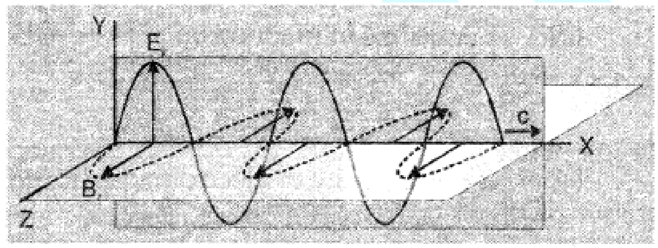
ELECTROMAGNETIC WAVES

INDUCTION

A changing electric field produces a changing magnetic field and vice versa which gives rise to a transverse wave known as electromagnetic waves. The time varying electric field and magnetic field mutually perpendicular to each other also perpendicular to the direction of propagation.

Thus the electromagnetic waves consist of sinusoidally time varying electric and magnetic field acting at right angles to each other as well as at right angles to the direction of propagation.

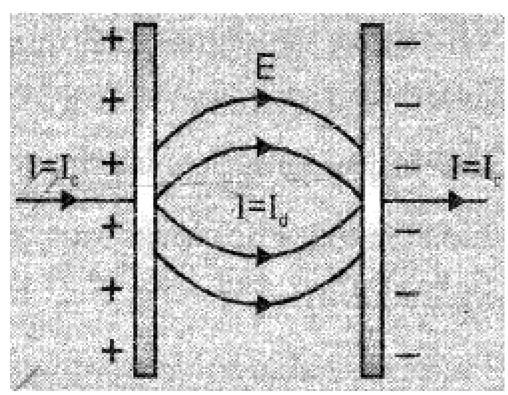


HISTORY OF ELECTROMAGNETIC WAVES

- In the year 1865, Maxwell predicted the electromagnetic waves theoretically. According to him, an accelerated charge sets up a magnetic field in its neighborhood.
- In 1887, Hertz produced and detected electromagnetic waves experimentally at wavelength of about 6m .
- Seven year later, J.C. Bose became successful in producing electromagnetic waves of wavelength in the range 5mmto25mm.
- In 1896, Marconi discovered that if one of the spark gap terminals is connected to an antenna and the other terminal is earthed, the electromagnetic waves radiated could go upto several kilometers.
- The antenna and the earth wires from the two plates of a capacitor which radiates radio frequency waves. These waves could be received at a large distance by making use of an antenna earth system as detector.
- Using these arrangements, in 1899 Marconi first established wireless communication across the English channel i.e., across a distance of about 50 km.

CONCEPT OF DISPLACEMENT CURRENT

When a capacitor is allowed to charge in an electric circuit, the current flows through connecting wires. As capacitor charges, charge accumulates on the two plates of capacitor and as a result, a changing electric field is produced across between the two plate of the capacitor.



According to maxwell changing electric field intensity is equivalent to a current through capacitor that current is known as displacement current (I_d). If + q and – q be the charge on the left and right plates of the capacitor respectively at any instant if σ be the surface charge density of plate of capacitor the electric field between the plate is given by

$$E = \frac{\sigma}{\epsilon_0} = \frac{q}{\epsilon_0 A}$$

charge on the plates of the capacitor increased by dq in time dt then dq = I dt

change in electric field is $dE = \frac{dq}{\epsilon_0 A} = \frac{Idt}{\epsilon_0 A} \Rightarrow \frac{dE}{dt} = \frac{I}{\epsilon_0 A}$

$$I = \epsilon_0 A \frac{dE}{dt} = \epsilon_0 \frac{d}{dt}(EA) = \epsilon_0 \frac{d\phi_E}{dt} \qquad (\because \phi_E = EA) \qquad \boxed{I_d = \epsilon_0 \frac{d\phi_E}{dt}}$$

The conduction cogent is the current due to the flow of charges in a conductor and is denoted **as Ic and displacement current** is the current due to changing electric field between the plate of the capacitor and denoted as Id so the total current I is sum of I_c and I_d i.e. $I = I_c + I_d$
Ampere's circuital law can be written as

$$\oint \vec{B}.\vec{dl} = \mu_0(I_c + I_d) \quad \Rightarrow \quad \oint \vec{B}.\vec{dl} = \mu_0\left(I_c + \epsilon_0 \frac{d\phi_E}{dt}\right)$$

MAXWELL'S EQUATION

There are four Maxwell's equations are given below

(1) Gauss law in electrostatics: $\oint \vec{E}.\vec{ds} = \frac{q}{\epsilon_0}$ …(i)

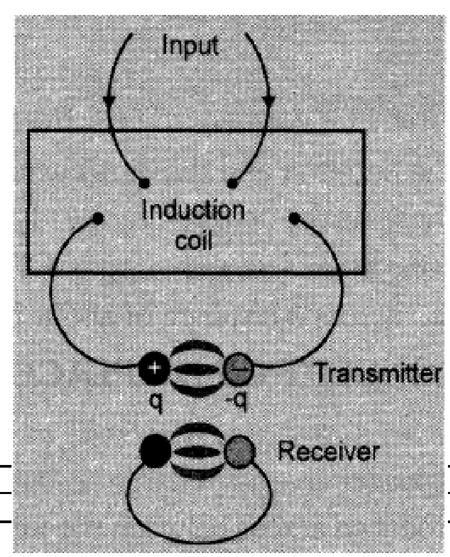
(2) Gauss law in magnetism : $\oint \vec{B}.\vec{ds} = 0$ …(ii)

(3) Faraday's law of electromagnetic induction : emf = $\oint \vec{E}.\vec{dl} = -\frac{d\phi_B}{dt}$ …(iii)

(4) Maxwell- Ampere's circuital law: $\oint \vec{B}.\vec{dl} = \mu_0\left[I_c + \epsilon_0 \frac{d\phi_E}{dt}\right]$ …(iv)

HERTZ EXPEIDMENT: (Practical production of EM waves)

- In 1888, Hertz demonstrated the production of electromagnetic waves by oscillating charge. His experimental apparatus is shown schematically in fig.



- An induction coil is connected to two spherical electrodes with a narrow gap between them. It acts as a transmitter. The coil provides short voltage surges to the spheres making one positive and the other negative. A spark is generated between the spheres when the voltage between them reaches the breakdown voltage for air. As the air in the gap is ionised, it conducts more rapidly and the discharge between the spheres becomes oscillatory.

- The above experimental arrangement is equivalent to an LC circuit, where the inductance is that of the loop and the capacitance is due to the spherical electrodes.
- Electromagnetic waves are radiated at very high frequency (≈ 100 MHz) as a result of oscillation of free charges in the loop.
- Hertz was able to detect these waves using a single loop of wire with its own spark gap (the receiver).
- Sparks were induced across the gap of the receiving electrodes when the frequency of the receiver was adjusted to match that of the transmitter.

PROPERTIES OF ELECTROMAGNETIC WAVES

- The electric and magnetic fields satisfy the following wave equations, which can be obtained from Maxwell's third and fourth equations.

$$\frac{\partial^2 E}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} \quad \text{and} \quad \frac{\partial^2 B}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2}$$

- Electromagnetic waves travel through vacuum with the speed of light c, where

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \, m/s$$

- The electric and magnetic fields of an electromagnetic wave are perpendicular to each other and also perpendicular to the direction of wave propagation. Hence, these are transverse waves.
- The instantaneous magnitudes of $\vec{E}$ and $\vec{B}$ in an electromagnetic wave are related by the expression.

$$\frac{E}{B} = c$$

- Electromagnetic waves carry energy. The rate of flow of energy crossing a unit area is described by the Poynting vector $\vec{S}$. where $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$
- Electromagnetic waves carry momentum and hence can exert pressure (P) on surfaces, which is known as radiation pressure. For an electromagnetic wave with Poynting vector $\vec{S}$, incident upon a perfectly absorbing surface $P = \frac{S}{c}$

  and if incident upon a perfectly reflecting surface $P = \frac{2S}{c}$
- The electric and magnetic fields of a sinusoidal plane electromagnetic wave propagating in the positive x-direction can also be written as

$$E = E_m \sin(kx - \omega t) \quad \text{and} \quad B = B_m \sin(kx - \omega t)$$

  where ω is the angular frequency of the wave and k is wave number which are given by

$$\omega = 2\pi f \qquad \text{and} \qquad k = \frac{2\pi}{\lambda}$$

- The intensity of a sinusoidal plane electro-magnetic wave is defined as the average value of Poynting vector taken over one cycle.

$$S_{av} = \frac{E_m B_m}{2\mu_0} = \frac{E_m^2}{2\mu_0 c} = \frac{c}{2\mu_0} B_m^2$$

- The fundamental sources of electromagnetic waves are accelerating electric charges. For examples radio waves emitted by an antenna arises from the continuous oscillations (and hence acceleration) of charges within the antenna structure.
- Electromagnetic waves obey the principle of superposition.
- The electric vector of an electromagnetic field is responsible for all optical effects. For this reason electric vector is also called a light vector.

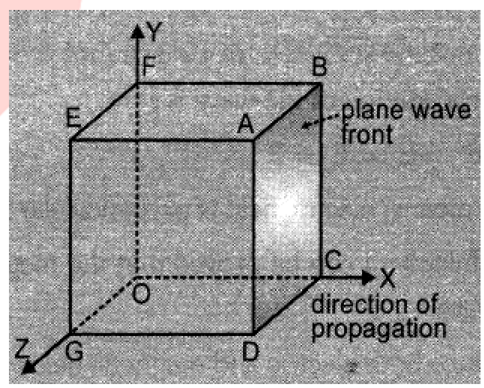
TRANSVERSE NATURE OF ELECTROMAGNETIC WAVES

Maxwell showed that a changing electric field produces a changing magnetic field and vice-versa. This alternate production of time 'varying electric and magnetic fields gives rise to the propagation of electromagnetic waves.

The variation of electric field ($\vec{E}$) and magnetic field ($\vec{B}$) are mutually perpendicular to each other as well as the direction of the propagation of the wave i.e., the electromagnetic waves are transverse in nature.

Proof :

Consider a plane electromagnetic wave travelling along X-direction with its wave front in the Y-Z plane and ABCD is its portion at time t. The values of electric field and magnetic field to the left of ABCD will depend on x and t (and not on y and z as the wave under consideration is a plane wave propagating in x direction.



According to Gauss' law, the total electric flux across the parallelopiped' ABCDOEFG is zero because it does not enclose any charge. i.e. $\oint \vec{E}.\vec{dS} = 0$

or $\oint_{ABCD} \vec{E}.\vec{dS} + \oint_{EFOG} \vec{E}.\vec{dS} + \oint_{ADGE} \vec{E}.\vec{dS} + \oint_{BCOF} \vec{E}.\vec{dS} + \oint_{OCDG} \vec{E}.\vec{dS} + \oint_{FBAE} \vec{E}.\vec{dS} = 0$ ....(i)

since electric field $\vec{E}$ does not depend on y and z, so the contribution to the electric flux coming from the faces normal toy and z axes cancel out in pairs.

i.e. $\oint_{OCDG} \vec{E}.\vec{dS} + \oint_{FBAE} \vec{E}.\vec{dS} = 0$ ....(ii)

and $\oint_{ADGE} \vec{E}.\vec{dS} + \oint_{BCOF} \vec{E}.\vec{dS} = 0$ ....(iii)

Using equation (ii) and (iii) in equation (i), we get

$\oint_{ABCD} \vec{E}.\vec{dS} + \oint_{EFOG} \vec{E}.\vec{dS} = 0$ ....(iv)

Now $\oint_{ABCD} \vec{E}.d\vec{S} + \oint_{ABCD} E_x . dS\cos 0 = \oint_{ABCD} E_x dS = E_x \oint_{ABCD} dS$ $\quad$ ($\because$ $\vec{E}_x$ is parallel to $d\vec{S}$)

$= E_x \times$ area of face ABCD $= E_x S$ ….(v)

and $\oint_{EFOG} \vec{E}'.d\vec{S} = \oint_{EFOG} E'_x\, dS\cos 180° = E'_x \oint_{EFOG} dS$ $\quad$ ($\because$ E'_x is antiparallel to $d\vec{S}$)

$= E'_x \times$ area of face EFOG $= E'_x S$ ….(vi)

where, E_x and E'_x are the x-components of electric field on the faces ABCD and EFOG respectively.

Substituting the values of equations (v) and (vi) in equation (iv), we get

$E_x S - E'_x S = 0$ $\quad$ or $\quad$ $S(E_x - E'_x) = 0$

$\because$ $\quad S \neq 0$

$\therefore$ $\quad Ex - E'_x = 0$ $\quad$ or $\quad$ $\boxed{E'_x = E_x}$

This equation shows that the value of the x-component of electric field does not change with time. In other words, electric field along x-axis is static.

Since the static electric field cannot propagate the wave, hence the electric field parallel to the direction of the propagation of the wave is zero.

i.e. $\quad E'_x = E_x = 0$

It means, electric field is perpendicular to the direction of propagation of the wave.

similarly, it can be proved that the magnetic field is perpendicular to the direction of the propagation of the wave.

Since both electric and magnetic fields are perpendicular to the direction of the propagation of the wave, so electromagnetic wave is transverse in nature.

GOLDEN KEY POINTS

- When a capacitor is connected across the battery through the connecting wires there is flow of conduction current I_c while through the gap between the plates of capacitor, there is flow of displacement current I_d.
- Maxwell's equation are mathematical formaulation of (I) Gauss' law in electrostatics (II) Gauss' law in electromagnetism (III) Faraday's law of electromagnetic induction and (IV) Ampere's circuital law.
- Frequency of electromagnetic waves is its inherent characteristic when an electromagnetic wave travels from one medium to another, its wavelength changes but frequency remains unchanged.
- Ozone layer: absorbs the ultra-violet rays from the sun and these prevents them from producing harmful effect on living organisms on the earth. Further it traps the infra–red rays and prevents them from escaping the surface of earth. It helps to keeps the earth's atmosphere warm.

ILLUSTRATIONS

Illustration 1.

A point source of electromagnetic radiation has an average power output of 800W. The maximum value of electric field at a distance 3.5 m from the source will be-

Solution.

Intensity of electromagnetic wave given is by $I = \frac{P_{av}}{4\pi r^2} = \frac{E_m^2}{2\mu_0 c}$

$$E_m = \sqrt{\frac{\mu_0 c P_{av}}{2\pi r^2}} = \sqrt{\frac{(4\pi\times10^{-7})\times(3\times10^{8})\times800}{2\pi\times(3.5)^2}} = 62.6 \text{ V/m}$$

Illustration 2.

In the above problem, the maximum value of magnetic field will be –

Solution.

The maximum value of the magnetic field is given by $B_m = \frac{E_m}{c}\frac{62.6}{3\times10^8} = 2.09 \times 10^{-7}$ T

Illustration 3.

In an electromagnetic wave, the amplitude f electric field is I V /m. The frequency of wave is 5×10^{14} Hz. The wave is propagating along z-axis. The average energy density of electric field, in Joule/m^3, will be –

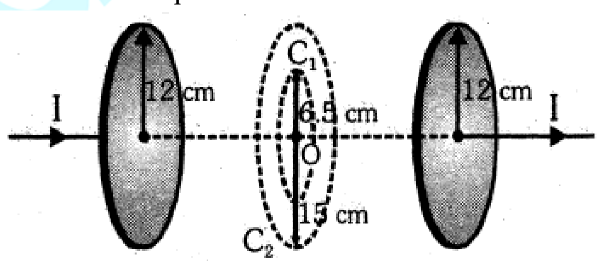
Solution.

Average energy density is given by

$$u_E = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0\left(\frac{E_0}{\sqrt{2}}\right)^2 = \frac{1}{4}\varepsilon_0 E_0^2 = \frac{1}{4}\times8.85\times10^{-12}\times(1)^2 = 2.2 \times 10^{-12} \text{ J/m}^2$$

Illustration 4.

Fig. shows a capacitor made of two circular plates, each of radius 12 cm, and separated by 5.0 mm. The capacitor is being charged by an external source (not shown in the figure). The charging current is constant and equal to 0.15 A.



(a) Calculate the capacitance and the rate of change of potential difference between the plates.

(b) Obtain the displacement current across the plates.

Solution.

(a) Area of one of the plates.

$A = \pi(12 \times 10^{-2}m)^2$

Distance between' the plates, d = 5.0 mm = 5×10^{-3} m

Capacitance of the capacitor, $C = \in_0 A/d$

or $$C = \left(\frac{1}{4\pi \times 9 \times 10^9} F/m\right) \times \frac{\pi(12 \times 10^{-2} m)^2}{5 \times 10^{-3} m}$$

$= 80 \times 10^{-12}$ F = 80 pF

Charging current, $I = \frac{dQ}{dt} = \frac{d}{dt}(CV)$

or $I = C\frac{dV}{dt}$ or $\frac{dV}{dt} = \frac{I}{C}$

Rate of change of potential difference $= \frac{dV}{dt} = \frac{I}{C}$

$$= \frac{0.15A}{80 \times 10^{-12} F} = 1.87 \times 10^9 \text{ V/s}$$

(b) Displacement current $I_d = \in_0 A\left(\frac{dE}{dt}\right)$

For a parallel-plate capacitor,

$$E = \frac{\sigma}{\in_0} = \frac{Q/A}{\in_0} = \frac{Q}{\in_0 A}$$

Where σ is surface density of charge.

Thus, $I_d = \in_0 A\frac{d}{dt}(E) = \in_0 A\frac{d}{dt}\left(\frac{Q}{\in_0 A}\right)$

$$= \frac{dQ}{dt} = I = 0.15A$$

(or simply, $I_d = I = 0.15$ A)

Illustration 5.

In a plane electromagnetic wave, the electric field oscillates sinusoidally at a frequency of 2.0×10^{10} Hz and amplitude 48 V /m,

(a) What is the wavelength of the wave ?

(b) What is the amplitude of the oscillating magnetic field ?

(c) Find the total average energy density of the electromagnetic field of the wave,

Solution.

We are given that;

$E_0 = 48$ V/m, $v = 2.0 \times 10^{10}$ Hz and $c = 3 \times 10^8$ V/m

(a) Wavelength of the wave,

$$\lambda = \frac{c}{v} = \frac{3\times10^{8}\,m/s}{2.0\times10^{10}\,s^{-1}} = 1.5\times10^{-2}\,m$$

(b) Amplitude of the oscillating magnetic field,

$$B_0 = \frac{E_0}{c} = \frac{48V/m}{3\times10^{8}\,m/s} = 1.6\times10^{-7}\,T$$

(c) Total average energy density,

$$u_{av} = \frac{1}{2}\in_0 E_0^2$$

$$= \frac{1}{2}(8.85 \times 10^{-12})\,(48)^2\ J/m^3 = 1.0 \times 10^{-8}\ J/m^3$$

Illustration 6.

A plane light wave in the visible region is moving along the Z-direction. The frequency of the wave is 0.5×10^{15} Hz and the electric field at any point is varying sinusoidally with time with an amplitude of 1 V /m. Calculate the energy densities of the electric and magnetic fields.

Solution.

Total average energy density (due to both electric and magnetic fields)

$$= \frac{1}{2}\in_0 E_0^2 = \frac{1}{2}(8.85 \times 10^{-12})\,(1)^2 = 4.42 \times 10^{-12}\ J/m^3$$

Since the energy is shared equally by the electric and magnetic fields,
average energy density of the electric field

$$= \frac{1}{2}(4.42 \times 10^{-12}\ J/m^3) = 2.21 \times 10^{-12}\ J/m^3$$

average energy density of the magnetic field

$$= \frac{1}{2}(4.42 \times 10^{-12}\ J/m^3) = 2.21 \times 10^{-12}\ J/m^3$$

Illustration

Radio receiver receives a message at 300m band, If the available inductance is 1 mH, then calculate required capacitance.

Solution

Radio recives EM wavs (velocity of EM waves $c = 3 \times 10^8$ m/s)

$$\therefore\ c = f\lambda \quad \Rightarrow \quad f = \frac{3\times10^{8}}{300} = 10^{6}\ Hz$$

$$\text{Now} \quad f = \frac{1}{2\pi\sqrt{LC}} = 1\times10^{6} \ \Rightarrow\ C = \frac{1}{4\pi^2\times10^{-3}\times10^{12}} = 25\ pF$$

Various parts of electromagnetic spectrum

S. No.	Radiation	Discover	How produced	Wavelength Range	Frequency range	Energy range	Properties	Application
1.	γ–Rays	Henry Becquerel and Madame Curie	Due to decay of radioactive nuclei.	10^{-14} m to 10^{-10} m	3×10^{22} Hz to 3×10^{-18}	10^{7} eV – 10^{4} eV	(a) High penetrating power (b) Unchanged (c) Low ionising power	(a) Gives information on nuclear structure (b) Medical treatment etc.
2.	X-Ray	Roentgen	Due to collisions of high energy electrons with heavy targets	6×10^{-12} m to 10^{-9} m	5×10^{19} Hz to 3×10^{17}Hz	2.4×10^{5} eV to 1.2×10^{3} eV	(a) Low Penetrating power (b) other properties similar to γ–rays except wavelength	(a) Medical diagnosis and treatment (b) Study of crystal structure (c) Industrial radiography
3.	Ultraviolet Rays	Ritter	By ionised gases, sun lamp spark etc.	6×10^{-10} m to 3.8×10^{-7} m	5×10^{17} Hz to 7×10^{14} Hz	2×10^{3} eV to 3eV	(a) All properties of light (b) Photoelectric effect	(a) To detect adulteration, writing and signature (b) Sterilization of water due to its destructive action on bacteria
4.	Visible light Subparts of visible spectrum (a) Violet (b) Blue (c) Green (d) Yellow (e) Orange (f) Red	Newton	Outer orbit electron, transitions in atoms, gas discharge tube, incandescent solids and liquid.	3.8×10^{-7} m to 7.8×10^{-7}m 3.9×10^{-7}m to 4.55×10^{-7}m 4.55×10^{-7}m to 4.92×10^{-7}m 4.92×10^{-7}m to 5.77×10^{-7}m 5.77×10^{-7}m to 5.97×10^{-7}m 5.97×10^{-7}m to 6.22×10^{-7}m 6.22×10^{-7}m to 7.80×10^{-7}m	8×10^{14} Hz to 4×10^{14} Hz 7.69×10^{14}Hz to 6.59×10^{14}Hz 6.59×10^{14}Hz to 6.10×10^{14}Hz 6.10×10^{14}Hz to 5.20×10^{14}Hz 5.20×10^{14}Hz to 5.03×10^{14} Hz 5.03×10^{14}Hz to 4.82×10^{14}Hz 4.82×10^{14}Hz to 3.84×10^{14}Hz	3.2 eV to 1.6 eV	(a) Sensitive to human eye	(a) To see objects (b) To study molecular structure
5.	Infra–Red waves	William Herschell	(a) Rearrangement of outer orbital electrons in atoms (b) Change E of molecular vibrational and rotational energies (c) By bodies at high temperature.	7.8×10^{-7} m to 10^{-3} m	4×10^{14} Hz to 3×10^{11} Hz	1.6eV to 10^{-3} eV	(a) Thermal effect (b) All properties similar to those of light except λ	(a) used in industry, medicine and astronomy (b) Used for fog or haze photography (c) Elucidating molecular structure.
6.	Microwaves	Hertz	Special	10^{-3} to 0.3 m	3×10^{11} Hz to 10^{9} Hz	10^{-3} ev to	(a)	(a) Radar and

			electronic devices such as klystron tube			10^{-5} eV	Phenomenon of reflection, refraction and diffraction	telecommunication. (b) Analysis of fine details of molecular structure
7. (A)	Radio waves Subparts of Radio–spectrum	Marconi	Oscillating circuits	0.3 to few kms.	10^9 Hz to few Hz	10^{-3} eV to ≈ 0	(a) Exhibit waves like properties more than particle like properties.	(a) Radio communication
(B)	Super High Frequency (a) SHF Ultra High Frequency (b) UHF Very High Frequency (c) VHF			0.01m to 0.1m 0.1 m to 1m 1m to 10 m	3×10^{10}Hz to 3×10^{9}Hz 3×10^{9}Hz to 3×10^{8}Hz 3×10^{8}Hz to 3×10^{7} Hz		Radar, Radio and satelite communication (Microwaves), Radar and Television broadcast short distance communication, Television communication	
	High Frequency (HF) Medium Frequency (MF) Low Frequency (LF) Very Low Frequency (VLF)			10m to 100 m 100 m to 1000 m 1000 m to 10000 m 10000 m to 30000 m	3×10^{7}Hz to 3×10^{6}Hz 3×10^{6}Hz to 3×10^{5} Hz 3×10^{5} Hz to 3×10^{4} Hz 3×10^{4} Hz to 10^{4} Hz		Medium distance communication Telephone communication, Marine and navigation use, long range communication. Long distance communication.	