Properties of Subtraction of Rational Numbers

Subtraction means finding the difference between two rational numbers.

If a and b are rational numbers, then a – b is also a rational number.

Subtraction Rule using Addition:

We can rewrite subtraction as:

a−b = a + (−b)a – b = a + (–b)

This means:

To subtract a rational number, add its additive inverse.

Properties of Subtraction of Rational Numbers

Unlike addition, subtraction of rational numbers does not follow all properties.

1. Closure Property

Statement:

If a and b are rational numbers, then a – b is also a rational number.

This property holds true for subtraction.

Example 1:

$\frac{2}{3} - \frac{1}{6} = \frac{(4-1)}{6} = \frac{3}{6} = \frac{1}{2}$ → a rational number

Example 2:

$-\frac{3}{4} - \frac{1}{4} = -\frac{4}{4} = -1$ → a rational number

2. Commutative Property

Statement:

a – b ≠ b – a

This property does NOT hold true for subtraction.

Example 1:

$\frac{3}{5} - \frac{2}{5} = \frac{1}{5}$

$\frac{2}{5} - \frac{3}{5} = -\frac{1}{5}$ → Not equal

Example 2:

$\frac{5}{6} - \frac{1}{2} = \frac{(5-3)}{6} = \frac{2}{6} = \frac{1}{3}$

$\frac{1}{2} - \frac{5}{6} = \frac{(3-5)}{6} = -\frac{2}{6} = -\frac{1}{3}$ → Not equal

3. Associative Property

Statement:

$(a - b) - c \neq a - (b - c)$

Subtraction is NOT associative.

Example 1:

Let $a = \frac{3}{4}$, $b = \frac{1}{4}$, $c = \frac{1}{2}$

$(\frac{3}{4} - \frac{1}{4}) - \frac{1}{2} = (\frac{2}{4}) - \frac{1}{2} = 0$

$\frac{3}{4} - (\frac{1}{4} - \frac{1}{2}) = \frac{3}{4} - (-\frac{1}{4}) = \frac{3}{4} + \frac{1}{4} = 1$ → Not equal

Example 2:

$(\frac{5}{6} - \frac{1}{3}) - \frac{1}{6} = \frac{(5-2)}{6-1} = \frac{3}{6} - \frac{1}{6} = \frac{2}{6}$

$\frac{5}{6} - (\frac{1}{3} - \frac{1}{6}) = \frac{5}{6} - (\frac{1}{6}) = \frac{4}{6}$ → Not equal

4. Additive Identity (0)

Statement:

$a - 0 = a$, but $0 - a \neq a$ (It equals the additive inverse of a)

Holds true in one direction.

Example 1:

$\frac{3}{7} - 0 = \frac{3}{7}$

$0 - \frac{3}{7} = -\frac{3}{7}$ → not same

Example 2:

$-\frac{5}{8} - 0 = -\frac{5}{8}$

$0 - (-\frac{5}{8}) = \frac{5}{8}$ → not same

5. Additive Inverse

Statement:

Subtracting a number is the same as adding its additive inverse.

$a - b = a + (-b)$

This is a useful rule for solving.

Example 1:

$$\frac{2}{3} - \frac{1}{4} = \frac{2}{3} + \left(-\frac{1}{4}\right) = \frac{(8-3)}{12} = \frac{5}{12}$$

Example 2:

$$-\frac{5}{6} - \left(\frac{2}{3}\right) = -\frac{5}{6} + \left(-\frac{2}{3}\right) = -\frac{5}{6} - \frac{4}{6} = -\frac{9}{6} = -\frac{3}{2}$$

Summary

Property	Does It Hold for Subtraction?
Closure Property	Yes
Commutative Property	No
Associative Property	No
Additive Identity (0)	Partially (a – 0 = a)
Additive Inverse usage	Yes (a – b = a + (–b))