Set & Relations

1. Set Theory

Collection of well defined objects which are distinct and distinguishable. A collection is said to be well defined if each and every element of the collection has some definition.

1.1 Notation of a set : Sets are denoted by capital letters like A, B, C or { } and the entries within the bracket are known as elements of set.

1.2 Cardinal number of a set : Cardinal number of a set X is the number of elements of a set X and it is denoted by n(X) e.g. $X = [x_1, x_2, x_3] \therefore n(X) = 3$

2. Representation of Sets

2.1 Set Listing Method (Roster Method) :

In this method a set is described by listing all the elements, separated by commas, within braces { }

2.2 Set builder Method (Set Rule Method) :

In this method, a set is described by characterizing property P(x) of its elements x. In such case the set is described by {x : P(x) holds} or {x | P(x) holds}, which is read as the set of all x such that P(x) holds. The symbol '|' or '**:**' is read as such that.

3. Type of Sets

3.1 Finite set :

A set X is called a finite set if its element can be listed by counting or labeling with the help of natural numbers and the process terminates at a certain natural number n. i.e. n(X) = finite no. eg (a) A set of English Alphabets (b) Set of soldiers in Indian Army

3.2 Infinite set :

A set whose elements cannot be listed counted by the natural numbers (1, 2, 3.......n) for any number n, is called a infinite set. e.g.

(a) A set of all points in a plane

(b) $X = \{x : x \in R, 0 < x < 0.0001\}$

(c) $X = \{x : x \in Q, 0 \le x \le 0.0001\}$

3.3 Singleton set :

A set consisting of a single element is called a singleton set. i.e. n (X) = 1,

e.g. $\{x : x \in N, 1 < x < 3\}$, {{}} : Set of null set, $\{\phi\}$ is a set containing alphabet ϕ.

3.4 Null set :

A set is said to be empty, void or null set if it has no element in it, and it is denoted by ϕ. i.e. X is a null set if n(X) = 0. e.g. : $\{x : x \in R \text{ and } x^2 + 2 = 0\}$, $\{x : x > 1 \text{ but } x < 1/2\}$, $\{x : x \in R, x^2 < 0\}$

3.5 Equivalent Set :

Two finite sets A and B are equivalent if their cardinal numbers are same i.e. n(A) = n(B).

3.6 Equal Set :

Two sets A and B are said to be equal if every element of A is a member of B and every element of B is a member of A. i.e. A = B, if A and B are equal and $A \neq B$, if they are not equal.

4. Universal Set

It is a set which includes all the sets under considerations i.e. it is a super set of each of the given set. Thus, a set that contains all sets in a given context is called the universal set. It is denoted by U. e.g. If A = {1, 2, 3}, B = {2, 4, 5, 6} and C = {1, 3, 5, 7}, then U = {1, 2, 3, 4, 5, 6, 7} can be taken as the universal set.

5. Disjoint Set

Sets A and B are said to be disjoint iff A and B have no common element or $A \cap B = \phi$. If $A \cap B \neq \phi$ then A and B are said to be intersecting or overlapping sets.

e.g. : (i) If A = {1, 2, 3}, B = {4, 5, 6} and C = {4, 7, 9} then A and B are disjoint set where B and C are intersecting sets. **(ii)** Set of even natural numbers and odd natural numbers are disjoint sets.

6. Complementary Set

Complementary set of a set A is a set containing all those elements of universal set which are not in A. It is denoted by $\overline{A}$, A^C or A′. So $A^C = \{x : x \in U \text{ but } x \notin A\}$. e.g. If set A = {1, 2, 3, 4, 5} and universal set U = {1, 2, 3, 4,.......50} then $\overline{A}$ = {6, 7, ...50}

NOTE :

All disjoint sets are not complementary sets but all complementary sets are disjoint.

7. Subset

A set A is said to be a subset of B if all the elements of A are present in B and is denoted by $A \subset B$ (read as A is subset of B) and symbolically written as : $x \in A \Rightarrow x \in B \Leftrightarrow A \subset B$

7.1 Number of subsets :

Consider a set X containing n elements as $\{x_1, x_2, \ldots\ldots, x_n\}$ then the total number of subsets of $X = 2^n$

Proof : Number of subsets of above set is equal to the number of selections of elements taking any number of them at a time out of the total n elements and it is equal to 2^n

$\because \quad {}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots\ldots + {}^nC_n = 2^n$

7.2 Types of Subsets :

A set A is said to be a **proper subset** of a set B if every element of A is an element of B and B has at least one element which is not an element of A and is denoted by $A \subset B$.

The set A itself and the empty set is known as **improper subset** and is denoted as $A \subseteq B$.

e.g. If $X = \{x_1, x_2, \ldots\ldots, x_n\}$ then total number of proper sets $= 2^n - 2$ (excluding itself and the null set). The statement $A \subset B$ can be written as $B \supset A$, then B is called the **super set** of A and is written as $B \supset A$.

8. Power Sets

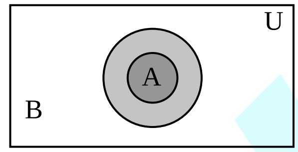
The collection of all subsets of set A is called the power set of A and is denoted by P(A)

i.e. $P(A) = \{x : x \text{ is a subset of } A\}$. If $X = \{x_1, x_2, x_3, \ldots\ldots\ldots\ldots x_n\}$ then $n(P(X)) = 2^n$; $n(P(P(x))) = 2^{2^n}$.

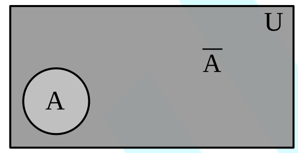
9. Venn(Euler) Diagrams

The diagrams drawn to represent sets are called Venn diagram or Euler-Venn diagrams. Here we represents the universal U as set of all points within rectangle and the subset A of the set U is represented by the interior of a circle. If a set A is a subset of a set B, then the circle representing A is drawn inside the circle representing B. If A and B are not equal but they have some common elements, then to represent A and B by two intersecting circles.

e.g. If A is subset of B then it is represented diagrammatically in fig.



e.g. If A is a set then the complement of A is represented in fig.



10. Operations on Sets

10.1 Union of sets :

If A and B are two sets then union ($\cup$) of A and B is the set of all those elements which belong either to A or to B or to both A and B. It is also defined as $A \cup B = \{x : x \in A \text{ or } x \in B\}$. It is represented through Venn diagram in fig.1 & fig.2

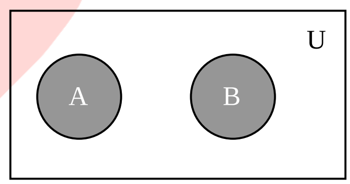


Fig. (1)

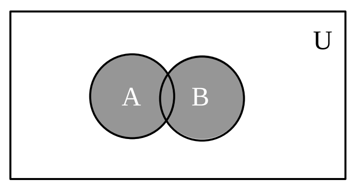
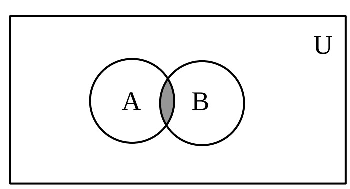


Fig. (2)

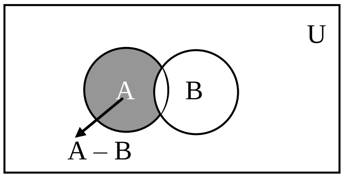
10.2 Intersection of sets :

If A and B are two sets then intersection ($\cap$) of A and B is the set of all those elements which belong to both A and B. It is also defined as $A \cap B = \{x : x \in A \text{ and } x \in B\}$ represented in Venn diagram (see fig.)



10.3 Difference of two sets :

If A and B are two sets then the difference of A and B, is the set of all those elements of A which do not belong to B.



Thus, $A - B = \{x : x \in A \text{ and } x \notin B\}$

or $A - B = \{x \in A ; x \notin B\}$

Clearly $x \in A - B \Leftrightarrow x \in A$ and $x \notin B$

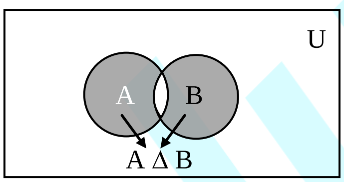
It is represented through the Venn diagrams.

10.4 Symmetric difference of two sets :

Set of those elements which are obtained by taking the union of the difference of A & B is (A – B) & the difference of B & A is (B – A), is known as the symmetric difference of two sets A & B and it is denoted by $(A \Delta B)$.

Thus $A \Delta B = (A - B) \cup (B - A)$

Representation through the venn diagram is given in the fig.



11. Number of Elements in Different Sets

If A, B & C are finite sets and U be the finite universal set, then

(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(ii) $n(A \cup B) = n(A) + n(B)$ (if A & B are disjoint sets)

(iii) $n(A - B) = n(A) - n(A \cap B)$

(iv) $n(A \Delta B) = n[(A - B) \cup (B - A)]$

$= n(A) + n(B) - 2n(A \cap B)$

(v) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B)$

$- n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

(vi) $n(A' \cup B') = n(A \cap B)' = n(U) - n(A \cap B)$

(vii) $n(A' \cap B') = n(A \cup B)' = n(U) - n(A \cup B)$

12. Cartesian Product of two Sets

Cartesian product of A to B is a set containing the elements in the form of ordered pair (a, b) such that $a \in A$ and $b \in B$. It is denoted by $A \times B$.

i.e. $A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$

$= \{(a_1, b_1),(a_1, b_2),(a_2, b_1),(a_2, b_2),(a_3, b_1),(a_3, b_2)\}$

If set $A = \{a_1, a_2, a_3\}$ and $B = \{b_1, b_2\}$ then

$A \times B$ and $B \times A$ can be written as :

$A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$ and

$B \times A = \{(b, a) ; b \in B \text{ and } a \in A\}$

Clearly $A \times B \neq B \times A$ until A and B are equal

Note :

1. If number of elements in A : $n(A) = m$ and $n(B) = n$ then number of elements in $(A \times B) = m \times n$
2. Since $A \times B$ contains all such ordered pairs of the type (a, b) such that $a \in A$ & $b \in B$, that means it includes all possibilities in which the elements of set A can be related with the elements of set B. Therefore, $A \times B$ is termed as largest possible relation defined from set A to set B, also known as universal relation from A to B.

13. Algebraic Operations on Sets

13.1 Idempotent operation :

For any set A, we have (i) $A \cup A = A$ and (ii) $A \cap A = A$

Proof :

(i) $A \cup A = \{x : x \in A \text{ or } x \in A\} = \{x : x \in A\} = A$

(ii) $A \cap A = \{x : x \in A \ \& \ x \in A\} = \{x : x \in A\} = A$

13.2 Identity operation :

For any set A, we have

(i) $A \cup \phi = A$ and

(ii) $A \cap U = A$ i.e. ϕ and U are identity elements for union and intersection respectively

Proof :

(i) $A \cup \phi = \{x : x \in A \text{ or } x \in \phi\}$

$= \{x : x \in A\} = A$

(ii) $A \cap U = \{x : x \in A \text{ and } x \in U\}$

$= \{x : x \in A\} = A$

13.3 Commutative operation :

For any set A and B, we have

(i) $A \cup B = B \cup A$ and (ii) $A \cap B = B \cap A$

i.e. union and intersection are commutative.

13.4 Associative operation :

If A, B and C are any three sets then

(i) $(A \cup B) \cup C = A \cup (B \cup C)$

(ii) $(A \cap B) \cap C = A \cap (B \cap C)$

i.e. union and intersection are associative.

13.5 Distributive operation :

If A, B and C are any three sets then

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

i.e. union and intersection are distributive over intersection and union respectively.

13.6 De-Morgan's Principle :

If A and B are any two sets, then

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

Proof : (i) Let x be an arbitrary element of $(A \cup B)'$. Then $x \in (A \cup B)'$ $\Rightarrow x \notin (A \cup B)$

$\Rightarrow x \notin A$ and $x \notin B$ $\Rightarrow x \in A' \cap B'$

Again let y be an arbitrary element of $A' \cap B'$. Then $y \in A' \cap B'$

$\Rightarrow y \in A'$ and $y \in B'$ $\Rightarrow y \notin A$ and $y \notin B$

$\Rightarrow y \notin (A \cup B)$ $\Rightarrow y \in (A \cup B)'$

$\therefore A' \cap B' \subseteq (A \cup B)'$.

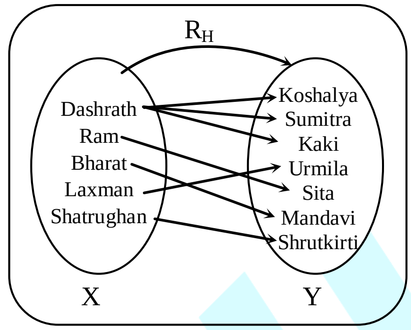
Hence $(A \cup B)' = A' \cap B'$

Similarly (ii) can be proved.

14. Relation

A relation R from set X to Y ($R : X \to Y$) is a correspondence between set X to set Y by which some or more elements of X are associated with some or more elements of Y. Therefore a relation (or binary relation) R, from a non-empty set X to another non-empty set Y, is a subset of $X \times Y$. i.e. $R_H : X \to Y$ is nothing but subset of $A \times B$.

e.g. Consider a set X and Y as set of all males and females members of a royal family of the kingdom Ayodhya X = {Dashrath, Ram, Bharat, Laxman, shatrughan} and Y = {Koshaliya, Kakai, sumitra, Sita, Mandavi, Urmila, Shrutkirti} and a relation R is defined as "was husband of "from set X to set Y.



Then R_H = {(Dashrath, Koshaliya), (Ram, sita), (Bharat, Mandavi), (Laxman, Urmila), (Shatrughan, Shrutkirti), (Dashrath, Kakai), (Dashrath, Sumitra)}

Note :

(i) If a is related to b then symbolically it is written as a R b where a is pre-image and b is image

(ii) If a is not related to b then symbolically it is written as a $\not R$ b.

14.1 Domain, Co-domain & Range of Relation :

Domain : of relation is collection of elements of the first set which are participating in the correspondence i.e. it is set of all pre-images under the relation R. e.g. Domain of R_H : {Dashrath, Ram, Bharat, Laxman, Shatrughan}

Co-Domain : All elements of set Y irrespective of whether they are related with any element of X or not constitute co-domain. e.g. Y = {Koshaliya, Kakai, Sumitra, Sita, Mandavi, Urmila, Shrutkirti} is co-domain of R_H.

Range : of relation is a set of those elements of set Y which are participating in correspondence i.e. set of all images. Range of R_H : {Koshaliya, Kakai, Sumitra, Sita, Mandavi, Urmila, Shrutkirti}.

15. Types of Relations

15.1 Reflexive Relation

$R : X \to Y$ is said to be reflexive iff $x R x \; \forall \; x \in X$. i.e. every element in set X, must be a related to itself therefore $\forall \; x \in X; (x, x) \in R$ then relation R is called as reflexive relation.

15.2 Identity Relation :

Let X be a set. Then the relation $I_x = \{(x, x) : x \in X\}$ on X is called the identity relation on X. i.e. a relation I_x on X is identity relation if every element of X related to itself only. e.g. y = x

Note : All identity relations are reflexive but all reflexive relations are not identity.

15.3 Symmetric Relation

R : X → Y is said to be symmetric iff $(x, y) \in R \Rightarrow (y, x) \in R$ for all $(x, y) \in R$ i.e. $x R y \Rightarrow y R x$ for all $(x, y) \in R$. e.g. perpendicularity of lines in a plane is symmetric relation.

15.4 Transitive Relation

R : X → Y is transitive iff $(x, y) \in R$ and $(y, z) \in R \Rightarrow (x, z) \in R$ for all (x, y) and $(y, z) \in R$. i.e. $x R y$ and $y R z \Rightarrow x R z$. e.g. The relation "being sister of" among the members of a family is always transitive.

Note :

(i) Every null relation is a symmetric and transitive relation.

(ii) Every singleton relation is a transitive relation.

(iii) Universal and identity relations are reflexive, symmetric as well as transitive.

15.5 Anti-symmetric Relation

Let A be any set. A relation R on set A is said to be an antisymmetric relation iff $(a, b) \in R$ and $(b, a) \in R \Rightarrow a = b$ for all $a, b \in A$ e.g. Relations "being subset of"; "is greater than or equal to" and "identity relation on any set A" are antisymmetric relations.

15.6 Equivalence Relation

A relation R from a set X to set Y (R : X → Y) is said to be an equivalence relation iff it is reflexive, symmetric as well as transitive. The equivalence relation is denoted by ~ e.g. Relation "is equal to" Equality, Similarity and congruence of triangles, parallelism of lines are equivalence relation.

16. Inverse of a Relation

Let A, B be two sets and let R be a relation from a set A to B. Then the inverse of R, denoted by R^{-1}, is a relation from B to A and is defined by $R^{-1} = \{(b, a) : (a, b) \in R\}$, Clearly,

$(a, b) \in R \Leftrightarrow (b, a) \in R^{-1}$ Also,

Dom of R = Range of R^{-1} and

Range of R = Dom of R^{-1}