Atomic Structure

VARIOUS MODELS FOR STRUCTURE OF ATOM

1. Dalton's Theory

Every material is composed of minute particles known as atom. Atom is indivisible i.e. it cannot be subdivided.

It can neither be created nor be destroyed.

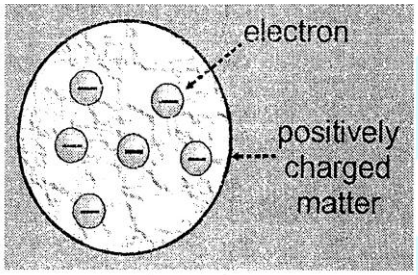
All atoms of same element are identical (Physically as well as chemically), whereas atoms of different elements are different in properties.

The atoms of different elements are comparable to hydrogen atoms. (The radius of the heaviest atom is about 10 times that of hydrogen atom and its mass is about 250 times that of hydrogen).

The atom is stable and electrically neutral.

2. Thomson's Atom Model

The atom as a whole is electrically neutral because the positive charge present on the atom (sphere) is equal to the negative charge of electrons present in the sphere.



Atom is a positively charged sphere of radius 10^{-10} m. in which electron are embedded in between. The positive charge and the whole mass of the atom is uniformly distributed throughout the sphere.

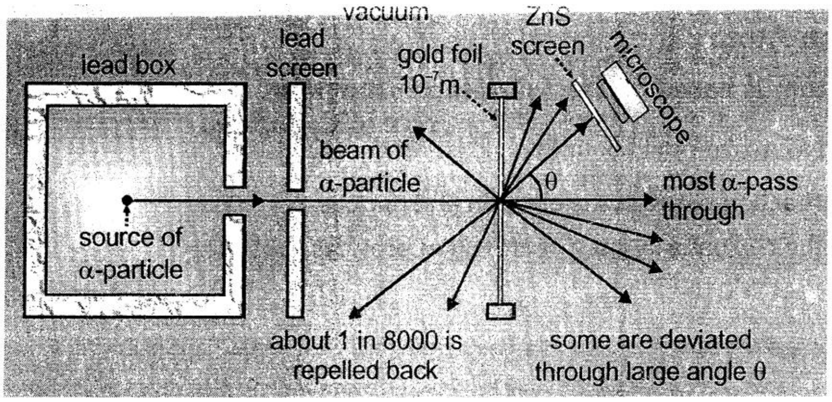
3. Shortcomings of Thomson's model

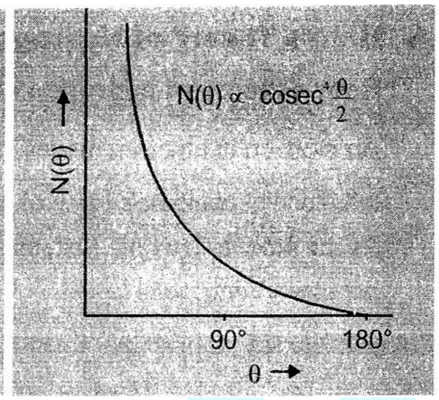
(i) The spectrum of atoms cannot be explained with the help of this model.

(ii) Scattering of α-particles cannot be explained with the help of this model.

4. Rutherford experiments on scattering of α-particles by thin gold foil

The experimental arrangement is shown in figure. α-particles are emitted by some radioactive material (polonium), kepi inside a thick lead box. A very fine beam of a-particles pass through a small hole in the lead screen. This well collimated beam is then allowed to fall on a thin gold foil. While passing through the gold foil, α-particles are scattered through different angles. A zinc sulphide screen was placed on the other side of the gold foil. This screen was movable, so as to receive the a-particles, scattered from the gold foil at angles varying from 0° to 180°. When an α-particle strikes the screen, it produces a flash of light and it is observed by the microscope. It was found that :





(i) Most of the α-particles went straight through the gold foil and produced flashes on the screen as if there were nothing inside gold foil. Thus the atom is hollow.

(ii) Few particles collided with the atoms of the foil which have scattered or deflected through considerable large angles. Few particles even turned back towards source itself.

(iii) The entire positive charge and almost whole mass of the atom is concentrated in small centre called a nucleus.

(iv) The electrons could not deflected the path of a α-particles i.e. electrons are very light.

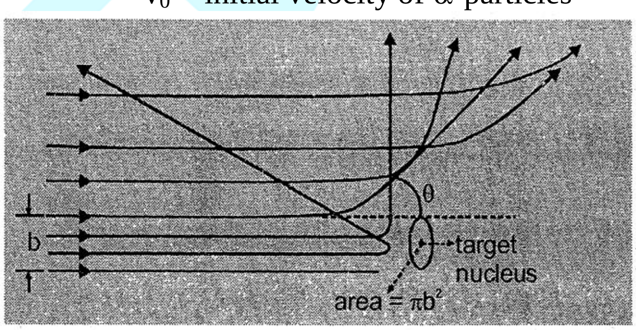
(v) Electrons revolve round the nucleus in circular orbits.

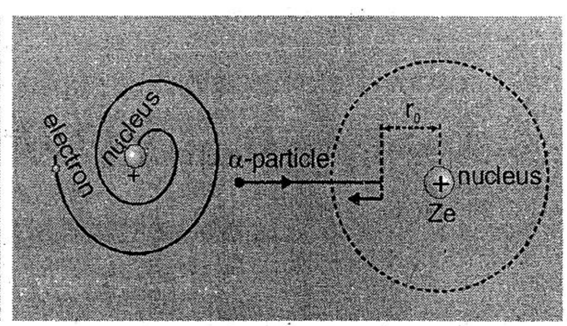
So, Rutherford 1911, proposed a new type of model of the atom. According to this model, the positive charge of the atom, instead of being uniformly distributed throughout a sphere of atomic dimension is concentrated in a very small volume (Less than 10^{-13} cm is diametre) at it centre. This central core, now called nucleus, is surrounded by clouds of electron makes the entire atom electrically neutral.

According to Rutherford scattering formula, the number of α-particle scattered at an angle e by a target are given by

$$N_\theta = \frac{N_0 nt(2Ze^2)^2}{4(4\pi\varepsilon_0)^2 r^2 (mv_0^2)^2} \times \frac{1}{\sin^4 \frac{\theta}{2}}$$

Where N_0 = number of α-particles that strike the unit area of the scatter
n = number of target atom per m^3
t = thickness of target
Ze = charge on the target nucleus
2e = charge on α-particle
r = distance of the screen from target
v_0 = initial velocity of α-particles





Now closest approach distance is $(r_0) = \frac{1}{4\pi\varepsilon_0} \times \frac{(2Ze)^2}{\left[\frac{1}{2}mv_0^2\right]} = \frac{1}{4\pi\epsilon_0}\frac{(2Ze)^2}{E_K}$

Where E_K = K.E. of α-particle

5. Failure of Rutherford's Atomic model :

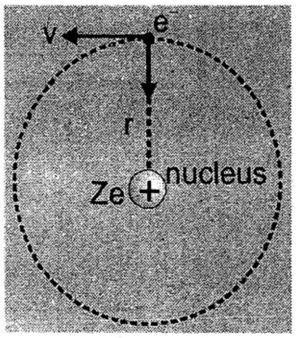
(i) It couldn't explain the stability of atom.

(ii) It couldn't explain discrete nature of hydrogen spectra.

6. Bohr's theory of Hydrogen Atom

Bohr's theory of hydrogen atom is based on the following assumption

- An electron in an atom moves in a circular orbit about the nucleus under the influence of coulomb's force of attraction between the electron and nucleus.



As the atom as a whole is stable the coulombian force of attraction provides necessary centripetal force:

$$\frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{mv^2}{r} \quad \text{.....(i)}$$

- Only those orbits are possible for which the angular momentum of the electron is equal to an integral multiple of $\frac{h}{2\pi}$ i.e.

$$mvr = n\frac{h}{2\pi} \quad \text{.....(ii)}$$

Where h is planck's constant.

- The electron moving in such allowed orbits does not radiate electromagnetic radiations. Thus the total energy of the electron revolving in any of the stationary orbits remains constant.
- Electromagnetic radiations are emitted if an electron jumps from stationary orbit of higher energy E_2 to another stationary orbit of lower energy, E_1. The frequency v of the emitted radiation is related by the equation.

$$E_2 - E_1 = hv \quad \text{..... (iii)}$$

7. Shortcomings of Bohr's model

- This model could not explain the fine structure of spectral lines, Zeeman effect and Stark effect.
- This model is valid only for single electron systems. (cannot explain electron-electron interaction)

- This model is based on circular orbits of electrons whereas in reality there is no orbit.
- Electron is presumed to revolve round the nucleus only whereas in reality motion of electron cannot be described.
- This model could not explain the intensity of spectral lines.
- It could not explain the doublets obtained in the spectra of some of the atoms.
- Bhor's model is semi quantum model, it means, it includes two quantum numbers (E and L) but unfortunately it consider circular motion of electron.

8. Merit of Bohr's model

Energy of electron obtained by it and quantum model are same.

CHARACTERISTICS OF BOHR MODEL

1. Radii of orbits

From equation $v = \frac{nh}{2\pi mr}$, Here n is number of orbit

Substituting value of v in equation $\frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{m}{r}\left[\frac{nh}{2\pi mr}\right]^2 \Rightarrow r = \frac{mn^2h^2.4\pi\varepsilon_0 r^2}{4\pi^2 m^2 r^2 e^2} = \frac{n^2h^2\varepsilon_0}{\pi me^2}$

In general $r_n = \frac{n^2h^2\varepsilon_0}{\pi me^2}$ .....(iv)

equation (iv) shows that the radii of the permitted orbits vary as the square of n. For the smallest orbit n = 1substituting the values of h, ε_0, m and e we have

radius of first orbit $r_1 = 0.529 \times 10^{-10}$m = 0.529 Å

This calculations shows that the atom is about 10^{-10} meter in diameter.

2. Velocity of Revolving Electron

To obtain the velocity of the revolving electron, we substitute the value of r from eq. (iv) in eq. (ii), we have

$$mv\left[\frac{n^2h^2\varepsilon_0}{\pi me^2}\right] = n\frac{h}{2\pi} \Rightarrow v = \frac{nh}{2\pi}\cdot\frac{\pi me^2}{n^2h^2\varepsilon_0}\cdot\frac{1}{m} = \frac{e^2}{2nh\varepsilon_0}$$

This expressions shows that the velocity of the electron is inversely proportional to n i.e. the electron in the inner most orbit has the highest velocity.

3. Frequency of Electron in an orbit

Frequency of electron is given by

$$v = \frac{1}{T} = \frac{v}{2\pi r}$$

$$\Rightarrow v = \frac{e^2}{2nh\varepsilon_0}\times\frac{1}{2\pi}\times\frac{\pi\mu\varepsilon^2}{n^2h^2\varepsilon_0} = \frac{me^4}{4\varepsilon_0^2h^3n^3} \quad .....(vi)$$

This expression shows that the frequency of an electron is inversely proportional to the cube of n.

4. Electron Energy

The electron energy consist of two types :

(i) Kinetic energy and (ii) Potential energy

(i) Kinetic energy is due to the motion of electron and its value is $\frac{1}{2}$ mv^2 where v is the velocity of the electron,

$$\therefore \quad K.E. = \frac{1}{2}mv^2 = \frac{1}{2}m\left[\frac{e^2}{2nh\varepsilon_0}\right]^2 \text{ from equation}$$

$$\therefore \quad K.E. = \frac{me^4}{8n^2h^2\varepsilon_0^2}$$

(ii) Potential energy is due to the fact that electron lies in. the electric field of positive nucleus. We know that potential at a distance r from the nucleus is: $v = \frac{e}{4\pi\varepsilon_0 r}$

The potential energy of election of charge e is P.E.= $V\times(-e) = \frac{-e^2}{4\pi\varepsilon_0 r} = \frac{-e^2\times\pi me^2}{4\pi\varepsilon_0 n^2h^2\varepsilon_0} = \frac{-me^4}{4n^2h^2\varepsilon_0^2}$

So, Total energy in nth orbit, E_n = K.E. + P.E.

$$\Rightarrow \quad E_n = \frac{me^4}{8n^2h^2\varepsilon_0^2} - \frac{me^4}{4n^2h^2\varepsilon_0^2} \qquad \Rightarrow \quad E_n = \frac{-me^4}{8n^2h^2\varepsilon_0^2}$$

5. Frequency of Emitted Radiation

The frequency of emitted radiations can be found from the following relation when electron jumps from higher orbit n_2 to lower orbit n_1.

$$h\nu = E_{n_2}E_{n_1} \quad \Rightarrow \quad \nu = \frac{me^4}{8\varepsilon_0^2h^3}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \quad .....(viii)$$

$$\frac{1}{\lambda} = \frac{me^4}{8\varepsilon_0^2h^3c}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \text{ where } R = \frac{me^4}{8\varepsilon_0^2h^3c};$$

R = Rydberg's constant $= 10.97 \times 10^6$ m$^{-1} \approx 1.1 \times 7$ m^{-1}

$$\bar{\nu} = \frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

6. Electron Energy Levels in Hydrogen Atom

Energy of an electron revolving in nth orbit is given by

$$E_n = \frac{-me^4}{8\varepsilon_0^2h^2n^2} = -\frac{(9.11\times10^{-31})(1.6\times10^{-19})^4}{8(8.854\times10^{-12})^2(6.62\times10^{-34})^2n^2}$$

$$= -\frac{21.7\times10^{-19}}{n^2}\text{ joule} = -\frac{21.7\times10^{-19}}{1.6\times10^{-19}}\times\frac{1}{n^2}\text{eV} \qquad (\because 1\text{eV} = 1.6\times10^{-19}\text{ J})$$

$$\therefore \quad E_n = \frac{-13.6}{n^2}\text{eV}$$

The negative sign in energy shows that is thus electron is bound to the nucleus by attractive forces and to seperate the electron from the nucleus energy must be supplied to it. Giving different values to n, we can calculate the energy of the electron in different orbits.

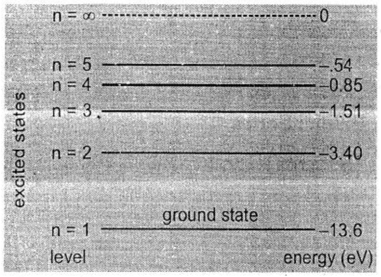
E_1 = –13.6 eV when n = 1 (K-shell)

E_2 = – 3.4 eV n = 2 (K-shell)

$E_3 = -1.5$ eV n = 3 (M-shell)

$E_\infty = 0$eV n = ∞ (Limiting case)

	Energy of electron	Binding energy or Ionisation energy
n = ∞	0	0
n = 4	–0.85 eV	+0.85 eV
n = 3	–1.51 eV	+1.51 eV
n = 2	–3.4 eV	+3.4 eV
n = 1	–13.6 eV	+13.6 eV



The diagram is known as energy level diagram. The lowest energy level (n = 1) correspond to normal unexcited state of hydrogen. This state is also called as ground state. In energy level diagram the lower energy (more negative) are at the bottom while higher energies (Less negative) are at the top. By such a consideration the various electron jumps between allowed orbit will be vertical arrows between different energy level. The energy of radiated photon is greater when the length of arrow is greater.

7. Spectral Series of Hydrogen Atom

It has been shown that the energy of the outer orbit is greater than the energy of the inner ones. When the Hydrogen atom is subjected to external energy, the electron jumps from lower energy State i.e. the hydrogen atom is excited. The excited state is not stable hence the electron return to its ground state in about 10^{-8} seconds. The excess of energy is now radiated in the form of radiations of different wavelength. The different wavelength constitute spectral series. Which are characteristic of atom emitting, then the wavelength of different members of series can be found from the following relations.

$$\bar{v} = \frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

This relation explain the complete spectrum of hydrogen. A detailed account of the important radiations are listed below.

(i) Lyman Series :

The series consist of wavelength which are emitted when electron jumps from an outer orbits to the first orbit i.e. the electron jumps to K orbit give rise to Lyman series.

Here $n_1 = 1$ and $n_2 = 2, 3, 4, ... \infty$.

The wavelengths of different members of Lyman series are :

(a) First member

In this case $n_1 = 1$ and $n_2 = 2$ hence

$$\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = \frac{3R}{4}$$

or $$\lambda = \frac{4}{3R}$$

or $$\lambda = \frac{4}{3 \times 10.97 \times 10^6} = 1216 \times 10^{-10}\,\text{m} = 1216\text{Å}$$

(b) Second member

In this case n_1 = 1 and n_2 = 3 hence

$$\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{3^2}\right] = \frac{8R}{9}$$

or $\lambda = \frac{9}{8R}$

or $\lambda = \frac{9}{8 \times 10.97 \times 10^6} = 1016 \times 10^{-10}\,m = 1026Å$

Similarly the wavelength of the other members can be calculated.

(c) Limiting member

In this case n_1 = 1 and n_2 = ∞ hence

$$\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{\infty^2}\right] = R$$

or $\lambda = \frac{1}{R}$

or $\lambda = \frac{1}{10.97 \times 10^6} = 912 \times 10^{-10}\,m = 912Å$

This series lies in ultraviolet region.

(ii) Balmer Series

This series is consist of all wavelengths which are emitted when an electron jumps from an outer orbit to the second orbit i. e. the electron jumps to L orbit give rise to Balmer series.

Here n_1 = 2 and n_2 = 3, 4, 5 .... ∞

The wavelength of different members of Balmer series.

(a) First member

In this case n_1 = 2 and n_2 = 3 hence

$$\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5R}{36}$$

or $\lambda = \frac{36}{5R}$

or $\lambda = \frac{36}{5 \times 10.97 \times 10^6} = 6563 \times 10^{-10}\,m = 6563Å$

(b) Second member

In this case n_1 = 2 and n_2 = 4 hence

$$\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{4^2}\right] = \frac{3R}{16}$$

or $\lambda = \frac{16}{3R}$

or $\lambda = \frac{16}{3 \times 10.97 \times 10^6} = 4816 \times 10^{-10}\,m = 4861Å$

(c) Limiting member

In this case n_1 = 2 and n_2 = ∞ hence

$$\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{\infty}\right] = \frac{R}{4} = \lambda = \frac{4}{R} = 3646Å$$

This series lies in visible and near ultraviolet region.

(iii) Paschen Series

This series consist of all wavelengths are emitted when an electron jumps from an outer orbit to the third orbit i.e. the electron jumps to M orbit give rise to Paschen series.

Here $n_1 = 3$ and $n_2 = 4, 5, 6 .... \infty$

The different wavelengths of this series can be obtained from the formula

$$\frac{1}{\lambda} = R\left[\frac{1}{3^2} - \frac{1}{n_2^2}\right] \qquad \text{where } n_2 = 4, 5, 6 ......\infty$$

For the first member, the wavelength is 18750Å. This series lies in infra-red region.

(iv) Brackett Series

This series is consist of all wavelengths which are emitted when an electron jumps from an outer orbits to the fourth orbit i.e. the electron jumps to N orbit give rise to Brackett series.

Here $n_1 = 4$ and $n_2 = 5, 6, 7, ... \infty$

The different wavelengths of this series' can be obtained from the formula

$$\frac{1}{\lambda} = R\left[\frac{1}{4^2} - \frac{1}{n_2^2}\right] \qquad \text{where } n_2 = 5, 6, 7 .... \infty$$

This series lies in infra-red region of spectrum.

(v) Pfund series

The series consist of all wavelengths which are emitted when an electron jumps from an outer orbit to the fifth orbit i.e. the electron jumps to O orbit give right to Pfund series.

Here $n_1 = 5$ and $n_2 = 6, 7, 8 .... \infty$

The different wavelengths of this series can be obtained from the formula

$$\frac{1}{\lambda} = R\left[\frac{1}{5^2} - \frac{1}{n_2^2}\right] \qquad \text{where } n_2 = 6, 7, 8 ....\infty$$

This series lies in infra-red region of sepectrum.

Conclusion

S.No.	Series Observed	Value of n_1	Value of n_2	Position in the spectrum
1.	Lyman Series	1	2, 3, 4….∞	Ultra Violet
2.	Balmer Series	2	3, 4, 5….∞	Visible
3.	Paschen Series	3	4, 5, 6….∞	Infra-red
4.	Brackett Series	4	5, 6, 7….∞	Infra-red
5.	Pfund Series	5	6, 7, 8….∞	Infra-red

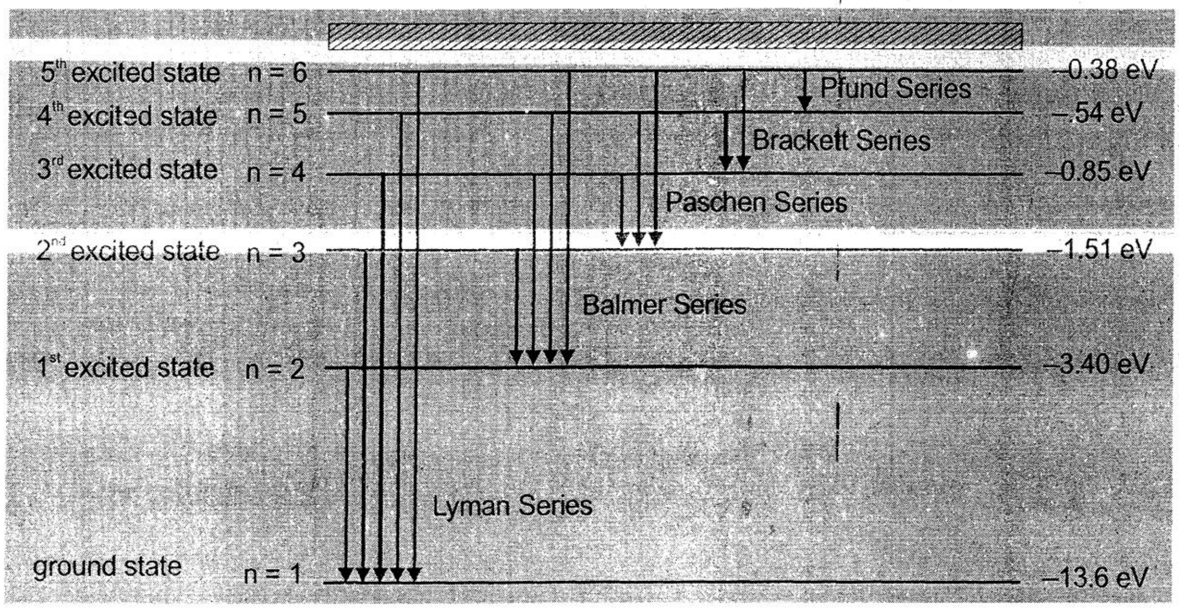
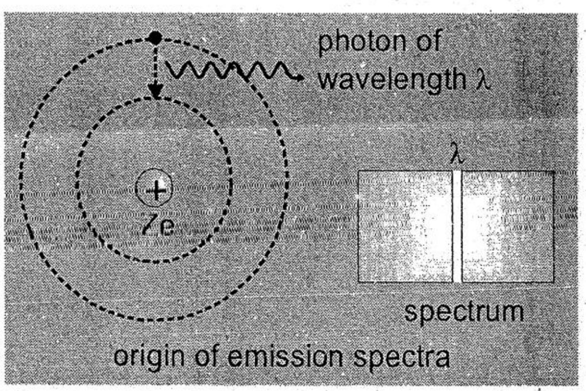
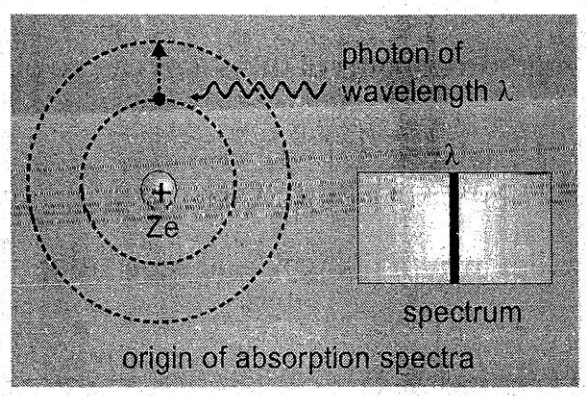


Figure : Conclusion in spectral form

3. EXCITATION AND IONISATION OF ATOMS

Consider the case of simplest atom i.e. hydrogen atom. this has one electron in the innermost orbit i.e.; (n = 1) and is said to be in the unexcited or normal state. If by some means, sufficient energy is supplied to the electron, it moves to higher energy states. When the atom is in a state of a high energy it is said to be excited. The process of raising or transferring the electron from lower energy state is called excitation. When by the process of excitation, the electron is completely removed from the atom, then the atom is said ionized. Now the atom has left with a positive charge. Thus the process of raising the atom from the normal state to the ionized state is called ionisation. The process of excitation and ionisation both are, absorption phenomena. The excited state is not stationary state and lasts in a very short interval of time (10^{-8} sec) because the electron under the attractive force of the nucleus jumps to the lower permitted orbit. This is accompanied by the emission of radiation according to BOHR'S frequency condition.





The energy necessary to excite an atom can be supplied in a number of ways. The most commonly kinetic energy (Wholly or partly) of the electrons is transferred to the atom. The atom is in now excited state.

The various values of potential to cause excitation of higher state called **excitation potential.** The potential necessary to accelerate the bombarding electrons to cause ionisation is called the **ionization potential**. We have seen that the energy required to excite the electron from first to second state is 13.6 – 3.4 = 10.2 eV, from first to third state is 13.6 – 1.5 = 12.1 eV., and so on. The energy required to ionise hydrogen atom is 0 – (–13.6) = 13.6 eV. Hence ionization potential of hydrogen atom is 13.6 volt.

4. RESULTS OF BOHR MODEL

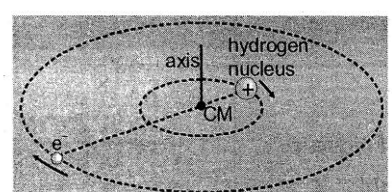
Physical Quantity	Formula	Ratio Formulae for hydrogen atom	Max. value	Min. value
Radius of Bohr orbit (r_n)	$r_n = \frac{n^2h^2}{4\pi^2 mKZe^2}$ $r_n = 0.53\frac{n^2}{Z}$ Å $r_n \propto \frac{n^2}{Z}$	$r_1 : r_2 : r_3 ..... r_n$ $= 1 : 4 : 9 ..... n^2$	$n = \infty$	$n = 1$
Velocity of electron in n^{th} Bohr orbit (v_n)	$v_n = \frac{2\pi KZe^2}{nh}$ $v_n \propto \frac{Z}{n}$ $v_n = 2.2\times10^6 \frac{Z}{n}$	$v_1 : v_2 : v_3 ..... v_n$ $= 1 : \frac{1}{2} : \frac{1}{3} ..... \frac{1}{n}$	$n = 1$	$n = \infty$
Momentum of electron (P_n)	$P_n = \frac{2\pi mKZe^2}{nh}$ $P_n \propto \frac{Z}{n}$	$P_1 : P_2 : P_3 ..... P_n$ $= 1 : \frac{1}{2} : \frac{1}{3} ..... \frac{1}{n}$	$n = 1$	$n = \infty$
Angular velocity of electron (ω_n)	$\omega_n = \frac{8\pi^3 K^2 Z^2 me^4}{n^3h^3}$ $\omega_n \propto \frac{Z^2}{n^2}$	$\omega_1 : \omega_2 : \omega_3 ..... \omega_n$ $= 1 : \frac{1}{8} : \frac{1}{27} ..... \frac{1}{n^3}$	$n = 1$	$n = \infty$
Time Period of electron (T_n)	$T_n = \frac{n^3h^3}{4\pi^2 K^2 Z^2 me^4}$ $T_n \propto \frac{n^3}{z^2}$	$T_1 : T_2 : T_3 ..... T_n$ $= 1 : 8 : 27 ..... n^3$	$n = \infty$	$n = 1$
Frequency (f_n)	$f_n = \frac{4\pi^2 K^2 Z^2 e^4 m}{n^3h^3}$	$f_1 : f_2 : f_3 ..... f_n$	$n = 1$	$n = \infty$

	$f_n \propto \frac{Z^2}{n^3}$	$= 1 : \frac{1}{8} : \frac{1}{27} \ldots\ldots \frac{1}{n^3}$		
Orbital current (I_n)	$I_n = \frac{4\pi^2 K^2 Z^2 e^5 m}{n^3 h^3}$ $I_n \propto \frac{Z^2}{n^3}$	$I_1 : I_2 : I_3 \ldots\ldots I_n$ $= 1 : \frac{1}{8} : \frac{1}{27} \ldots\ldots \frac{1}{n^3}$	n = 1	n = ∞
Angular momentum (J_n)	$J_n = \frac{nh}{2\pi}$ $J_n \propto n$	$I_1 : I_2 : I_3 \ldots\ldots I_n$ $= 1 : 2 : 3 \ldots\ldots n$	n = ∞	n = 1
Centripetal acceleration (a_n)	$a_n = \frac{16\pi^4 K^3 Z^3 m e^6}{n^4 h^4}$ $a_n \propto \frac{Z^3}{n^4}$	$a_1 : a_2 : a_3 \ldots\ldots a_n$ $= 1 : \frac{1}{16} : \frac{1}{81} \ldots\ldots \frac{1}{n^4}$	n = 1	n = ∞
Kinetic energy (E_{K_n})	$E_{K_n} = \frac{RchZ^2}{n^2}$ $E_{K_n} \propto \frac{Z^2}{n^2}$	$E_{K_1} : E_{K_2} : E_{K_3} .. E_{K_n}$ $= 1 : \frac{1}{4} : \frac{1}{9} \ldots\ldots \frac{1}{n^2}$	n = 1	n = ∞
Potential energy (U_n)	$U_n = \frac{-2RchZ^2}{n^2}$ $U_n \propto \frac{Z^2}{n^2}$	$U_1 : U_2 : U_3 \ldots\ldots U_n$ $= 1 : \frac{1}{4} : \frac{1}{9} \ldots\ldots \frac{1}{n^2}$	n = ∞	n = 1
Total Energy (E_n)	$E_n = \frac{-RchZ^2}{n^2}$ $E_n \propto \frac{Z^2}{n^2}$ $E_n = -13.6\frac{Z^2}{n^2}\,eV$ $E_n = -\frac{KZe^2}{2r_n}$	$E_1 : E_2 : E_3 \ldots\ldots E_n$ $= 1 : \frac{1}{4} : \frac{1}{9} \ldots\ldots \frac{1}{n^2}$ (but remember it negative sign)	n = ∞	n = 1

GOLDEN KEY POINTS

- **Recoil energy** $E_r = \frac{p^2}{2M}$ $\quad E_2 - E_1 = pc = \frac{hc}{\lambda}$
- **Reduced mass μ =** $\frac{m_e m_H}{m_e + m_H}$ $\quad \mu = \frac{Rch^3}{2\pi^2 K^2 e^4}$

For proton-meson system $\mu = \frac{207 m_e \times 1840 m_e}{(207 + 1840) m_e} = 186 m_e$



Both the electron and nucleus of hydrogen atom revolve around a common centre of mass

For electron-positron system $\mu = \frac{m}{2}$

ILLUSTRATIONS

Illustrations 1

A hydrogen atom in the ground state is excited by radiations of wavelength 975 Å.

Find : (a) the energy state to which the atom is excited.

(b) how many lines will be possible in emission spectrum

Solution

(a) $\lambda = 975$ Å $= 975 \times 10^{-10}$ m

$$\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{n^2}\right]$$

$$\therefore \quad \frac{1}{975 \times 10^{-10}} = 1.1 \times 10^7 \left[\frac{1}{1^2} - \frac{1}{n^2}\right] \quad \text{or} \quad n = 4$$

(b) n = 4

Q Number of spectral lines (N) = $\frac{n(n-1)}{2}$

$$\therefore \quad N = \frac{4 \times (4-1)}{2} = 6$$

Possible transition 4→3, 4→2, 4→1, 3→2, 3→1, 2→1

Illustration 2

Find the first and second excitation potentials of an atom when its ionisation potential is 122.4 V.

Solution

I.P. = 122.4 V

$$E_{ex1} = 122.4 - \frac{122.4}{4} = 91.8V$$

$$\therefore \quad E_{ex2} = 122.4 - \frac{122.4}{9} = 108.8V$$

Illustration 3

Find the atomic number of atom when given that it ionization potential is equal to 122.4 V.

Solution

I.P. = 122.4 V

$E = Z^2 E_H$

$$Q \quad Z = \sqrt{\frac{E}{E_{H_2}}} = \sqrt{\frac{122.4}{13.6}} = 3$$

Illustration 4

Find the maximum wavelength of Brakett series of hydrogen atom.

Solution

$n_1 = n_2 = 5$

$$\therefore \quad \frac{1}{\lambda_{max}} = R\left[\frac{1}{4^2} - \frac{1}{5^2}\right]$$

or $\lambda_{max} = \frac{25 \times 16 \times 10^{10}}{9 \times 1.1 \times 10^{7}} = 40400$ Å

Illustrations 5

Find the value of magnetic induction at the proton due to electron motion, if the radius of the first orbit of hydrogen atom is 0.5 A and the speed of electron in it is 2.2×10^6 m/sec.

Solution

r = 0.5 Å and V = 2.2×10^6 m/sec

$$B = \frac{\mu_0}{4\pi}\frac{ev}{r^2} = \frac{10^{-7} \times 1.6 \times 10^{-19} \times 2.2 \times 10^6}{25 \times 10^{-22}} = 14.08 \text{ Tesla}$$

Illustration 6

Find the ratio of equivalents current due to electron motion in first and second orbits of hydrogen atom.

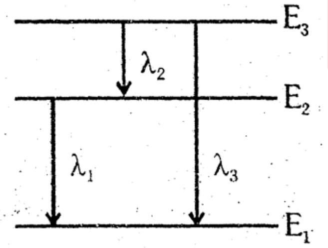
Solution

$$I_n \propto \frac{1}{n^3}$$

$$\therefore \quad \frac{I_1}{I_2} = \left[\frac{n_2}{n_1}\right]^3 = \left[\frac{2}{1}\right]^3 = 8:1$$

Illustration 7

For the given transitions of electron, obtain the relation between λ_1, λ_2 and λ_3. **[AIPMT 2004]**



Solution

For given condition $E_3 - E_1 = (E_3 - E_2) + (E_2 - E_1)$

$$\Rightarrow \quad \frac{hc}{\lambda_3} = \frac{hc}{\lambda_2} + \frac{hc}{\lambda_1}$$

$$\Rightarrow \quad \frac{1}{\lambda_3} = \frac{1}{\lambda_2} + \frac{1}{\lambda_1}$$

Therefore $\lambda_3 = \frac{\lambda_2\lambda_1}{\lambda_2 + \lambda_1}$

Illustration 8

A hydrogen atom is in a state of ionization energy0.85 eV. If it makes a transition to the ground state, what is the energy of the emitted photon.

Solution

Energy of emitted photon = 13.6 – 0.85 = 12.75 eV

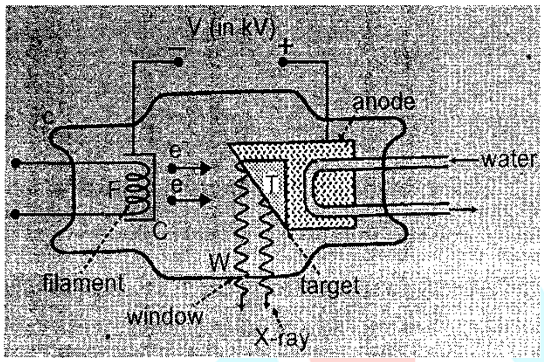
BEGINNER'S BOX-1

1. Find the longest and shortest wavelength when a hydrogen atom in the ground state is excited by radiations of wavelength 975 Å.
2. How many lines will be possible in the absorption spectrum when a hydrogen atom in the ground state is excited by radiations of wavelength 975 Å.
3. Find the ratio of wavelength of first line of Lyman series of doubly ionised lithium atom to that of the first line of Lyman series of deuterium$({}_1H^2)$.
4. Find the ratio of the area of orbit of first excited state of electron to the area of orbit of ground level for hydrogen atom.
5. If the ionisation potential in the ground state for hydrogen is 13.6 e.V., then find the excitation potential of third orbit.
6. When an electron jump from second orbit to ground state of hydrogen atom then calculate the wavelength of emitted photon.

X-Ray

COOLIDGE METHOD OF X-RAY PRODUCTION

Coolidge developed thermionic vacuum X-ray tube in which electrons are produced by thermionic emission method. Due to high potential difference electrons (emitted due to thermionic method) move towards the target and strike from the atom of target due to which X-ray are produced. Experimentally it is observed that only 1% or 2% kinetic energy of electron beam is used to produce X-ray and rest of energy is wasted in form of heat.



CONTROL ON X-RAY

There are two types of control on X-ray -

(i) Intensity control

The intensity of X-ray depend on number of electrons striking the target and number of electron depend on temperature of filament which can be controlled by filament current.

Thus intensity of X-ray depend on current flowing through filament.

(ii) Penetrating Power control

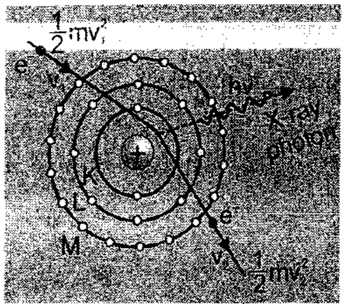
The Penetrating power of X-ray depend on the energy of incident electron. The energy of electron can be controlled by applied potential difference. Thus penetrating power of X-ray depend on applied potential difference.

Thus the intensity of X-ray depend on current flowing through filament while penetrating power depend on applied potential difference.

TYEPS OF X-RAY

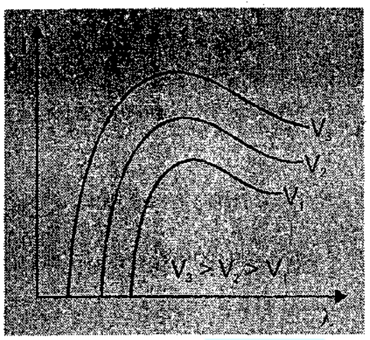
	Soft X-ray	**Hard X-ray**
Wavelength	10Å to 100Å	0.1 Å – 10Å
Energy	$\frac{12400}{\lambda}$ eV – Å	$\frac{12400}{\lambda}$ eV – Å
Penetrating power	Less	More
Use	Radiography	Radiotherapy

CONTINUOUS SPECTRUM OF X-RAY



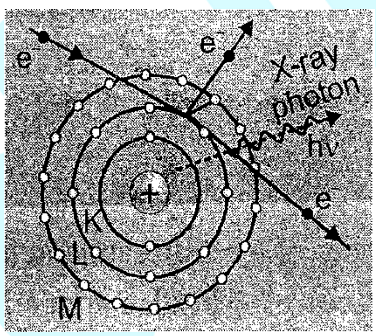
It is produced due to retardation of electron passing through the lattice of metal.

$$\lambda_{min} = \frac{hc}{eV_a} = \frac{12400eV - Å}{eV_a} = \frac{12400V - Å}{V_a}$$

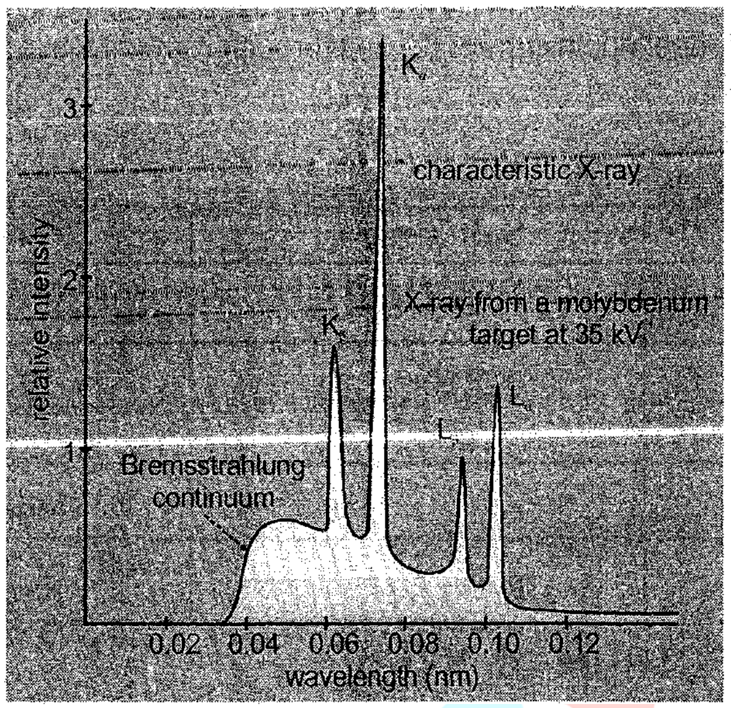


Continuous X-rays are also known as white X-ray. Minimum wavelength of these spectrum only depend on applied potential and don't depend on atomic number.

CHARACTERISTIC SPECTRUM OF X-RAY



When highly accelerated electron strike with the atom of target then it knockout the electron of orbit, due to this a vacancy is created. To -fill this vacancy electron jump from higher energy level and electromagnetic radiation are emitted which form characteristic spectrum of X-ray.



MOSELEY'S LAW

Moseley studied the characteristic spectrum of number of many elements and observed that the square root of the frequency of a K-line is closely proportional to atomic number of the element. This is called Moseley's law.

$$\sqrt{v} \propto (Z-b) \quad \Rightarrow \quad v \propto (Z-b)^2 \quad v = a(Z-b)^2 \quad .....(i)$$

Z = atomic number of target, v = frequency of characteristic spectrum

b =screening constant (for K-series b = 1) a = proportionality constant

- For X-ray production, moseley formulae are used because heavy metal are used.

ILLUSTRATIONS

Illustrations 1

Find out wave length of K_α X-ray.

Solution

K_a means transition from n_2 = 2 to n_1 = 2 and b = 1 for K series

$$\frac{1}{\lambda_{K\alpha}} = R(Z-1)^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \qquad \frac{1}{\lambda_{K\alpha}} = R(Z-1)^2\left[\frac{1}{1^2} - \frac{1}{2^2}\right]$$

$$\frac{1}{\lambda_{K\alpha}} = \frac{3R(Z-1)^2}{4} \qquad R = 1.097 \times 10^7\ m^{-1} \text{ and } \frac{1}{R} = 912Å$$

$$\lambda_{K\alpha} = \frac{4}{3R(Z-1)^2} \qquad \lambda_{K\alpha} = \frac{1216}{(Z-1)^2}Å$$

Similarly

$v_{K\alpha} = 2.47 \times 10^{15}\ (Z-1)^2$ Hz and $E_{K\alpha} = 10.2\ (Z-1)^2$ eV

Graph

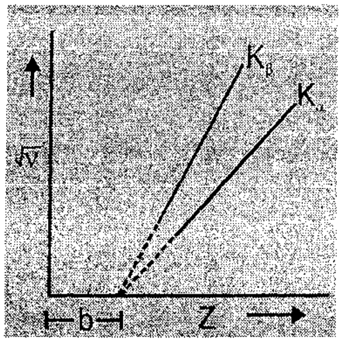


Illustration 2

For tungsten, atomic energy level of K, L & M are given 69.5 keV, 11.3 keV and 2.30 keV respectively. For obtaining characteristic K_β & K_α lines for tungsten, what should be the required minimum accelerating potential and λ_{min} ? Also calculate λ_α and λ_β.

Solution

Required minimum accelerating potential $= \dfrac{\text{ionisation energy}}{e} = 69.5\,\text{kV}$

For this accelerating potential, $\lambda_{min} = \dfrac{hc}{eV_{max}} = \dfrac{12400}{69.5\times10^3}\,\text{Å} = 0.178\text{Å}$

Wavelength for K_α $\quad \lambda_\alpha = \dfrac{12400}{(69.5-11.3)\times10^3}\,\text{Å} = 0.213\text{Å}$

Wavelength for K_β $\quad \lambda_\beta = \dfrac{12400}{(69.5-2.30)\times10^3}\,\text{Å} = 0.184\text{Å}$

Illustration 3

An X-ray tube operates at 20 kV. A particular electron loses 5% of its kinetic energy to emit an X-ray photon at the first collision. Find the wavelength corresponding to this photon.

Solution

Kinetic energy acquired by the electron = 20×10^3 eV

The energy of the photon = $\dfrac{5}{100}\times 20\times10^3\,\text{eV} = 10^3\,\text{eV}$

Thus, $\dfrac{hc}{\lambda} = 10^3\,\text{eV}$ or $\lambda = \dfrac{hc}{10^3\,\text{eV}} = \dfrac{12400\text{eV}-\text{Å}}{10^3\,\text{eV}} = 12.4\text{Å}$

Illustration 4

The K_α X-ray of molybdenum has wavelength 71 pm. If the energy of a molybdenum atom with p. K electron knocked out is 23.32 keV. What will be the energy of this atom when an L-electron is knocked out?

Solution

Given, $\lambda_{K\alpha}$ = 71 pm = 0.71 Å

$$E_K - E_L = \frac{hc}{\lambda_{k\alpha}} = \frac{12400\text{ eV}-\text{Å}}{0.71\text{Å}} = 17.46\text{ keV}$$

Thus, $E_L = E_K$ – 17.46 keV = 23.32 keV – 17.46 keV = 5.86 keV

Illustration 5

The electron current in an X-ray tube operating at 40 kV is 10 mA. Assume that on an average 1% of the total kinetic energy of electrons hitting the target is converted into X-rays.

(a) What is total power carried by X-rays ?

(b) How much heat is produced at the target per second ?

Solution

Power drawn by X-ray tube is $P = i \times V = 10 \times 10^{-3} \times 40 \times 10^{3} = 400$ W

(a) $P_{X\text{-ray}} = 1\%$ of 400 W $= 400 \times \dfrac{1}{100} = 4$W

(b) Remaining 99% power is convert into heat at target.

$\therefore$ Heat produced = 400 – 4 = 396 W

Illustration 6

In the experiment of Coolidge tube, wavelength of electron striking at the target is 0.01 nm. What will be value of minimum wavelength of X-rays obtained from the tube?

Solution

Wavelength of a moving electron $\lambda_e = \dfrac{12.27}{\sqrt{V_a}}$ Å

or V_a = accelerating potential of electron = $\dfrac{150}{\lambda_e^2} = \dfrac{150}{(0.1)^2} = 15000$ volt

Minimum wavelength of X-rays $\lambda_{min} = \dfrac{hc}{eV_a} = \dfrac{6.62\times10^{-34}\times3\times10^{8}}{1.6\times10^{-19}\times15000} = 0.826$Å

Illustration 7

An X-ray beam of wavelength 1.0Å is incident on a crystal of lattice spacing 2.8 Å. Calculate the value of Bragg's angle for first order diffraction.

Solution

From Bragg's equation $2d \sin\theta = n\lambda$

$2 \times 2.8 \times 10^{-10} \times \sin\theta = 1 \times 10^{-10}$

$\sin\theta = \dfrac{1}{5.6} \Rightarrow \sin\theta = 0.1786$ or $\theta = \sin^{-1}(0.1786)$

BEGINNER'S BOX-2

1. If wavelength of K_α radiation of Mo (Z = 42) is 0.71 Å then calculate the wavelength of the corresponding radiation of Cu(Z = 29).

2. X-rays are produced in a X-ray tube py electrons accelerated through an electric potential difference of 50.0 kV. An electron makes three collisions in the target before coming to rest and loses half of its remaining kinetic energy in each of the first two collisions. Determine the wavelength of the resulting photons, (Neglect the recoil of the heavy target atoms)

3. Voltage applied across the coolidge tube is 25 kV. What is kinetic energy of electron striking at the target and cut-off wavelength of X-ray obtained from the tube ?

4. If applied potential across the X-ray tube is made $\frac{2}{5}$ time then minimum wavelength of X-rays is shifted by 1 Å. Find the original values of applied potential and minimum wavelength.

5. Find the
(a) maximum frequency, and
(b) minimum wavelength of X-rays produced by 30 kV electrons.

6. (a) An X -ray tube produces a continuous spectrum of radiation with its short wavelength end at 0.45 A. What is the maximum energy of a photon in the radiation?
(b) From your answer to (a), guess what order of accelerating voltage (for electrons) is required in such a tube?

7. The intensity of X-rays of wavelength 0.5Å reduces to one fourth on passing through 3.5 mm thickness of a metal foil. The coefficient of absorption of metal will be.

DIFFRACTION OF X-RAY

Diffraction of X-ray is possible by crystals because the interatomic spacing in a crystal lattice is order of wavelength of X-rays. It was first verified by Lauve.

Diffraction of X-ray take place according to Bragg's law

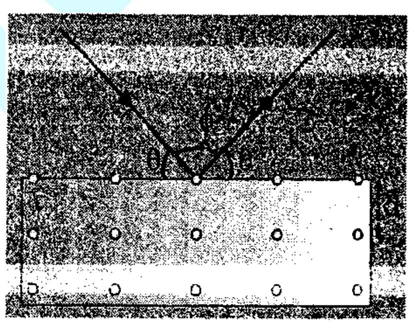
$$\boxed{2d\sin\theta = n\lambda}$$

d = Spacing of crystal plane or lattice constant or distance between adjacent atomic plane

θ = Bragg's angle or glancing angle

ϕ = Diffracting angle

n = 1, 2, 3, ……..



For Maximum Wavelength

$\sin\theta = 1$, n = 1

so if $\lambda > 2d$ diffraction is not possible i.e. solution of Bragg's equation is not possible.

ABSORPTIONOF X-RAY

As electromagnetic waves pass through any medium (except vacuum) its energy is partially absorbed, increasing the internal energy in the material and the intensity is correspondingly attenuated.

When a beam of X-ray (or any electromagnetic wave) passes through a thin sheet of material of thickness dx, the decrease dI in its intensity I is found to be proportional to the initial intensity I and to the thickness dx. Thus $dI = -\mu I\, dx$

The proportionality constant α, which depends on the material is called the absorption coefficient. The intensity after passage through a slab of finite thickness x can be obtained by integrating above equation.

$I = I_0 e^{-\mu x}$

Where I_0 is the intensity at x = 0. The above equation is called Lambert's law.

PROPERTIES AND USES OF X-RAY

Property

- X-ray always travel with the velocity of light in straight line because their wavelength is very small.
- X-ray is electromagnetic radiation it show particle and wave both nature.
- In reflection, diffraction, interference, refraction X-ray shows wave nature while in photoelectric effect it shows particle nature.
- There is no charge on X-ray thus these are not deflected by electric field and magnetic field.
- X-ray are invisible.
- X-ray affect the photographic plate
- When X-ray incident on the surface of substance it exert force and pressure and transfer energy and momentum
- Characteristic X-ray cannot obtained from hydrogen because the difference of energy level in hydrogen is very small.

Uses

(a) In study of crystal structure (b) In surgery

(c) In radiography (d) In Engineering

ANSWERS

BEGINNER'S BOX-1

1.	$\lambda_{max} = 18787.8Å$, $\lambda_{min} = 973Å$	**2.**	n = 4	**3.**	1 : 9
4.	16 : 1	**5.**	0.66 eV	**6.**	122 nm

BEGINNER'S BOX-2

1.	1.52 Å	**2.**	49.6 pm, 99.2 pm
3.	Kinetic energy = 25 keV, λ_{min} = 0.5 Å	**4.**	λ_{min} = 0.66 Å, V_a = 18.6 kV
5.	(a) 7.26×10^{18} Hz (b) 0.413 Å	**6.**	(a) 27.55 keV, (b) 27.5 kV
7.	0.4 mm^{-1}		