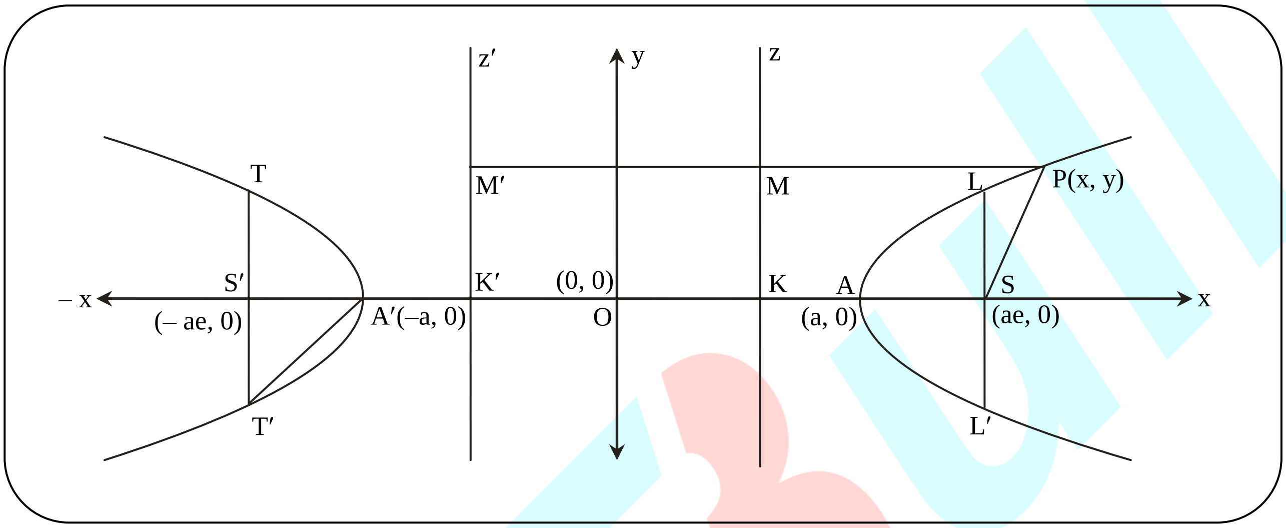
Hyperbola

1. Standard Equation and Definitions

Standard Equation of hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$



(i) Definition hyperbola :

A **Hyperbola** is the locus of a point in a plane which moves in the plane in such a way that the ratio of its distance from a fixed point (called focus) in the same plane to its distance from a fixed line (called directrix) is always constant which is always greater than unity.

(ii) Vertices :

The point A and A′ where the curve meets the line joining the foci S and S′ are called vertices of hyperbola.

(iii) Transverse and Conjugate axes :

The straight line joining the vertices A and A′ is called transverse axes of the hyperbola. Straight line perpendicular to the transverse axes and passes through its centre called conjugate axes.

(iv) Latus Rectum :

The chord of the hyperbola which passes through the focus and is perpendicular to its transverse axes is called latus rectum. Length of latus rectum = $\frac{2b^2}{a}$.

(v) Eccentricity :

For the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, $b^2 = a^2 (e^2 - 1)$

$$e = \sqrt{1+\left(\frac{2b}{2a}\right)^2} = \sqrt{1+\left(\frac{\text{Conjugate axes}}{\text{Transverse axes}}\right)^2}$$

(vi) Focal distance :

The distance of any point on the hyperbola from the focus is called the focal distance of the point.

Note : The difference of the focal distance of a point on the hyperbola is constant and is equal to the length of the transverse axes. $|S'P - SP| = 2a$ (const.)

2. Conjugate Hyperbola

The hyperbola whose transverse and conjugate axes are respectively the conjugate and transverse axes of a given hyperbola is called conjugate hyperbola.

Equation of conjugate hyperbola $-\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Note :

(i) If e_1 and e_2 are the eccentricities of the hyperbola and its conjugate then $\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1$

(ii) The focus of hyperbola and its conjugate are concyclic.

S.No.	Particulars	Hyperbola	Conjugate Hyperbola
		$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$-\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
1.	Co-ordinate of the centre	(0, 0)	(0, 0)
2.	Co-ordinate of the vertices	(a, 0) & (–a, 0)	(0, b) & (0, –b)
3.	Co-ordinate of foci	(± ae, 0)	(0, ± be)
4.	Length of the transverse axes	2a	2b
5.	Length of the conjugate axes	2b	2a
6.	Equation of directrix	x = ± a/e	y = ± b/e
7.	Eccentricity	$e = \sqrt{1+\frac{b^2}{a^2}}$	$e = \sqrt{1+\frac{a^2}{b^2}}$
8.	Length of latus rectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$
9.	Equation of transverse axes	y = 0	x = 0
10.	Equation of conjugate axes	x = 0	y = 0

3. Parametric equation of the Hyperbola

Let the equation of ellipse in standard form will be given by $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Then the equation of ellipse in the parametric form will be given by x = a sec ϕ, y = b tan ϕ where ϕ is the eccentric angle whose value vary from $0 \le \phi < 2\pi$. Therefore coordinate of any point P on the ellipse will be given by (a secϕ, b tan ϕ).

4. Position of a point P(x_1, y_1) with respect to Hyperbola

The quantity $\frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} = 1$ is positive, zero or negative according as the point (x_1, y_1) lies inside on or outside the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

5. Line and Hyperbola

‘’The straight line y = mx + c is a sacant, a tangent or passes outside the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ according as $c^2 > = < a^2m^2 - b^2$

6. Equation of Tangent

(i) The equation of tangents of slope m to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are $y = mx \pm \sqrt{a^2m^2 - b^2}$ and the co-ordinates of the point of contacts are

$$\left(\pm\frac{a^2m}{\sqrt{a^2m^2-b^2}}, \pm\frac{b^2}{\sqrt{a^2m^2-b^2}}\right)$$

(ii) Equation of tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point (x_1, y_1) is $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$

(iii) Equation of tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point (a secθ, b tanθ) is $\frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta = 1$

Note : In general two tangents can be drawn from an external point (x_1, y_1) to the hyperbola and they are $y - y_1 = m_1 (x - x_1)$ and $y - y_1 = m_2 (x - x_1)$, where m_1 and m_2 are roots of

$(x_1^2 - a^2) m^2 - 2x_1y_1 + y_1^2 + b^2 = 0$