# NEWTON'S & LAWS OF MOTION

## ARISTOTLE'S FALLACY

According to Aristotle, a constant continuous force is required to keep a body in uniform motion. This is called Aristotle's Fallacy.

## l. NEWTON'S FIRST LAW OF MOTION (OR GALILEO'S LAW OF INERTIA)

Every body continues its state of rest or uniform motion in a straight line unless compelled by an external force to change its state.

- This law defines the force and states that "force is a factor which can change the state of object."

**Definition of force from Newton's first law of motion**

"Force is the push or pull which changes or tends to change the state of rest or of uniform motion".

## 2. FORCE

Any push or pull which either changes or tends to change the state of rest or of uniform motion (constant velocity) of a body is known as force.

**Effects of Resultant force :-**

A non zero resultant force may produce the following effects on a body :

(i) It may change the speed of the body.
(ii) It may change the direction of motion.
(iii) It may change both the speed and direction of motion.
(iv) It may change the size or/and the shape of the body.

**Units for measurement of force :-**

| **Absolute units** | **Other units** |
|---|---|
| (i) N (M.K.S) | kg-wt or kg-f (kg-force) |
| (ii) dyne (C.G.S) | g-wt or g-f |

**Relation between above units :-**

1 kg-wt = 9.8 N
1 g-wt = 980 dyne
1 N = $10^5$ dyne

## 3. INERTIA

Inertia is the property of a body due to which it opposes any change in its state. Mass of a body is the measure of its inertia of translational motion. It is difficult to change the state of rest or uniform motion of a body of heavier mass and vice-versa.

- Mass of a body is quantitative or numerical measure of a body's inertia.
- Larger the inertia of a body, more will be its mass.

**Inertia of rest :** It is the inability of a body to change its state of rest by itself.

**Examples :**

- When we shake a branch of a mango tree, the mangoes fall down.
- When a bus or train starts suddenly, the passengers sitting inside tends to fall backwards.
- When a horse starts off suddenly, its rider falls backwards.
- A coin is placed on cardboard and this cardboard is placed over a tumbler such that coin is above the mouth of tumbler. Now if the cardboard is removed with a sudden jerk, then the .coin falls into the tumbler.
- The dust particles in a blanket fall off when it is beaten with a stick.

**Inertia of motion :** It is the inability of a body to change its state of uniform motion by itself.

**Examples :**

- When a bus or train stops suddenly, the passengers sitting inside lean forward.
- A person who jumps out of a moving train may fall in the forward.
- A bowler runs with the ball before throwing it, so that his speed of running gets added to the speed of the ball at the time of throw.
- An athlete run through a certain distance before taking a long jump because the velocity acquired during the running gets added to the velocity of athlete at the time of jump and hence he can jump over a longer distance.
- A ball is thrown in the upward direction by a passenger sitting inside a moving train. The ball will fall :-
  - back to the hands of the passenger, if the train is moving with constant velocity.
  - ahead of the passenger, if the train is retarding (slowing down)
  - behind of the passenger, if the train is accelerating (speeding up)

**Inertia of direction :** It is the inability of a body to change its direction of motion by itself

**Examples :**

- When a straight running car turns sharply, the person sitting inside feels a force radially outwards.
- Rotating wheels of vehicle throw out mud, mudguard fitted over the wheels prevent this mud from spreading.
- When a knife is pressed against a grinding stone, the sparks produced move in the tangential direction.

## 4. MOMENTUM

The total quantity of motion possessed by a moving body is known as the momentum of the body. It is the product of the mass and velocity of a body. It is a vector quantity whose direction is along the instantaneous velocity.

momentum $\vec{p} = m\vec{v}$

SI Unit : kg-m/s

Dimension: $[M\,LT^{-1}]$

## 5. NEWTON'S SECOND LAW OF MOTION

According to Newton, the rate of change of momentum of any system is directly proportional to the applied external force and this change in momentum takes place in the direction of the applied force.

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) \quad \text{or} \quad \vec{F} = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt} \quad \text{(general form)}$$

If m = constant then $\frac{dm}{dt} = 0$

$$\Rightarrow \boxed{\vec{F} = m\frac{d\vec{v}}{dt} = m\vec{a}}$$

If $\vec{v}$ = constant then $\frac{d\vec{v}}{dt} = 0$

$$\Rightarrow \boxed{\vec{F} = \vec{v}\frac{dm}{dt}}$$ (e.g. conveyor belt, rocket propulsion)

## Illustrations

**Illustration 1.**

A force $\vec{F} = (6\hat{i} - 8\hat{j} + 10\hat{k})$ N produces an acceleration of $\sqrt{2}$ m/s$^2$ in a body. Calculate the mass of the body

**Solution:**

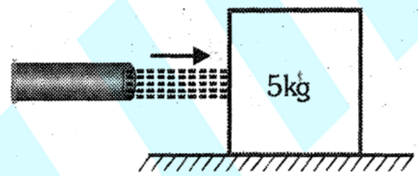
From Newton's II$^{nd}$ law $|\vec{F}| = |m\vec{a}| = ma$

$\Rightarrow$ Acceleration $a = \frac{|\vec{F}|}{m}$ $\qquad \Rightarrow m = \frac{|\vec{F}|}{a} = \frac{\sqrt{6^2 + 8^2 + 10^2}}{\sqrt{2}} = 10$ kg

**Illustration 2.**

A 5kg block is resting on a frictionless plane. It is struck by a jet, releasing water at the rate of 3 kg/s emerging with a speed of 4 m/s. Calculate the initial acceleration of the block.

**Solution:**



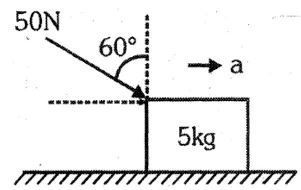

Force exerted on block $F = v\frac{dm}{dt} = 4 \times 3 = 12$ N

so acceleration of the block $a = \frac{f}{m} = \frac{12}{5} = 2.4$ m/s$^2$

**Illustration 3.**

A force of 50 N acts in the direction as shown in figure. The block is of mass 5 kg, resting on a smooth horizontal surface. Find out the acceleration of the block.

**Solution:**




Horizontal component of the force = 50 sin 60° = $\frac{50\sqrt{3}}{2}$

Acceleration of the block, a

$= \frac{\text{component of force in the direction of acceleration}}{\text{mass}}$

$= \frac{50\sqrt{3}}{2} \times \frac{1}{5} = 5\sqrt{3}$ m/s$^2$

**6. IMPULSE**

When a large for is applied on a body for a very short interval of time, then the product of force and time interval is known as impulse.

$$d\vec{I} = \vec{F}dt = d\vec{p} \qquad \left[\because \frac{d\vec{p}}{dt} = \vec{F}\right]\text{-}$$

Unit of impulse = N – s or kg-m/s.

Dimensions of impulse = [F] [t] = [m] [a] [t] = $[M^1L^1T^{-2}T^1] = [M^1L^1T^{-1}]$

**Case-I :** If this force is working from time $t_1$ to $t_2$, then integrating the above equation, we get-

$$\vec{I} = \int_{t_1}^{t_2} \vec{F}dt = \int_{\vec{p}_1}^{\vec{p}_2} d\vec{p} = \vec{p}_2 - \vec{p}_1$$

$\Rightarrow$ Impulse = Change in momentum

**Case-II :** If a constant or average force acts on body, then :-

$$\vec{I} = \int_{t_1}^{t_2} \vec{F}_{avg}\, dt = \int_{\vec{p}_1}^{\vec{p}_2} d\vec{p} \Rightarrow \vec{I} = \vec{F}_{avg}\int_{t_1}^{t_2} dt = \int_{\vec{p}_1}^{\vec{p}_2} d\vec{p}$$

$$\vec{I} = \vec{F}_{avg}[t]_{t_1}^{t_2} = [\vec{p}]_{\vec{p}_1}^{\vec{p}_2} \Rightarrow \vec{I} = \vec{F}_{avg}(t_2 - t_1) = (\vec{p}_2 - \vec{p}_1)$$

$$\boxed{\vec{I} = \vec{F}_{avg}\,\Delta t = \Delta\vec{p}}$$

### 6.1 Impulse-Momentum Theorem

The impulse of force is equal to the change in momentum. This relation is known as impulse-momentum theorem.

### 6.2 Law of Conservation of linear momentum

If net external force on a system is zero then the linear momentum of the system remains constant. According to Newton's II$^{nd}$ Law.

$$\vec{F}_{ext} = \frac{d\vec{p}}{dt}$$

If $\vec{F}_{ext} = \vec{0}$ then

$$\frac{d\vec{p}}{dt} = \vec{0} \Rightarrow \boxed{\vec{p} = \text{constant}} \text{ or } \boxed{\vec{p}_{initial} = \vec{p}_{final}}$$

$\Rightarrow \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \vec{p}_4 + ..... + \vec{p}_n = \text{constant}$ (Conservation of linear momentum)

$\Rightarrow \Delta(\vec{p}_1 + \vec{p}_2 + ..... + \vec{p}_n) = 0$

$\Rightarrow \Delta\vec{p}_1 + \Delta\vec{p}_2 + ..... + \Delta\vec{p}_n = 0$

For two-particles system $\vec{p}_1 + \vec{p}_2 = \text{constant}$

$\Delta\vec{p}_1 + \Delta\vec{p}_2 = 0$

$\boxed{\Delta\vec{p}_1 + \Delta\vec{p}_2}$ (change in momentum of I$^{st}$ particle = –change in momentum of II$^{nd}$ particle)

## GOLDEN KEY POINTS

**Important points about Newton's Second Law of Motion**

- Newton's first law of motion defines force and second law of motion measures force. It gives the units, dimensions and magnitude of the force.
  Unit of force = (unit of mass) × (unit of acceleration) = 1kg × 1 m/s$^2$ = 1N
  1N = 1 kg-m/s$^2$
  1 dyne = 1 g-cm/s$^2$
  Dimensions of force :- = [m] [a] = $[M^1][L^1T^{-2}] = [M^1L^1T^{-2}]$
- If a particle moves uniformly, means velocity = constant

  $$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d(\text{constant})}{dt} = \vec{0} \quad \text{so} \quad \vec{F} = m\vec{a} = m \times \vec{0} = \vec{0}$$

  It means that, in the absence of external force, a particle moves uniformly. This is Newton's first law of motion. it means that, we can derive mathematically Newton's first law of motion with the help of Newton's second law of motion,

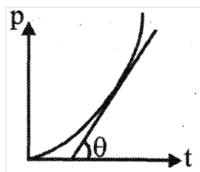
- Accelerated motion is always due to an external force.
- Law of Conservation of Linear Momentum (COLM) is applicable in the direction in which external force is zero, i.e.

  If $F_x = 0 \Rightarrow \frac{dp_x}{dt} = 0$, then $p_x$ = constant

  If $F_y = 0 \Rightarrow \frac{dp_y}{dt} = 0$, then $p_y$ = constant

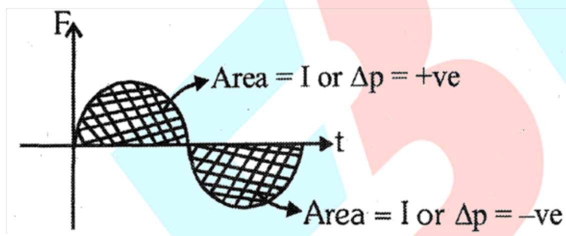
  If $F_z = 0 \Rightarrow \frac{dp_z}{dt} = 0$, then $p_z$ = constant

- The slope of momentum-time graph is equal to the force on the particle
  e.g.




Here, $F = \frac{dp}{dt} = \text{slope} = \tan\theta$

- Area under the force-time graph represents impulse or change in momentum.
  e.g.




Here, $I \text{ or } \Delta p = \int F dt = \text{Area under F-t graph}$

**Note :-** Since $F_{avg} = \frac{\Delta p}{\Delta t}$

therefore, for a certain momentum change if the time interval is increased, then the average force exerted on body will decrease.

**Examples**

(i) A cricketer lowers his hands while catching a ball so as to increase the time interval of momentum change consequently the average reaction force on his hands decreases. So he can save himself from getting hurt.

(ii) Shockers are provided in vehicles to avoid jerks.

(iii) Buffers are provided in boggies of train to avoid jerks.

(iv) A person jumping on a hard cement floor receives more injurie than a person jumping on muddy or sandy road.

## Illustrations

**Illustration 4.**

A hammer of mass 1 kg moving with a speed of 6 m/s strikes a wall and comes to rest in 0.1 s. Calculate.

(a) Impulse of the force

(b) average retarding force that stop the hammer.

(c) average retardation of the hammer

**Solution:**

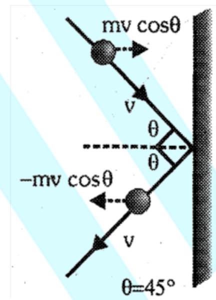
(a) Impulse = F × Δt = m (v – u) = 1 (0 – 6) = 6 N–s

(b) Average retarding force that stops the hammer $F = \dfrac{\text{Impulse}}{\text{time}} = \dfrac{6}{0.1} = 60$ N

(c) Average retardation $a = \dfrac{F}{m} = \dfrac{60}{1} = 60\ \text{m/s}^2$

**Illustration 5.**

A ball of 0.20 kg hits a wall with a velocity of 25 m/s at an angle of 45º. If the ball rebounds at 90º to the direction of incidence, calculate the magnitude of change in momentum of the ball.




**Solution:**

Change in momentum = (–mv cos 45º) – (mv cos 45º) = –2mv cos 45º

$|\Delta\vec{p}| = 2mv\cos 45^\circ = 2 \times 0.2 \times 25 \times \dfrac{1}{\sqrt{2}} = 5\sqrt{2}$ N–s

**Illustration 6.**

A cricket ball of mass 150 g is moving with a velocity of 12 m/s and is hit by a bat so that the ball gets turned back With a velocity of 20 m/s. If the duration of contact between the ball and bat is 0.01 s, find the impulse and the average force exerted on the half by the bat.

**Solution:**

According to given problem change in momentum of the ball

$\Delta p == P_f - p_i = m(v - u) = 150 \times 10^{-3}\,[20 - (-12)]$

So by impulse-momentum theorem, Impulse I = Δp = 4.8 N–s

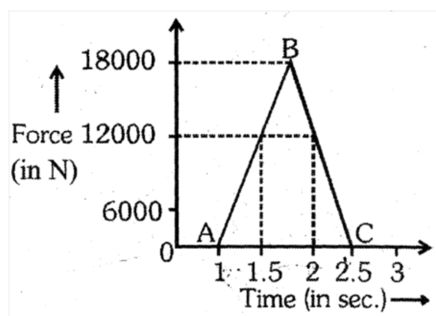
And by time average definition of force in case of impulse $F_{av} = \dfrac{I}{\Delta t} = \dfrac{\Delta p}{\Delta t} = \dfrac{4.80}{0.01} = 480$ N

**Illustration 7.**

Figure shows an estimated force-time graph for a base ball struck by a bat. From this curve, determine

(a) Impulse delivered to the ball

(b) Average force exerted on the ball.




**Solution:**

(a) Impulse = Area under F-t curve

$= \text{Area of } \Delta ABC = \dfrac{1}{2}\ 18000 \times (2.5 - 1)$

$= 1.35 \times 10^4$ kg-m/s

(b) Average force $= \dfrac{\text{Impulse}}{\text{Time}} = \dfrac{135 \times 10^4}{(25 - 1)} = 9000$ N

## BEGINNER'S BOX - 1

**1.** A force of 72 dynes is inclined at an angle of 60° to the horizontal, find the acceleration in a mass of 9 g which moves under the effect of this force in the horizontal direction.

**2.** A constant retarding force of 50N is applied to a body of mass 20 kg moving initially with a speed of 15 m/s. What time does the body take to stop ?

**3.** A constant force acting on a body of mass 3 kg changes its speed from 2 m/s to 3.5 m/s in 25 s. In the direction of the motion of the body what is the magnitude and direction of the force.

**4.** A body of mass 5kg is acted upon by two perpendicular force of 8N and 6N, find the magnitude and direction of the acceleration.

**5.** A boll of mass 1 kg dropped from 9.8 m height, strikes the ground and rebounds to a height of 4.9 m. If the time of contact between ball and ground is 0.1 s, then find impulse and average force acting on ball.

**6.** A machine gun fires a bullet of mass 50 gm with velocity 1000 m/s. If average force acting on gun is 200 N then find out the maximum number of bullets fired per minute.

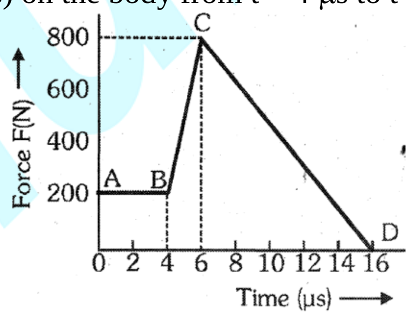
**7.** A machine gun has a mass of 20 kg. It fires 35 g bullets at the rate of 400 bullets per minute with a speed of 400 m/s. What average force must be applied to the gun to keep it in position?

**8.** A force of 10N acts on a body for 3 μs. If mass of the body is 5 g, calculate the impulse and the change in velocity.

**9.** A body of mass 0.25 kg moving with velocity 12m/s is stopped by applying a force of 0.6 N. Calculate the time taken to stop the body. Also calculate the impulse of this force.

**10.** A batsman deflects a ball by an angle of 90° without changing its initial speed, which is equal to 54 km/hr. What is the impulse imparted to the ball? (Mass of the ball is 0.15 kg)

**11.** The magnitude of the force (in newton) acting on a body varies with time t (in microsecond) as shown in fig. AB, BC and CD are straight line segments. Find the magnitude of the total impulse of the force (in N–s) on the body from t = 4 μs to t = 16 μs.




**7. ROCKET PROPULSION**

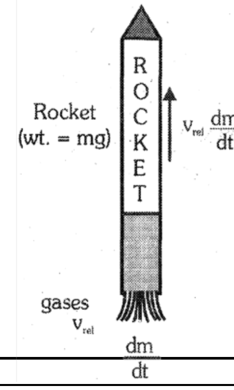
**Case-1 :** If rocket is accelerating upwards, then-

Net upwards force on rocket = ma

$$v_{rel}\frac{dm}{dt} - mg = ma$$

Where $v_{rel}$ is the relative velocity of the ejected mass w.r.t rocket

**Case-II :** If rocket is moving with constant velocity, then a = 0



$$v_{rel}\frac{dm}{dt} = mg$$

**Illustration 8.**

A 600 kg rocket is set for a vertical firing. If the exhaust speed of gases is 1000 m/s, then calculate the mass of gas ejected per second to supply the thrust needed to overcome the weight of rocket.

**Solution:**

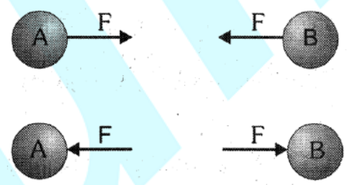
Force required to overcome the weight of rocket F = mg and thrust needed = $v_{rel}\frac{dm}{dt}$

so $v_{rel}\frac{dm}{dt} = mg \Rightarrow \frac{dm}{dt} = \frac{mg}{v_{rel}} = \frac{600\times9.8}{1000} = 5.88$ kg/s

**8. NEWTON'S THIRD LAW OF MOTION**

According to Newton's third law, to every action, there is always an equal (in magnitude) and opposite (in direction) reaction. This law is also known as action-reaction law. Here, $\vec{F}_{12}$ (force on first body due to second body) is equal in magnitude and opposite in direction to $\vec{F}_{21}$ (force on second body due to first body).

$$\boxed{\vec{F}_{12} = -\vec{F}_{21}}$$



The forces between two objects A and B are equal and opposite, whether they are attractive or repulsive.

**Important points about Newton's Ill law**

(i) We cannot produce a single isolated force in nature. Forces are always produced in action reaction pair.

(ii) There is no time gap in between action and reaction. So we cannot say that action is the cause and reaction is its effect. Any one force can be action and the other reaction.

(iii) Action - reaction law is applicable on both the states either at rest or in motion.

(iv) Action and reaction is also applicable between bodies which are not in physical contact.

(v) Action - reaction law is applicable to all the interaction forces eg. gravitational force, electrostatic force, electromagnetic force, tension, friction, viscous force, etc.

(vi) Action and reaction never cancel each other because they act on two different bodies.

**Examples:** Walking, Swimming, Recoiling of gun when a bullet is fired from it, Rocket propulsion.

## GOLDEN KEY POINTS

- **First law :** If no net force acts on a particle, then it is possible to select a set of reference frames, called inertial reference frames, observed from which the particle moves without any change in velocity.
- **Second law :** Observed from an inertial reference frame, the net force on a particle is equal to the rate of change of its linear momentum w.r.t. time: $\frac{d(mv)}{dt}$

- **Third law :** Whenever a particle A exerts a force on another particle B, simultaneously B exerts a force on A with the same magnitude in the opposite direction.
- **Newton's III law can be derived from principle of conservation of linear momentum**.
  If two particles of masses $m_1$ and $m_2$ are moving under the action of their mutually interacting forces with each other, such that no external force acts on the system, then
  ∴ momentum of system remains constant.

  i.e. $\Delta\vec{p}_1 + \Delta\vec{p}_2 = 0 \Rightarrow \Delta\vec{p}_1 = -\Delta\vec{p}_2 \Rightarrow \frac{\Delta\vec{p}_1}{\Delta t} = -\frac{\Delta\vec{p}_2}{\Delta t}$

  $\boxed{\vec{F}_{12} = -\vec{F}_{21}}$

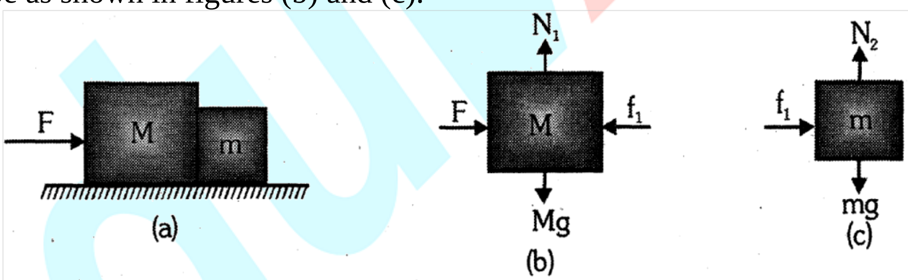
  Force on 1st due to 2nd = –Force on 2nd due to 1st
- **Inertial and Gravitational mass**

  (1) The ratio of force applied on a particle to its acceleration is known as inertial mass $m_i = \frac{F}{a}$

  (2) The ratio of gravitational force to gravitational acceleration is known as gravitational mass $m_g = \frac{F}{g}$. It is experimentally proved that both masses are equal i.e. $m_i = m_g$

## 9. FREE BODY DIAGRAM

A diagram showing all external forces acting on an object is called "Free Body Diagram" (F.B.D.)
In a specific problem, we are required to choose a body first and then we access the different forces acting on it, and all the forces are drawn on the body, The resulting diagram is known as free body diagram (FBD).
For example, if two bodies of masses m and M resting on a smooth floor are in contact and a force F is applied on M from the left as shown in figure (a), the free body diagrams of M and m will be as shown in figures (b) and (c).




**Important Point :**
Two forces in Newton's third law never occur in the same free-body diagram. This is because a free-body diagram shows forces acting on a single object, and the action-reaction pair in Newton's third law always act on different objects.

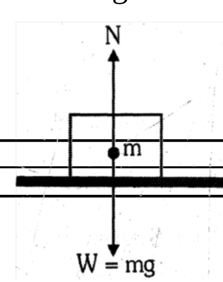
## 10. NORMAL REACTION

When a stationary body is placed on a surface then, that surface exerts a contact force on that body which is **perpendicular to the surface and towards the body.** This force is known as Normal Reaction

e.g. A block of mass "m" is placed on a horizontal table. Then the forces exerted on it are:

(1) Downward gravitational force of attraction on body due to earth, means weight of the block (=mg).

(2) Upward normal reaction exerted by the surface of table on the block.
Since the body is in equilibrium, therefore net force on it is zero.



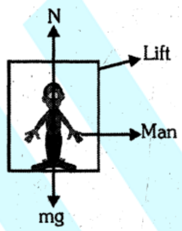
$\therefore \quad \vec{N} + \vec{W} = \vec{0}$

$\vec{N} = -\vec{W}$

$N = W \qquad [\because |\vec{N}| = |-\vec{W}|]$

$\therefore \quad N = mg \qquad [\because W = mg]$

### 10.1 Effective or Apparent weight of a man in lift

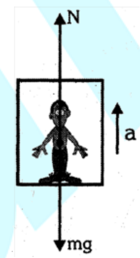



**Case-1 :** If the lift is at rest or moving uniformly (a = 0), then

$N = mg$

$$\boxed{\begin{aligned} W_{app} &= N \\ W_{actual} &= mg \end{aligned}}$$

So, $W_{app} = W_{actual}$




**Case-II :** If the lift is accelerating upwards, then -

Net upward force on man = ma

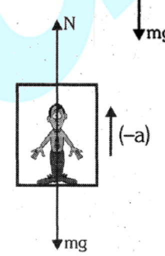
$N - mg = ma$

$N = mg + ma$

$\therefore \quad N = m(g + a)$

$W_{app}$ or $N = m(g + a)$

So, $W_{app} > W_{actual}$



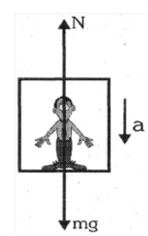

**Case-III :** If the lift is retarding upwards, then N

$N - mg = m(-a)$

$N = mg - ma$

$\therefore \quad W_{app}$ or $N = m(g - a)$

So, $W_{app} < W_{actual}$



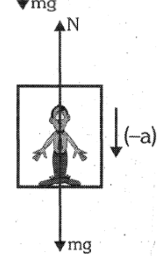

**Case-IV :** If the lift is accelerated downwards, then -

$Mg - N = ma$

$N = mg - ma$

$w_{app}$ or $N = m(g - a)$

So, $W_{app} < W_{actual}$




**Case-V :** If the lift is retarding downwards, then -

$mg - N = m(-a)$

$mg - N = -ma$

$N = ma + mg$

$W_{app}$ or $N = m(g + a)$

So, $W_{app} > W_{actual}$

**Two Special Cases of downward acceleration (IV Case)**

**<u>I Special Case</u> :-** If the lift is falling freely, it implies that its acceleration is equal to the acceleration due to gravity. i.e. a = g, then-

$W_{app.} = m(g - a) = m(g - g) = m \times 0 = 0$

$\therefore \quad w_{app.} = 0$

It means the man will feel weightless. This condition is known as Condition of Weightlessness.

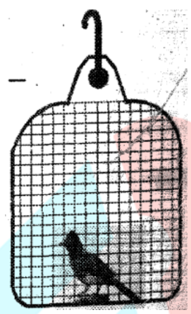
The apparent weight of any freely falling body is zero.

**II Special Case :-** If the lift is accelerating downwards with an acceleration which is greater than 'g', then the man will move up with respect to the lift and stick to the ceiling.

**10.2 Bird-Cage Problem :**

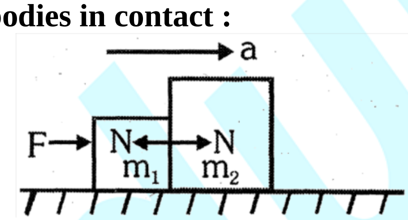
**I.** A bird is sitting on the base in an air tight cage. Now, if the bird starts flying, then-
(i) Weight of system will not change, if the bird flies with constant velocity.
(ii) Weight of system will increase, if the bird flies with upward acceleration.
(iii) Weight of system will decrease, if the bird flies with downward acceleration.

**II.** A bird is sitting on the base in a. wire cage. Now if the birds flies upward its weight will in all the cases.

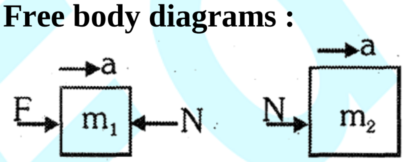


**11. MOTION OF BODIES IN CONTACT (CONTACT FORCE)**

**Two bodies in contact :**




**Free body diagrams :**




$F - N = m_1 a$ .....(1)

$N = m_2 a$ .....(2)

On adding the above equations

$F = m_1a + m_2a$

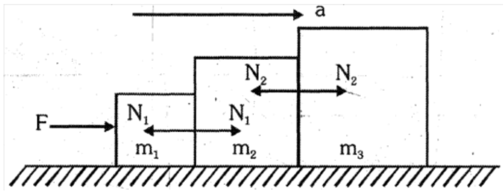
$F = a(m_1 + m_2)$

$\Rightarrow$ $\boxed{a = \dfrac{F}{m_1 + m_2}}$ .....(3) OR $\boxed{a = \dfrac{F_{net}}{m_{total}}}$

Putting the value of 'a' from equation (3) in (2), we get

$\boxed{N = \dfrac{m_2 F}{m_1 + m_2}}$ (here N = contact force)

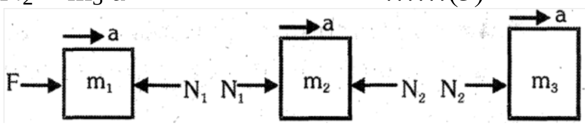
**Three bodies in contact :**




$F - N_1 = m_1 a$ ......(1)

$N_1 - N_2 = m_2 a$ ......(2)

$N_2 = m_3 a$ ......(3)




On adding the above equations

$F = m_1a + m_2a + m_3a$

$F = a (m_1 + m_2 + m_3)$

$\Rightarrow \boxed{a = \dfrac{F}{m_1 + m_2 + m_3}}$ ......(4) or $\boxed{a = \dfrac{F_{Net}}{m_{total}}}$

Putting the value of 'a' from equation (4) in (3), we get

$$\boxed{N_2 = \frac{Fm_3}{m_1 + m_2 + m_3}}$$

Putting the value of 'a' from (4) in (1);

$F - m_1a = N$

$N_1 = F - \dfrac{Fm_1}{m_1 + m_2 + m_3} \Rightarrow \boxed{N_1 = \dfrac{(m_2 + m_3)F}{m_1 + m_2 + m_3}}$

## Illustrations

**Illustration 9.**

Consider a box of mass 10 kg resting on a horizontal table and acceleration due to gravity to be 10 m/s$^2$.

(a) Draw the free body diagram of the box. J

(b) Find value of the force exerted by the table on the box.

(c) Find value of the force exerted by the box on the table.

(d) Does force exerted by table on the box and weight of the box form third law action-reaction pair?

**Solution:**

(a) $N_1$ : Force exerted by table on the box.

(b) The block is in equilibrium. $\sum \vec{F} = \vec{0} \Rightarrow mg - N_1 = 0 \Rightarrow N_1 = 100$ N

(c) $N_1 = 100$ N : Because force by table on the box and force by box on table make Newton's third law pair.

(d) No

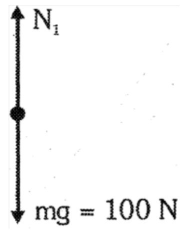
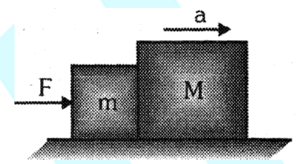


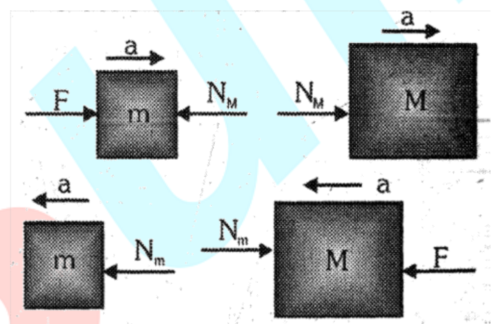

**Illustration 10.**

Two blocks of masses m = 2 kg and M = 5 kg are in contact on a frictionless table. A horizontal force F (= 35 N) is applied to m. Find the force of contact between the blocks. Will the force of contact remain same if F is applied to M ?




**Solution:**

As the blocks are rigid both will move with same acceleration under the action of a force F




$a = \dfrac{F}{m+M} = \dfrac{35}{2+5} = 5 \text{ m/s}^2$

Force of contact $N_M = Ma = 5 \times 5 = 25$ N

If the force is applied to M then its action on m will be

$N_m = ma = 2 \times 5 = 10$ N.

**Note :-** From this problem it is clear that acceleration does not depend on the fact that whether the force is applied to m or M, but force of contact does.

**Illustration 11.**

A spring weighing machine inside a stationary lift reads 50 kg when a man stands on it. What would happen to the scale reading if the lift is moving upward with (i) constant velocity, and (ii) constant acceleration ?

**Solution:**

(i) In the case of constant velocity a = 0

Therefore, N = mg so $W_{app} = W_{act}$

hence reading = 50 kg .

(ii) In case with upward acceleration

$N - mg = ma$

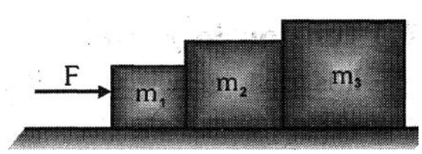
So $N = mg + ma = m(g + a)$

So $W_{app.} > W_{act}$

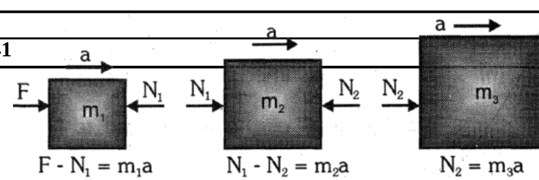
Hence scale reading will be $= \dfrac{50(g+a)}{g} = 50\left(1+\dfrac{a}{g}\right)$ kg.

**Illustration 12.**

Three blocks of masses $m_1$ = 1 kg, $m_2$ = 1.5 kg and $m_3$ = 2 kg are in contact with each other on a frictionless surface as shown in fig. Find the (a) horizontal force F needed to push the blocks as a single unit with an acceleration of 4 m/s$^2$ (b)





resultant force on each block and (c) magnitude of contact forces between the blocks.

**Solution:**

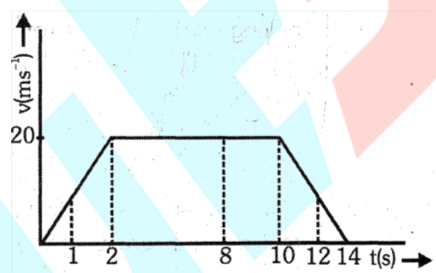
(a) $F = (m_1 + m_2 + m_3)\, a$
$= (1 + 1.5 + 2) \times 4$
$= 4.5 \times 4 = 18$ N

(b) For $m_1$
$F - N_1 = m_1 a = 1 \times 4$
$\Rightarrow F - N_1 = 4$ N ……(i)
for $m_2$,
$N_1 - N_2 = m_2\, a = 1.5 \times 4 = 6$
$\Rightarrow N_1 - N_2 = 6$N ……(ii)
for $m_3$,
$N_2 = m_3 a = 2 \times 4$
$\Rightarrow N_2 = 8$N ……(iii)

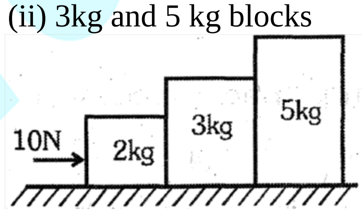
(c) Contact force between $m_2$ and $m_3$ is $N_2 = 8$ N
and contact force between $m_1$ and $m_2$ is $N_1 = N_2 + 6 = 8 + 6 = 14$N.

**BEGINNER'S BOX - 2**

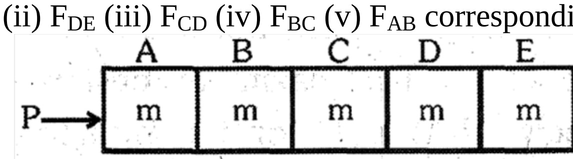
**1.** A person of mass M kg is standing on a lift. If the lift moves t vertically upwards according to given v-t graph then find out the weight of man· at the following instants : ($g = 10$ m/s$^2$)
(i) t = 1 second
(ii) t = 8 seconds
(iii) t = 12 seconds




**2.** Find the acceleration of the system and the contact force between
(i) 2kg and 3 kg blocks (ii) 3kg and 5 kg blocks



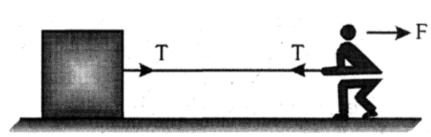

**3.** Calculate : (i) $a_{system}$ (ii) $F_{DE}$ (iii) $F_{CD}$ (iv) $F_{BC}$ (v) $F_{AB}$ corresponding to the following diagram.




## 12. SYSTEM OF MASSES TIED BY STRINGS

### 12.1 Tension in a String

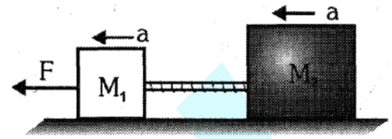
It is the intermolecular forces between the molecules of a string, which become active when the string is stretched.



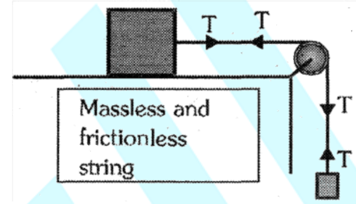
Important points about the tension in a string :

(a) Force of tension act on a body in the direction away from the point of contact or tied end of the string.

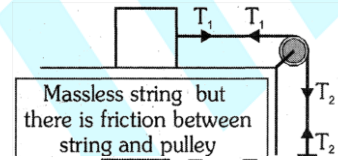
(b) String is assumed to be inextensible so that the magnitude of accelerations of the blocks tied to the strings are always same.



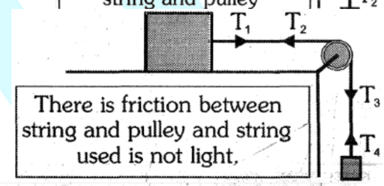

(c) (i) If the string is massless and frictionless, tension throughout the string remains same.



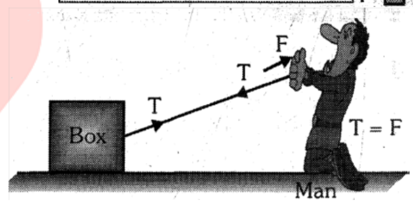

(ii) If the string is massless but not frictionless, at every contact tension changes.



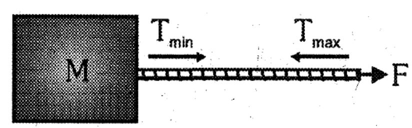

(iii) If the string is not light, tension at each point of the string will be different depending on the acceleration.




(d) If a force is directly applied to a string, say a man is pulling a string from the other end with some force, then tension will be equal to the applied force irrespective of the motion of the pulling agent, irrespective of whether the box moves or not, man moves or not.




(e) String is assumed to be massless unless stated, hence tension in it remains the same every where and equal to the applied force. However, if a string has a mass, tension at different points will be different being maximum (= applied force) at the end through which force is applied and minimum at the other end connected to a body.

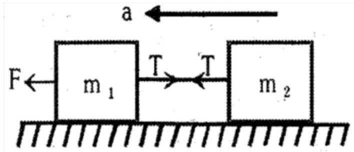



(f) In order to produce tension in a string two equal and opposite stretching forces must be applied. The tension thus produced is equal in magnitude to either applied force (i.e., T = F) and is directed inwards opposite to F.

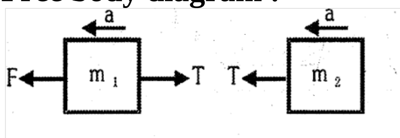

(g) Every string can bear a maximum tension, i.e. if the tension in a string is continuously increased it will break beyond a certain limit. The maximum tension which a string can bear without breaking is called its "breaking strength". It is finite for a string and depends on its material and dimensions.

## 12.2 Motion of Connected Bodies

**Two Connected Bodies :**




**Free body diagram :**




$F - T = m_1 a$

$T = m_2 a$

On adding the above equations

$F = m_1a + m_2a$

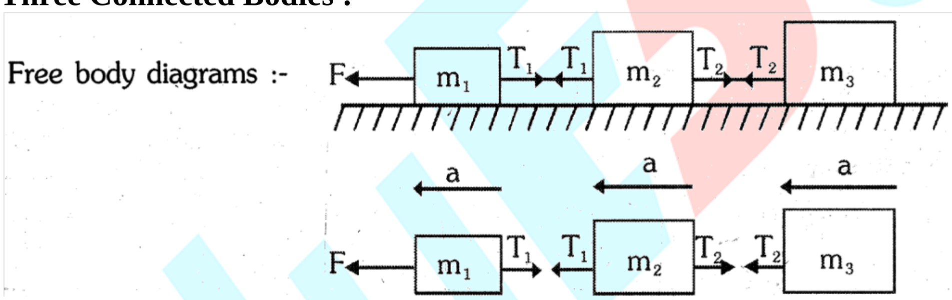
$F = a\,(m_1 + m_2)$

$$a = \frac{F}{m_1 + m_2} \quad \text{.....(3) Or} \quad a = \frac{F_{net}}{m_{total}}$$

Putting the value of 'a' from equation (3) in (2), we get-

$$T = \frac{F}{m_1 + m_2}$$

**Three Connected Bodies :**




$F - T_1 = m_1a$ ......(1)

$T_1 - T_2 = m_2a$ ......(2)

$T_2 = m_3a$ ......(3)

On adding above equations

$F = (m_1 + m_2 + m_3)\,a$

$$\therefore \quad a = \frac{F}{m_1 + m_2 + m_3} \quad \text{or} \quad a = \frac{F_{net}}{m_{total}}$$

Put the value of 'a' in equation (1)

$T_1 = F - m_1a$

$$\Rightarrow \quad T_1 = F - \frac{m_1F}{m_1 + m_2 + m_3} = \frac{m_1F + m_2F + m_3F - m_1F}{m_1 + m_2 + m_3}$$

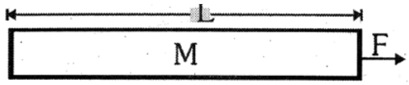
$$T_1 = \frac{(m_2 + m_3 F)}{m_1 + m_2 + m_3}$$

Similarly, putting the value of 'a' in equation (3)

$$T_2 = \frac{m_3F}{m_1 + m_2 + m_3}$$

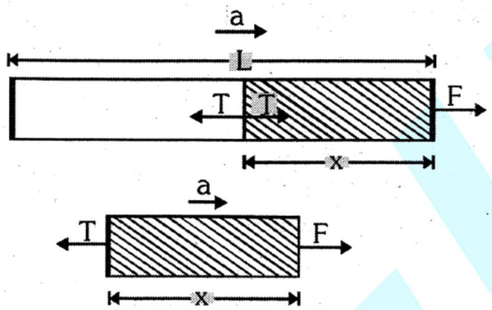
## 12.3 Tension in a Rod

Given




Mass of Rod = M

Length of Rod = L




F − T = ma

⇒ T = F − ma

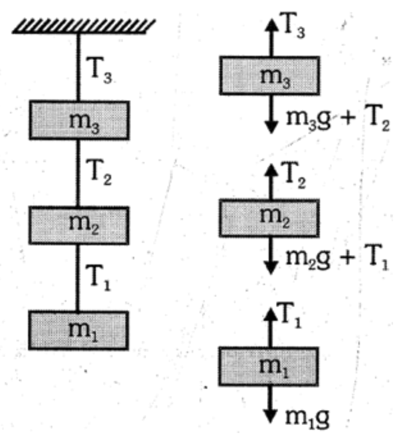
$$\Rightarrow T = F - m\frac{F}{M} \qquad \left(\because a = \frac{F}{M}\right)$$

$\because$ Mass of length 'L' = M $\quad\therefore$ Mass of unit length = $\frac{M}{L}$ $\quad\therefore$ Mass (m) of length 'x' = $\frac{M}{L}x$

Put this value of 'm' in equation (1)

$$T = F - \left(\frac{M}{L}x\right)\frac{F}{M} \Rightarrow \boxed{T = F\left(1 - \frac{x}{L}\right)}$$

## 12.4 Bodies Hanging Vertically




Since all the bodies are in equilibrium, therefore net force on each body is zero

$T_1 = m_1g$ ....(1)

$T_2 = T_1 + m_2g$

$\Rightarrow T_2 = (m_1 + m_2)g$ ....(2)

$T_3 = T_2 + m_3g$

$\Rightarrow T_3 = (m_1 + m_2 + m_3)g$

## 12.5 Bodies Accelerating Vertically Upwards

For $m_1$ $\quad T_1 - m_1g = m_1a$

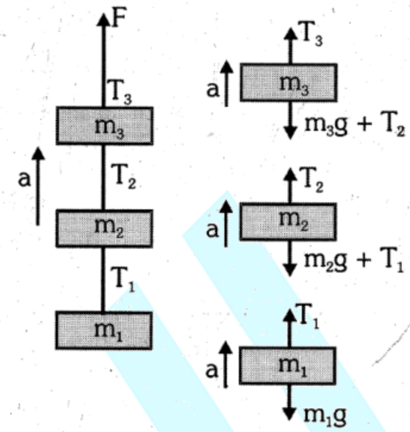
$\Rightarrow \quad T_1 = m_1(g+a)$ ....(1)

For $m_2$ $\quad T_2 - m_2g - T_1 = m_2a$

$\Rightarrow \quad T_2 = m_2a + m_2g + T_1$

$\Rightarrow \quad T_2 = m_2(g+a) + m_1(g+a)$ (from equation 1)

$\Rightarrow \quad T_2 = (m_1 + m_2)(g+a)$ ....(2)



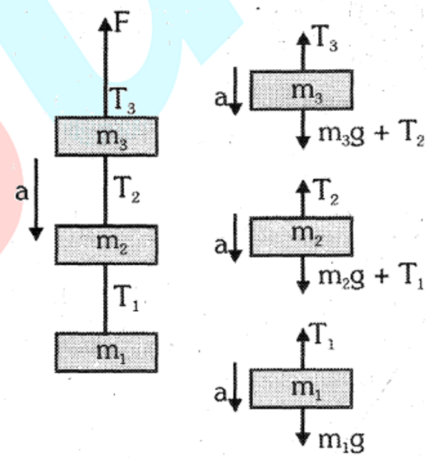

For $m_3$ $\quad F = T_3$ and $T_3 - m_3g - T_2 = m_3a$

$\Rightarrow \quad T_3 = T_2 + m_3g + m_3a$

$\Rightarrow \quad T_3 = (m_1 + m_2)(g+a) + m_3(g+a)$ (from equation 2)

$\Rightarrow \quad T_3 = (m_1 + m_2 + m_3)(g+a)$ ....(3)

## 12.6 Bodies Accelerating Vertically Downwards




For $m_1$ $\quad m_1g - T_1 = m_1a$

$\Rightarrow \quad T_1 = m_1(g-a)$ ....(1)

For $m_2$ $\quad m_2g + T_1 - T_2 = m_2a$

$\Rightarrow \quad T_2 = m_2g - m_2a + T_1$

$\Rightarrow \quad T_2 = m_2(g-a) + m_1(g-a)$ (from equation 1)

$\Rightarrow \quad T_2 = (m_1 + m_2)(g-a)$ ....(2)

For $m_3$ $\quad F = T_3$ and $m_3g + T_2 - T_3 = m_3a$

$\Rightarrow \quad T_3 = m_3g - m_3a + T_2$

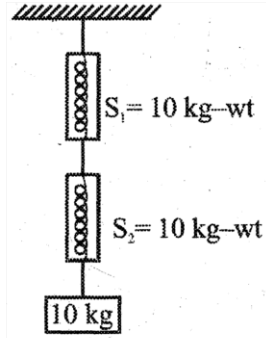
$\Rightarrow \quad T_3 = m_3(g-a) + (m_1 + m_2)(g-a)$ (from equation 2)

$\Rightarrow \quad T_3 = (m_1 + m_2 + m_3)(g-a)$ ......(3)

## GOLDEN KEY POINTS

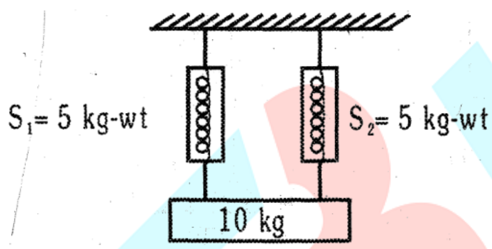
- If several spring balances are connected in series, then the reading of each balance is the same and is equal to the applied load (Note : Spring balances have negligible mass so they are assumed massless)

  e.g.




- If several spring balances are connected in parallel and symmetrically to the load, then the applied load is applied Load equally divided in all the balances; so the reading of each balance will be = $\frac{\text{applied load}}{\text{no. of balances}}$

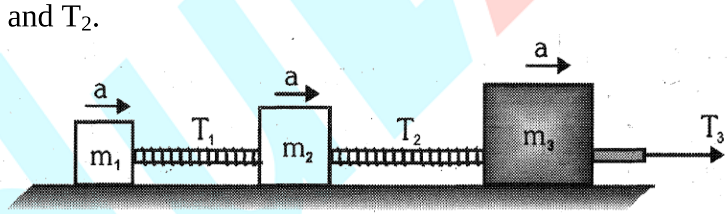
e.g.




## Illustrations

**Illustration 13.**

Three blocks, are connected by strings as shown in the figure below, and are pulled by a force $T_3 = 120$ N. If $m_1 = 5$ kg, $m_2 = 10$ kg and $m_3 = 15$ kg. Calculate the acceleration of the system and tensions $T_1$ and $T_2$.



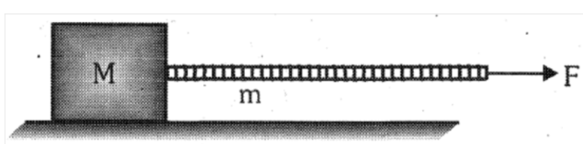

**Solution:**

(i) Acceleration of the system $a = \frac{F}{m_1 + m_2 + m_3} = \frac{120}{5+10+15} = 4 \text{ m/s}^2$

(ii) $T_1 = m_1 a = 5 \times 4 = 20$ N $\quad T_2 = (m_1 + m_2)a = (5 + 10)\,4 = 60$ N

**Illustration 14.**

A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m as shown in fig. A horizontal force F is applied to one end of the rope. Find the (i) Acceleration of the rope and the block (ii) Force that the rope exerts on the block. (iii) Tension in the rope at its mid point.

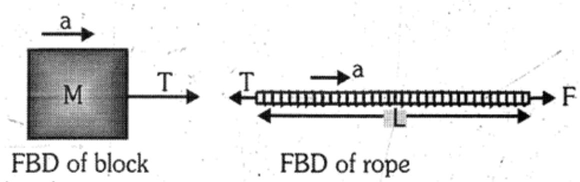



**Solution:**

(i) Acceleration $a = \frac{M.F}{(m + M)}$

(ii) Force exerted by the rope on the block is

$$T = Ma = \frac{M.F}{(m+M)}$$




(iii) $T_1 = \left(\frac{m}{2}+M\right)a = \left(\frac{m+2M}{2}\right)\left(\frac{F}{m+M}\right)$

Tension in rope at midpoint is $T_1 = \frac{(m+2M)F}{2(m+M)}$

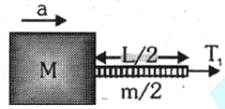
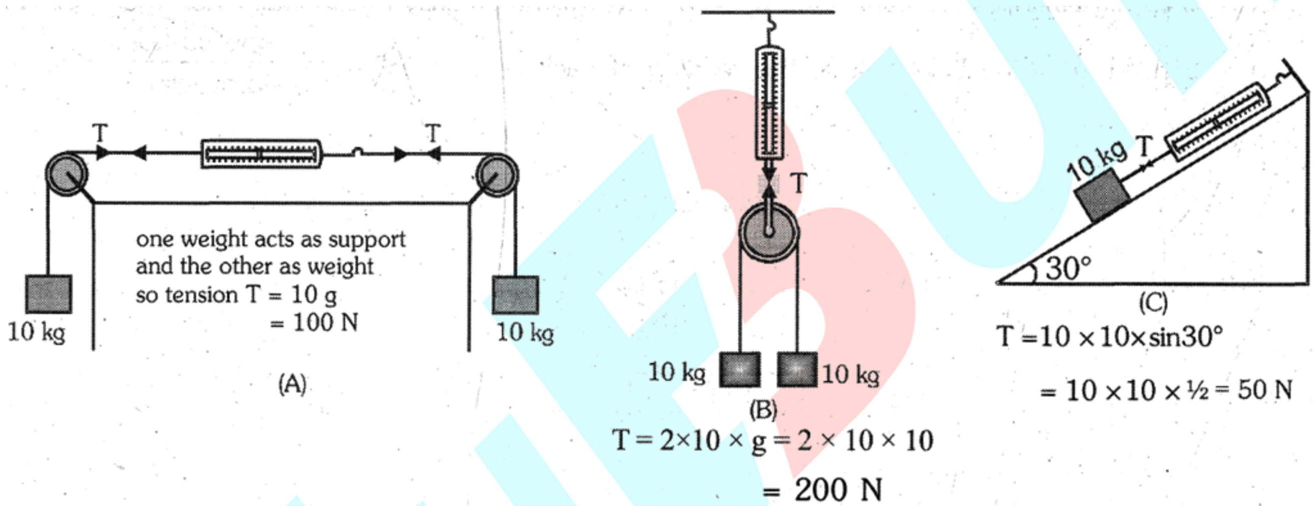



**Illustration 15.**

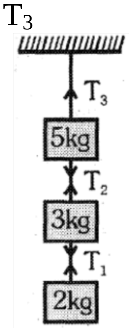
The system shown in fig. is in equilibrium. If the spring balance is calibrated in newtons, what does it record in each case? (g = 10 m/s$^2$)

**Solution:**

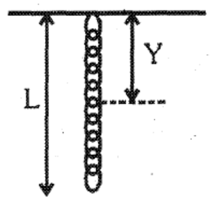



## BEGINNER'S BOX - 3

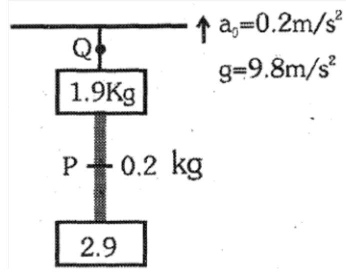
**1.** In the given figure determine $T_1 : T_2 : T_3$



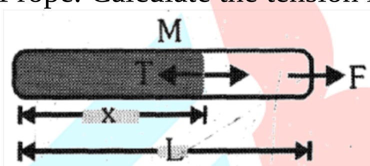

**2.** Find the tension in the chain at a distance Y from the support. Mass of chain is M.



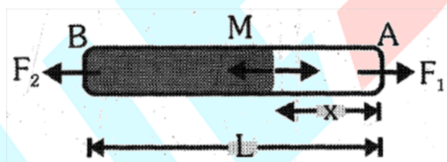

**3.** Calculate $T_P$ and $T_Q$ (P is mid point)




**4.** A uniform rope of mass M and length L is placed on a smooth horizontal surface. A horizontal force F is acting at one end of rope. Calculate the tension in the rope at a distance x as shown.

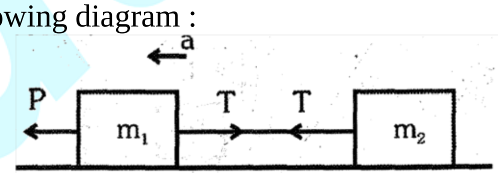



**5.** Calculate the tension T in the rope at a distance x from end A in the following diagram assuming $F_1 > F_2$.

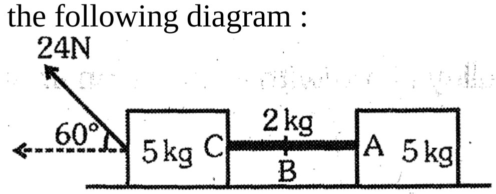



**6.** A man in a lift carrying a 5 kg bag. If the lift moves vertically downwards with g/2 acceleration. Find the tension in the handle of the bag.

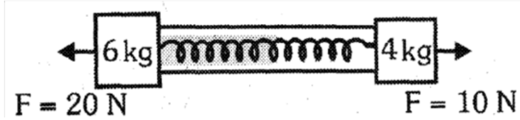
**7.** Calculate T for the following diagram :




**8.** Calculate $T_A$, $T_B$, $T_C$ for the following diagram :




**9.** A dynamometer is attached to two block of masses 6 kg and 4 kg. Forces of 20 N and 10 N are applied on the blocks as shown in figure. Find the dynamometer reading in the steady state.




**13. PULLEY SYSTEM**

- Ideal pulley is considered massless and frictionless.
- Ideal string is massless and inextensible.
- A pulley may change the direction of force in the string but not the tension.

The only function of pulley (which has no friction on its axle to retard rotation) is to change the direction of force through the cord that joins the two blocks.

**Some Cases of Pulley**

**Case I :**

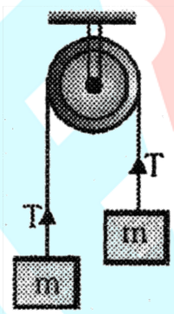
$m_1 = m_2 = m$

Tension in the string $T = mg$

Acceleration 'a' = zero

Reaction at the point of suspension of the pulley or thrust on pulley.

$R = 2T = 2\ mg$




**Case II :**

$m_1 > m_2$ now for mass $m_1$,

$m_1 g - T = m_1 a$ ……(i)

for mass $m_2$

$T - m_2 g = m_2 a$ ……(ii)

adding (i) and (ii)

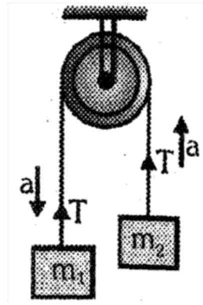
$$a = \frac{(m_1 - m_2)}{(m_1 + m_2)} g \text{ and } T = \frac{2m_1 m_2}{(m_1 + m_2)} g = \frac{2W_1W_2}{W_1 + W_2}$$

$$\text{acceleration} = \frac{\text{net pulling force}}{\text{total mass to be pulled}}$$

$$\text{Tension} = \frac{2 \times \text{Product of masses}}{\text{Sum of masses}} g$$

Reaction at the suspension point of pulley (or thrust on pulley)

$$R = 2T = \frac{4m_1 m_2 g}{(m_1 + m_2)} = \frac{4W_1W_2}{w_1 + W_2}$$



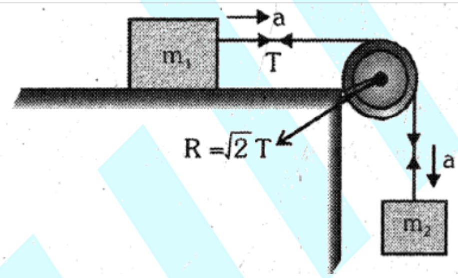

**Case III :**

For mass $m_1$ : $T = m_1 a$

For mass $m_2$ : $m_2 g - T = m_2 a$

acceleration $a = \frac{m_2 g}{(m_1 + m_2)}$ and $T = \frac{m_1 m_2}{(m_1 + m_2)} g$

Reaction at suspension point of pulley $R = \sqrt{2}T$




**Case IV :** $(m_1 > m_2)$

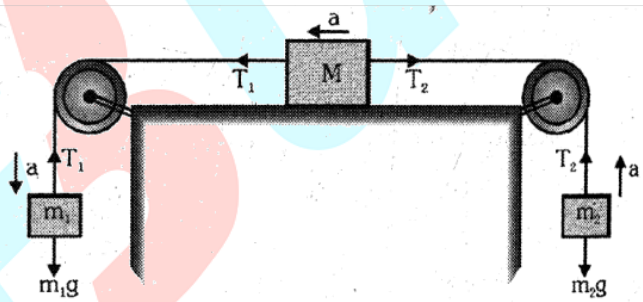
$m_1 g - T_1 = m_1 a$ (i)

$T_2 - m_2 g = m_2 a$ (ii)

$T_1 - T_2 = Ma$ (iii)

adding (i), (ii) and (iii)

$$a = \frac{(m_1 - m_2)}{(m_1 + m_2 + M)} g$$



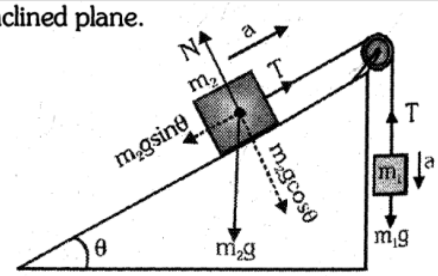

**Case V :** Mass suspended over a pulley along with another on an inclined plane.

For mass $m_1$ : $m_1 g - T = m_1 a$

For mass $m_2$ : $T - m_2 g \sin\theta = m_2 a$

acceleration $a = \frac{(m_1 - m_2 \sin\theta)}{(m_1 + m_2)} g$

$$T = \frac{m_1 m_2 (1 + \sin\theta)}{(m_1 + m_2)} g$$



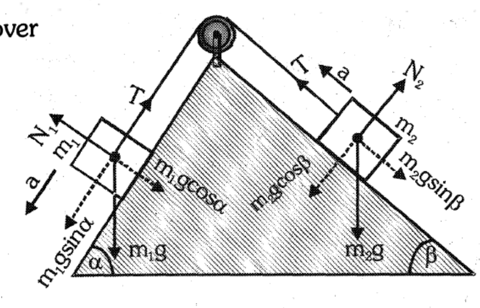

**Case VI :** Masses $m_1$ and $m_2$ are connected by a string passing over a pulley $m_1 \sin\alpha > m_2 \sin\beta$

$m_1 g \sin\alpha - T = m_1 a$ .......(i)

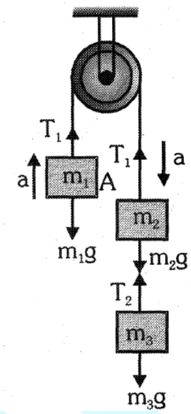
$T - m_2 g \sin\beta = m_2 a$ .......(ii)

After solving equation (i) and (ii)

Acceleration $a = \frac{(m_1 \sin\alpha - m_2 \sin\beta)}{(m_1 + m_2)} g$



Tension $T = \frac{m_1 m_2 (\sin\alpha + \sin\beta)}{(m_1 + m_2)} g$




**Case VII :** For mass $m_1$ : $T_1 - m_1 g = m_1 a$

For mass $m_2$ : $m_2 g + T_2 - T_1 = m_2 a$

For mass $m_3$ : $m_3 g - T_2 = m_3 a$

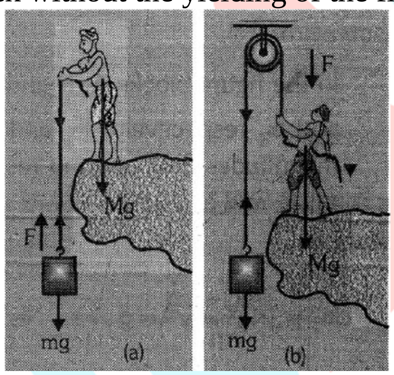
$$\Rightarrow \quad a = \frac{(m_2 + m_3 - m_1)}{(m_1 + m_2 + m_3)} g$$

we can calculate tensions $T_1$ and $T_2$ from above equations

## Illustration

**Illustration 16.**

A block of mass 25 kg is raised in two different ways by a 50 kg man as shown in fig. What is the action in the two cases? If the floor yields to a normal force of 700 N, which mode should the man adopt to lift the block without the yielding of the floor?




**Solution:**

Mass of the block, m = 25 kg ;
mass of the man, M = 50 kg
Force applied to lift the block
$F = mg = 25 \times 9.8 = 245$ N
Weight of the man,
$Mg = 50 \times 9.8 = 490$ N

(a) When the block is raised by the man by applying a force F in the upward direction, reaction being equal and opposite to F will act on the floor in addition to the weight of the man.

$\therefore$ Action on the floor $Mg + F = 490 + 245 = 735$ N

(b) When the block is raised by the man applying force F over the rope (passing over the pulley) in the downward direction, reaction being equal and opposite to F will act on the floor against the weight of the man.

$\therefore$ Action on the floor $Mg - F = 490 - 245 = 245$ N

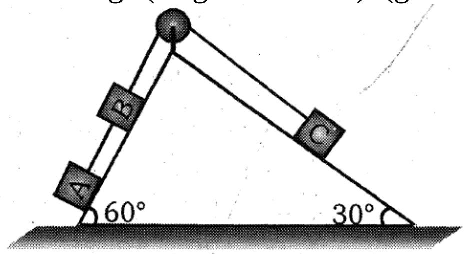
since floor yields to a normal force of 700 N, mode (b) should be adopted by the man to lift the block

**Illustration 17.**

In the adjacent figure, masses of A, B and C are 1 kg, 3 kg and 2 kg respectively.

Find (a) the acceleration of the system and

(b) the tensions in the strings (Neglect friction). ($g = 10\ m/s^2$)




**Solution:**

(a) In this case net pulling force

$= m_A g \sin60^o + m_B g \sin60^o - m_C g \sin30^o = (m_A + m_B)\, g \sin 60^o - m_C\, g \sin 30^o$

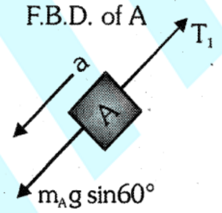
$= (1 + 3) \times 10 \times \dfrac{\sqrt{3}}{2} - 2 \times 10 \times \dfrac{1}{2} = 20\sqrt{3} - 10 = 20 \times 10.732 - 10$

$= 24.64$ N

Total mass being pulled = 1 + 3 + 2 = 6 kg

$\therefore$ acceleration of the system $a = \dfrac{24.64}{6} = 4.1\ m/s^2$

F.B.D. of A



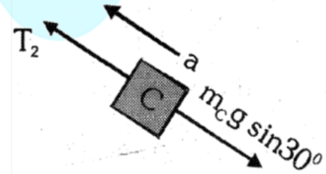

(b) For the tension in the string between A and B.

$m_A g \sin60^o - t_1 = (m_A)\,(a)$

$\therefore$ $T_1 = m_A\, g \sin60^o - m_A\, a = m_A\, (g \sin60^o - a)$

$\therefore$ $T_1 = (1)\left(10 \times \dfrac{\sqrt{3}}{2} - 4.1\right) = 4.56$ N

F.B.D. of C



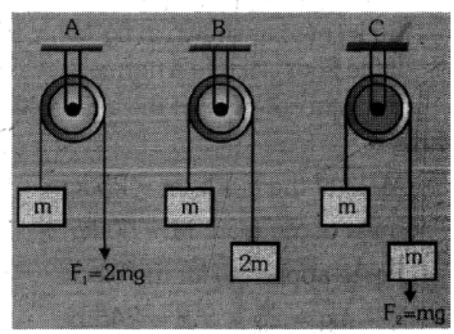

For the tension in the string between B and C.

$T_2 - m_C\, g \sin30^o = m_C a$

$\therefore$ $T_2 = m_C\, (a + g\, \text{sing}30^o) = 2\left[4.1 + 10\left(\dfrac{1}{2}\right)\right] = 18.2$ N

**Illustration 18.**

In the figure blocks A, B and C have accelerations $a_1$, $a_2$ and $a_3$ respectively. $F_1$ and $F_2$ are external forces of magnitudes 2mg and mg respectively. Find the value of $a_1$, $a_2$ and $a_3$.




**Solution**

$a_1 = \dfrac{2mg - mg}{m} = g$ ; $a_2 = \dfrac{2m - m}{2m + m} g = \dfrac{g}{3}$

$a_3 = \dfrac{mg + mg - mg}{2m} = \dfrac{g}{2}$

**Illustration 19.**

A 12 kg monkey climbs a light rope as shown in fig. The rope passes over a pulley and is attached to a 16 kg bunch of bananas. Mass and friction in the pulley are negligible so that the effect of pulley is only to reverse the direction of force of the rope. What maximum acceleration can the monkey have without lifting the bananas? (Take g = 10 m/s$^2$)

**Solution**

For Monkey

$T - 120 = 12 \times a$ ....(i)

For Bananas

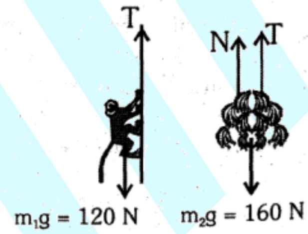
$160 - T = N$

Condition for just loosing the contact is N = 0

$160 - T = 0 \Rightarrow T = 160$ ....(ii)

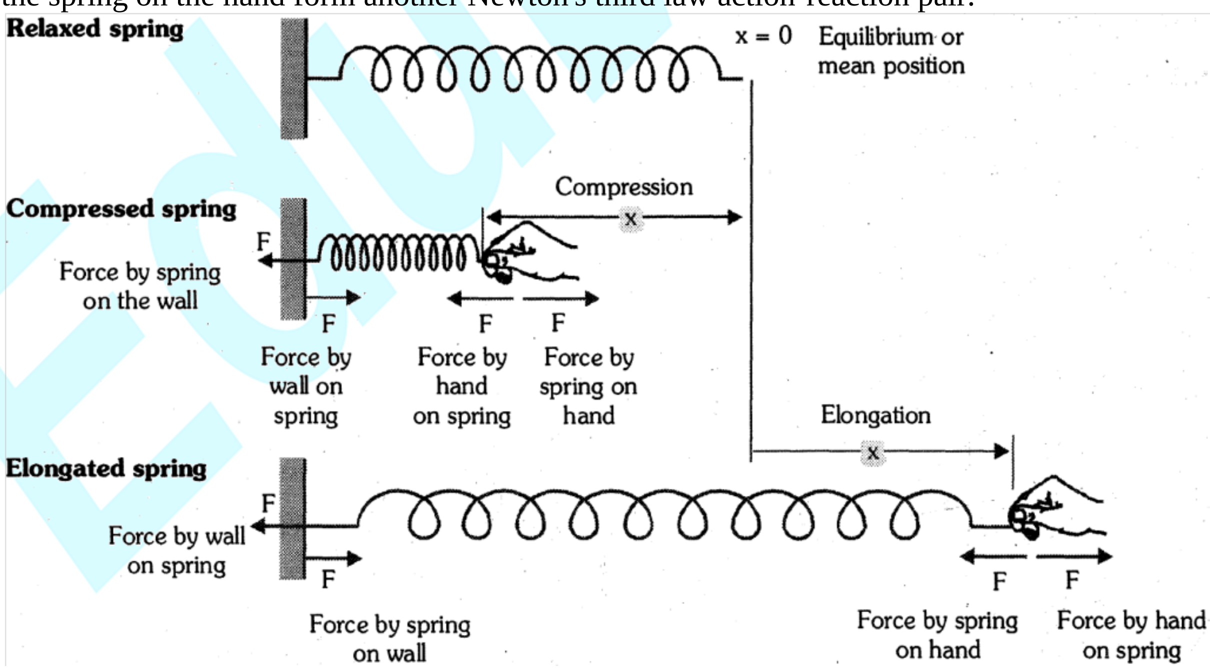
from equation (i) & (ii)

$160 - 120 = 12 \times a \Rightarrow a = 3.33 \text{ m/s}^2$



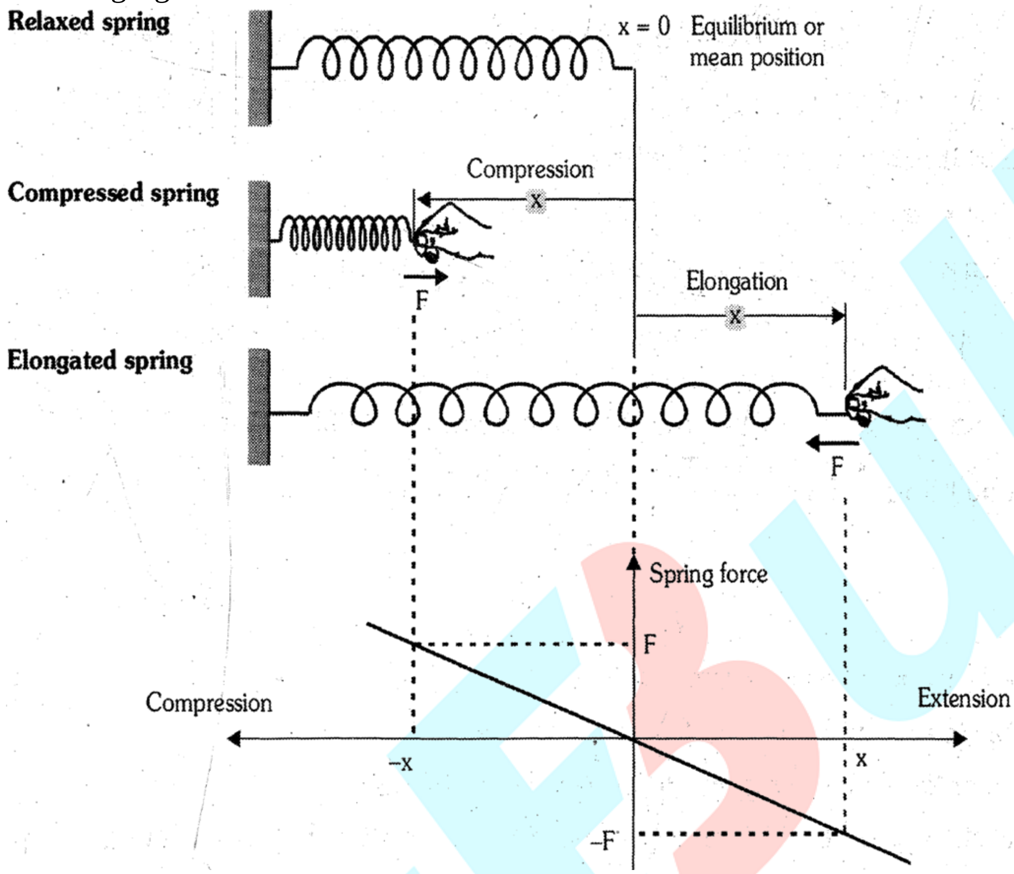

## 14. SPRING FORCE :

When no force acts on a spring, it is in relaxed condition i.e. neither compressed nor elongated. Consider a spring attached to a fixed support at one of its ends and the other end being free. If we neglect gravity, it remains in a relaxed state. When it is pushed by a force F, it is compressed and displacement x of its free end is called compression. When the spring is pulled by a force F, it is elongated and displacement x of its free end is called elongation. Various forces developed in these situations are shown in the following figure. The force & applied by the spring on the wall and the force applied by the wall on the spring form a Newton's third law action-reaction pair. Similarly, force by hand on the spring and force by the spring on the hand form another Newton's third law action-reaction pair.




### Hooke's Law :

How spring force varies with deformation in length x of the spring is also shown in the following figure.




The force F varies linearly with x and acts in a direction opposite to x. Therefore, it is expressed by the following equation

$$F = -kx$$

Here, the minus (–) sign represents the fact that force F is always opposite to x.

The constant of proportionality k is known as force constant of the spring or simply as spring constant. The modulus of slope of the graph equals the spring constant.

SI unit of spring constant is newton per meter or (N/m).

Dimensions of spring constant are $[MT^{-2}]$.

**For a pulley - spring system (at steady state) :-**

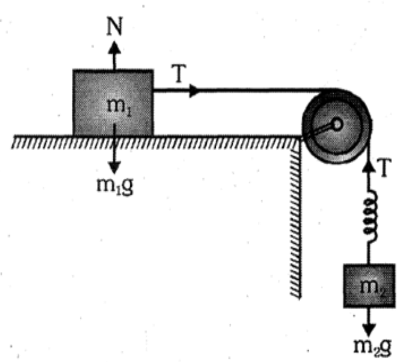
Acceleration $a = \dfrac{m_2 g}{m_1 + m_2}$

tension $T = \dfrac{m_1 m_2}{(m_1 + m_2)} g$

If x is the extension in the spring,

then $T = kx$

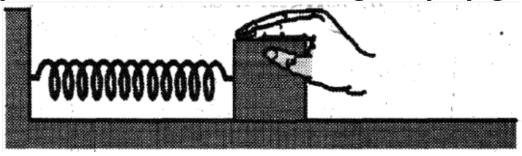
$$x = \frac{T}{k} = \frac{m_1 m_2 g}{k(m_1 + m_2)}$$




## Illustration

**Illustration 20.**

Consider a spring attached to a fixed support at one of its ends and to a box at other end, which rest on a smooth floor as shown in the figure. Denote mass of the box by m, force constant of the spring by k and acceleration due to gravity by g.



The box is pushed horizontally displacing it by a distance x towards the fixed support and held at rest.

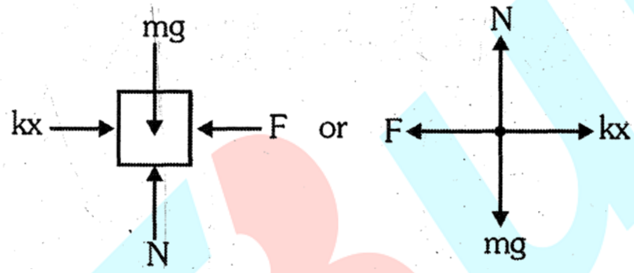
(a) Draw the free body diagram of the box.

(b) Find the force exerted by the hand on the box.

(c) Write all the action-reaction pairs corresponding to Newton's III$^{rd}$ law.

**Solution:**

(a) F is push by hand.




(b) Since the block is in equilibrium $\Sigma F_x = 0 \Rightarrow F = kx$

(c) (i) Force on box by hand and force on hand by box.

(ii) Force on box by spring and force on spring by box.

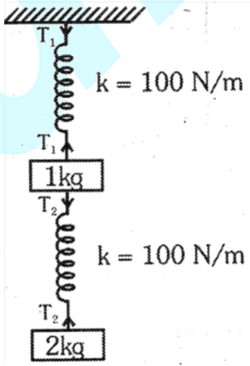
(iii) Normal reaction by box on floor and normal reaction by floor on box.

(iv) Weight of the box and the gravitational force by which box pulls the earth.

(v) Force by spring on support and force by support on spring.

**Illustration 21.**

Find extension in both the springs




**Solution:**

From figure (a)

$T_2 - 20 = 0 \Rightarrow T_2 = 20\text{ N}$ ......(1)

From figure (b)

$T_1 = T_2 + 10$ ......(2)

From equation (1) & (2)

$T_1 = 20 + 10 = 30\text{ N}$

For I[st] Spring

$T_1 = kx_1 \quad \Rightarrow 30 = 100 \times x_1 \;\Rightarrow x_1 = 0.3\text{ m}$

For 2[nd] Spring

$T_2 = kx_2 \quad \Rightarrow 20 = 100 \times x_2 \;\Rightarrow x_2 = 0.2\text{ m}$

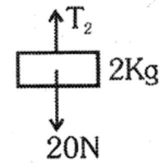


Figure (a)

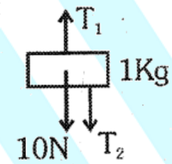


Figure (b)

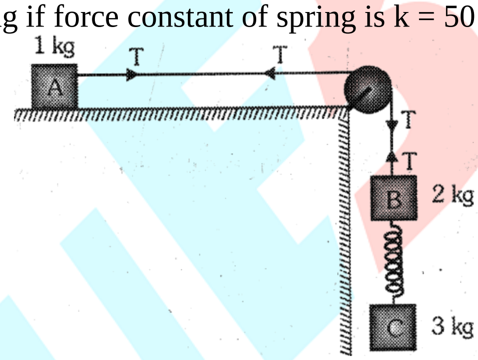
**Illustration 22.**

In the system shown in figure all surfaces are smooth, string is massless and inextensible. (in steady state) Find the

(a) acceleration of the system

(b) tension in the string and

(c) extension in the spring if force constant of spring is k = 50 N/m (Take $g = 10\text{ m/s}^2$)



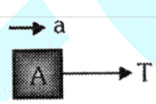

**Solution:**

(a) $3g - kx = 3a$ ......(1)

$2g + kx - T = 2$ ......(2)

$T = a$ ......(3)

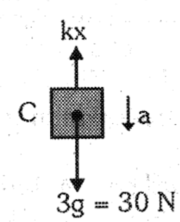
$\therefore$ Acceleration of the system is $a = \dfrac{50}{6}\text{ m/s}^2$

(b) 

Free body diagram of 1 kg block gives $T = ma = (1)\left(\dfrac{50}{6}\right)\text{N} = \dfrac{50}{6}\text{N}$

(c) Free body diagram of 3 kg block gives

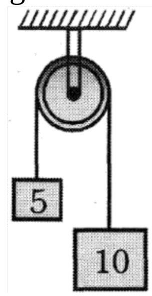
$30 - kx = ma$ but $ma = 3 \times \dfrac{50}{6} = 25\text{ N}$




$x = \dfrac{30-25}{k} = \dfrac{5}{50} = 0.1\text{ m} = 10\text{ cm}$

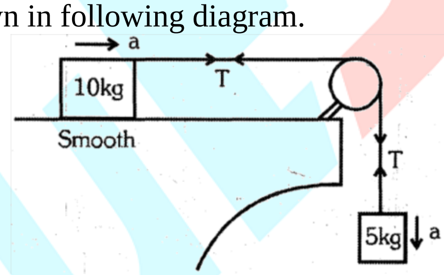
**BEGINNER'S BOX - 4**

1. The respective masses of the blocks are shown in the diagram. in appropriate units. Find acceleration of system, tension in the string and thrust on the pulley (in terms of g).

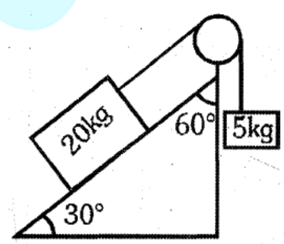



2. Find the reading of the spring balance
**Note :** Spring balance reads thrust on the pulley which is calibrated in kg-wt.

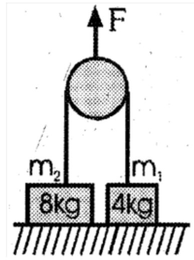

4. Calculate the acceleration of the system, tension in the string and thrust on the pulley in terms of g for the situation shown in following diagram.



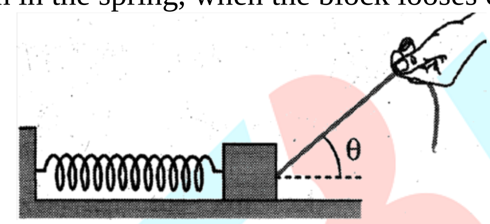

5. Calculate the acceleration of the system and tension in the string for the situation shown in following diagram.




6. Two blocks of masses 8 kg and 4 kg respectively are connected by a string as shown. Calculate their accelerations if they are initially at rest on the floor, after a force of l00N is applied on the pulley in the upward direction ($g = 10\ m/s^2$)




**7.** A block of mass m placed on a smooth floor is connected to a fixed support with the help of a spring of force constant k. It is pulled by a rope as shown in the figure. Tension T of the rope is increased gradually without changing its direction, until the block losses contact the floor. The increase in rope tension T is so gradual that acceleration in the block can be neglected.
(a) Draw its free body diagram, well before the block losses contact with the floor.
(b) What is the necessary tension in the rope so that the block looses contact with the floor?
(c) What is the extension in the spring, when the block looses contact with the floor?




## 15. FRAME OF REFERENCE

A system with respect to which the position or motion of a particle is described is known as a frame of reference. We can classify frames of reference into two categories:

**(i) Inertial Frame of Reference:**
The frame for which law of inertia is applicable is known as inertial frame of reference. All the frames which are at rest or moving uniformly with respect to an inertial frame, are inertial frame.

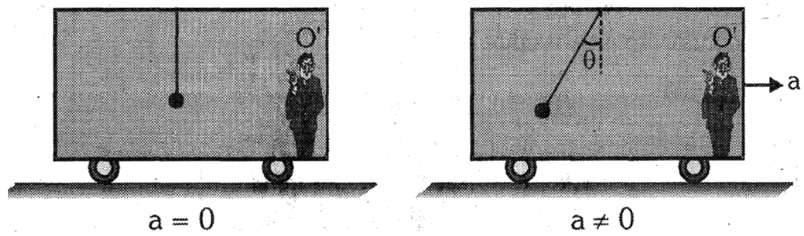
**(ii) Non-Inertial Frame of Reference:**
The frame for which law of inertia is not applicable is known as non-inertial frame of reference. All the frames which are accelerating or rotating with respect to an inertial frame will be non inertial frames.

## 16. PSEUDO FORCE

It is a fictitious force or an apparent force or a correction force which is used to explain the motion of objects in non inertial reference frames.
This force always works in the direction opposite to that of acceleration of frame and its magnitude is equal to the product of mass of the body and the acceleration of the non-inertial reference frame.

$$\boxed{\vec{F} = m\vec{a}}$$

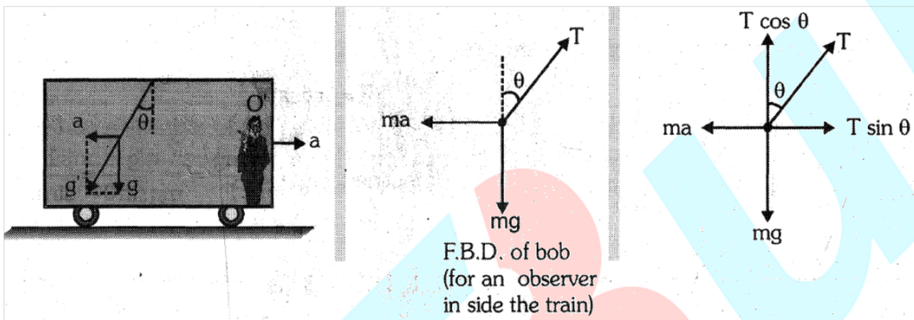
Pseudo force does not follow action-reaction law.
**Examples :**




## Illustrations

**Illustration 23.**

A pendulum of mass m is suspended from the ceiling of a train moving with an acceleration 'a' as shown in figure. Find the angle e in the equilibrium position. Also calculate tension in the string.




**Solution:**

With respect to train, the bob is in equilibrium

$\therefore \quad \Sigma F_y = 0 \Rightarrow T\cos\theta = mg$ ……(1)

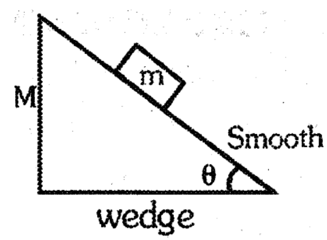
and $\quad \Sigma F_x = 0 \Rightarrow T\sin\theta = mg$ ……(2)

Dividing eqns. (2) by (1) $\tan\theta = \dfrac{a}{g} \Rightarrow \theta = \tan^{-1}\left(\dfrac{a}{g}\right)$

Squaring and adding eqns. (1) and (2) $T = m\sqrt{a^2 + g^2}$

**Illustration 24.**

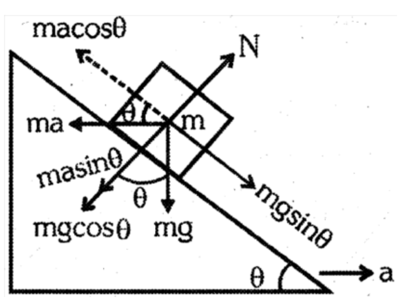
What horizontal acceleration should be provided to the wedge so that the block of mass m kept on wedge remains at rest w.r.t. wedge?




**Solution:**

For equilibrium along wedge $ma\cos\theta = mg\sin\theta$

$$\Rightarrow a = g\frac{\sin\theta}{\cos\theta} \quad \Rightarrow \quad a = g\tan\theta$$





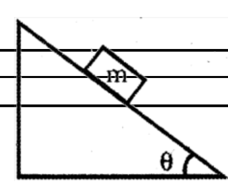



**Illustration 25.**

What horizontal acceleration should be provided to the wedge so that the block of mass m placed on the wedge falls freely?

**Solution:**

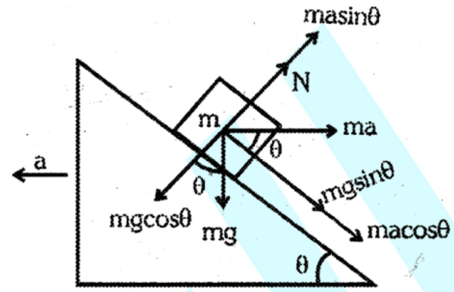
For free fall, normal reaction should be zero

$N = 0$

For equilibrium perpendicular to the wedge

$0 + ma\sin\theta = mg\cos\theta$

$\Rightarrow \quad a = \dfrac{g}{\tan\theta}$




**17. MECHANICAL ADVANTAGE:**

The ratio of load to effort is called mechanical advantage (M.A.)

Thus, mathematically, M.A.= $\dfrac{\text{Load}}{\text{effort}}$

e.g. : Mechanical advantage of an inclined plane.

$F = mg\sin\theta$

M.A. of inclined plane = $\dfrac{mg}{F} = \dfrac{mg}{mg\sin\theta} = \dfrac{1}{\sin\theta}$ = cosesθ

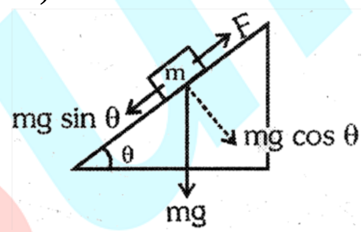



**Illustration 26.**

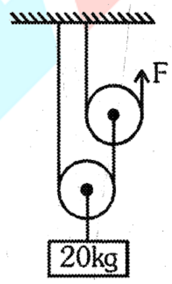
A students is able to lift a bag containing books of 20 kg-wt by applying a force of 5 kg-wt. Find the mechanical advantage.

**Solution:**

W = 20 kg-wt

F = 5 kg-wt

M.A. = $\dfrac{20}{5} = 4$




**18. TRANSLATIONAL EQUILIBRIUM**

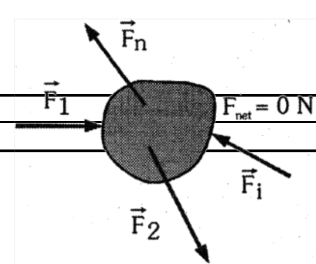
A body in a state of rest or moving with constant velocity is said to be in translational equilibrium. Thus if a body is in translational equilibrium in a certain inertial frame of reference, it must have no linear acceleration.

When it is at rest, it is in static equilibrium, whereas if it is moving with constant velocity it is in dynamic equilibrium.

**Conditions for translational equilibrium**

For a body to be in translational equilibrium, no net force must act on it i.e. vector sum of all the forces acting on it must be zero.

If several external forces $\vec{F}_1, \vec{F}_2 \ldots\ldots \vec{F}_i \ldots\ldots$ and $\vec{F}_n$ act simultaneously on a body and the body is in translational equilibrium, the resultant of these forces must be zero.

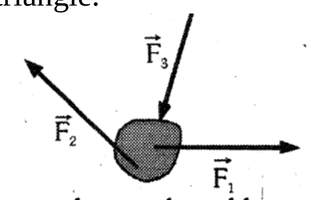


$$\sum \vec{F}_i = \vec{0}$$

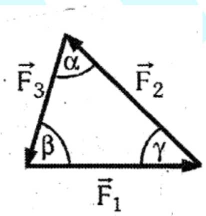
If the forces $\vec{F}_1, \vec{F}_2 .....\vec{F}_i ...... \text{and } \vec{F}_n$ are expressed in Cartesian components, we have:

$$\sum F_{ix} = 0 \qquad \sum F_{iy} = 0 \qquad \sum F_{iz} = 0$$

If a body is acted upon by a single external force, it cannot be in equilibrium.
If a body is in equilibrium under the action of only two external forces, the forces must be equal and opposite.
If a body is in equilibrium under the action of three forces, their resultant must be zero. Therefore, according to the triangle law of vector addition they must be coplanar and should form a closed triangle.




$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{0} \Rightarrow$$

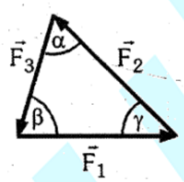


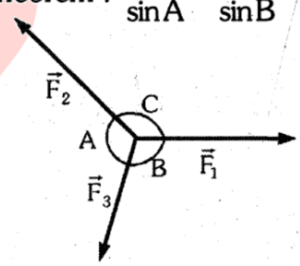

The situation can be analyzed by either graphical or analytical method.

- **Lami's theorem :-**
  Graphical method makes use of sine rule or Lami's theorem.

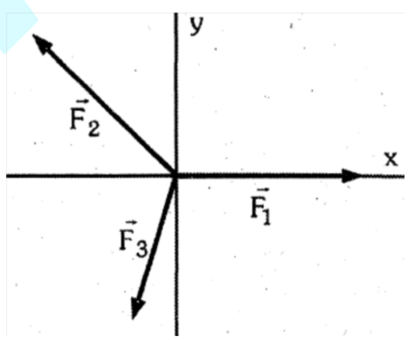
*Sine rule :* $\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$

*Lami's theorem :* $\frac{F_1}{\sin A} = \frac{F_2}{\sin B} = \frac{F_3}{\sin C}$




- Analytical method makes use of Cartesian components. Since the forces involved form a closed triangle, they lie in a plane and a two-dimensional Cartesian frame can be used to resolve the forces. As far as possible orientation of the x-y frame is selected in such a manner that angles made by different forces with the axes should have convenient values.




$$\sum F_x = 0 \Rightarrow F_{1x} + F_{2x} + F_{3x} = 0$$

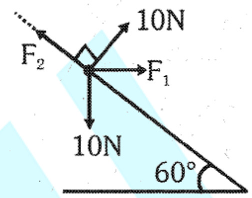
$$\sum F_y = 0 \Rightarrow F_{1y} + F_{2y} + F_{3y} = 0$$

Problems involving more than three forces should be analyzed by analytical method. However, in some situations, there may be some parallel or anti-parallel forces and they should be combined first to minimize the number of forces. This may sometimes reduce a system involving more than three forces to a three-force system.

## Illustrations

**Illustration 27.**

In the given figure if all the forces are in equilibrium then calculate $F_1$ and $F_2$.




**Solution:**

Resolve the forces along x-direction & y-direction

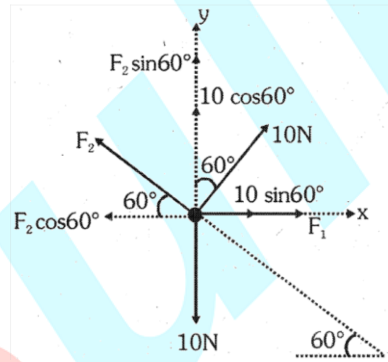
**In x-direction**

$\Sigma F_x = 0$ (for equilibrium)

$F_1 + 10 \sin 60^\circ = F_2 \cos 60^\circ$

$\Rightarrow \quad F_1 + 10\left(\frac{\sqrt{3}}{2}\right) = F_2\left(\frac{1}{2}\right)$

$\Rightarrow \quad 2F_1 - F_2 = -10\sqrt{3}$ ……(i)




**In y-direction**

$\Sigma F_y = 0$

$F_2 \sin 60^\circ + 10 \cos 60^\circ = 10$

$\Rightarrow \quad F_2\left(\frac{\sqrt{3}}{2}\right) + 10\left(\frac{1}{2}\right) = 10 \Rightarrow \quad F_2\left(\sqrt{3}\right) + 10 = 20$

$\Rightarrow \quad F_2 = \left(\frac{10}{\sqrt{3}} N\right)$

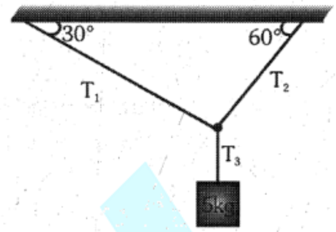
Now from equation (i)

$2F_1 - F_2 = -10\sqrt{3} \quad \Rightarrow \quad 2F_1 - \frac{10}{\sqrt{3}} = -10\sqrt{3}$

$\Rightarrow \quad 2\sqrt{3}F_1 - 10 = -30 \quad \Rightarrow \quad 2\sqrt{3}F_1 = -20 \Rightarrow \quad \boxed{F_1 = \frac{-10}{\sqrt{3}} N}$

**Illustration 28.**

Calculate the tensions $T_1$, $T_2$ and $T_3$ in the massless strings shown in figure ($g = 10\ m/s^2$)



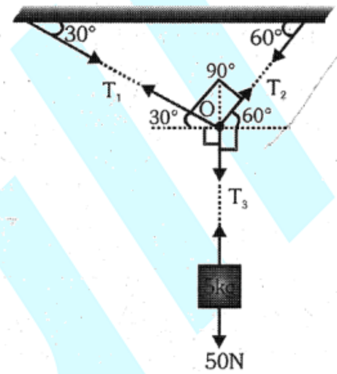

**Solution**

Considering the adjoining figure

$T_3$ = wt. of the 5 kg block (mg)

$T_3 = 5 \times 10 = 50$ N

Now applying Lami's theorem at point O.




$$\frac{T_1}{\sin(90^\circ + 60^\circ)} = \frac{T_2}{\sin(90^\circ + 30^\circ)} = \frac{T_3}{\sin(180^\circ - 60^\circ - 30^\circ)}$$

$$\Rightarrow \quad \frac{T_1}{\cos 60^\circ} = \frac{T_2}{\cos 30^\circ} = \frac{50}{\sin 90^\circ}$$

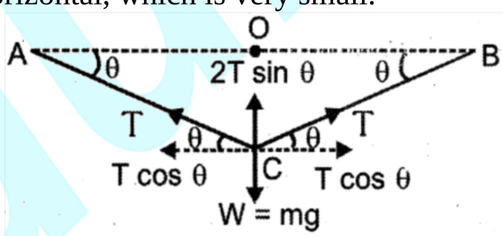
$$T_1 = 50\,\frac{\cos 60^\circ}{\sin 90^\circ} = 25\text{ N and } \quad T_2 = 50\,\frac{\cos 30^\circ}{\sin 90^\circ} = 25\sqrt{3}\text{N}$$

**Illustration 29.**

A bird of mass m perches at the middle of a stretched string Show that the tension in the string is given by $T = \frac{mg}{2\sin\theta}$. Assume that each half of the string is straight.


**Solution:**

Initial position of string is = AOB. Final position of string is = ACB Let θ be the angle made by the string with the horizontal, which is very small.




Resolving tension T of string in horizontal and vertical directions, we note that the horizontal components cancel while vertical components add and balance the weight. For equilibrium

$2\,T \sin\theta = W = mg$

$$\Rightarrow T = \frac{mg}{2\sin\theta}$$

**Illustration 30.**

(a) A box of weight $10\sqrt{3}$ N is held in equilibrium with the help of two strings OA and OB as shown in figure-1. The string OA is horizontal. Find the tensions in both the strings.

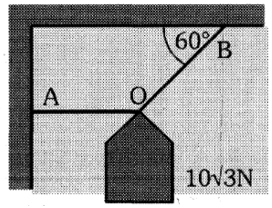
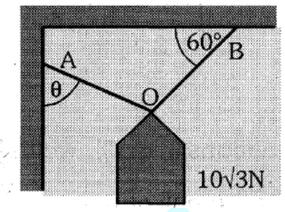
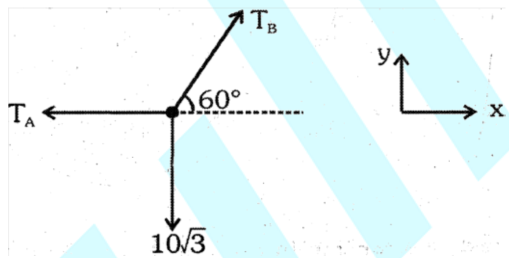
Fig. I 


Fig. II 


(b) If you can change the location of point A on the wall and hence the orientation of the string OA without altering the orientation of the string OB as shown in figure-II. What angle should the string OA make with the wall so that a minimum tension is developed in it?

**Solution:**




(a) FBD of box

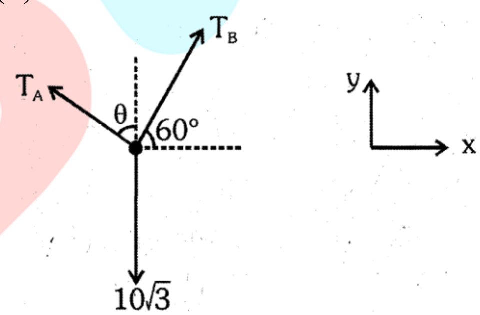
$\Sigma F_x = 0 \Rightarrow T_B \cos 60° - T_A = 0$ .....(i)

$\Sigma F_y = 0 \Rightarrow T_B \cos 60° - 10\sqrt{3} = 0$ .....(ii)

Solving equations (i) and (ii)

We have, $T_A = 10$ N and $T_B = 20$ N

(b) FBD of box




$\Sigma F_x = 0 \Rightarrow T_B \cos 60° - T_A \sin\theta = 0$ .....(i)

$\Sigma F_y = 0 \Rightarrow T_A \cos\theta + T_B \sin 60° - 10\sqrt{3} = 0$ .....(ii)

From equation (i) and (ii)

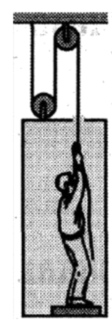
We have, $T_A = \dfrac{10\sqrt{3}}{\sqrt{3}\sin\theta + \cos\theta}$

$T_A$ is minimum when $(\sqrt{3}\sin\theta + \cos\theta)$ is maximum.

Now, $(\sqrt{3}\sin\theta + \cos\theta)_{max} = \sqrt{(\sqrt{3})^2 + (1)^2} = 2$

Therefore, $(T_A)_{min} = \dfrac{10\sqrt{3}}{2}$ N $= 5\sqrt{3}$ N

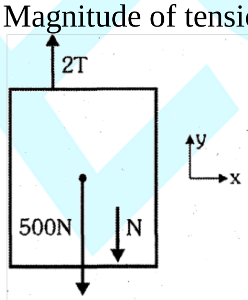
**Illustration 31.**



A 70 kg man standing on a weighing machine in a 50 kg lift Pulls on the rope, which supports the lift as shown in the figure. Find the force with which the man should pull the rope to keep the lift stationary. Also, find the weight of the man as shown by the weighing machine.

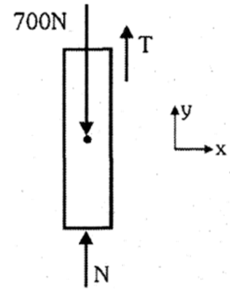
**Solution:**

Magnitude of tension everywhere in the string is same. For equilibrium of the lift.




$\sum F_y = 0 \Rightarrow 500 + N = 2T$ ...(i)

To analyse the equilibrium of the man let us assume him as a block

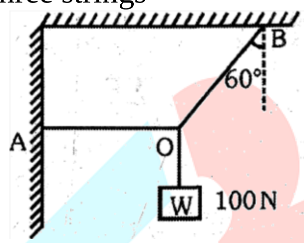



$$\sum F_y = 0 \Rightarrow N + T = 700 \quad \ldots(ii)$$

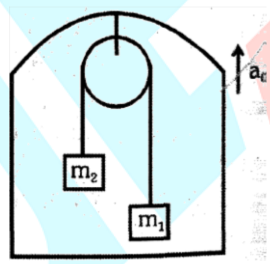
From equations (i) & (ii), we have T = 400 N and N = 300 N
Here, T is the pull of mass and N is the reading of the weighing machine.

**BEGINNER'S BOX - 5**

**1.** A block of weight 100 N is suspended (as shown) with the help of three strings. Find the tension in each of the three strings




**2.** Determine the acceleration of the masses w.r.t. lift and tension in the string if the whole system is moving vertically upwards with uniform acceleration $a_0$. ($m_1 > m_2$)



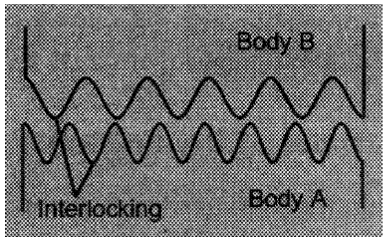

## 19. FRICTION

### 19.1 Introduction

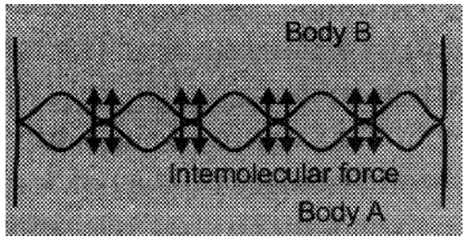
It is that component of total contact force which acts parallel to the contact surface. The friction force always opposes the relative slipping or tendency of relative slipping between the two contact surfaces.

### 19.2 Cause of Sliding Friction

**Old View :** When two bodies are in contact with each other, the irregularities in the surface of one body get interlocked with the irregularities in the surface of the other. This inter locking opposes the tendency of relative motion.

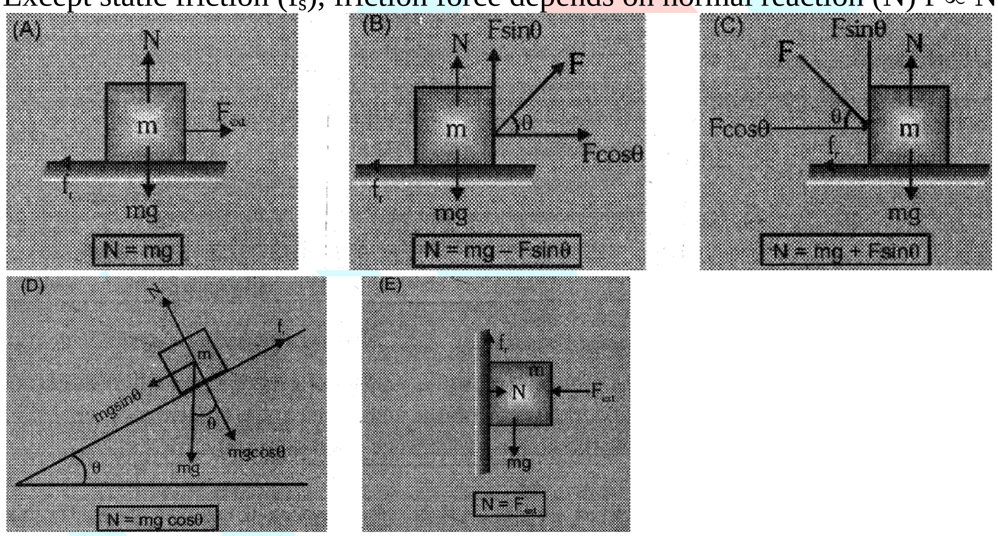



**Modem View :** Friction arises on account of strong inter atomic or inter molecular forces of attraction between the two· surfaces at the Point of actual contact.




**Friction depends on the following factors :**

1. Friction force depends only on the area covered by contact particles of contact surfaces (actual contact area) it does not depend on the area covered by the body (apparent contact area)
2. Except static friction ($f_s$), friction force depends on normal reaction (N) $f \propto N$




## 20. TYPES OF FRICTION

Before we proceed further into the details of frictional phenomena, it is advisable to become familiar with different types of frictional forces. All types of frictional phenomenon can be categorized into dry friction, fluid friction, and internal friction.

**Dry Friction**

It exists when two solid un-lubricated surfaces are in contact under the condition of sliding or tendency of sliding. It is also known as Coulomb friction.

**Fluid Friction**

Fluid friction is developed when adjacent layers of a fluid move at different velocities and gives rise to a phenomena, which we call viscosity of the fluid. Resistance offered to the motion of a solid body in a fluid also comes in this category and is commonly known as viscous drag. We will study this kind of friction in fluid mechanics.

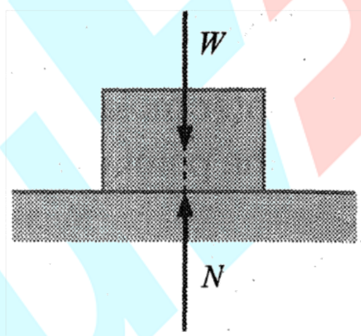
**Internal Friction**

When solid materials are subjected to deformation, internal resistive forces are developed because of relative movement of different parts of the solid. These internal resistive forces constitute a system which is defined as internal friction. They always cause loss of energy.

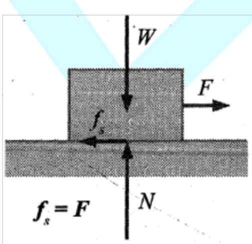
Frictional forces exist everywhere in nature and result in loss of energy that is primarily dissipated in the form of heat. Wear and tear of moving bodies is another unwanted result of friction. Therefore, sometimes, we try to reduce their effects - such as in bearings of all types, between piston and the inner walls of the cylinder of an internal combustion engine, flow of fluid in pipes, and aircraft and missile propulsion through air. Though these examples create a negative picture of frictional forces, yet there are other situations where frictional) forces become essential and we try to maximize its effect. It is the friction between our feet and the ground, which enables us to walk and run. Both the acceleration and braking of wheeled vehicles depend on friction.

**Types of Dry Friction**

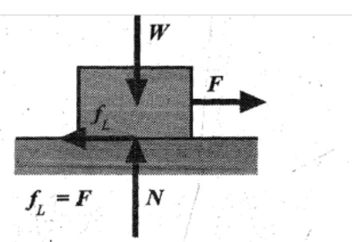
In mechanics of non-deformable bodies, we are always concerned with the dry friction. Therefore, we often drop the word "dry" and simply call it friction. To understand the nature of friction let us consider a box of weight W placed on a rough horizontal surface. The forces acting on the box are its weight and reaction from the horizontal surface. They are shown in the figure. The weight does not have any horizontal component, so the reaction of the horizontal surface on the box is normal to the surface. It is represented by N in the figure. The box is in equilibrium therefore both W and N are equal in magnitude, opposite in direction, and collinear.




Now suppose the box is being pulled by a gradually increasing horizontal force F to slide the box. Initially when the force F is small enough, the box does not slide. This can only be explained if we assume a frictional force, which is equal in magnitude and opposite in direction to the applied force facts on the box. The force F produces a tendency of relative sliding in the box and the friction force is opposing this tendency of relative sliding. The frictional force developed before relative sliding initiates is defined as static friction. It opposes tendency of relative sliding.

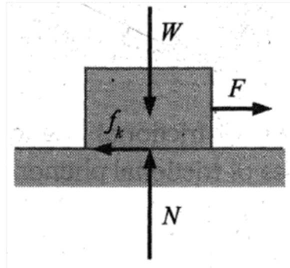



**Static Friction**



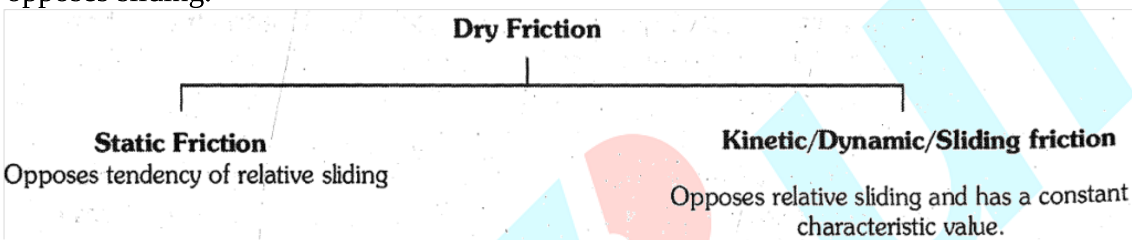

**Limiting friction: The maximum Static Friction**

As we increase F, the box remain stationary until a value off F is reched when the box starts sliding. Before the box starts sliding, the static friction increases with F and counterbalances F until the static friction reaches its maximum value known as the limiting friction or maximum static friction $f_L$.




**Kinetic friction**

When the box starts sliding, a force F is needed to overcome frictional force to maintain its sliding. This frictional force is known as sliding or dynamic or kinetic friction ($f_k$). It always opposes sliding.




**Static Friction**

- It is the frictional force which is effective before relative motion begins between two surfaces in contact with each other.
- It's nature is self adjusting (in direction and magnitude upto certain limit)
- Numerical value of static friction is equal to the external force tendency to generate the relative motion of the body.
- Maximum value of static friction is called limiting friction.

## 21. LAWS OF LIMITING FRICTION

- The magnitude of the force of limiting friction ($f_L$) between any two bodies in contact is directly proportional to the normal reaction (N) between them

$$f_L \propto N$$

- The direction of the force of limiting friction is always opposite to the direction in which the body is on the verge of moving over the other.
- The force of limiting friction is independent of the apparent contact area, as long as normal reaction between the two bodies in contact remains the same.
- Limiting friction between any two bodies in contact depends on the nature of material of the surfaces in contact and their roughness and smoothness.
- Its value is more than other types of friction force.

## 22. LAWS OF KINETIC FRICTION

If the body is in relative motion, the friction opposing its relative motion is called dynamic or kinetic friction.

- This is always slightly less than the limiting friction
- It depends on N.
- Its value does not depend on types of motion of body such as accelerated motion, retarded motion or moving with constant velocity because it is a constant friction.
- Its numerical value is $f_k = \mu_k N$, where $\mu_k$ = coefficient of kinetic friction.
- **Coefficient of friction :-**

Coefficient of Friction

| Static friction coefficient | Kinetic/Sliding friction coefficient |
|---|---|
| $\mu_s = \frac{f_L}{N}$ | $\mu_k = \frac{f_k}{N}$ |

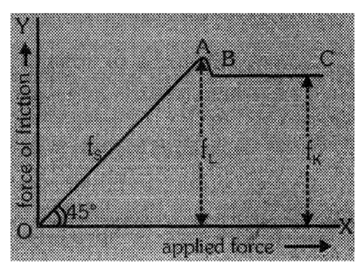
The values of $\mu_s$ and $\mu_k$ depend on the nature of both the surfaces in contact.
The values of $\mu$ depend on material of the surfaces in contact.
$\mu_k$ and $\mu_s$ are dimensionless and unitless.

## GOLDEN KEY POINTS

- Friction always oppose the tendency of relative motion or the relative motion of contact surface.
- The force of static friction exactly balances the applied force during the stationary state of a body.
- Static friction is a self-adjusting force whereas kinetic friction is not a self adjusting force.
- The frictional force is a contact force parallel to the surfaces in contact and directed so as to oppose the relative motion or attempted relative motion of the surfaces.
- **In exceptional cases $\mu_s$ and $\mu_k$ can exceed unity,** although their commonly used values are less then 1 in numerical.
- When two highly polished surfaces are pressed hard, then a situation, similar to welding, occurs. It is called **cold welding.**
- When two copper plates are highly polished and placed in contact with each other, then, the force of friction increases instead of decreasing. This arises due to the fact that for two highly polished surfaces in contact, the number of molecules coming in contact increases and as a result the cohesive/adhesive forces increases. This in turn, increases the force of friction.

### 22.1 Graph Between Applied Force and Force of Friction

The part OA of the curve represents static friction, ($f_s$) which goes on increasing, with the applied force. At A, the static friction is maximum. This represents the limiting friction. Beyond A, the force of friction is seen to decrease slightly. The portion BC of the curve, therefore, represents the kinetic friction ($f_K$).

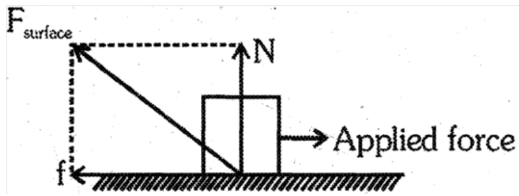



As the portion BC of the curve is parallel to OX, therefore, kinetic friction does not change with the applied force, It remains constant, whatever be the applied force.

### 22.2 Contact force

Let f be the force of friction and N the normal reaction, then the net contact force by the surface on the object is $F_{surface} = \sqrt{N^2 + f^2}$. Its minimum value (when f = 0) is N and maximum value (when $f = \mu N$) is $N\sqrt{1+\mu^2}$

Therefore $N \le F_{surface} \le N\sqrt{1+\mu^2}$

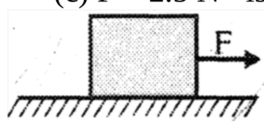



## Illustrations

**Illustration 32.**

A block of mass 1 kg is at rest on a rough horizontal surface having coefficient of static friction 0.2 and kinetic friction 0.15. Find the frictional force if a horizontal force,

(a) F = 1N (b) F = 1. 96 N (c) F = 2.5 N is applied on the block




**Solution:**

Maximum force of friction $f_{max} = 0.2 \times 1 \times 9.8$ N = 1.96 N

(a) For $F_{ext} = 1$ N

$F_{ext} < f_{max}$

So, body is at rest which implies that static friction is present and hence $f_s = F_{ext} = 1$N

(b) For $F_{ext} = 1,96$ N

$F_{ext} = f_{max} = 1.96$ N

so, f = 1.96N

(c) For $F_{ext} = 2.5$ N

so $F_{ext} > f_{max.}$

Now the body is in motion

$\therefore f_{max.} = f_k = \mu_k N = \mu_k mg = 0.15 \times 1 \times 9.8 = 1.47$ N

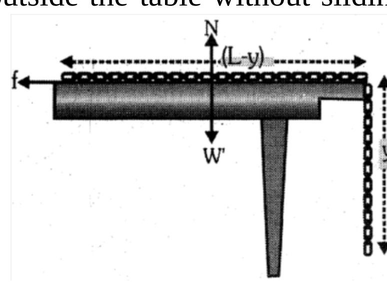
**Illustration 33.**

Length of a chain is L and coefficient of static friction is μ. Calculate the maximum length of the chain which can hang from the table without sliding.

**Solution:**

Let y be the maximum length of the chain that can be held outside the table without sliding.

Length of chain on the table = (L – y)




Weight of the part of the chain on table $W = \frac{M}{L}(L - y\ g)$

Weight of hanging part of the chain $W = \frac{M}{L}yg$

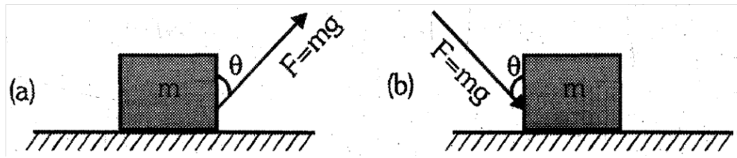
For equilibrium :

limiting force of friction on (L–y) length = weight of hanging part of the chain of y length

$$\mu N = W \Rightarrow \mu W = W \Rightarrow \mu\frac{M}{L}(L - y\ g) = \frac{M}{L}yg \Rightarrow \mu L - \mu y = y \Rightarrow y = \frac{\mu L}{1+\mu}$$

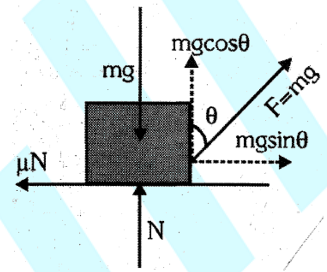
**Illustration 34.**

A block of mass m rests on a rough horizontal surface as shown in figure (a) and (b). Coefficient of friction between the block and surface is μ. A force F = mg act at an angle θ with the vertical side of the block. Find the condition for which the block will move along the surface.




**Solution:**

**For (a) :** normal reaction N = mg – mg cos θ, limiting frictional force = μN = μ(mg–mg cos θ) Now, block can be pulled when : Horizontal component of force ≥ limiting frictional force i.e. mg sinθ ≤ μ(mg – mg cosθ)




$\Rightarrow \quad 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} \geq (1-\cos\theta)$

$\Rightarrow \quad 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} \geq \sin^2\frac{\theta}{2} \Rightarrow \cot\frac{\theta}{2} \geq \mu$

**For (b) :** Normal reaction N = mg + mg cos θ = mg (1 + cosθ)
Hence, block can be pushed along the horizontal surface when.
horizontal component of force ≥ limiting frictional force
i.e. mg sinθ ≥ μ mg(1 + cosθ)

$\Rightarrow \quad 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} \geq \mu\cos^2\frac{\theta}{2} \Rightarrow \tan\frac{\theta}{2} \geq \mu$

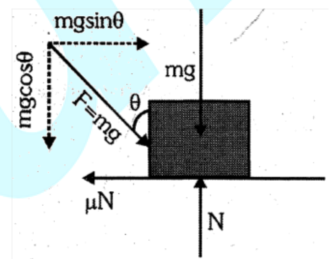



**Illustration 35.**

A block of mass 2 kg is placed on a plane inclined at an angle of 37° with the horizontal. The coefficient of friction between the block and the surface is 0.7.

(i) What will be the frictional force acting on the block ?

(ii) What is the force applied by inclined plane on block ?

**Solution:**

(i) Normal reaction of the surface N = mg cos 37° = 16 N

∴ Limiting force of friction $f_L = \mu_s N = \mu_s\, mg\cos 37° = 0.7 \times 2 \times 10 \times \frac{4}{5} = 11.2$ N

∴ $mg\sin 37° = 2 \times 10 \times \frac{3}{5} = 12$ N ∴ force of friction= 11.2 N

(ii) $F = \sqrt{(f_L)^2 + (N)^2} = \sqrt{(11.2)^2 + (16)^2} = \sqrt{125.44 + 256} = \sqrt{381.44} = 19.53$ N

**Illustration 36.**

A body of mass m rests on a horizontal floor with which it has a coefficient of static friction μ. It is desired to make the body move by applying the minimum possible force F. Find the magnitude of F and the direction in which it has to be applied.

**Solution:**

Let the force F be applied at an angle θ with the horizontal as shown in figure.

For vertical equilibrium,

$N + F\sin\theta = mg$ i.e $N = mg - F\sin\theta$ .......(i)

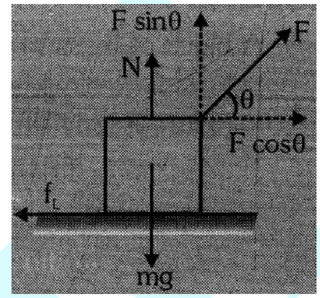
for horizontal motion

$F\cos\theta \geq f_L$ i.e. $F\cos\theta \geq \mu N$ [as $f_L = \mu N$] ......(ii)

substituting expression for N from equation (i) in (ii),

$$F\cos\theta \geq \mu(mg - F\sin\theta) \Rightarrow F \geq \frac{\mu mg}{(\cos\theta + \mu\sin\theta)}$$

......(iii)

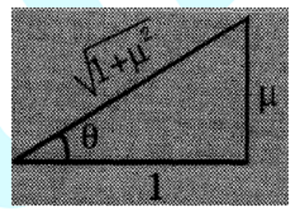



For the force F to be minimum $(\cos\theta + \mu\sin\theta)$ must be maximum,

$$\frac{d}{d\theta}(\cos\theta + \mu\sin\theta) = 0$$

or $-\sin\theta + \mu\cos\theta = 0$ i.e., $\tan\theta = \mu$ ......(iv)

$\therefore$ $\sin\theta = \frac{\mu}{\sqrt{1+\mu^2}}$ and $\cos\theta = \frac{1}{\sqrt{1+\mu^2}}$




substituting these in equation (iii)

$$F \geq \frac{\mu mg}{\frac{1}{\sqrt{1+\mu^2}} + \frac{\mu^2}{\sqrt{1+\mu^2}}} \text{ i.e. } F \geq \frac{\mu mg}{\sqrt{1+\mu^2}}$$

so that $F_{min} = \frac{\mu mg}{\sqrt{1+\mu^2}}$ with $\theta = \tan^{-1}(\mu)$

**Note :** As $-\sqrt{A^2+B^2} \leq A\sin\theta + B\cos\theta \leq \sqrt{A^2+B^2}$

So $(\cos\theta + \mu\sin\theta)_{max} = \sqrt{1+\mu^2}$ Therefore $F_{min} = \frac{\mu mg}{\sqrt{1+\mu^2}}$

**Illustration 37.**

A horizontal force of 49 N is just able to move a block of wood weighing 10 kg on a rough horizontal surface. Calculate the coefficient of friction and angle of friction.

**Solution:**

Here, $F = 49$ N, $N = W = mg = 10 \times 9.8$N so $\mu = \frac{49}{10\times 9.8} = 0.5$

$\Rightarrow$ $\tan\theta = \mu = 0.5 \Rightarrow$ Angle of friction $\theta = \tan^{-1}(0.5)$

**BEGINNER'S BOX - 6**

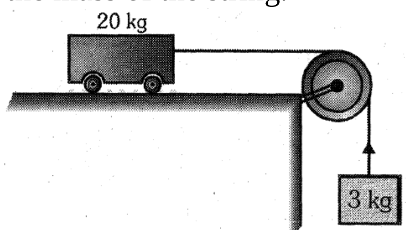
**1.** A body of mass 5 kg is placed on a rough horizontal surface. If coefficients of static and kinetic friction are 0.5 and 0.4 respectively, then find value of force of friction when external applied horizontal force is (i) 15 N (ii) 25 N and (iii) 35 N.

**2.** A body of mass 5 kg is placed on a rough horizontal surface. If coefficient of friction is $\frac{1}{\sqrt{3}}$, find what pulling force should act on the body at an angle 30° to the horizontal so that the body just begins to move.

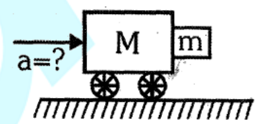
**3.** A body of mass 0.1 kg is pressed against a wall with a horizontal force 5 N. If coefficient of friction is 0.5 then find force of friction.

**4.** A body sliding on ice with a velocity of 8 m/s comes to rest after travelling 40 m. Find the coefficient of friction ($g = 9.8\ m/s^2$).

**5.** What is the acceleration of the block and trolley system shown in fig. if the coefficient of kinetic friction between the trolley and the surface is 0.04? What is the tension in the string? (Take $g = 10\ m/s^2$). Neglect the mass of the string.




**6.** A cart of mass M has a block of mass min contact with it as shown in fig. The coefficient of friction between the block and the cart is $\mu$. What should be the minimum acceleration of the cart so that the block of mass m does not fall ?




**7.** Is it unreasonable to expect the coefficient of friction to exceed unity ?

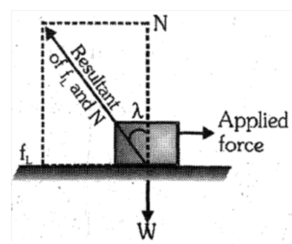
**8.** It is known that polishing a surface beyond a certain limit increases (rather than decreases) the frictional force. Explain.

**9.** Why do we slip on a muddy road ?

**10.** State whether the following statement is true or false:
When a person walks on a rough surface, the frictional force exerted by the surface on the person is opposite to the direction of his motion.

## 22.3 Angle of friction ($\lambda$)

The angle which the resultant of the force of limiting friction $f_L$ and normal reaction N makes with the direction of normal reaction N.




$$\tan\lambda = \frac{f_L}{N} = \mu_s \quad \Rightarrow \quad \lambda = \tan^{-1}\mu_s$$

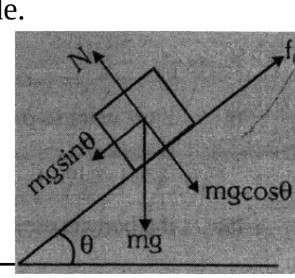
## 22.4 Angle of Repose or Angle of Sliding

It is defined as the minimum angle of inclination of a plane with the horizontal at which a body placed on it just begins to slide down or equivalently the maximum angle of inclination of plane with the horizontal at which a body placed on it does not slide.

$f_L = mg\sin\theta$ ..... (i) and $N = mg\cos\theta$ ..... (ii)

Dividing (i) by (ii)

so $$\mu_s = \frac{f_L}{N} = \frac{mg\sin\theta}{mg\cos\theta} = \tan\theta \Rightarrow \quad \theta = \tan^{-1}\mu_s$$



This fact is used for finding the coefficient of static friction in the laboratory.

Angle of repose (θ) = Angle of friction (λ)

**22.5 Pulling, is Easier Than Pushing**

Case of pulling :

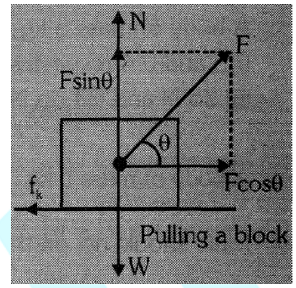
Force F is applied to pull a block of weight W.

F can be resolved into two rectangular components: F cosθ and F sinθ.

the normal reaction $N = W - F \sin\theta$

Force of kinetic friction $f_k = \mu_k N$

$f_k = \mu_k (W - F \sin\theta)$ .... (i)




Case of pushing :

Force F is applied to push a block

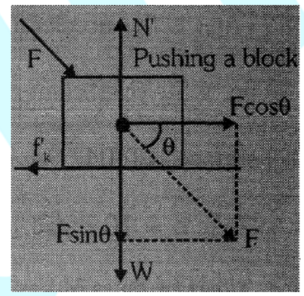
normal reaction $N' = W + F \sin\theta$

Force of kinetic friction $f'_k = \mu_k N'$

or $f'_k = \mu_k (W + F \sin\theta)$ .... (ii)

from (i) and (ii) $f'_k > f_k$

The opposing frictional force is more in the case of push. Hence it is easier to pull than to push a body.




**22.6 Acceleration of a Body Down a Rough Inclined Plane**

If angle of inclination is greater than the angle of repose, then the body accelerates down the incline.

Net force on the body down the inclined plane

$F_{net} = mg \sin\theta - f_r$

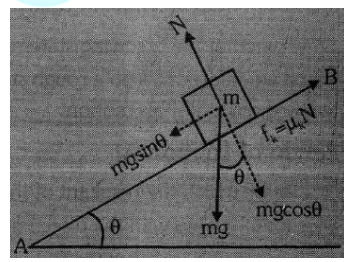
applying Newton's second laws of motion

$ma = mg \sin\theta - \mu_k N = mg \sin\theta - \mu_k\, mg \cos\theta$

$ma = mg [\sin\theta - \mu_k \cos\theta]$

$a = g [\sin\theta - \mu_k \cos\theta]$ hence $a < g$

acceleration of a body down a rough inclined plane is always less than 'g'.




**Note:**

(i) If we want to prevent the downward slipping of body then minimum upward force required is

$= mg \sin\theta - \mu_k\, mg \cos\theta$

(ii) If a body is projected in upward direction along the inclined plane then retardation of body is

$a = g [ \sin\theta + \mu_k \cos\theta]$

retardation of a body up a rough inclined plane may be greater than 'g'

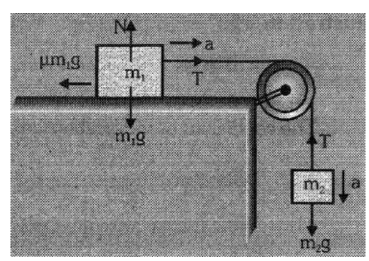
**22.7 Pulling with friction between block and surface**

**Case-I:** For mass $m_1$ : $T - \mu m_1 g = m_1 a$

For mass $m_2$ : $m_2 g - T = m_2 a$

on solving,

$$\text{Acceleration } a = \frac{(m_2 - \mu m_1)g}{(m_1 + m_2)} \Rightarrow T = \frac{m_1 m_2 (1+\mu)g}{(m_1 + m_2)}$$



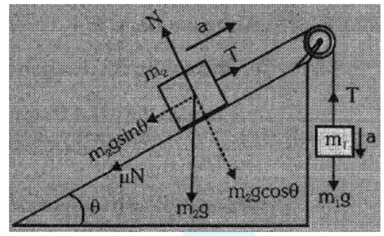

**Case-II :** For mass $m_1$ : $m_1 g - T = m_1 a$ and $N = m_2 g \cos\theta \Rightarrow \mu N - \mu m_2\, g \cos\theta$

For mass $m_2$ : $T - \mu m_2 g \cos\theta - m_2 g \sin\theta = m_2 a$

on solving.

Acceleration $a = \left[\frac{m_1 - m_2(\sin\theta + \mu\cos\theta)}{(m_1 + m_2)}\right] g$

Tension $T = \frac{m_1 m_2 (1 + \sin\theta + \mu\cos\theta) g}{(m_1 + m_2)}$




## Illustrations

**Illustration 38.**

If the coefficient of friction between an insect and a hemispherical bowl is μ and the radius of the bowl is r, find the maximum height to which the insect can crawl up in the bowl.

**Solution:**

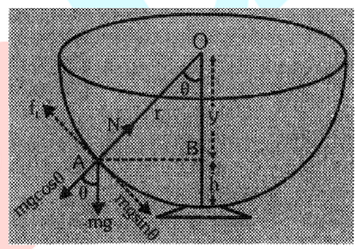
The insect will crawl up the bowl till the component of its weight along the bowl is balanced by limiting frictional force. So, resolving weight perpendicular to the surface of bowl and along the surface of bowl,

$N = mg\cos\theta$ .....(i)

$f_L = mg\sin\theta$ ….(ii)

Dividing (ii) by (i),

$\tan\theta = \frac{f_L}{N} = \frac{\mu N}{N} = \mu$




$\frac{AB}{OB} = \mu \quad \Rightarrow \quad \frac{\sqrt{r^2 - y^2}}{y} = \mu$

$\Rightarrow \quad r^2 - y^2 = \mu^2 y^2 \quad \Rightarrow \quad y^2(1 + \mu^2) = r^2$

$\Rightarrow \quad y = \frac{r}{\sqrt{1+\mu^2}}$

So $\quad h = r - y = r\left[1 - \frac{1}{\sqrt{1+\mu^2}}\right]$

**Illustration 39.**

A block of mass 2 kg slides down an inclined plane which makes an angle 30° with the horizontal. The coefficient of friction between the block and the surface is $\frac{\sqrt{3}}{2}$.

(i) What force must be applied to the block so that it moves down the plane without acceleration?

(ii) What force should be applied to the block so that it moves up without any acceleration?

**Solution:**

Make a 'free-body' diagram of the block. Take the force of friction opposite to the direction of motion.

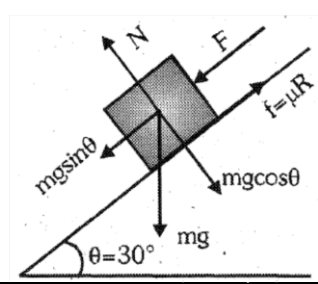
(i) Project forces along and perpendicular to the plane. Perpendicular to plane N=mg cosθ

Along the plane $F + mg\sin\theta - f = 0$

(Θ there is no acceleration along the plane)

$F + mg\sin\theta - \mu N = 0 \Rightarrow F + mg\sin\theta = \mu mg\cos\theta$

$F = mg(\mu\cos\theta - \sin\theta) = 2 \times 9.8\left(\frac{\sqrt{3}}{2}\cos 30^\circ - \sin 30^\circ\right)$



$$= 19.6\left(\frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}-\frac{1}{2}\right) = 19.6\left(\frac{3}{2}-\frac{1}{2}\right) = 4.9 \text{ N}$$

(ii) This time the direction of F is reversed and that of the frictional force is also reversed.

$\therefore$ N = mg cosθ; F = mg sinθ + f

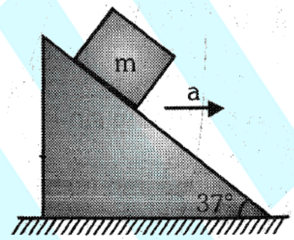
$$\Rightarrow F = mg(\mu\cos\theta + \sin\theta) = 19.6\left(\frac{3}{4}+\frac{1}{2}\right) = 24.5 \text{ N}$$

**Illustration 40.**

A block of mass 1 kg rests on an incline as shown in figure.

(a) What must be the frictional force between the block and the incline if the block is not to slide along the incline when the incline is accelerating to the right at 3 $m/s^2$ ?

(b) What is the least value of $\mu_s$ which can have for this to happen ?




**Solution:**

N = m (g cos 37° + a sin 37°) = 1(9.8 × 0.8 + 3 × 0.6) = 9.64 N

mg sin 37° = ma cos 37° + f

(a) f = 1(9.8 × 0.6 – 3 × 0.8) = 3.48

(b) Θ f = μN $\quad\therefore\ \mu = \frac{f}{N} = \frac{3.48}{9.64} = 0.36$

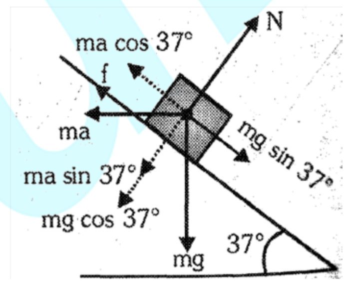



**Illustration 41.**

A body placed on a tough inclined plane just begins to slide. Calculate the coefficient of friction if the gradient (slope) of the plane is 1 in 4.

**Solution:**

Here $\sin\theta = \frac{1}{4}$

So, $\tan\theta = \frac{1}{\sqrt{15}} \quad\Rightarrow\quad \mu = \tan\theta = \frac{1}{\sqrt{15}} = 0.258$

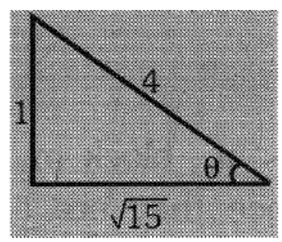
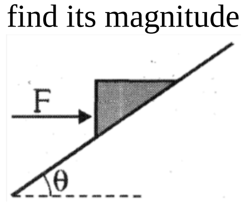


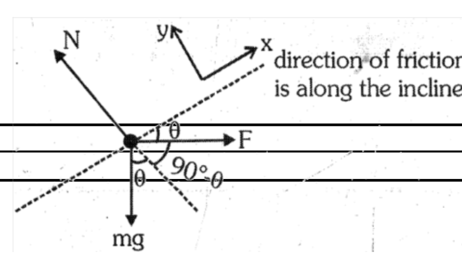

**Illustration 42.**

A block rests on a rough inclined plane as shown in fig. A horizontal force F is applied to it (a) Find the force of normal reaction, (b) Can the force of friction be zero, if yes when? and (c) Assuming that friction is not zero find its magnitude and direction of its limiting value.




**Solution:**

(a) $\sum F_y = 0 \rightarrow \qquad N = mg\cos\theta + F\sin\theta$



(b) $\sum F_x = 0 \rightarrow$ $F\cos\theta = mg\sin\theta \Rightarrow F = mg\tan\theta$

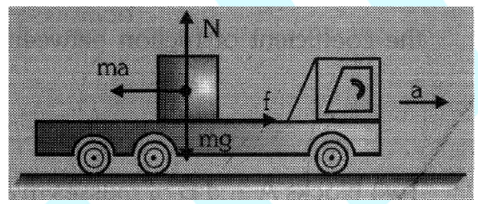
(c) Limiting friction $f_{sm} = \mu N = m\,(mg\cos\theta + F\sin\theta)$;

It acts down the plane if body has tendency to slide up and acts up the plane if body has tendency to slide down.

**Illustration 43.**

A block of mass 1kg lies on a horizontal surface of a truck; the coefficient of static friction between the block and the surface is 0.6, What is the force of friction on the block, if the acceleration of the truck is 5 m/s$^2$.

**Solution:**




Fictitious force {pseudo force) on the block opposite to the acceleration of the block

$F = ma = 1 \times 5 = 5N$

While the limiting friction force

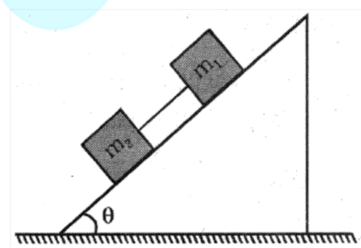
$f_L = \mu_s N = \mu_S mg$

$= 0.6 \times 1 \times 9.8 = 5.88$ newton

As applied force F is less than the limiting friction force, the block will remain at rest in the truck and the force of friction will be equal to 5 N and in the direction of acceleration of the truck.

**Illustration 44.**

Two blocks with masses $m_1 = 1$ kg and $m_2 = 2$ kg connected by a string, slide down an plane inclined at an angle $\theta = 45°$with the horizontal. The coefficient of sliding friction between $m_1$ and plane is $\mu_1 = 0.4$ and that between $m_2$ and plane is $\mu_2 = 0.2$. Calculate the common acceleration of the two blocks and the tension in the string.




**Solution:**

As $\mu_2 < \mu_l$, block $m_2$ has greater acceleration than $m_1$ if we separately consider the motion of blocks. But since they are connected they move together as a system with common acceleration.

So acceleration of the blocks is :

$$a = \frac{(m_1+m_2)g\sin\theta - \mu_1 m_1 g\cos\theta - \mu_2 m_2 g\cos\theta}{m_1+m_2}$$

$$= \frac{(1+2)(10)\left(\frac{1}{\sqrt{2}}\right) - 0.4\times1\times10\times\frac{1}{\sqrt{2}} - 0.2\times2\times10\times\frac{1}{\sqrt{2}}}{1+2} = \frac{22}{3\sqrt{2}}\ \text{m/s}^2$$

**For block of mass $m_2$ :** $m_2 g\sin\theta - \mu_2 m_2 g\cos\theta - T = m_2 a \Rightarrow T = m_2 g\sin\theta - \mu_2 m_2 g\cos\theta - m_2 a$

$$= 2\times10\times\frac{1}{\sqrt{2}} - 0.2\times2\times10\times\frac{1}{\sqrt{2}} - 2\times\frac{22}{3\sqrt{2}} = \frac{4}{3\sqrt{2}}\text{N}$$

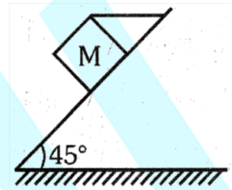
## BEGINNER'S BOX - 7

**1.** A cubical block rests on a plane of $\mu = \frac{1}{\sqrt{3}}$ determine the angle through which the plane be inclined to the horizontal, so that the block just slides down.

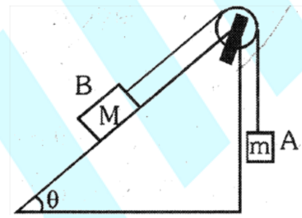
**2.** A body is in limiting equilibrium on a rough inclined plane .at angle of 30° with horizontal. Calculate the acceleration with which the body will slide down, when inclination of the plane is changed to 60°. (Take $g = 10\ m/s^2$)

**3.** A weight W rests on rough horizontal plane. If the angle of friction be θ, then calculate the least horizontal force that will move the body along the plane.

**4.** A block of mass 15 kg is resting on a rough inclined plane as shown in figure. The block is tied up by a horizontal string having a tension of 50 N. Calculate the coefficient of friction between the block and the inclined plane.

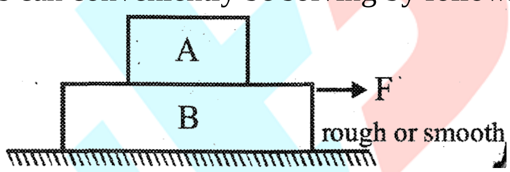



**5.** Two blocks A and B of masses in m and M are connected to the two ends of a string passing over a pulley. B lies on plane inclined at an angle θ with the horizontal and A is hanging freely as shown. The coefficient of static friction between B and the plane is $\mu_S$. Find the minimum and maximum values of m so that the system is at rest.




**23. TWO BLOCKS SYSTEM IN FRICTION**

Consider two blocks A and B placed one above the other, resting on a horizontal surface. A horizontal force F applied on either blocks, tend to move the system of blocks. Problems involving such situations can conveniently be solving by following mentioned step.




**Step 1:** Draw the FBD of the combined block system. If friction appears in the FBD, then take its limiting value (maximum static friction)

If applied force > limiting friction then motion is possible, otherwise not.

If movement occurs then, either the blocks move together or separately depending on the fact that whether frictional forces are able to support the combined motion or not.

**Step 2:** Assuming combined motion, find the common acceleration $a_c$ Draw the FBD of the body on which external force is not applied. Find the frictional force f required to make it move combinedly with the other block. Compare the above calculated force with the limiting value $f_L$ (maximum static friction).

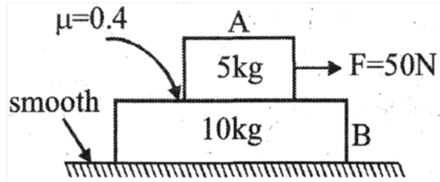
If $f \le f_L$, then both move together with common acceleration $a_c$. Otherwise, they move separately.

**Step 3:** For separate motion, draw the individual FBD's of both blocks with kinetic friction forces acting wherever applicable. Find the individual accelerations of the two blocks using Newton's second law.

## Illustrations

**Illustration 45.**

Calculate the accelerations of the blocks and the force of friction between them.

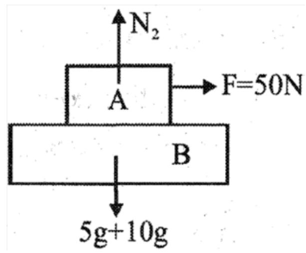



**Solution:**

**Step 1:** Draw FBD of the combined blocks system.

Obviously, there is an unbalanced force (F = 50N) in the horizontal direction.

∴ Movement occurs.




**Step 2:** Assuming that the blocks move together with common acceleration

$$a_c = \frac{50}{5+10} = 3.3.\ 3\ m/s^2$$

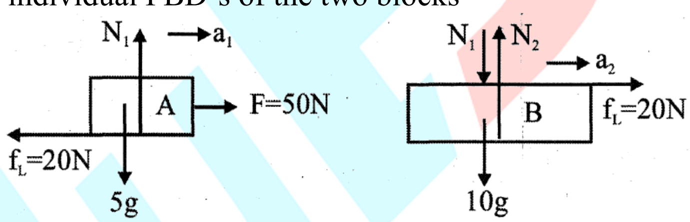
draw the FBD of block B (on which external force is not applied)

Required frictional force $f = m_B a_c = 10 \times 3.33 = 33.3$ N

Now limiting friction (maximum available static friction) $f_L = \mu N_1 = 0.4 \times 5 \times 10 = 20$ N

Obviously, f exceeds $f_L$. ∴ The two blocks move separately.

**Step 3:** Draw the individual FBD's of the two blocks



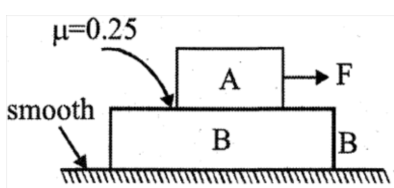

Applying Newton's II law, in the horizontal direction

$$a_1 = \frac{15-20}{5} = 6\ m/s^2 \text{ and } a_2 = \frac{20}{10} = 2\ m/s^2$$

Force of friction between the blocks = 20 N

**Illustration 46.**

In the figure shown $m_A$ = 10 kg, $m_B$ = 15 kg. Find the maximum value of F, below which the blocks move together.




**Solution:**

Assuming that both blocks move together, their common acceleration is $a_C = \frac{F}{10+15} = \frac{F}{25}$

FBD of block B (on which no externally applied force acts).

The required friction force f is equal to $f = m_B a_C = \frac{15F}{25} = \frac{3F}{5}$

Now, maximum static friction available is $f_L = \mu N_1 = 0.25(100) = 25$ N (here $N_1 = m_A g = 100N$)

$$\therefore\ f \le f_L \Rightarrow \frac{3F}{5} \le 25 \Rightarrow F \le \frac{125}{3} N \Rightarrow F_{max} = \frac{125}{3} N$$

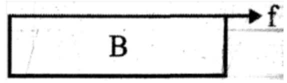
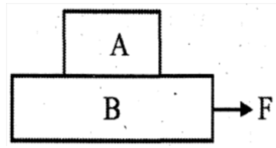



**Illustration 47.**

For the figure shown, $m_A = 10$ kg, $m_B = 20$ kg. F = 90 N. Find the accelerations of the two blocks and the frictional force between them.




**Solution:**

**Step 1:**Draw the FBD of the two block system combindly. Obviously there is an unbalanced horizontal force F = 90 N, so motion begins.

**Step 2:**Assuming bot the blocks to move together, their common acceleration is

$$a_C = \frac{F}{m_A + m_B} = \frac{90}{10+20} = 3\ m/s^2$$

Draw the FBD of block A (the body on which no externally applied force acts).

The required frictional force $f = m_A a_C = 10 \times 3 = 30$ N

Now, maximum static friction (limiting friction) available is $f_L = \mu N_1 = 0.4 \times 10g = 40N$

Obviously, $f < f_L$, so both move together with a common acceleration = $3 m/s^2$. The force of friction between them = 30 N.

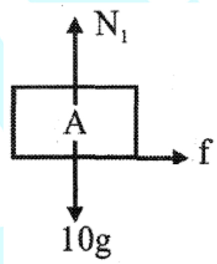
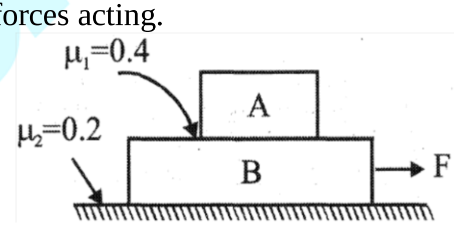



**Illustration 48.**

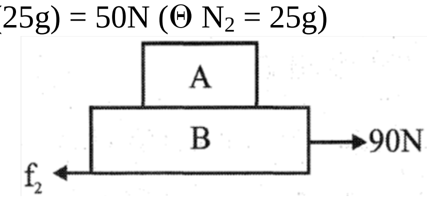
For the figure shown $m_A = 10$ kg, $m_B = 15$ kg and F = 90 N. Find the accelerations of the blocks and the frictional forces acting.




**Solution:**

**Step 1:**Draw the FBD of the combined blocks system.

$(f_2)_L = \mu_2 N_2 = 0.2\ (25g) = 50N\ (\because N_2 = 25g)$



Since 90 N > 50N, net unbalanced forces appear and hence movement begins.

**Step 2:** Assuming that both the blocks move together, their combined acceleration is

$a_C = \frac{90-50}{25} = 1.6 \text{ m/s}^2$

Draw the FBD of block A (on which externally applied force does not act).

The force required is $f_1 = m_A a_C = 10 \times 1.6 = 16$ N

Now, $(f_1)_L = \mu_1 N_1 = 0.4\ (10g) = 40$ N. Clearly, $f_1 < (f_1)_L$

$\therefore$ The frictional force is strong enough to support the combined motion.

$\therefore$ common acceleration is $a_C = 1.6 \text{ m/s}^2$ and $f_1 = 16$ N and $f_2 = 50$ N

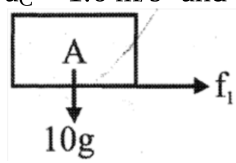
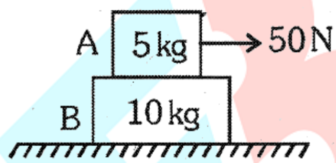



**Illustration 49.**

Blocks A and B of masses 5 kg and 10 kg are placed as shown in figure. If block A is pulled with 50 N. Find out the accelerations of A and B. If coefficient of friction between A and B is 0.5 and between B and ground is 0.4.




**Solution:**

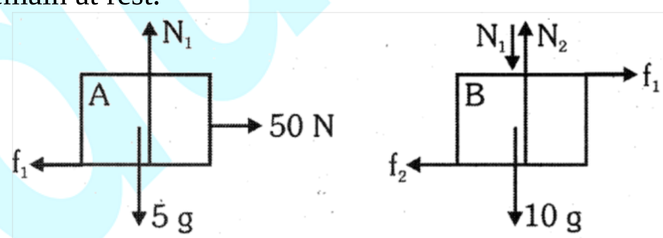
FBDs of blocks :

Limiting friction between A and B, $f_{1L} = \mu_1 N_1 = 0.5 \times 5g = 24.5$ N

Limiting friction between B and ground $f_{2L} = \mu_2 N_2 = 0.4 \times 15g = 58.8$ N

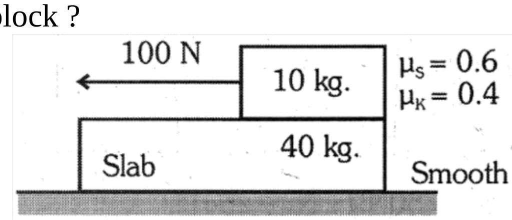
For block A; $50 - f_1 = 5 \times a \Rightarrow 50 - 24.5 = 5 \times a \Rightarrow a = 5.1 \text{ m/s}^2$

For block B accelerating force $f_1 = 24.5$ N is less than the limiting friction $f_2 = 58.8$ N so block B will remain at rest.



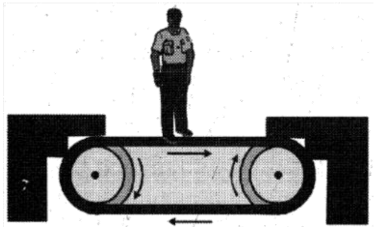

**BEGINNER'S BOX - 8**

**1.** If 100 N force is applied to the 10 kg block as shown in diagram. What is the acceleration produced for slab and block ?



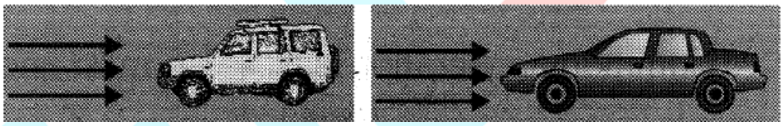
**2.** A 3 kg block (A) is placed on a 6 kg block (B) which rests on a table. Coefficient of friction between (A) and (B) is 0.3 and between (B) and table is 0.6. A 30 N horizontal force is applied on the block (B), then calculate the frictional force between the blocks (A) and (B).

**3.** Figure shows a man standing stationary with respect to a horizontal conveyor belt which is accelerating with 1 $m/s^2$. What is the net force on the man?
If the coefficient of static friction between the man's shoes and the belt is 0.2, upto what acceleration of the belt can the man continue to remain stationary relative to the belt? (Mass of the man = 65 kg)



**24. METHODS OF REDUCING RRICTION**

- By polishing the surface. (But extreme polishing increase friction)
- By lubrication.
- By proper selection of materials.
- By avoiding moisture
- By use of alloys
- By streamlining the shape
- By using ball bearings or roller bearings



**25. ADVANTAGES AND DISADVANTAGES OF FRICTION**

**Disadvantages**

(i) A significant amount of energy of moving objects is wasted in the form of heat energy to overcome the force of friction.
(ii) Friction restricts the speed of moving vehicles like buses, trains, aeroplanes, rockets etc.
(iii) The efficiency of machines decrease due to the presence of force of friction.
(iv) Friction causes lot of wear and tear in the moving parts of a machine.
(v) Sometimes, machines gets burnt due to the friction between different moving parts.

**Advantages**

(i) The force of friction helps us to move on the surface of earth. In the absence of friction, we cannot think of walking on the surface. That is why, we fall down while moving on a smooth surface.
(ii) The force of friction between the tip of a pen and the surface of paper helps us to write on the paper. It is not possible to· write on a glazed paper as there is no force of friction.
(iii) The force of friction between the tyres of a vehicle and the road aids the vehicle to stop when brakes are applied. In the absence of friction, the vehicle skids off the road after brakes are applied.

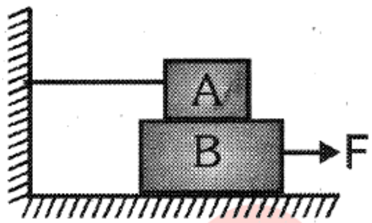
(iv) Moving belts remain on the rim of a wheel because of friction.

(v) The force of friction between a chalk and a black board helps us to write on the board. Thus, we observe that inspite of various disadvantages associated with friction, it is very difficult to part with it. Hence, it is commonly said that, friction is. a necessary evil.

## Illustration

**Illustration 50.**

A is a 100 kg block and B is a 200 kg block. As shown in figure, block A is attached to a string tied to a wall. The coefficient of friction between A and B is 0.2 and the coefficient of friction between B and floor is 0.3. Then calculate the minimum force required to move the block B. (take g =10 $m/s^2$).




**Solution:**

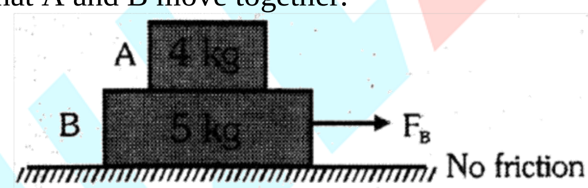
When B is made to move, by applying a force F, the frictional forces acting on it are $f_1$ and $f_2$ with limiting values, $f_1 = (\mu_S)_A g$ and $f_2 = (\mu_S)_B (m_A + m_B)g$

Then minimum value of F should be (such as to overcome these limiting values),

$F_{min} = f_l + f_2 = 0.2 \times 100g + 0.3 \times 300g = 110g = 1100$ N

**Illustration 51.**

12 N of fore required to be applied on A to slip on B. Find the maximum horizontal force F to be applied on B so that A and B move together.




**Solution:**

Let $\mu$ be friction coefficient between A and B.

As 12 N force on A is required for slipping so $\dfrac{\mu m_A g}{m_A} = \dfrac{12}{m_A + m_B} \Rightarrow \mu = \left(\dfrac{12}{m_A + m_B}\right)\left(\dfrac{m_B}{m_A g}\right) = \dfrac{1}{6}$

Maximum force ($F_B$) required to be applied on B so that A & B move together :

$F_B = (m_A + m_B)a$ where $a = \mu g \Rightarrow F_B = (m_A + m_B)\,\mu g = (4 + 5)\,(1/6)\,(10) = 15$ N

## ANSWERS

### BEGINNER'S BOX - 1

| | | | | | |
|---|---|---|---|---|---|
| **1.** | 4 $cm/s^2$ **2.** 6 s | **3.** | 0.18N, in the direction of motion | | |
| **4.** | a = 2$m/s^2$, 37° from 8N | **5.** | 23.89 Ns, 238.9 N | **6.** | 240 |
| **7.** | 93.3N | **8.** | $3\times10^{-5}$Ns, $6\times10^{-3}$m/s | **9.** | 5 s, 3Ns |
| **10.** | 3.18 Ns | **11.** | $5.0\times10^{-3}$N-s | | |

### BEGINNER'S BOX - 2

**1.** (i) 20M, (ii) 10M, (iii) 5M

**2.** 1 $ms^{-2}$, (i) 8N (ii) 5 N

**3.** (i) (P/5m) (ii) P/5 (iii) 2P/5 (iv) 3P/5 (v) 4P/5

## BEGINNER'S BOX - 3

**1.** 2 : 5 : 10 **2.** $T = \frac{M(L-Y)}{L}g$ **3.** 30N, 50N **4.** T = F(x/L)

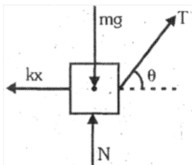
**5.** $T = F_1 + (F_2 - F_1)(x/L)$ **6.** 25 N **7.** $T = \frac{Pm_2}{m_1 + m_2}$

**8.** 5N, 6N, 7N **9.** 14 N

## BEGINNER'S BOX - 4

**1.** $\frac{g}{3}, \frac{20g}{3}, \frac{40g}{3}$ **2.** $\frac{10}{3}$ kg-wt **3.** $T_1 = \frac{8g}{3}$, $T_2 = \frac{4g}{3}$

**4.** $\frac{g}{3}, \frac{10g}{3}, \frac{10\sqrt{2g}}{3}$ **5.** 2 m/s$^2$, 60N **6.** $a_{4\,kg} = 2.5$ m/s$^2$, $a_{8\,kg} = 0$

**7.** (i) 

, (ii) mg cosecθ, (iii) $\frac{mg\cot\theta}{K}$

## BEGINNER'S BOX - 5

**1.** $T_{OB} = 200N$, $T_{OA} = 100\sqrt{3}$ N and $T_{OW} = 100N$

**2.** $a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)(g + a_0)$, $T = \left(\frac{2m_1m_2}{m_1 + m_2}\right)(g + a_0)$

## BEGINNER'S BOX - 6

**1.** 15N, 25N, and 20N **2.** 25N **3.** 1 N **4.** 0.0816

**5.** $\frac{22}{23}$ m/s$^2$, 27.13 N **6.** $\frac{g}{\mu}$

**7.** For normal plane surfaces the coefficient of friction is less than unity. But when the surfaces are so irregular that sharp minute projections and cavities exist in the surfaces, the coefficient of friction may be greater than one.

**8.** When a surface is polished beyond a certain limit, the molecules of both surfaces come closer to each other to such an extent that inter molecular forces become appreaciable, which exert strong attractive forces on each other. This is called surface adhesion. To overcome these forces, additional force, is required. Hence the frictional force increases..

**9.** Water on a muddy road provides a thin layer in between our feet and road. This layer breakes the interlocking and decreases the friction.

**10.** False, when a person walks on a rough surface the man exerts a backward frictional force on the surface. As a result, the surface exerts a forward frictinal force, according to Newton's III law.

## BEGINNER'S BOX - 7

**1.** 30° **2.** $\frac{10}{\sqrt{3}}$ m/s$^2$ **3.** W tan θ **4.** $\mu = \frac{1}{2}$

**5.** $m_{min} = M(\sin\theta - \mu\cos\theta)$ & $m_{max} = M(\sin\theta + \mu\cos\theta)$

## BEGINNER'S BOX - 8

**1.** 0.98 m/s$^2$, 6.08 m/s$^2$ **2.** Zero **3.** 65 N; 1.96 m/s$^2$