Properties of Addition of Rational Numbers

In rational numbers, addition follows certain rules or properties that help us solve problems easily and understand how numbers behave.

Closure Property of Addition

Statement:

If we add any two rational numbers, the result will also be a rational number.

Example 1:

$\frac{2}{3} + \frac{1}{6} = \frac{(4+1)}{6} = \frac{5}{6} \rightarrow$ a rational number

Example 2:

$-\frac{5}{7} + \frac{3}{7} = -\frac{2}{7} \rightarrow$ a rational number

Conclusion: Rational numbers are closed under addition.

Commutative Property of Addition

Statement:

Changing the order of the numbers does not change the sum.

i.e., a + b = b + a

Example 1:

$\frac{1}{4} + \frac{3}{5} = \frac{(5+12)}{20} = \frac{17}{20}$

$\frac{3}{5} + \frac{1}{4} = \frac{(12+5)}{20} = \frac{17}{20}$

Example 2:

$-\frac{2}{3} + \frac{4}{7} = \frac{(-14+12)}{21} = -\frac{2}{21}$

$\frac{4}{7} + (-\frac{2}{3}) = \frac{(12-14)}{21} = -\frac{2}{21}$

Conclusion: Addition of rational numbers is commutative.

Associative Property of Addition

Statement:

When adding three rational numbers, the way we group them does not affect the result.

i.e., (a + b) + c = a + (b + c)

Example 1:

Let a = $\frac{1}{2}$, b = $\frac{1}{3}$, c = $\frac{1}{6}$

$(\frac{1}{2} + \frac{1}{3}) + \frac{1}{6} = (\frac{5}{6}) + \frac{1}{6} = \frac{6}{6} = 1$

$\frac{1}{2} + (\frac{1}{3} + \frac{1}{6}) = \frac{1}{2} + (\frac{1}{2}) = 1$

Example 2:

$(-\frac{3}{4} + \frac{1}{2}) + \frac{1}{4} = (-\frac{1}{4}) + \frac{1}{4} = 0$

$-\frac{3}{4} + (\frac{1}{2} + \frac{1}{4}) = -\frac{3}{4} + \frac{3}{4} = 0$

Conclusion: Addition of rational numbers is associative.

Additive Identity

Statement:

When we add 0 to any rational number, the result is the same number.

i.e., a + 0 = a

Example 1:

$\frac{4}{9} + 0 = \frac{4}{9}$

Example 2:

$-\frac{7}{8} + 0 = -\frac{7}{8}$

Conclusion: 0 is the additive identity for rational numbers.

Additive Inverse

Statement:

For every rational number, there is another rational number such that their sum is zero.

i.e., a + (–a) = 0

Example 1:

$\frac{3}{5} + (-\frac{3}{5}) = 0$

Example 2:

$-\frac{9}{11} + \frac{9}{11} = 0$

Conclusion: The number which gives 0 when added is called the additive inverse.