Numerical Methods

1. Introduction

The study of **numerical analysis** is aimed at providing convenient methods for obtaining useful solutions to problems of advanced learning in science and technology. Besides this, analytical solutions to a large number of problems are not available. **Numerical methods** provides methods for obtaining approximate solution to such problem also.

Numerical method, in general, are of repetitive nature. In each step, better approximation to the exact solution of the problem is obtained and the process is continued till accuracy to the desired degree is arrived at.

2. Iterative method of solving equations

2.1 Successive Bisection method:

If f(x) is a continuous function in the interval [a, b] and f(a), f(b) are of opposite sign, then there exist at least one value of x say $\alpha \in (a, b)$ such that $f(\alpha) = 0$ and $a < \alpha < b$.

Working Rule:

(i) Find f(a) and f(b).

(ii) Let f(a) be negative and f(b) be positive then take $\alpha = \frac{a+b}{2}$.

(iii) If $f(\alpha) = 0$, Then α is a required root. If $f(\alpha) > 0$ then roots lie between a and α. If $f(\alpha) < 0$ then roots lies between α and b.

(iv) Repeat the process until we get the root correct up to desired level of accuracy.

2.2 False Position Method (Regula-falsi method) :

If the root of f(x) = 0 belongs to the interval (x_0, x_1) and $f(x_0)$, $f(x_1)$ are of opposite sign (say $f(x_0) < 0$, $f(x_1) > 0$), then

$$x_2 = x_0 - \frac{(x_1 - x_0)f(x_0)}{f(x_1) - f(x_0)}$$

Working Rule :

(i) Calculate $f(x_0)$ and $f(x_1)$, if these are of opposite sign then the root lies between x_0 & x_1.

(ii) Calculate x_2 by the above formula.

(iii) Now if $f(x_2) = 0$, then x_2 is the required root.

(iv) If $f(x_2)$ is negative, then the root lies in (x_2, x_1)

(v) If $f(x_2)$ is positive, then the root lies in (x_0, x_2).

(vi) Repeat it until we get the root nearest to the real root.

2.3 Newton- Raphson Method :

If f(x) = 0 and f(a) and f(b) are of opposite sign, then to find the root in interval (a, b)-

(i) Find | f(a) |, | f(b)|. If |f(a) | < |f (b)| then assume $a = x_0$ otherwise $b = x_0$

(ii) Find $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

(iii) If $f(x_1) = 0$, then x_1 is the required root, otherwise by taking x_1 as starting point find x_2

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

This process is continued till we get a value for the root up to the desired level of accuracy.

3. Numerical Integration Rules

3.1 Trapezoidal Rule :

Let y = f(x) be a function defined on [a,b] which is subdivided into n equal subintervals each of width h so that b – a = nh.

$$\int_a^b f(x)\,dx = \frac{h}{2}[(y_0 + y_n) + 2(y_1 + y_2 + \ldots\ldots + y_{n-1})]$$

where n is any positive integer and y_r is the value of f(x) for x = a + r h.

3.2 Simpson's One Third Rule :

Let y = f(x) be a function defined in the interval [a, b] which is divided into n(an even number) equal parts of width h so that b – a = nh and y_r, is the value of f(x) for x = a + rh, Then

$$\int_a^b f(x)\,dx = \frac{h}{3}[(y_0 + y_n) + 4(y_1 + y_3 + \ldots + y_{n-1}) + 2(y_2 + y_4 + \ldots + y_{n-2})]$$