QUADRATIC EQUATION

An algebraic statement of the second degree in x is called a quadratic equation. A quadratic equation has the conventional form ax2 + bx + c = 0, where a and b are coefficients, x is the variable, and c is the constant term.

1. Linear Equations: In linear equations, both X and Y have only one value. So relations can be established easily.
4X+3Y=18, 7x+5Y= 12
(4X+3Y= 18)× 5, (7X+5Y=12)× 3
20X+15Y=90……..(i)
21X+15Y=36……..(ii)
subtracting equation (i) from equation (2)
we get, X = -54, Y = 78
Hence, Y > X

2. Squares: In this, solutions have both negative and positive values.
X^2=1600 and Y^2=3600
X = ±40 and Y = ±60
+60 is greater than both -40 and +40, but -60 is less than both -40 and +40. So, the answer will be Cannot be determined.

TRICK: Whenever both equations are given in square form, our ANSWER will be 'Can't be determined.'

3. Squares and Square root case.
X^2=1600 and Y = √3600
We know that the square root always gives a positive value. So, Y will have **ONLY +60 NOT -60**.
X = ±40 and Y = +60
+60 is greater than both +40 and -40. Hence Y > X.

4. Cubes Case.
If X^3=1331, Y^3=729
then, X=11 and Y = 9
X is greater than Y, so the relation is X > Y.
If X^3= -1331 and Y^3= 729
then, X= -11 and Y = 9
X is greater than Y, so the relation is X < Y.
Note: Can you see something common in the above example? Common thing is that when X^3 > Y^3, the relationship is X > Y, and when X^3 < Y^3, the relation is X < Y.
TRICK: When both equations are in cube form. If X^3 > Y^3, then X > Y and X^3 < Y^3, then X < Y.

5. Square and cube cases.
If X^2=16 and Y^3=64
then X = +4, -4 and Y=4
So, Y = 4 is equal to X = 4 and Y = 4 is greater than X = -4.
So, Y ≥ X

If $X^2=25$ and $Y^3=64$
then X = +5, -5 and Y = 4
So, Y = 4 is greater than X = -5 and less than X = +5, So the relation Can't be Determined.

Table Method to Solve Quadratic Equations Easily

1. Write down the table (given below) before the exam starts, in your rough sheet, to use during the exam, Analyse the (+, -) signs in the problem, and refer to the table of signs.

2. Write down the new (solution) signs, and see if a solution is obtained instantly. If not, then go to step 3.

3. Obtain the two possible values for X & Y, from both equations,

4. Rank the values and get the solution,

STEP 1

Firstly, when you enter the exam hall, you need to write down the following **master table** in your rough sheet instantly (only the signs):-

Let us consider that the equations are $AX^2+BX+C = 0$ and $AY^2+BY+C = 0$

Type of Equation	**$AX^2+BX+C = 0$ or $AY^2+BY+C = 0$**		**Roots in X or Y equation**	
	Sign of BX or BY	**Sign of C**	**Sign of bigger root**	**Sign of smaller root**
P	+	+	-	-
Q	-	+	+	+
R	+	-	-	+
S	-	-	+	-

Now we will discuss the cases as mentioned below in the table.

CASE	ROOTS OF X /Y	ROOTS OF X/Y	CONCLUSION
I	**+,+ (Q)**	**+,+ (Q)**	**Easy**
II	**+,+ (Q)**	**+,- (R or S)**	**Will discuss**
III	**+,+ (Q)**	**-,- (P)**	**Left>Right**
IV	**+,- (R or S)**	**-,- (P)**	**Will discuss**
V	**+,- (R or S)**	**+,- (R or S)**	**Cannot be defined**
VI	**-,- (P)**	**-,- (P)**	**Easy**

CASE I: When the result of both equations are Q-type having both roots (+).

(i) If $X^2-5X+6 = 0$
both roots will be positive i.e. +3 and +2
$Y^2-17Y+66 = 0$

both roots will be positive i.e. +11 and +6
We can see that both roots of X are less than both roots of Y. So, X < Y.

(ii) If X^2-17X+42
both roots will be positive i.e. +14 and +3.
Y^2-17Y+66 = 0
both roots will be positive i.e. +11 and +6.

Here, +14 > +11
but +14 < +6
also +3 < +11
and +3 < +6

As we can see in the comparison above, there are TWO relations between X and Y which are both > and <. So **relation cannot be defined.**

Note 1: When both equations have BX (-) and C(+), You have to go into detail.

CASE II: When the result of one equation is Q type and another is either R type or S type.
(i) Q type: Y^2-49Y+444, Roots are 37,12
R type: X^2+14X-1887, Roots are -51,37

Now let us compare the values in the table below -

X	RELATION	Y
-51	<	37
-51	<	12
37	=	37
37	>	12

When we compared the values of X and Y in the table above, we found that there are THREE relations between X and Y i.e. =, > and <. So, a relation **cannot be defined**.

(ii) Q type: X^2-5X+6 = 0, Roots are 3,2
R type: Y^2-Y-6 = 0, Roots are 3,-2

Now let us compare the values in the table below -

X	RELATION	Y
3	=	3
3	>	-2
2	<	3
2	>	-2

When we compared the values of X and Y in the table above, we found that there are TWO relations between X and Y i.e. >, =. **So the relation CANNOT BE DEFINED.**

CASE III: When one equation is P-type having both roots (-) another Q-type has both roots (+).

(i) P-type: $X^2+5X+6=0$, Roots are -3, -2
Q type: $Y^2-7Y+12=0$, Roots are 4,3

Comparing the values in the below table -

X	RELATION	Y
-3	<	3
-3	<	2
-2	<	3
-2	<	2

On comparing, we saw that the So roots of the Y equation are greater than the roots of X.

Note: In this case, the roots of the equation Q-type will always be greater than the P-type.

CASE IV: When the result of one equation is P-type having both roots negative and another is either R type or S type having one root (-) and another one (+)

(i) P-type: X^2+5X+6, Roots are -3,-2
R type: X^2-X-6, roots are 3,-2

Comparing the values in the below table -

X	RELATION	Y
-3	<	3
-3	<	-2
-2	<	3
-2	=	-2

In a comparison of X and Y values, There are THREE relations between X and Y i.e. =, > and <. So relation **cannot be defined**.

(ii) P type: X^2+5X+6, Roots are -3, -2
R type: X^2-X-6, Roots are 3,-2

Now let us compare the values in the table below -

X	RELATION	Y
-3	<	3
-3	<	-2
-2	<	3
-2	=	-2

When we compared the values of X and Y in the table above, we found that there are TWO relations between X and Y i.e. <, =. **So, the relation is $X \le Y$.**

Case V: When the result of both equations is either R type or S type or one equation is R type and another is S type having one root (-) and another root (+).

(i) If $X^2+X-6 = 0$

Roots are -3 and +2.

$Y^2+5Y-66 = 0$

Roots are -11 and +6.

Comparing the values in the below table -

X	**RELATION**	**Y**
-3	>	-11
-3	<	+6
+2	>	-11
+2	<	+6