglected)

4. An air bubble of radius 1 mm is allowed to rise through a long cylindrical column of a viscous liquid of radius 5 cm and travels at a steady rate of 2.1 cm per second. If the density of the liquid is 1.47 g/cc, find its viscosity.
Assume g = 980 cm/s and neglect the density of air.

5. A liquid flows through two capillary tubes connected in series. Their lengths are λ and 2λ and radii rand 2r respectively, then the pressure difference across the first and second tubes are in the ratio.....

5. SURFACE TENSION

Surface tension is basically a property of liquid. The liquid surface behaves like a stretched elastic membrane which has a natural tendency to contract and tends to have a minimum possible area. This property of liquid is called surface tension.

5.1 Intermolecular forces

Forces of attraction or repulsion acting among the molecules are known as **intermolecular forces.** The nature of intermolecular force is electromagnetic.

There are two types of intermolecular attractive forces.

• **Cohesive Force :-** The force of attraction acting between the molecules of same material is defined as cohesive force.
Ex. : force acting between water molecules, Hg molecules.

• **Adhesive Force :-** The force of attraction acting between molecules of two different materials is defined as adhesive force.
Ex. : Force acting between the molecules of paper and ink, black board and chalk etc.

- Intermolecular forces are different from gravitational forces in the sense that the former does not obey inverse-square law.
- The distance upto which these forces remains effective, is called molecular range. This distance is nearly 10^{-9} m. Within this limit the forces increases very rapidly as the distance decreases.
- Molecular range depends on the nature of the substance.

Examples:

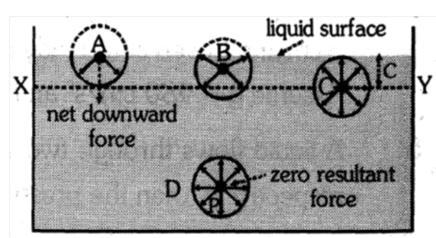
- **Water Glass :** Water wets glass surface but mercury does not because when water comes in contact with glass adhesive force acting between water and glass molecules is greater than the cohesive force of water molecules, so water molecules, cling to the glass surface and surface becomes wet. In case of mercury adhesive force is less than that of cohesive force and mercury molecules do not cling to the glass and consequently, mercury does not wet galss.

- **Oil-water :** Since cohesive force of water > Adhesive force oil-water > Cohesive force of oil.
(i) If water is poured on the surface of oil, it contracts in the form of globule.
(ii) If oil drop is poured on the surface of water it spreads to a larger area in the form of a thin film.

- **Ink-paper :** Since adhesive force between ink-paper > cohesive force on ink, so ink sticks to the paper.

5.2 EXPLANATION OF SURFACE TENSION (Molecular theory of surface Tension)

Laplace explained the phenomenon of surface tension on the basis of intermolecular forces. According to him surface tension is a molecular phenomenon and its root cause is electromagnetic force. He explained the cause of surface tension as described below. If the distance between two molecules is less than the molecular range C ($\approx 10^{-9}$ m) then they attract each other, but if the distance is more than this the attraction becomes negligible.
If a sphere of radius C With a molecule at centre is drawn, then only those molecules which are enclosed within this sphere can attract or be attracted by the molecule at the centre of the sphere. This sphere is called sphere of molecular activity or sphere of influence. In order to understand the tension acting at the free surface of liquid, let us consider four liquid molecules A, B, C and D along with their spheres of molecular activity.



(a) According to figure sphere D is completely inside liquid. So molecule is attracted equally in all directions and hence resultant cohesive force is equal to zero.

(b) According to figure, sphere of molecule C is just below the liquid surface. So resultant cohesive force is equal to zero.

(c) The molecule B which is a little below the liquid surface is attracted downwards due to excess of molecules present below. Hence the resultant cohesive force is acting downwards.

(d) Molecule A is situated at the surface so that its sphere of molecular activity is half outside the liquid and half inside. Only lower portion has liquid molecules. Hence it experiences a maximum downward force. Thus all the molecules situated between the surface and a plane XY, distant C below the surface, experience a resultant downward cohesive force.

When the surface area of liquid is increased molecules from the interior of the liquid rise to the surface. As these molecules reach close to the surface, work is done against the downward cohesive force. This work is stored in the molecules in the form of potential energy. Thus the potential energy of the molecules lying close to the surface is greater than that of the molecules in the interior of the liquid. A system is in stable equilibrium when its potential energy is minimum. Hence in order to have minimum potential energy the liquid surface tends to have minimum number of molecules. In other words any surface tends to contract to a minimum possible area.
This tendency is exhibited as surface tension.

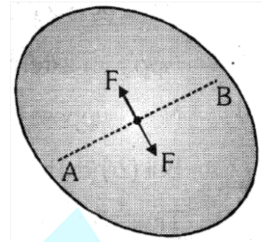
- **Effects of surface tension**
  - (i) Small liquid drops and soap bubbles are spherical
  - (ii) The hairs of the brush remain separated from each other inside water, but when the brush is taken out, the hairs stick together.
  - (iii) Floatation of needle on water.
  - (iv) Formation of lead shots.
  - (v) Dirty clothes become clean in hot detergent solution in comparison to pure water at room temperature.

Dependency of Surface Tension

- **On Cohesive Force**
  Those factors which increase the cohesive force between molecules increase the surface tension and those which decrease the cohesive force between molecules decrease the surface tension.
- **On Impurities**
  If the impurity is completely soluble then on dissolving it in the liquid, its surface tension increases. e.g., on dissolving ionic salts in small quantities in a liquid, its surface tension increases. If the impurity is partially soluble in a liquid then its surface tension decreases because adhesive force between insoluble impurity molecules and liquid molecules decreases cohesive force effectively, e.g.
  On mixing detergent in water its surface tension decreases.
- **On Temperature**
  On increasing temperature surface tension decreases. At critical temperature and boiling point it becomes zero.
  Note : Surface tension of water is maximum at 4°C.
- **On Contamination**
  Dust particles or lubricating materials on the liquid surface decreases its surface tension.

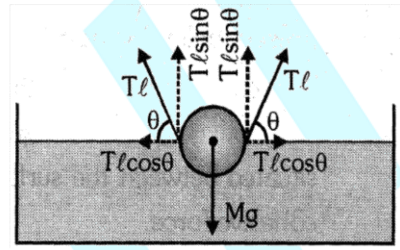
5.3 Definition of surface tension

The force acting per unit length on one side of an imaginary line drawn on the free liquid surface at right angles to the line and in the plane of liquid surface, is defined as surface tension.

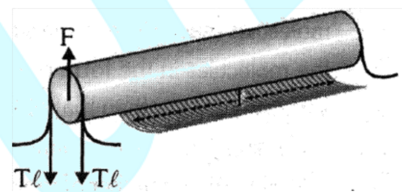


Let an imaginary line AB be drawn in any direction on a liquid surface. The surface on either side of this line exerts a pulling force, which is perpendicular to line AB. If force is F and length of AB is L then $T = \frac{F}{L}$

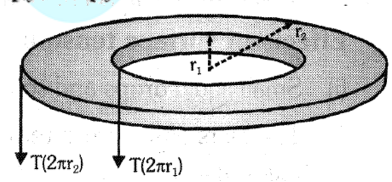
SI UNITS: N/m and J/m^2 **CGS UNITS:** dyne/cm and erg/cm^2 **Dimensions:** $[M^1L^0T^{-2}]$



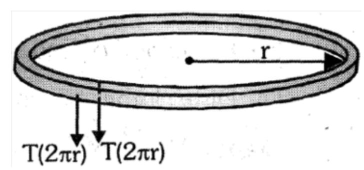
- When a needle floats on the liquid surface then $2T\lambda \sin\theta = Mg$
  Ex. A mosquito sitting on a liquid surface.



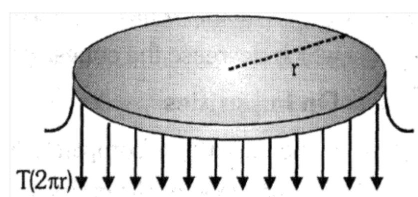
- If the needle is lifted from the liquid surface then required excess force will be $F_{excess} = 2T\lambda$
  Minimum force required $F_{min} = Mg + 2T\lambda$



- Required excess force for a circular thick ring (or annular ring) having internal and external radii r_1 and r_2 is dipped in and taken out from liquid.
  $F_{excess} = F_1 + F_2 = T(2\pi r_1) + T(2\pi r_2) = 2\pi T(r_1 + r_2)$.



- Required excess force for a circular ring ($r_1 = r_2 = r$)
  $F_{excess} = 2\pi\, T(r + r) = 4\pi rT$.

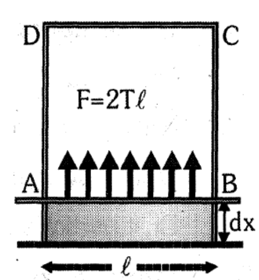


- Required excess force for a circular disc ($r_1 = 0$, $r_2 = r$)
  $F_{excess} = 2\,\pi\, rT$

5.4 Surface Energy

According to molecular theory of surface tension the molecules on the surface have certain additional energy due to their position. This additional energy of the surface is called 'Surface energy'.

Let a liquid film be formed on a wire frame and a straight wire of length λ can slide on this wire frame as shown in figure. The film has two surfaces and both the surfaces are in contact with the sliding wire and hence, exert forces of surface tension on it. If T be the surface tension of the solution, each surface will pull the wire parallel to itself with a force $T\lambda$. Thus, the net force on the wire due to both the surfaces is $2T\lambda$.



Apply an external force F equal and opposite to it to keep the wire in equilibrium. Thus, $F=2T\lambda$

Now, suppose the wire is moved through a small distance dx, then work done by the force is, $dW = F\,dx = (2T\lambda)\,dx$

But $(2\lambda)(dx)$ is the total increase in area of both the surfaces of the film. Let it be dA, then $dW = T\,dA \Rightarrow T = dW/dA$

Thus, the surface tension T can also be defined as the work done in increasing the surface area by unity. Further, since there is no change in kinetic energy, work done by the external force is stored as potential

energy of the new surface. $T = \dfrac{dU}{dA}$ [as dW = dU]

Special Cases

- Work done (surface energy) in formation of a drop of radius r = Work done against surface tension

$$W = \text{Surface tension } T \times \text{change in area } \Delta A = T \times 4\pi r^2 = 4\pi r^2 T$$

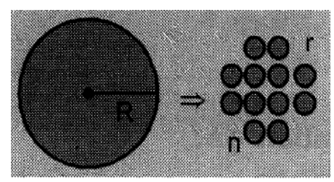
- Work done (surface energy) in blowing of a soap bubble of radius r :

$W = T \times \Delta A$ or $W = T \times 2 \times 4\pi r^2 = 8\pi r^2 T$ [$\because$ soap bubble has two surfaces]

- Work done in blowing of small bubble of radius r_1 to large bubble of radius r_2.

$$W = T \times \Delta A \text{ or } W = T \times 2 \times (4\pi r_2^2 - 4\pi r_1^2) = 8\pi T(r_2^2 - {}_1^2)$$

- Splitting of big drop into smaller droplets



If a big a drop is split into smaller droplets then in this process volume of liquid always remain conserved. Let the big drop have a radius R. It is splitted into n smaller drops of radius r then by conservation of volume

(i) $\dfrac{3}{4}\pi R^3 = n\left[\dfrac{4}{3}\pi r^3\right] \Rightarrow n = \left[\dfrac{R}{r}\right]^3 \Rightarrow r = \dfrac{R}{n^{1/3}}$

(ii) Initial surface area = $4\pi R^2$ and final surface area= $n(4\pi r^2)$

Therefore initial surface energy $E_i = 4\pi R^2 T$ and final surface energy $E_f = n(4\pi r^2 T)$

Change in area $\Delta A = n4\pi r^2 - 4\pi R^2 = 4\pi\,(nr^2 - R^2)$.

Therefore the amount of surface energy absorbed i.e. $\Delta E = E_f - E_i = 4\pi T(nr^2 - R^2)$

$\therefore$ Magnitude of work done against surface tension i.e. $W = 4\pi(nr^2 - R^2)T$

$$W = 4\pi T\,(nr^2 - R^2) = 4\pi R^2 T\,(n^{1/3} - 1) = 4\underline{\pi} R^2 T\left[\frac{R}{r} - 1\right] \Rightarrow W = 4\pi R^3 T\left[\frac{1}{r} - \frac{1}{R}\right]$$

In this process temperature of the system decreases as energy gets absorbed during the increase surface area.

$$W = J\,ms\Delta\theta = 4\pi R^3 T\left[\frac{1}{r} - \frac{1}{R}\right] \Rightarrow \Delta\theta = \frac{4\pi R^3 T}{\frac{4}{3}\pi R^3 J\rho s}\left[\frac{1}{r} - \frac{1}{R}\right] = \frac{3T}{J\rho s}\left[\frac{1}{r} - \frac{1}{R}\right]$$

Where ρ = liquid density, s = specific heat of liquid

Thus, in this process area increases, surface energy increases, internal energy decreases, temperature decreases, and energy is absorbed.

GOLDEN KEY POINTS

- Surface tension is a scalar quantity.

- Force due to surface tension acts tangential to the liquid surface.
- Surface tension is due to cohesive fore.
- More is the cohesive force, stronger is the surface tension.
- When surface area of liquid is increased, molecules from the interior of the liquid rises to the surface. In this process, work is done against the downward cohesive forces.

Illustrations

Illustration 43.

The length of a needle floating on water is 2.5 cm. Calculate the additional force required to pull the needle out of water. [$T = 7.2 \times 10^{-2}$ N/m]

Solution:

Force of surface tension $F = T \times 2\lambda$ ($\because$ Two free surfaces are there)

$$\Rightarrow F = 7.2 \times 10^{-2} \times 2 \times 2.5 \times 10^{-2} = 3.6 \times 10^{-3} \text{ N.}$$

Illustration 44.

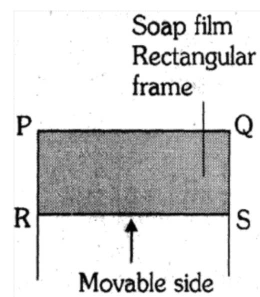
A paper disc of radius R from which a hole of radius r is cut out, is floating on liquid of surface tension, T. What will be force on the disc due to surface tension ?

Solution:

$$T = \frac{F}{L} = \frac{F}{2\pi(R+r)} \qquad \therefore F = 2\pi(R+r)T$$

Illustration 45.

PQSR is a rectangular frame of copper wire shown in fig. The side RS frame of the frame is movable. If a soap film is formed on it then what is the diameter of the wire to maintain equilibrium ? (Given that surface tension of soap solution = 0.045 N/m and density of copper = 8.96×10^3 kg/m^3)



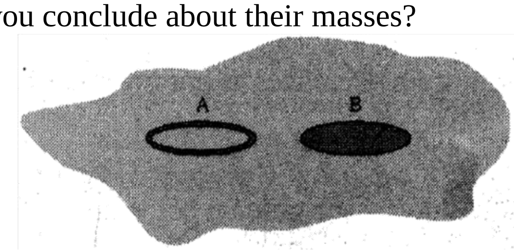
Solution:

Force due to surface tension = weight of wire $\Rightarrow 2T\lambda = mg = \pi r^2 \lambda dg \Rightarrow r = \sqrt{\dfrac{2T}{\pi dg}}$

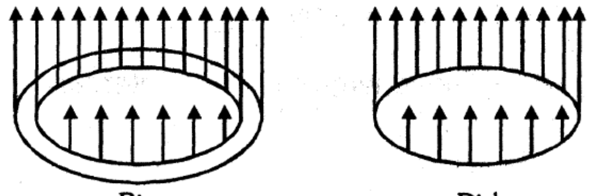
$$\Rightarrow \quad r = \sqrt{\frac{2\times0.045}{3.14\times8.96\times10^3\times9.8}} = 0.57 \text{ mm} \qquad \therefore \text{ Required diameter } d = 2r = 1.14 \text{ mm.}$$

Illustration 46.

A rigid ring A and a thin rigid disc B both made of same material, when gently placed on water, just manage to float due to surface tension as shown in the figure. Both the ring and the disc have same radius. What can you conclude about their masses?



Solution:



Ring has double the surface length in contact with liquid than that of disc. So ring has double the mass compared to the mass of disc.

Illustration 47.

A water film is formed in between two parallel wires whose lengths are 10 cm and placed at a distance 0.5 cm apart. Find the work done in order to increase the distance between the wires by 1 mm. (surface tension of water is 72 dyne/cm) :-

Solution:

$W = T \times \Delta A = 72 \times [2 \times (10 \times 0.6 - 10 \times 0.5)] = 144$ erg.

Illustration 48.

Calculate the work done against surface tension in blowing a soap bubble from a radius 10 cm to 20 cm if the surface tension of soap solution is 25×10^{-3} N/m. Then compare it with a liquid drop for same radii.

Solution:

(i) For soap bubble : Extension in area $= 2 \times (4\pi r_2^2 - 4\pi r_1^2) = 8\pi\,[(0.2)^2 - (0.1)^2] = 0.24\pi\ m^2$

Work done w_1 = surface tension × extension in area $= 25 \times 10^{-3} \times 0.24\pi = 6\pi \times 10^{-3}$ J.

(ii) For Liquid Drop : in case of liquid drop there is only one free surface, so extension in area will be half that of soap bubble

$$\therefore W_2 = \frac{W_1}{2} = 3\pi \times 10^{-3}\ J$$

Illustration 49.

If W is the amount of work done in forming a soap bubble of volume V, then calculate the amount of work done in forming a bubble of volume 2V from the same solution.

Solution:

Volume of bubble $V = \frac{4}{3}\pi r^3 \quad \Rightarrow \quad V \propto r^3$

$$\frac{2V}{V} = \left(\frac{r_2}{r_1}\right)^3 \Rightarrow \quad \frac{r_2}{r_1} = 2^{1/3}$$

Work done in forming the bubble $W = 8\pi r^2 T \Rightarrow \quad W \propto r^2$

$$\frac{W_2}{W_1} = \left(\frac{r_2}{r_1}\right)^2 = (2^{1/3})^2 = 2^{2/3}$$

$$W_2 = 2^{2/3}\ W$$

Illustration 50.

A big drop is formed by coalescing 1000 small droplets of water. What will be the change in the surface energy? What will be the ratio between the total surface energy of the droplets and the surface energy of the big drop?

Solution:

By conservation of volume $\frac{4}{3}\pi R^3 = 1000 \times \frac{4}{3}\pi r^3 \Rightarrow r = \frac{R}{10}$

Surface energy of 1000 droplets = $1000 \times T \times 4\pi\left[\frac{R}{10}\right]^2 = 10\,(T \times 4\pi R^2)$

Surface energy of the big drop = $T \times 4\pi R^2$
Change in the surface energy = $36\pi R^2 T$
Surface energy will decrease in the process of formation of bigger drop, hence energy is released and temperature increases.

$$\therefore \qquad \frac{\text{total surface energy of 1000 droplets}}{\text{surface energy of big drop}} = \frac{10(T \times 4\pi R^2)}{R \times 4\pi R^2} = \frac{10}{1}$$

Illustration 51.

A water drop of radius 1mm is split into 10^6 identical drops. Surface tension of water is 72 dynes/cm. Find the energy spent in this process.

Solution:

As volume of water remains constant, so $\frac{4}{3}\pi R^3 = n\frac{4}{3}\pi r^3 \Rightarrow r = \frac{R}{n^{1/3}}$

Increase in surface area $\Delta A = n\,(4\pi r^2) - 4\pi R^2 = 4\pi(n^{1/3} - 1)\,R^2 = 4\pi(100 - 1)\,10^{-6}$

$\therefore$ Energy spent= $T\Delta A = 4\pi \times 99 \times 10^{-6} \times 72 \times 10^{-3} = 89.5 \times 10^{-6}$ J

BEGINNER'S BOX - 6

1. A stick of length 4 cm is floating on the surface of water. If one side of it has surface tension of 80 dyne/ cm and on the other side by keeping a piece of camphor the surface tension is reduced to 60 dyne/ cm, then find the resultant force (in dyne) acting on the stick.

2. A thin wire ring of radius 1m is situated on the surface of a liquid. If the excess force required to lift it upwards (before the liquid film breaks) from the liquid surface is 8 N. calculate the surface tension of liquid ?

3. A circular frame made of 20 cm long thin wire is floating on the surface of water. The surface tension of water is 70 dyne/cm. Calculate the required excess force to separate this frame from water?

4. A metallic wire of density d floats horizontally in water. Find out the maximum radius of the wire so that the wire may not sink. (surface tension of water = T)

5. A rectangular film of liquid is 5 cm long and 3 cm wide. If the work done in increasing its area to 9 cm × 5 cm is 6×10^{-4} joule. find the surface tension of the liquid ?

6. Calculate the surface energy of ring floating on the liquid surface?
(surface tension of liquid is 75 N/m and area of ring is 0.04 m^2)

7. Find the work done in increasing the volume of a soap bubble by 700% if its radius is R and surface tension is T.

8. A bubble of radius 2 cm is blown inside a cold drink using a straw. If the surface tension of the liquid is 60 dyne/cm find the workdone (in ergs) in blowing the bubble.

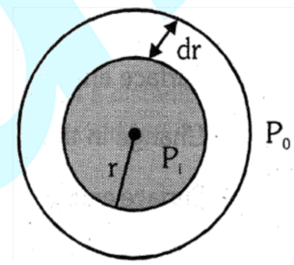
9. 10^6 tiny drops coalesce to form a big drop. The surface tension of liquid is T. Calculate the percentage fractional energy loss.

10. A liquid drop of radius 'r' is divided into 64 similar tiny droplets. If the surface tension of liquid is T, calculate the increase in energy ?

5.5 Excess pressure inside a curved liquid surface

The pressure on the concave side of curved liquid surface is greater than that on the convex side. Therefore a pressure difference exists across two sides of a curved surface. This pressure difference is called excess pressure.

(i) Excess pressure inside the drop



Let a drop of radius r have internal and external pressures P_i and P_0 respectively,

so that the excess pressure $P_{ex} = (P_i - P_0)$.

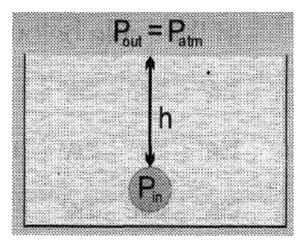
If the radius of the drop is changed from r to (r + dr) then

Work done = F.dr = $(P_{ex}\ A)dr = P_{ex}\ 4\pi r^2 dr$

Change in surface area = $4\pi(r + dr)^2 - 4\pi r^2 = 8\pi r dr$ ($\because dr^2$ is neglizible)

So by definition of surface energy $T = \dfrac{W}{\Delta A} = \dfrac{4\pi r^2 P_{ex} dr}{8\pi r dr} \Rightarrow P_{ex} = (P_1 - P_0) = \dfrac{2T}{r}$

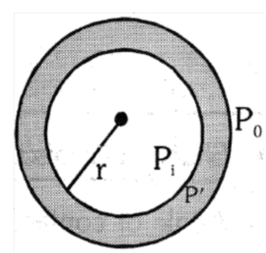
(ii) Excess pressure inside a cavity or air bubble in liquid –



$$P_{excess} = P_{in} - P_{out} = \rho gh + \frac{2T}{R} \qquad (\rho = \text{density of liquid})$$

$$P_{inside} = P_{atm.} + \rho gh + \frac{2T}{R}$$

(iii) Excess pressure inside a soap bubble :



Since a soap bubble has two surfaces, excess pressure will get doubled as compared to a drop

$$P_i - P' = \frac{2T}{r},\ P' - P_0 = \frac{2T}{r} \Rightarrow \text{excess pressure} = P_i - P_0 = \frac{4T}{r}$$

5.6 Angle of contact (θ_C)

The angle enclosed between the tangent plane at the liquid surface and the tangent plane at the solid surface at the point of contact inside the liquid is defined as the angle of contact. The angle of contact depends on the nature of the solid and liquid in contact.

- **Effect of Temperature on angle of contact**

On increasing temperature surface tension decreases, thus $\cos\theta_c$ increases $\left[\because \cos\theta_c \propto \frac{1}{T}\right]$ and θ_c decreases.

So on increasing temperature, θ_c decreases.

- **Effect of Impurities on angle of contact**

  (a) Soluble impurities increase surface tension, so $\cos\theta_c$ decreases and angle of contact θ_c increases.

  (b) Partially soluble impurities decrease the surface tension, so angle of contact θ_c decreases.

- **Effect of Water Proofing Agent**

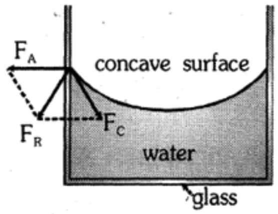
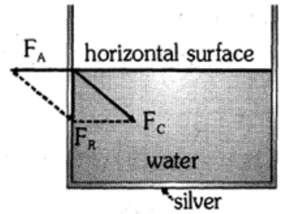
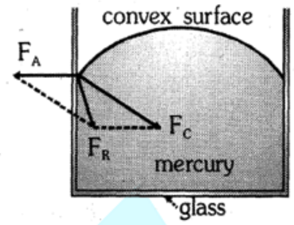
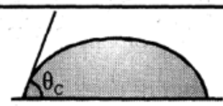
  Angle of contact increases due to the presence of water proofing agent. It changes from acute to obtuse angle.

Shape of Liquid Surface (Shape of meniscus)

When a liquid is brought in contact with a solid surface, the surface of the liquid becomes curved near the place of contact. The shape of the surface (concave or convex) depends upon the relative magnitudes of cohesive force between the liquid molecules and the adhesive force between the molecules of the liquid and the solid.

The free surface of a liquid which is near the walls of a vessel and which is curved because of surface tension is known as meniscus. The cohesive force acts at an angle of 45º from liquid surface whereas the adhesive force acts at right angles to the solid surface.

The relation between the shape of liquid surface, cohesive/adhesive forces, angle of contact, etc are summarized in the table below :

• Relation between cohesive and adhesive force	 $F_A > \frac{F_C}{\sqrt{2}}$	 $F_A = \frac{F_C}{\sqrt{2}}$	 $F_A < \frac{F_C}{\sqrt{2}}$
• Shape of meniscus	Concave	Plane	Convex
• Angle of contact	$\theta_C < 90°$ (Acute angle)	$\theta_C = 90°$ (Right angle)	$\theta_C > 90°$ (Obtuse angle)
• Shape of liquid drop			
• Level of liquid	Liquid rises up	Liquid neither rises nor falls	Liquid falls
• Wetting property	Liquid wets the solid surface	Liquid does not wet the solid surface	Liquid does not wet the solid surface
• Example	Glass – Water	Silver – Water	Glass – Mercury

5.7 Capillary Tube and Capillarity

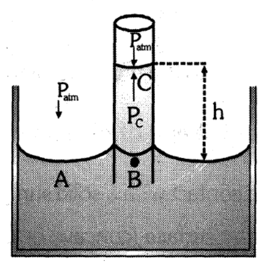
A glass tube with a fine bore and open at both ends is known as a **capillary tube**. The property by virtue of which a liquid rises or gets depressed in a capillary tube is known as **capillarity**. Rise or fall of liquid in tubes of narrow bore (capillary tube) is called **capillary action.**

Ex. : (i) Kerosene oil in lanterns rise upward due to capillarity.
(ii) Working of fountain pen's nib split due to capillarity.

Calculation of Capillary Rise

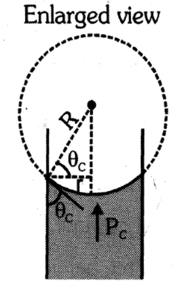
(i) Pressure Balance Method :

When a capillary tube is first dipped in a liquid as shown in the figure, the liquid climbs up the walls curving the surface. Let the radius of the meniscus be R and the radius of the capillary tube be r. Angle of contact is θ_C, surface tension is T, density of liquid is ρ and the liquid rises to a height h.



R= Radius of the meniscus

Enlarged view



$R = \frac{r}{\cos\theta}$

Now let us consider two points A and B at the same horizontal level as shown. By Pascal's law

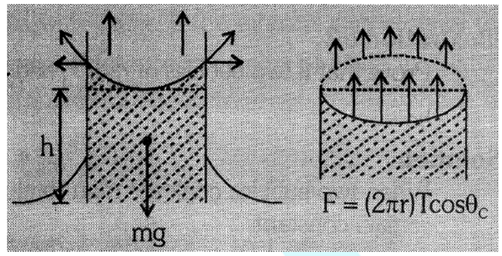
$$P_A = P_B \Rightarrow \quad P_A = P_C + \rho gh$$

Now, point C is on the curved meniscus which has P_{atm} and P_C as the pressures on its concave and convex sides respectively.

$$\therefore\ P_{atm} = (P_{atm} - \frac{2T}{R}) + h\rho g \Rightarrow h = \frac{2T}{R\rho g} = \frac{2T\cos\theta_C}{r\rho g}$$

(ii) Force Balance Method :-

The liquid continues to rise in the capillary tube until the weight of the liquid column becomes equal to force due to surface tension. On liquid force due to surface tension = $(2\pi r)\, T\cos\theta_C$



In equilibrium : force due to S.T = weight of rise liquid

$$(2\pi r)T\cos\theta_C = mg$$

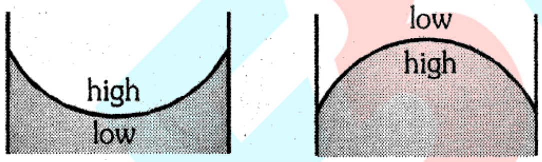
$$(2\pi r)T\cos\theta_C = (\pi r^2 h\rho)g$$

$$h = \frac{2T\cos\theta_C}{r\rho g}$$

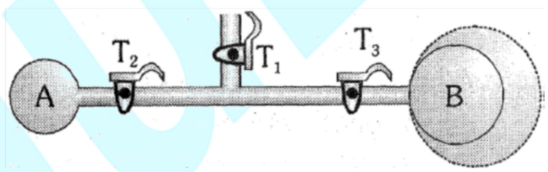
- **Zurin's Law :**
  The height of rise of liquid in a capillary tube is inversely proportional to the radius of the capillary tube, if T, θ, ρ and g are constant $h \propto \frac{1}{r}$ or rh = constant. It implies that liquid will rise more in capillary tube of less radius and vice versa.

GOLDEN KEY POINTS

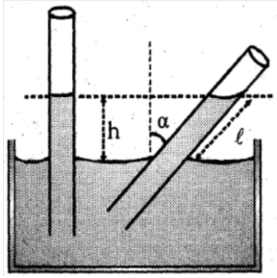
- For a liquid surface, pressure on concave side is always higher than on the convex side



- If a bubble is formed inside a liquid, pressure inside the bubble is more than the pressure outside the bubble.
- In the following arrangement, air will flow from bubble A to B if T_2 and T_3 are opened, because pressure in A is greater than that in B.



- For pure water and clean glass capillary $\theta_c \approx 0° \Rightarrow$ Radius of meniscus = radius of capillary
- If angle of contact θ_c is acute then $\cos\theta_c$ is positive, so h is positive and liquid rises. If θ_c is obtuse then cosec is negative, so h is negative, therefore liquid gets deprersed.
- Rise of liquid in a capillary tube does not obey the law of conservation of mechanical energy.
- Inside a satellite, water will rise upto the top level but will not overflow, Radius of curvature (R') increases in such a way that final height h' is reduced and given by $h' = \frac{hR}{R'}$. (It is in accordance with Zurin's law).
- If a capillary tube is dipped into a liquid and tilted at an angle α from vertical then the vertical height of the liquid column remains same whereas the length of liquid column in the capillary tube increases.



$h = \ell\cos\alpha \Rightarrow \ell = \dfrac{h}{\cos\alpha}$

- The height 'h' is measured from the lowest point of the meniscus. However, there exists some liquid above this line also. If correction is applied then the formula will be $T = \dfrac{r\rho g\left[h + \frac{1}{3}r\right]}{2\cos\theta}$
- If a hollow sphere which has a fine hole of radius r is plunged into a liquid upto h depth, then liquid will not enter upto a critical height h, given by $h\rho g = \dfrac{2R\cos\theta}{r}$ [normally $\theta \approx 0°$ therefore $\cos\theta \approx 1$]

Illustrations

Illustration 52.

Prove that If two bubbles of radii r_1 and r_2 coalesce isothermally in vacuum then radius of the new bubble will be $r = \sqrt{r_1^2 + r_2^2}$

Solution:

When two bubbles coalesce then total number of molecules of air will remain same and temperature will also remain constant

so $n_1 + n_2 = n \Rightarrow P_1V_1 + P_2V_2 = PV \Rightarrow \dfrac{4T}{r_1}\left(\dfrac{4}{3}\pi r_1^3\right) + \dfrac{4T}{r^2}\left(\dfrac{4}{3}\pi r_2^3\right) = \dfrac{4T}{r}\left(\dfrac{4}{3}\pi r^3\right) \Rightarrow r = \sqrt{r_1^2 + r_2^2}$

Illustration 53.

Prove that if two bubbles of radii r_1 and r_2 ($r_1 < r_2$) come in contact with each other then the radius of curvature of the common surface $r = \dfrac{r_1 r_2}{r_2 - r_1}$.

Solution:

Θ $r_1 < r_2$

Θ $P_1 > P_2$ Small portion of bubbles is in contact and in equilibrium

$\Rightarrow P_1 - P_2 = \dfrac{4T}{r} \Rightarrow \dfrac{4T}{r_1} - \dfrac{4T}{r_2} = \dfrac{4T}{r} \Rightarrow r = \dfrac{r_1 r_2}{r_2 - r_1}$

Illustration 54.

A bubble of air of radius 0.1 mm in water. If the bubble had been formed 10 cm below the water surface on a day when the atmospheric pressure was 1.013×10^5 Pa, then what would have been the total pressure inside the bubble? (Surface tension of water = 73×10^{-3} N/m)

Solution:

Excess pressure $P_{in} = P_{atm} + hdg + \dfrac{2T}{r} = (1.013 \times 10^5) + (10 \times 10^{-2} \times 10^3 \times 9.8) + 1460$

$= 101300 + 980 + 1460 = 103740 = 1.037 \times 10^5$ Pa.

Illustration 55.

Calculate the height to which water will rise in a capillary tube of diameter 1×10^{-3} m. [Given : surface tension of water is 0.072 N/m, angle of contact is 0°, g = 9.8 m/s^2 and density of water = 1000 kg/m^3]

Solution:

Height of capillary rise $h = \frac{2T\cos\theta}{r\rho g} = \frac{2 \times 0.072 \times \cos 0^\circ}{5 \times 10^{-4} \times 1000 \times 9.8} = 2.94 \times 10^{-2}$ m.

Illustration 56.

Water rises to a height of 20 mm in a capillary. If the radius of the capillary is made one third of its previous value then what is the new value of the capillary rise?

Solution:

$\therefore \frac{h_2}{h_1} = \frac{r_1}{r_2} = \frac{1}{(1/3)} = 3$ hence $h_2 = 3h_1 = 3 \times 20$ mm = 60 mm.

Illustration 57.

A hollow sphere which has a small hole in its bottom is immersed in water to a depth of 30 cm before any water enters in it. If the surface tension of water is 75 dynes/cm then find the radius of the hole in meters (taking g = 10 m/s^2)

Solution:

Radius of the hole $r = \frac{2T}{hdg} = \frac{2 \times 75 \times 10^{-3}}{30 \times 10^{-2} \times 10^3 \times 10} = 5 \times 10^{-5}$ m.

Illustration 59.

AU - tube is supported with its limbs vertical and is partly filled with water. If the internal diameters of the limbs are 1×10^{-2} m and 1×10^{-4} m respectively. What will be the difference in heights of water in the two limbs? (Surface tension of water is 0.07 N/m.)

Solution:

Let h_1 and h_2 be the heights of water columns in the limbs of radis r_1 and r_2.

Then $h_1 = \frac{2T\cos\theta}{r_1 dg} = \frac{2 \times 0.07 \times \cos 0^\circ}{0.5 \times 10^{-4} \times 1000 \times 9.8} = 2.8 \times 10^{-3}$ m $= 0.028 \times 10^{-1}$ m

Similarly $h_2 = \frac{2T\cos\theta}{r_2 dg} = \frac{2 \times 0.07 \times \cos 0^\circ}{0.5 \times 10^{-2} \times 1000 \times 9.8} = 2.8 \times 10^{-1}$ m

Therefore difference in heights $= h_2 - h_1 = (2.8 - 0.028) \times 10^{-1}$ m $= 2.772 \times 10^{-1}$ m.

BEGINNER'S BOX - 7

1. When a cylindrical tube is dipped vertically into a liquid the angle of contact is 80º. When the tube is dipped with an inclination of 40°, find the angle of contact ?

2. If A liquid rises in a capillary tube, then what can be the angle of contact ?

3. The excess pressure in a soap bubble is three times the excess pressure in another bubble. Find the ratio of their volumes?

4. An air bubble of radius 0.02 mm is at a depth of 25 cm in oil of density 0.8 gm/cm^3. If the surface tension of oil is 25×10^{-3} N/m find the pressure inside the bubble (atmospheric pressure = 10^5 N/m^2)?

5. The pressure inside two soap bubbles is 1.01 and 1.02 atmosphere respectively. Find the ratio of their volumes?

6. On dipping one end of a capillary in a liquid and inclining the capillary at angles 30° and 60° with the vertical, the lengths of liquid columns in it are found to be λ_1 and λ_2 respectively. Find the ratio of λ_1 and λ_2 ?

7. Water rises in two capillaries of same material up to heights of 40 and 60 mm. Find the ratio of their radii ?

8. When a capillary tube is dipped inside water, water rises inside the capillary tube up to 0.015 m. If the surface tension of water is 75×10^{-3} N/m calculate the radius of the capillary tube?

ANSWERS

BEGINNER'S BOX - 1

1.	5 N/m^2, $5\sqrt{3}$ N/m^2	**2.**	0.004 radians	**3.**	6 mm
4.	Break	**5.**	10.2 km	**6.**	20 kg-wt

BEGINNER'S BOX - 2

1.	2.0×10^9 N/m^2	**2.**	27 : 1	**3.**	4
4.	0.25%	**5.**	0.4 cm^3	**6.**	2×10^{-7} m
7.	4.8×10^{-5} J				

BEGINNER'S BOX - 3

1.	$\rho_1 = 2$ kg/m^3, $\rho_2 = 6$ kg/m^3 or $\rho_1 = 6$ kg/m^3, $\rho_2 = 2$ kg/m^3			**2.**	83.33 cm
3.	25 N	**4.**	1 cm	**5.**	(i) 5 (ii) 0.67
6.	(i) 0.67×10^3 kg/m^3 (ii) 0.74×10^3 kg/m^3			**7.**	300 cm^3
8.	500 g-f				

BEGINNER'S BOX - 4

1.	1 m/s	**2.**	8 m/s	**3.**	$\frac{6}{\sqrt{10}} \times 10^{-3}$ m^3/s

BEGINNER'S BOX - 5

1.	2×10^{-2} N	**2.**	2 mm	**3.**	0.012 m/s
4.	1.52 poise	**5.**	8 : 1		

BEGINNER'S BOX - 6

1.	80 dynes	**2.**	$\frac{2}{\pi}$ N/m	**3.**	2800 dynes
4.	$\sqrt{\frac{2T}{\pi dg}}$	**5.**	0.1 N/m	**6.**	3 J
7.	$24\pi R^2T$	**8.**	960π erg	**9.**	99%
10.	$12\ \pi r^2T$				

BEGINNER'S BOX - 7

1.	80°	**2.**	acute	**3.**	1 : 27
4.	1.045×10^5 Pa	**5.**	8 : 1	**6.**	$1 : \sqrt{3}$
7.	3 : 2	**8.**	1 mm		