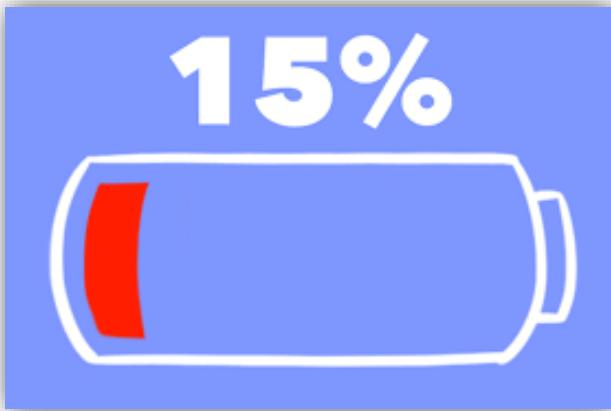


PERCENTAGE



'When first time you heard this word percentage??



May be your parents asked you how much percent you got in your exams or may be recently in news time you heard that the sale of Maruti is increased by 35%.

So in this study note we have to discuss the chapter which is the pillar of banking maths i.e. percentage.



BASIC DEFINITION:

Percent implies “for every hundred” and the sign % is read as percentage and $x\%$ is read as x per cent. In other words, a fraction with denominator 100 is called a per cent. For example, 20 % means $20/100$ (i.e. 20 parts from 100). This can also be written as 0.2.



Percentage is a fraction whose denominator is always 100.
 x percentage is represented by $x\%$.

Percentage means per hundred when we say 50% of a number it means half of the number. Similarly, 25% of a number is one fourth of the number; 20% means one fifth of the number, 16.67% means one sixth of a number etc. Hence percentage indicates the part or fraction of a number.

PERCENTAGE CHANGE FORMULA

$$\frac{\text{NEW VALUE} - \text{OLD VALUE}}{\text{OLD VALUE}} \times 100$$

BASIC FORMULA:

In order to calculate p % of q, use the formula:

$$\frac{p}{100} \times q = \frac{pq}{100}$$

Also remember: p % of q = q % of p

Example:

$$100\% \text{ of } 60 = 60\% \text{ of } 100 = 60 \times (100/100) =$$

$$\frac{60}{100} \times 100 = 60$$

BASIC CONCEPTS OF PERCENTAGES

Expressing One Quantity as a Per Cent with respect to the other:

To express a quantity as a per cent with respect to other quantity, the following formula is used:

$$= \left[\frac{\text{The quantity to be expressed in per cent}}{\text{2nd quantity (in respect of which the per cent has to be obtained)}} \times 100 \right] \%$$



PERCENTAGE AS COMPARISON:

Percentage helps us to compare between different fractions when the denominator or the total number is different in each case. It is one of the simplest tools for the comparison of data.

A screenshot of a Microsoft Excel spreadsheet. The table has three columns: 'Month' (B), 'Sales, USD' (C), and 'Monthly Change, %' (D). The data shows monthly sales figures and the percentage change from the previous month. Row 2 (January) is the header. Row 3 (February) shows sales of \$31 and a change of -32.61%. Row 4 (March) shows sales of \$42 and a change of 35.48%. Row 5 (April) shows sales of \$23 and a change of -45.24%. Row 6 (May) shows sales of \$55 and a change of 139.13%. Row 7 (June) shows sales of \$40 and a change of -27.27%. Row 8 (July) shows sales of \$20 and a change of -50.00%. Row 9 (August) shows sales of \$58 and a change of 190.00%. The formula bar at the top shows the formula $= (C3-C2)/C2$.

Month	Sales, USD	Monthly Change, %
January	\$46	
February	\$31	-32.61%
March	\$42	35.48%
April	\$23	-45.24%
May	\$55	139.13%
June	\$40	-27.27%
July	\$20	-50.00%
August	\$58	190.00%

Take, for example, this table below which shows the marks obtained by a student in 3 different subjects

Subject	Marks Obtained
History	75
Math	33
English	64

From this data alone, we cannot compare the marks obtained for the various subjects.



Now suppose we have the data of the total marks obtained as follows

Subject	Marks Obtained	Total Marks	Marks Obtained/total marks × 100
History	75	100	75%
Math	33	50	66%
English	64	80	80%

Now as all the three subjects are represented on a scale of 100, it is easy to compare the marks for the three subjects and decide which subject has the student scored the maximum in.

Example: 60 % of a number is 360. What is 99 % of the same number?

Solution: Let the number be n.

$$\text{Given } (60/100) \times n = 360 \Rightarrow n = 600$$

$$99\% \text{ of } 600 = (99/100) \times 600 = 594$$

Or we can calculate directly

$$\frac{360}{60} \times 99 = 594$$

Example: 40 % of a number is 240. What is 112 % of the same number?

$$\text{Solution: } \frac{240}{40} \times 112 = 672$$

Example: What percent is 60 of 240?

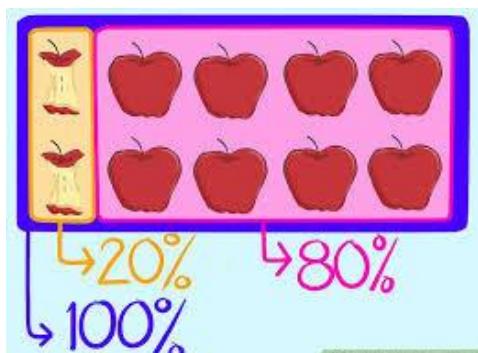
Solution: First write the given numbers in the fraction form:

$$60/240 = \frac{1}{4}$$

Multiply the numerator and denominator with 25 to make the denominator equal to 100

$$(1 \times 25)/(4 \times 25) = 25/100$$

CALCULATION OF PERCENTAGE



- To express x% as a fraction :
We know
 $x\% = x/100$
Thus $10\% = 10/100$ (means 10 parts out of 100 parts)
 $= 1/10$ (means 1 part out of 10 parts)
- To express x/y as a percentage :
We know that $x/y = (x/y \times 100)\%$
Thus $1/4 = (1/4 \times 100)\% = 25\%$
and $0.8 = (8/10 \times 100)\% = 80\%$
- To increase a number by a given percentage(x%):
Multiply the number by the following factor

$$= \left[\frac{100 + x}{100} \right]$$

- To decrease a number by a given percentage(x%):
Multiply the number by the following factor

$$= \left[\frac{100 - x}{100} \right]$$

- To find the % increase of a number:

$$\text{Percent increment} = \left[\frac{\text{Final Value} - \text{Initial Value}}{\text{Initial Value}} \right] * 100$$

- To find the % decrease of a number:

$$\text{Percent decreament} = \left[\frac{\text{Initial Value} - \text{Final Value}}{\text{Initial Value}} \right] * 100$$

PERCENTAGE AND RATIO

Decimal	Percentage	Fraction
0.5	50%	$\frac{1}{2}$
0.25	25%	$\frac{1}{4}$
0.75	75%	$\frac{3}{4}$
0.2	20%	$\frac{1}{5}$
0.1	10%	$\frac{1}{10}$
0.3	33.3%	$\frac{1}{3}$

$$x + y + \frac{xy}{100}$$

But always remember one thing x and y always come with sign. Means if quantity is increased we will take + and if it is decreased we will take -.

So the answer of the question asked will be-

$$20 - 20 - \frac{20 \times 20}{100} = -4\%$$

But what if there are three changes??

We also have formula for that

$$x + y + z + \frac{xy}{100} + \frac{yz}{100} + \frac{zx}{100} - \frac{xyz}{10000}$$

But using this formula is not a great idea instead this we can apply first formula thrice Like

Example:

Find the net percentage change of 40% increase.25% decrease and 10% increase?

Solution:

Very important point it doesn't matter which change is applied first final change will be always same.

$$40 - 25 - \frac{25 \times 40}{100} = 5$$

$$\text{Again } 5 + 10 + \frac{5 \times 10}{100} = 15.5$$

So final change will be decrease of 17.5%.

Example:

The capacity of a ground was 100000 at the end of 2012. In 2013, it increased by 10% and in 2014, it decreased by 18.18%. What was the ground's capacity at the end of 2014?

Solution:

When One percentage change is positive and the other is negative:

x is positive and y is negative, then net percentage change = $(x-y-xy/100)\%$

Final Percentage Change over the original value
 $= 10 - 18.18 - (10 \times 18.18/100) = -9.998$

(the difference above is cause by using exact values).

So the capacity of the ground is decreased by 9.998%

Hence, net capacity = 90002

Example:

A's salary is increased by 10% and then decreased by 10%.

The change in salary is

Solution:

$$10 - 10 - \frac{10 \times 10}{100} = -1$$

Example:

A number is first increased by 10% and then it is further increased by 20%. The original number is increased altogether by:

Answers:

Percentage change formula when both x and y are positive
 $= \{x + y + (xy/100)\}\%$

Here, x = 10 and y = 20

Hence net percentage change = $\{10 + 20 + (10 \times 20)/100\} = 32\%$

What is the Product Constancy Ratio?

In this concept, we essentially refer to the practice wherein two or more quantities make up a third quantity. With the variation in the numbers of one quantity, the other quantities need to undergo change in order to maintain the same product.

Let's take an example.

Let there be two quantities A and B that multiply to form a quantity P. We can say:

$$A \times B = P$$

Now if A is increased by a certain percentage, then B is required to be decreased by a certain percentage to keep the product P stable.

The following table illustrates the varying values of A and B that will maintain the same product P.

Change in A (INCREASE)	Change in B (DECREASE)	CHANGE IN P
100%	50%	0
50%	33.33%	0
33.33%	25%	0
25%	20%	0
20%	16.66%	0
16.66%	14.28%	0

The same table can also be expressed in the form of ratios:

Change in A (INCREASE)	Change in B (DECREASE)	Change in P
$\frac{1}{1}$	$\frac{1}{2}$	0
$\frac{1}{2}$	$\frac{1}{3}$	0
$\frac{1}{3}$	$\frac{1}{4}$	0
$\frac{1}{4}$	$\frac{1}{5}$	0

Application of Product Constancy: Expenditure Questions

If the price of a commodity increases or decreases by a%, then, the percentage decrease or increase in consumption, so as not to increase or decrease the expenditure is equal to:
 $(a/100+a) \times 100\%$

Example:

Length of a rectangle is increased by 33.33%. By what percentage should the breadth be decreased so that area remains constant?

Example:

Ram can buy 5 Kg more sugar in rupees 100 as the price of sugar has decreased by 10%. Find the actual price of the sugar?

$$\frac{10 \times 100}{(100 - 10) \times 5} = \frac{20}{9}$$

- (9) If in an election, a candidate got $\frac{100 \times y}{100 - 2x}$ total votes cast and still lose by y votes, the total number of votes cast –

$$\frac{100 \times y}{100 - 2x}$$

Example:

In an election contested by two candidates, one candidate got 40% of total votes and still lost by 500 votes, find the total number of votes casted?

$$TV = \frac{100 \times 500}{100 - (2 \times 40)} = 2500$$

- (10) If the population of a town is P and it increases or decreases at the rate of $R\%$ per annum then –

I. Population after ' n ' years :

$$= P * \left[1 \pm \frac{R}{100} \right]^n$$

II. Population ' n ' years ago :

$$= \frac{P}{\left[1 \pm \frac{R}{100} \right]^n}$$

- (11) If the value of a number is first increased by and again decreased by the net effect is always decreased by $\frac{x^2}{100}\%$
 Example: -The salary of a worker is first increased by 5% and then it is decreased by 5%. What is the change in his salary?

$$\text{Percent Decrease} = \frac{(5)^2}{100} = 0.25\%$$

$$15\% \text{ of } 80 = 12$$

$$\underline{85\% \text{ of } 620 =}$$

PROBLEM TYPE-1:

PROBLEMS BASED ON QUANTITY PURCHASED

Example:

A reduction of 21% in the price of an item enables a person to buy 3 kg more for 100. The reduced price of item per kg is:

(a) Rs. 5.50

(b) Rs. 7.50

(c) Rs. 10.50

(d) Rs. 7.00

Solution: (d)

Reduced price will be:

Rp/100y per kg

In our case R= Rs. 100 , x=21% , y=3kg

$\{(100 \times 21)/(100 \times 3)\} = \text{Rs. 7}$

Alternate method:

Expenditure = price per quantity \times consumption

$$E = P \times Q = 100 \dots\dots (1)$$

Now, as per the question,

$$E = P \left(\frac{100 - 21}{100} \right) (Q + 3) = 100 \dots\dots (2)$$

$$P \times Q = P \left(\frac{100 - 21}{100} \right) (Q + 3)$$

$$Q = \left(\frac{100 - 21}{100} \right) (Q + 3)$$

$$Q = \frac{79}{7} \text{ kg}$$

$$\text{Price}(p) = \frac{100}{79} \times 7 = \frac{700}{79} \text{ rs}$$

$$\text{reduced price will be} = \frac{700}{79} \left(\frac{100 - 21}{100} \right) = 7 \text{ rs}$$

PROBLEM TYPE-2:

PROBLEMS BASED ON MIXTURES

Example:

A vessel has 60 L of solution of acid and water having 80% acid. How much water is to be added to make it solution in which acid forms 60%?

(a) 48 L

(b) 20 L

(c) 36 L

(d) None of these

Solution: (b)

Given, percentage of acid = 80%

Then, percentage of water = 20%

In 60L of solution, water = $(60 \times 20)/100 = 12\text{L}$

Let p liter of water is to be added.

According to the question,

$$\Rightarrow \{(12 + p)/(60 + p)\} \times 100 = 40 \quad (\because 100 - 60 = 40\% \text{ water})$$

$$\Rightarrow 1200 + 100p = 2400 + 40p$$

$$\Rightarrow 60p = 1200$$

$$p = 20\text{L}$$

Rohit obtained 480 marks out of 600 and Mohit obtained 560 marks out of 800. Whose performance is better and how much %?

Solution:

$$\text{Rohit got} = \frac{480}{600} \times 100 = 80\%$$

$$\text{Mohit got} = \frac{560}{800} \times 100 = 70\%$$

Rohit performance is better.

$$\% \text{ better} = \frac{10}{70} \times 100 = 14 \frac{2}{7}\%$$

Question:

In an election between two candidates, 65 votes were declared invalid one candidate gets 52% votes and win by 98 votes. Find the total number of votes?

Winner 52% Looser 48%

$$4\% = 98$$

$$100\% = 2450$$

$$\text{Valid vote} = 2450$$

$$\text{Total vote} = 2450 + 65 = 2515$$

Question:

The length and breadth of the rectangle are increased by 30% and 20% respectively, so find the net % change in its area?

Solution:

$$\text{Net \% change} = x + y + \frac{X \times Y}{100} = 30 + 20 + \frac{30 \times 20}{100} = 56\%$$

Question:

A, B and C are three persons. A earned 40% more than B and B earned 20% less than C, so what % more earned A than C.

Solution:

A	B	C
112	80	100

$$\text{A more earned than C} = 112 - 100 = 12$$

$$\% \text{ more earned} = \frac{12 \times 100}{100} = 12\%$$

Question:

Due to reduction of 20% in the price of apples enables a person to purchase 16 apples more for Rs.320, so find the reduced rate of 10 apples?

Solution:

$$\text{Reduced rate} = \frac{320 \times 20}{100} = 64$$

$$\text{Reduced rate of 1 apple} = \frac{64}{16} = \text{Rs. 4}$$

$$\text{Reduced rate of 10 apples} = 4 \times 10 = \text{Rs.40}$$

Question:

A mixture of 40 litres of milk and water contains 10% water. How much water should be added, so water may be 20% in the new mixture?

Solution:

$$40 \times \frac{10}{100} = 4 \text{ lit., } 80\% = 36, 1\% = \frac{36}{80}$$

$$20\% = \frac{36}{80} \times 20 = 9 \text{ Lit.} \quad \text{Req. water} = 9 - 4 = 5 \text{ lit.}$$

Question:

Fresh fruit contains 68% water and dry fruit contains 20% water. How much dry fruit can be obtained from 100 kg. of fresh fruit?

Solution:

$$\text{Fruit content in 100 kg. of fresh fruits} = 100 \times \frac{32}{100} = 32 \text{ kg.}$$

Since, dry fruits contain 80% fruit content,

$$80\% = 32, 1\% = \frac{32}{80}, 100\% = \frac{32}{80} \times 100 = 40 \text{ kg.}$$

Question:

In an examination, 35% of the candidates failed in Mathematics and 38% failed in English. 28% passed in both subjects, then how much percent failed in both the subjects?

Solution:

$$\text{Failed in Maths} = 35\%$$

$$\text{Passed in Maths} = 100\% - 35\% = 65\%$$

$$\text{Failed in English} = 38\%$$

$$\text{Passed in English} = 100\% - 38\% = 62\%$$

$$\text{Failed in both subjects} = 100\% - (37\% + 28\% + 34\%) = 1\%$$

Question:

A number is mistakenly divided by 10 instead of being multiplied by 10. What is the percentage error in the result?

Solution:

Let the number is 10.

$$\text{Actual result} = 10 \times 10 = 100$$

$$\text{Wrong result} = \frac{10}{10} = 1$$

$$\% \text{ change} = \frac{100 - 1}{100} \times 100 = 99\%$$

Question:

The numerator of a fraction is increased by 400% and the denominator is increased by 500%, so the resultant fraction is 15/22. Find the original fraction?

Solution:

$$\frac{500\% \text{ of } x}{600\% \text{ of } y} = \frac{15}{22} \quad \frac{5x}{6y} = \frac{15}{22}$$

$$\text{Original Fraction} \frac{x}{y} = \frac{15 \times 6}{22 \times 5}$$