Exponential & Lograithmic Series

1. The number 'e'

The sum of the series $1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots\dots\dots + \infty$ is denoted by the number 'e'.

i.e. $e = \lim\limits_{n\to\infty}\left(1 + \frac{1}{n}\right)^n$

Note:

(i) The number 'e' lies between 2 and 3. Approximate value of e = 2.718281828.

(ii) 'e' is an irrational number.

2. Exponential Series

Expansion of any power 'x' to the number 'e' is the exponential series.

i.e. $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots\dots\dots \infty$

2.1 Exponential Theorem:

Let a > 0 then for all real value of x,

$a^x = 1 + x(\log_e a) + \frac{x^2}{2!}(\log_e a)^2 + \frac{x^3}{3!}(\log_e a)^3 + \dots\dots$

2.2 Some standard deduction from exponential series:

(i) $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} \dots\dots \frac{x^n}{n!} + \dots\dots \infty$

(ii) $e^{-x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots\dots \frac{(-1)^n}{n!}x^n + \dots\dots \infty$

(Replace x by –x)

(iii) $e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots\dots \infty$

{Putting x = 1 in (i)}

(iv) $e^{-1} = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots\dots\dots\dots \infty$

{Putting x = –1 in (ii)}

(v) $\frac{e^x + e^{-x}}{2} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots\dots\dots \infty$

(vi) $\frac{e^x - e^{-x}}{2} = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots\dots\dots\dots \infty$

(vii) $\frac{e + e^{-1}}{2} = 1 + \frac{1}{2!} + \frac{1}{4!} + \dots\dots\dots \infty$

(viii) $\frac{e - e^{-1}}{2} = 1 + \frac{1}{3!} + \frac{1}{5!} + \dots\dots\dots \infty$

3. Logarithmic Series

If $-1 < x \le 1$ then

$\log_e(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots\dots \infty$ is called as logarithmic series.

3.1 Some standard deductions from logarithmic series:

(i) $\log(1 - x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} + \dots\dots \infty$

(ii) $\log(1 + x) - \log(1 - x) = \log\left(\frac{1 + x}{1 - x}\right)$

$= 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \dots\right)$

(iii) $\log(1 + x) + \log(1 - x) = \log(1 - x^2)$

$= -2\left(\frac{x^2}{2} + \frac{x^4}{4} + \frac{x^6}{6} + \dots\right)$

Note:

(i) Naperian or Natural log can be converted into common by using following relation

$\log_{10}N = \log_e N \times 0.43429448$

(ii) $\log 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \dots\dots\dots$

$= \frac{1}{1.2} + \frac{1}{3.4} + \frac{1}{5.6} + \dots\dots\dots$