Heights & Distances

1. Introduction

One of the important application of trigonometry is in finding the height and distance of the point which are not directly measurable. This is done with the help of trigonometric ratios.

Definitions

2.1 Angle of elevation :

Let O and P be two points where P is at a higher level than O. Let O be at the position of the observer and P be the position of the object. Draw a horizontal line OM through the point O. OP is called the line of observation or line of sight. Then ∠POM $= \theta$ is called the angle of elevation of P as observed from O.



2.2 Angle of depression :

In the above figure, if P be at a lower level than O, then $\angle MOP = \theta$ is called the angle of depression.



Note :

- (1) The angle of elevation or depression is the angle between the line of observation and the horizontal line according as the object is at a higher or lower level than the observer.
- (2) The angle of elevation or depression is always measured from horizontal line through the point of observation.

2.3 Bearing :

In the figure, if the observer and the object i.e., O and P be on the same level then bearing is defined. To measure the 'Bearing', the four standard directions East, West, North and South are taken as the cardinal directions. Angle between the line of observation i.e. OP and any one standard directioneast, west, north or south is measured. Thus, $\angle POE$ $= \theta$ is called the bearing of point P with respect to O measured from east to north.



In other words the bearing of P as seen from O is the direction in which P is seen from O.

2.4 North-east :

North-east means equally inclined to north and east, south-east means equally inclined to south and east. ENE means equally inclined to east and north-east.



Some useful results 3.

3.1 In a triangle ABC,

$$\sin \theta = \frac{p}{h}, \ \cos \theta = \frac{b}{h}$$

1A

р

в

3.2 In any triangle ABC,



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad [By \text{ sine rule}]$$

or cosine formula



3.3 In any triangle ABC, if BD : DC = m : n and $\angle BAD = \alpha, \angle CAD = \beta$ and $\angle ADC = \theta$, then (m + n) cot $\theta = m \cot \alpha - n \cot \beta$



3.4 In a triangle ABC, if $DE \parallel AB$.



3.5 To find the shadow of line object AB with respect to the light source P, we first join the upper points P and B. Let O be the projection of P on the plane on which object AB is situated, join OA. The section AC obtained by the intersection of the lines PB and OA, extended represents the shadow of AB with respect to light source P.



4. Problems based on properties of a circle

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Whenever a line subtends equal angles at two points or the greatest angles at some points on a given line, such problems can be solved easily using the properties of a circle.

Mainly the following geometrical properties of a circle will be used :

(a) Angles on the same segment of a circle are equal.



In other words, we can say that if the angles APB and AQB subtended on the segment AB are equal, a circle will pass through the points A, B, Q, and P, i.e. the point A, B, Q and P will be concyclic.

(b) If AR be the tangent to the circle through the points P, Q, R then

$$\angle PRA = \angle PQR = \theta$$
.



[angle between any chord and the tangent to the circle is equal to the angle subtended by the chord at the circumference].

(c) Greatest Angle :



If PQ subtends greatest angle at R which lies on the line OR, then point R will be the point of contact of the tangent to the circle through P, Q, R.