Linear Programming Problems

1. Linear Inequation

If a, b, c ∈ R, then the equation ax + by = c is called a **linear equation** in two variables x, y whereas inequalities of the form ax + by ≤ c, ax + by ≥ c, ax + by < c & ax + by > c are called **Linear Inequations** in two variables x & y. We know that the graph of the equation ax + by = c is a straight line which divides the xy-plane into two parts (i) ax + by ≤ c (ii) ax + by ≥ c. These two are known as the **half spaces.**

In set form {(x, y) : ax + by = c} is the straight line whereas, sets {(x, y) : ax + by ≤ c} and {(x, y) : ax + by ≥ c} are **closed half spaces** and the sets {(x, y) : ax + by < c} and {(x, y) : ax + by > c} are **open half spaces**. These half spaces are also known as the solution sets of the corresponding inequation.

2. Graphs of linear inequations

Consider a linear inequation ax + by ≤ c. Drawing the graph of a linear inequation means finding its solution set.

Steps to draw the graph :

To draw the graph of an equation, following procedures are to be made-

(i) Write the inequation ax + by ≤ c into an equation ax + by = c which represent a straight line in xy-plane.

(ii) Put y = 0 in ax + by = c to get point where the line meets x- axis. Similarly, put x = 0 to obtain a point where the line meets y- axis. Join these two points to obtain the graph of the line.

(iii) If the inequation is > or <, then the points lie on this line does not consider and line is drawn dotted or discontinuous.

(iv) If the inequation is ≥ or ≤ , then the point lie on the line consider and line is drawn black (bold) or continuous.

(v) This line divides the plane XOY in two region.

To find the region that satisfies the inequation, *we apply the following rules-*

(a) Choose a point [If possible (0, 0)] not lying on this line.

(b) Substitute its coordinates in the inequation. If the inequation is satisfied, then shade the portion of the plane which contains the chosen point, otherwise shade the portion which does not contain this point. The shaded portion represents the solution set.

Note :

In case of inequations ax + by ≤ c and ax + by ≥ c points on the line are also a part of the shaded region while in case of ax + by < c and ax + by > c points on the line ax + by = c are not included in the shaded region.

3. Simultaneous linear inequations in two variables

Since the solution set of a system of simultaneous linear inequation is the set of all points in two dimensional space which satisfy all the inequations simultaneously. Therefore to find the solution set we find the region of the plane common to all the portions comprising the solution sets of given inequations. In case there is no region common to all the solution of the given inequations, we say that the solution set is void or empty.

4. Feasible Region

The limited (bounded) region of the graph made by two inequations is called **Feasible Region**. All the coordinates of the points in feasible region constitutes the solutions of system of inequations.

5. Linear Programming Problems

Linear Programming is a device to optimize the results which occurs in business under some restrictions. A general Linear Programming problem can be stated as follows:

Given a set of m linear inequalities or equations in n variables, we wish to find non- negative values of these variables which will satisfy these inequalities or equations and maximize or minimize some linear functions of the variables.

The general form of Linear Programming Problems (L.P.P.) is-

Maximize (Minimize) $z = c_1x_1 + c_2x_2 + ...+ c_nx_n$ subjected to

$a_{11}\, x_1 + a_{12}\, x_2 + ...+ a_{1n}\, x_n \ \{\le, =, \ge\}\ b_1$

$a_{21}\, x_1 + a_{22}\, x_2 + ...+ a_{2n}\, x_n \ \{\le, =, \ge\}\ b_2$

....

$a_{m1}\, x_1 + a_{m2}\, x_2 + ...+ a_{mn}\, x_n \{\le, =, \ge\}\ b_n$

and $x_1, x_2, x_3, ..., x_n \ge 0$

where $x_1, x_2, x_3, ...x_n$ are the variables whose values are to be determined and are called the **decision variables**. The inequation are called

constraints and the function to be maximized or minimized is called the **objective function**.

6. Some Definitions

(i) Solution : A set of values of the decision variables which satisfy the constraints of a Linear Programming Problem (L.P.P.) is called a solution of the L.P.P.

(ii) Feasible Solution : A solution of L.P.P. which also satisfy the non- negative restrictions of the problem is called the feasible solution.

(iii) Optimal Solution : A feasible solution which maximize or minimize i.e. which optimize the objective function of L.P.P. called an optimal solution.

Note :

A Linear Programming Problem may have many optimal solution. If a L.P.P. has two optimal solution, then there are an infinite number of optimal solutions.

(iv) Iso-Profit Line: The line is drawn in geometrical area of feasible region of L.P.P. for which the objective function remains constant at all the points lie on this line, is called iso-profit line.

7. Graphical method of solution of Linear Programming Problems

The graphical method for solving linear programming problems is applicable to those problems which involve only two variables. This method is based upon a theorem, called **extreme point theorem**, which is stated as follows-

Extreme Point Theorem: If a L.P.P. admits an optimal solution, then at least one of the extreme (or corner) points of the feasible region gives the optimal solution.

Working Rule:

(i) Find the solution set of the system of simultaneous linear inequations given by constraints and non- negativity restrictions.

(ii) Find the coordinates of each of corner points of the feasible region.

(iii) Find the values of the objective function at each of the corner points of the feasible region. By the extreme point theorem one of the corner points will provide the optimal value of the objective function. The coordinates of that corner point determine the optimal solution of the L.P.P.

Note :

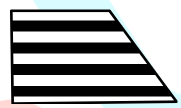
(i) If it is not possible to determine the point at which the suitable solution found, then the solution of problem is unbounded.

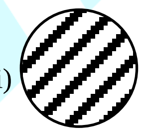
(ii) If feasible region is empty, then there is no solution for the problem.

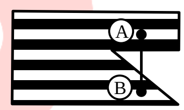
(iii) Nearer to the origin, the objective function is minimum and that of further from the origin the objective function is maximum.

8. Convex sets

In linear programming problem mostly feasible solution is a polygon in first quadrant this polygon is a convex. It means that if two points of polygon are connecting by a line then the line must be inside to polygon. For example-

(i) 

(ii) 

(iii) 

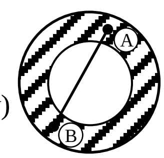
(iv) 

Figure (i) and (ii) are convex set while (iii) & (iv) are not convex set. It can be easily seen that the intersection of two convex sets is a convex set and the set of all feasible solutions of a LPP is also a convex set.