INTERFERENCE OF LIGHT

LIGHT

The physical cause, with the help of which our eyes experience the sensation of vision, is known as light or the form of energy, which excites our retina and produce the sensation of vision, is known as light.

PROPERTIES OF VISIBLE LIGHT

(i) No material medium is required for the propagation of light energy i.e. it travels even in vacuum.

(ii) Its velocity is constant in all inertial frames i.e. it is an absolute constant. It is independent of the relative velocity between source and the observer.

(iii) Its velocity in vacuum is maximum whose value is 3×10^8 m/s.

(iv) It lies in the visible region of electromagnetic spectrum whose wavelength range is from 4000 Å to 8000 Å.

(v) Its energy is of the order of eV.

(vi) It propagates in straight line.

(vii) It exhibits the phenomena of reflection, refraction, interference, diffraction, polarisation and double refraction.

(viii) It can emit electrons from metal surface i.e. it can produce photoelectric effect.

(ix) It produces thermal effect and exerts pressure when incident upon a surface. It proves that light has momentum and energy.

(x) Its velocity is different in different media. In rarer medium it is more and in denser medium it is less.

(xi) Light energy propagates via two processes.

(a) The particles of the medium carry energy from one point of the medium to another.

(b) The particles transmit energy to the neighbouring particles and in this way energy propagates in the form of a disturbance.

DIFFERENT THEORIES OF LIGHT

1. Newton's corpuscular theory of light.
2. Hygen's wave theory of light.
3. Maxwell's electromagnetic theory of light.
4. Plank's Quantum theory of light.
5. De-Broglie's dual theory of light.

1. Newton's corpuscular theory of light

This theory was enuciated by Newton.

(i) Characteristics of the theory

(a) Extremely minute, very light and elastic particles are being constantly emitted by all luminous bodies (light sources) in all directions

(b) These corpuscles travel with the speed of light..

(c) When these corpuscles strike the retina of our eye then they produce the sensation of vision.

(d) The velocity of these corpuscles in vacuum is 3×10^8 m/s.

(e) The different colours of light are due to different size of these corpuscles.

(f) The rest mass of these corpuscles is zero.

- **(g)** The velocity of these corpuscles in an isotropic medium is same in all directions but it changes with the change of medium.
- **(h)** These corpuscles travel in straight lines.
- **(i)** These corpuscles are invisible.

(ii) The phenomena explained by this theory

- **(a)** Reflection and refraction of light.
- **(b)** Rectilinear propagation of light.
- **(c)** Existence of energy in light.

(iii) The phenomena not explained by this theory

- **(a)** Interference, diffraction, polarisation, double refraction and total internal reflection.
- **(b)** Velocity of light being greater in rarer medium than that in a denser medium.
- **(c)** Photoelectric effect and Crompton effect.

2. WAVE THEORY OF LIGHT

This theory was enunciated by Hygen in a hypothetical medium known as luminiferrous ether.

Ether is that imaginary medium which prevails in all space, in isotropic, perfectly elastic and massless.

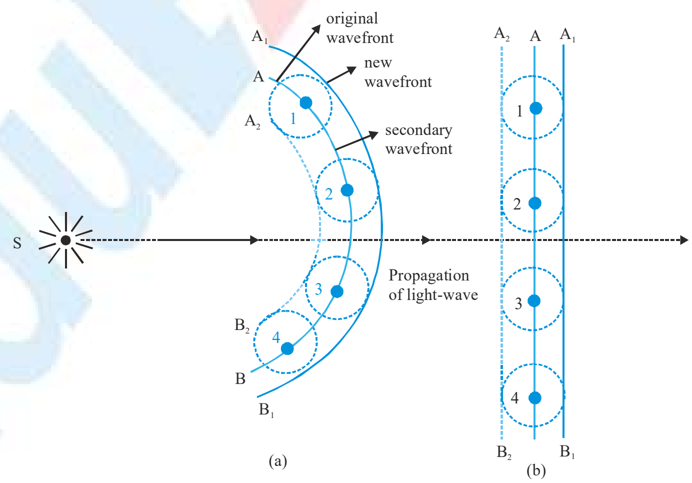
The different colours of light are due to different wave lengths of these waves.

The velocity of light in a medium is constant but changes with change of medium.

This theory is valid for all types of waves.

- **(i)** The locus of all ether particles vibrating in same phase is known as wavefront.
- **(ii)** Light travels in the medium in the form of wavefront.
- **(iii)** When light travels in a medium then the particles of medium start vibrating and consequently a disturbance is created in the medium.
- **(iv)** Every point on the wave front becomes the source of secondary wavelets. It emits secondary wavelets in all directions which travel with the speed of light **(v)**,

The tangent plane to these secondary wavelets represents the new position of wave front.



The phenomena explained by this theory

(i) Reflection, refraction, interference, diffraction, polarisation and double refraction.

(ii) Rectilinear propagation of light.

(iii) Velocity of light in rarer medium being grater than that in denser medium.

Phenomena not explained by this theory

(i) Photoelectric effect, Compton effect and Raman effect.

(ii) Backward propagation of light.

Wave front, various types of wave front and rays

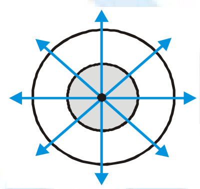
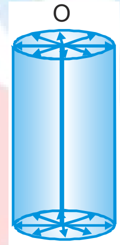
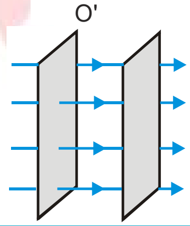
(a) Wavefront

The locus of all the particles vibrating in the same phase is known as wavefront.

(b) Types of wavefront

The shape of wavefront depends upon the shape of the light source originating that wavefront. On the basis of there are three types of wavefront.

Comparative study of three types of wavefront

S.No.	Wavefront	Shape of light source	Diagram of shape of wavefront	Variation of amplitude with distance	Variation of intensity with distance
1.	Spherical	Point source		$A \propto \frac{1}{d}$ or $A \propto \frac{1}{r}$	$I \propto \frac{1}{r^2}$
2.	cylindrical	Linear or slit		$A \propto \frac{1}{\sqrt{d}}$ or $A \propto \frac{1}{\sqrt{r}}$	$I \propto \frac{1}{r}$
3.	Plane	Extended large source situated at very large distance		A = constant	I = constant

Characteristic of Wavefront

(a) The phase difference between various particles on the wavefront is zero.
(b) These wavefronts travel with the speed of light in all directions in an isotropic medium.
(c) A point source of light always gives rise to a spherical wavefront in an isotropic medium.
(d) In an anisotropic medium it travels with different velocities in different directions.
(e) Normal to the wavefront represents a ray of light.
(f) It always travels in the forward direction of the medium.

PRINCIPLE OF SUPERPOSITION

When two or more waves simultaneously pass through a point, the disturbance of the point is given by the sum of the disturbances each wave would produce in absence of the other wave(s). In case of wave on string disturbance means displacement, in case of sound wave it means pressure change, in case of Electromagnetic Waves. it is electric field or magnetic field. Superposition of two light travelling in almost same direction results in modification in the distribution of intensity of light in the region of superposition. This phenomenon is called *interference*.

Superposition of two sinusoidal waves

Consider superposition of two sinusoidal waves (having same frequency), at a particular point.

Let, $x_1(t) = a_1 \sin \omega t$

and, $x_2(t) = a_2 \sin(\omega t + \phi)$

represent the displacement produced by each of the disturbances. Here we are assuming the displacements to be in the same direction. Now according to superposition principle, the resultant displacement will be given by,

$$x(t) = x_1(t) + x_2(t)$$
$$= a_1 \sin \omega t + a_2 \sin(\omega t + \phi)$$
$$= A \sin(\omega t + \phi_0)$$

where $A^2 = a_1{}^2 + a_2{}^2 + 2a_1 \,.\, a_2 \cos \phi$ .... **(1.1)**

and $\tan \phi_0 = \dfrac{a_2 \sin\phi}{a_1 + a_2 \cos\phi}$ ....**(1.2)**

Ex. S_1 and S_2 are two sources of light which produce individually disturbance at point P given by $E_1 = 3\sin \omega t$, $E_2 = 4\cos \omega t$. Assuming $\vec{E}_1$ & $\vec{E}_2$ to be along the same line, find the resultant after their superposition.

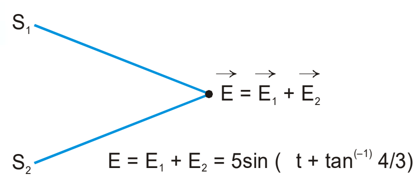


Figure 1.1

Sol.: $E = 3\sin\omega t + 4\sin(\omega t + \frac{\pi}{2}) \Rightarrow A^2 = 3^2 + 4^2 + 2(3)(4)\cos\frac{\pi}{2} = 5^2$

$$\tan\phi_0 = \frac{4\sin\frac{\pi}{2}}{3 + 4\cos\frac{\pi}{2}} = \frac{4}{3} \Rightarrow \phi_0 = 53^\circ \Rightarrow E = 5\sin[\omega t + 53^\circ]$$

Superposition of progressive waves; path difference :

Let S_1 and S_2 be two sources producing progressive waves (disturbance travelling in space given by y_1 and y_2)

At point P,

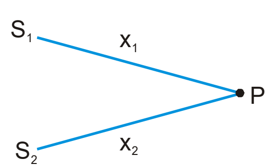


Figure: 1.3

$y_1 = a_1 \sin(\omega t - kx_1 + \theta_1)$

$y_2 = a_2 \sin(\omega t - kx_2 + \theta_2)$

$y = y_1 + y_2 = A\sin(\omega t + \Delta\phi)$

Here, the phase difference,

$\Delta\phi = (\omega t - kx_1 + \theta_1) - (\omega t - kx_2 + \theta_2)$

$= k(x_2 - x_1) + (\theta_1 - \theta_2) = k\Delta p - \Delta\theta$ where $\Delta\theta = \theta_2 - \theta_1$

Here $\Delta p = \Delta x$ is the path difference

Clearly, phase difference due to path difference = k (path difference)

where $k = \dfrac{2\pi}{\lambda}$

$\Rightarrow \quad \Delta\phi = k\Delta p = \dfrac{2\pi}{\lambda}\Delta x$ ..... **(1.3)**

For Constructive Interference :

$\Delta\phi = 2n\pi, \quad n = 0, 1, 2 ....$

or, $\Delta x = n\lambda$

$A_{max} = A_1 + A_2$

Intensity, $\sqrt{I_{max}} = \sqrt{I_1} + \sqrt{I_2} \Rightarrow I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$ ..... **(1.4)**

For Destructive interference :

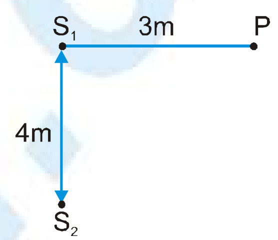
$\Delta\phi = (2n + 1)\pi, \quad n = 0, 1, 2 ....$

or, $\Delta x = (2n + 1)\lambda/2$

$A_{min} = |A_1 - A_2|$

Intensity, $\sqrt{I_{min}} = \sqrt{I_1} - \sqrt{I_2} \Rightarrow I_{min.} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$ ..... **(1.5)**

Ex. S_1 and S_2 are two coherent sources of frequency 'f' each. ($\theta_1 = \theta_2 = 0^o$)
$V_{sound} = 330 m/s$.
(i) so that constructive interference at 'p'
(ii) so that destructive interference at 'p'



Sol. For constructive interference

$K\Delta x = 2n\pi$

$\dfrac{2\pi}{\lambda} \times 2 = 2n\pi \Rightarrow \lambda = \dfrac{2}{n} \Rightarrow V = \lambda f \Rightarrow V = \dfrac{2}{n} f \Rightarrow f = \dfrac{330}{2} \times n$

For destructive interference

$K\Delta x = (2n + 1)\pi$

$\dfrac{2\pi}{\lambda} . 2 = (2n + 1)\pi \Rightarrow \dfrac{1}{\lambda} = \dfrac{(2n+1)}{4} \Rightarrow f = \dfrac{V}{\lambda} = \dfrac{330 \times (2n+1)}{4}$

Ex. Light from two sources, each of same frequency and travelling in same direction, but with intensity in the ratio 4 : 1 interfere. Find ratio of maximum to minimum intensity.

Sol. : $\dfrac{I_{max}}{I_{min}} = \left(\dfrac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = \left(\dfrac{\sqrt{\dfrac{I_1}{I_2}} + 1}{\sqrt{\dfrac{I_1}{I_2}} - 1}\right)^2 = \left(\dfrac{2+1}{2-1}\right)^2 = 9 : 1.$

INTERFERENCE OF LIGHT

When two light waves of same frequency with zero initial phase difference or constant phase difference superimpose over each other, then the resultant amplitude (or intensity) in the region of superimposition is different from the amplitude (or intensity) of individual waves.

This modification in intensity in the region of superposition is called interference.

(a) Constructive interference

When resultant intensity is greater than the sum of two individual wave intensities $[I > (I_1 + I_2)]$, then the interference is said to be constructive.

(b) Destructive interference

When the resultant intensity is less than the sum of two individual wave intensities $[I < (I_1 + I_2)]$, then the interference is said to destructive.

There is no violation of the law of conservation of energy in interference. Here, the energy from the points of minimum energy is shifted to the points of maximum energy.

TYPES OF SOURCES

(i) Coherent source

Two sources are said to be coherent if they emit light waves of the same wavelength and start with same phase or have a constant phase difference.

Note : Laser is a source of monochromatic light waves of high degree of coherence.

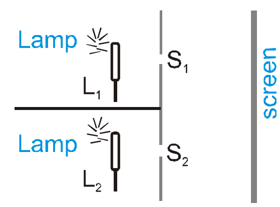
Main points

1. They are obtained from the same single source.
2. Their state of polarization is the same

(ii) Incoherent source

Two independent monochromatic sources, emit waves of same wavelength.

But the waves are not in phase. So they are incoherent. This is because, atoms cannot emit light waves in same phase and these sources are said to be incoherent sources. By using two independent laser beams it has been possible to record the interference pattern.



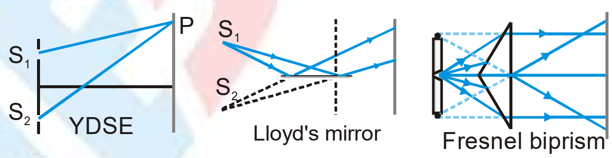
Independent Source (non-coherent source)

METHOD FOR OBTAINING COHERENT SOURCE

(i) Division of wave front

In this method, the wavefront is divided into two or more parts by use of mirrors, lenses or prisms.

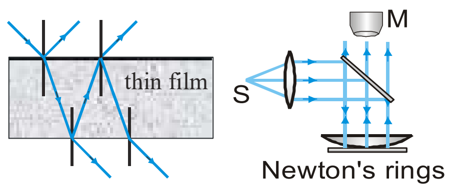
Ex. : Young's double slit experiment. Fresnel's Biprism and Lloyd's single mirror method.



(ii) Division of amplitude

The amplitude of incoming beam is divided into two or more parts by partial reflection or refraction. These divided parts travel different paths and are finally brought together to produce interference.

Ex. : The brilliant colour seen in a thin film of transparent material like soap film, oil film, Michelson's Interferro Meter, Newtons' ring etc.



Condition for sustained interference

To obtain the stationary interference pattern, the following conditions must be fulfilled :

(a) The two sources should be coherent, i.e., they should vibrate in the same phase or there should be a constant phase difference between them.

(b) The two sources must emit continuously waves of same wavelength and frequency.

(c) The separation between two coherent sources should be small.

(d) The distance of the screen from the two sources should be small.

(e) For good contrast between maxima and minima, the amplitude of two interfering waves should be as nearly equal as possible and the background should be dark.

(f) For a large number of fringes in the field of view, the sources should be narrow and monochromatic.

ANALYSIS OF INTERFERENCE OF LIGHT

When two light waves having same frequency and equal or nearly equal amplitude are moving in the same direction, They superimpose each other, at some point the intensity of light is maximum and at some point it is minimum this phenomenon is known as interference of light.

Let two waves having amplitude a_1 and a_2 and same frequency, same phase difference ϕ superpose. Let their displacement are : $y_1 = a_1 \sin \omega t$ and $y_2 = a_2 \sin(\omega t + \phi)$

By principle of superposition.

$y = y_1 + y_2 = a_1 \sin \omega t + a_2 \sin(\omega t + \phi) = a_1 \sin \omega t + a_2 [\sin \omega t \cos \phi + \cos \omega t \sin \phi]$

$= \sin \omega t (a_1 + a_2 \cos \phi) + a_2 \cos \omega t \sin \phi$

Let, $a_1 + a_2 \cos \phi = A \cos \theta$ and $a_2 \sin \phi = A \sin \theta$

Hence $y = A \sin \omega t \cos \theta + A \cos \omega t \sin \theta = A \sin(\omega t + \theta)$

Resultant amplitude $A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2 \cos\phi}$ and Phase angle $\theta = \tan^{-1} \dfrac{a_2 \sin\phi}{a_1 + a_2 \cos\phi}$

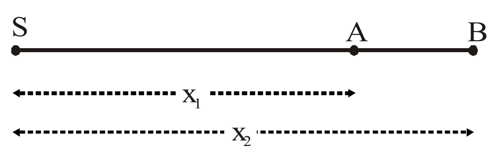
Intensity $\propto$ (Amplitude)^2 $\Rightarrow I \propto A^2 \Rightarrow I = KA^2$ So $I_1 = Ka_1^2$ & $I_2 = Ka_2^2$ $\therefore I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2} \cos\phi$

Here, $2\sqrt{I_1}\sqrt{I_2} \cos\phi$ is known as interference factor.

- If the distance of a source from two points A and B is x_1 and x_2 then

Path difference $\delta = x_2 - x_1$

Phase difference $\phi = \dfrac{2\pi}{\lambda}(x_2 - x_1) \Rightarrow \phi = \dfrac{2\pi}{\lambda}\delta$



Time difference $\Delta t = \dfrac{\phi}{2\pi} t$

$$\frac{\text{Phase difference}}{2\pi} = \frac{\text{Path difference}}{\lambda} = \frac{\text{Time difference}}{T} \Rightarrow \frac{\phi}{2\pi} = \frac{\delta}{\lambda} = \frac{\Delta t}{T}$$

TYPES OF INTERFERENCE

(i) Constructive Interference

When both waves are in same phase. So phase difference is an even multiple of $\pi \Rightarrow \phi = 2n\pi$; $n = 0,1,2$ ...

(a) When path difference is an even multiple of $\frac{\lambda}{2}$

$$\frac{\phi}{2\pi} = \frac{\delta}{\lambda} \Rightarrow \frac{2n\pi}{2\pi} = \frac{\delta}{\lambda} \Rightarrow \delta = 2n\left(\frac{\lambda}{2}\right) \Rightarrow \delta = n\lambda \text{ (where } n = 0,1,2...)$$

(b) When time difference is an even multiple of $\frac{T}{2}$ $\quad \therefore \Delta t = 2n\left(\frac{T}{2}\right)$

(c) In this condition the resultant amplitude and Intensity will be maximum.

$$A_{max} = (a_1 + a_2) \Rightarrow I_{max} = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2} = (\sqrt{I_1} + \sqrt{I_2})^2$$

(ii) Destructive Interference

When both the waves are in opposite phase. So phase difference is an odd multiple of π .
$\phi = (2n-1)\pi$; $n = 1, 2$ ...

(a) When path difference is an odd multiple of $\frac{\lambda}{2}$, $\delta = (2n-1)\frac{\lambda}{2}$, $n = 1, 2$ ...

(b) When time difference is an odd multiple of $\frac{T}{2}$, $\Delta t = (2n-1)\frac{T}{2}$, (n=1,2...)

In this condition the resultant amplitude and intensity of wave will be minimum.

$$A_{min} = (a_1 - a_2) \Rightarrow I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$$

ETOOS KEY POINTS

(i) Interference follows law of conservation of energy.

(ii) Average Intensity $I_{av} = \frac{I_{max} + I_{min}}{2} = I_1 + I_2 = a_1^2 + a_2^2$

(iii) Intensity $\propto$ width of slit $\propto$ (amplitude)$^2 \Rightarrow I \propto w \propto a^2 \Rightarrow \frac{I_1}{I_2} = \frac{w_1}{w_2} = \frac{a_1^2}{a_2^2}$

(iv) $$\frac{I_{max}}{I_{min}} = \left[\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right]^2 = \left[\frac{a_1 + a_2}{a_1 - a_2}\right]^2 = \left[\frac{a_{max}}{a_{min}}\right]^2$$

(v) Fringe visibility $V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \times 100\%$ when $I_{min} = 0$ then fringe visibility is maximum
i.e. when both slits are of equal width the fringe visibility is the best and equal to 100%.

Ex. If two waves represented by $y_1 = 4 \sin \omega t$ and $y_2 = 3 \sin (\omega t + \frac{\pi}{3})$ interfere at a point. Find out the amplitude of the resulting wave.

Sol. Resultant amplitude $A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi} = \sqrt{(4)^2 + (3)^2 + 2.(4)(3)\cos\frac{\pi}{3}} \Rightarrow A \simeq 6$

Ex. Two beams of light having intensities I and 4I interferer to produce a fringe pattern on a screen. The phase difference between the beam is $\frac{\pi}{2}$ at point A and 2π at point B. Then find out the difference between the resultant intensities at A and B.

Sol. Resultant intensity $I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi$

Resultant intensity at point A is $I_A = I + 4I + 2\sqrt{I}\sqrt{4I}\cos\frac{\pi}{2} = 5I$

Resultant intensity at point B, $I_B = I + 4I + 2\sqrt{I}\sqrt{4I}\cos 2\pi = 9I$ $\quad (\because \cos 2\pi = 1)\ \therefore I_B - I_A = 9I - 5I \Rightarrow 4I$

Ex. In interference pattern, if the slit widths are in the ratio 1:9. Then find out the ratio of minimum and maximum intensity.

Sol. Slit width ratio

$$\frac{w_1}{w_2} = \frac{1}{9} \because \frac{I_1}{I_2} = \frac{w_1}{w_2} = \frac{a_1^2}{a_2^2} = \frac{1}{9} \Rightarrow \frac{a_1}{a_2} = \frac{1}{3} \Rightarrow 3a_1 = a_2 \therefore \frac{I_{min}}{I_{max}} = \frac{(a_1 - a_2)^2}{(a_1 + a_2)^2} = \frac{(a_1 - 3a_1)^2}{(a_1 + 3a_1)^2} = \frac{4}{16} = 1:4$$

Ex. The intensity variation in the interference pattern obtained with the help of two coherent source is 5% of the average intensity. Find out the ratio of intensities of two sources.

Sol. $$\frac{I_{max}}{I_{min}} = \frac{105}{95} = \frac{21}{19} \Rightarrow \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{21}{19} \Rightarrow \frac{a_1 + a_2}{a_1 - a_2} = \sqrt{\frac{21}{19}} = 1.05 \Rightarrow a_1 + a_2 = 1.05\,a_1 - 1.05\,a_2$$

$$0.05\,a_1 = 2.05\,a_2 \Rightarrow \frac{a_1}{a_2} = \frac{2.05}{0.05} = \frac{41}{1} \quad \therefore \frac{I_1}{I_2} = \frac{a_1^2}{a_2} = \frac{1680}{1}$$

Ex. Waves emitted by two identical sources produces intensity of K unit at a point on screen where path difference between these waves is λ, calculate the intensity at that point on screen at which path difference is $\frac{\lambda}{4}$.

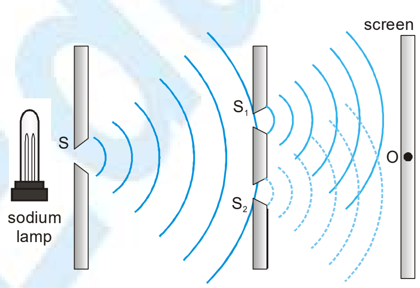
Sol. $$\phi_1 = \frac{2\pi\delta}{\lambda} \Rightarrow \frac{2\pi}{\lambda} \times \lambda = 2\pi \quad \text{and} \quad \phi_2 = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2} \quad I_1 = I_0 + I_0 + 2\sqrt{I_0}\sqrt{I_0}\cos 2\pi = 4I_0$$

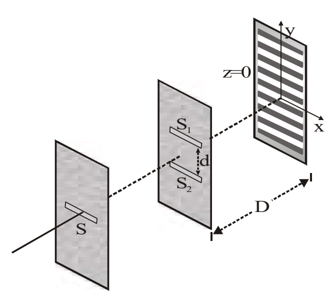
and $$I_2 = I_0 + I_0 + 2\sqrt{I_0}\sqrt{I_0}\frac{\cos 2\pi}{2} = 2I_0 \quad \therefore \frac{I_1}{I_2} = \frac{4I_0}{2I_0} = 2 \quad \Rightarrow I_2 = \frac{I_1}{2} = \frac{K}{2} \text{ unit} \quad [\because I_1 = K \text{ unit}]$$

YOUNG'S DOUBLE SLIT EXPERIMENT (YDSE)

According to Huygen, light is a wave. It is proved experimentally by YDSE.

S is a narrow slit illuminated by a monochromatic source of light sends wave fronts in all directions. Slits S_1 and S_2 become the source of secondary wavelets which are in phase and of same frequency. These waves are superimposed on each other gave rise to interference. Alternate dark and bright bands are obtained on a screen (calledinterference fringes) placed certain distance from the plane of slit S_1 and S_2. Central fringe is always bright (due to path from S_1O and S_2O centre is equal) called central maxima.





Energy is conserved in interference. This indicated that energy is redistributed from destructive interference region to the constructive interference region .

(i) If one of the two slit is closed. The interference pattern disappears. It shows that two coherent sources are required to produce interference pattern.

(ii) If white light is used as parent source, then the fringes will be coloured and of unequal width.

(a) Central fringe will be white.

(b) As the wave length of violet colour is least, so fringe nearest to either side of the central white fringe is violet and the fringe farthest from the central white fringe is red.

CONDITION FOR BRIGHT AND DARK FRINGES

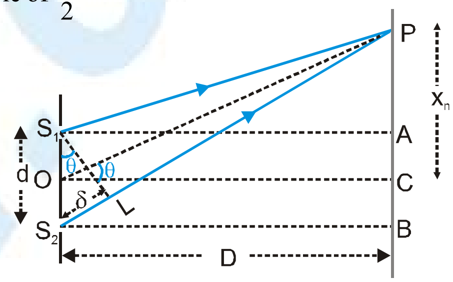
(i) Bright Fringe

D = distance between slit and screen, d = distance between slit S_1 and S_2

Bright fringe occurs due to constructive interference.

✈ For constructive interference path difference should be even multiple of $\frac{\lambda}{2}$

$\therefore$ Path difference $\delta = PS_2 - PS_1 = S_2L = (2n)\frac{\lambda}{2}$



In ΔPCO $\tan\theta = \frac{x_n}{D}$; In ΔS_1S_2L $\sin\theta = \frac{\delta}{d}$

$\delta = n\lambda$ for bright fringes

If θ is small then $\tan\theta \simeq \sin\theta$ $\Rightarrow$ $\frac{x_n}{D} = \frac{\delta}{d}$

The distance of n^{th} bright fringe from the central bright fringe $x_n = n\frac{D\lambda}{d}$

(ii) Dark Fringe

Dark fringe occurs due to destructive interference.

✈ For destructive interference path difference should be odd multiple of $\frac{\lambda}{2}$.

$\therefore$ Path difference $\delta = (2m-1)\,\frac{\lambda}{2}$

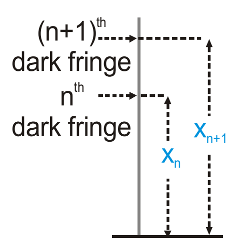
The distance of the m^{th} dark fringe from the central bright fringe $x_m = \frac{(2m-1)D\lambda}{2d}$

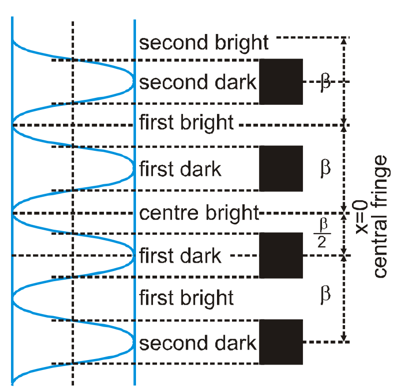
(iii) Fringe Width

The distance between two successive bright or dark fringe is known as fringe width.

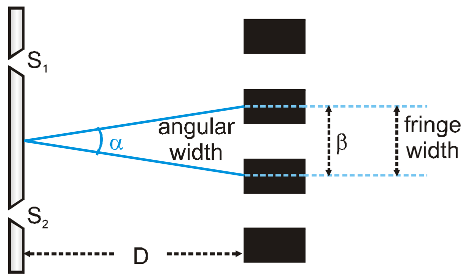
$$\beta = x_{n+1} - x_n = \frac{(n+1)D\lambda}{d} - \frac{nD\lambda}{d}$$

Fringe Width $\beta = \frac{D\lambda}{d}$





(iv) Angular Fringe width



Angular Fringe Width $\alpha = \frac{\beta}{D}, \ \alpha = \frac{\lambda}{d} \quad \left[\because \frac{\beta}{D} = \frac{\lambda}{d}\right]$

(a) The distance of n^{th} bright fringe from the central bright fringe $x_n = \frac{n\lambda D}{d} = n\beta$

(b) The distance between n_1 and n_2 bright fringe $x_{n_2} - x_{n_1} = n_2\frac{\lambda D}{d} - n_1\frac{\lambda D}{d} = (n_2 - n_1)\beta$

(c) The distance of m^{th} dark fringe from central fringe $x_m = \frac{(2m-1)D\lambda}{2d} = \frac{(2m-1)\beta}{2}$

(d) The distance of n^{th} bright fringe from m^{th} dark fringe $x_n - x_m = n\frac{D\lambda}{d} - \frac{(2m-1)D\lambda}{2d} = n\beta - \frac{(2m-1)\beta}{2}$

$$x_n - x_m = \left[n - \frac{(2m-1)}{2}\right]\beta$$

ETOOS KEY POINTS

(i) If the whole apparatus is immersed in a liquid of refractive index μ, then wavelength of light $\lambda' = \frac{\lambda}{\mu}$ since $\mu > 1$ so $\lambda' < \lambda \Rightarrow$ wavelength will decrease. Hence fringe width ($\beta \propto \lambda$) will decrease $\Rightarrow$ fringe width in liquid $\beta' = \beta/\mu$ angular width will also decrease.

(ii) With increase in distance between slit and screen D, angular width of maxima does not change, fringe width β increase linearly with D but the intensity of fringes decreases.

(iii) If an additional phase difference of π is created in one of the wave then the central fringe become dark.

(iv) When wavelength λ_1 is used to obtain a fringe n_1. At the same point wavelength λ_2 is required to obtain a fringe n_2 then $n_1\lambda_1 = n_2\lambda_2$

(v) When waves from two coherent sources S_1 and S_2 interfere in space the shape of the fringe is hyperbolic with foci at S_1 and S_2.

Ex. Laser light of wavelength 630 nm incident on a pair of slits produces an interference pattern in which the bright fringes are separated by 8.1 mm. A second light produces an interference pattern in which the fringes are separated by 7.2 mm. Calculate the wavelength of the second light.

Sol. Fringe separation is given by $\beta = \frac{\lambda D}{d}$ i.e. $\frac{\beta_2}{\beta_1} = \frac{\lambda_2}{\lambda_1} \Rightarrow \lambda_2 = \frac{\beta_2}{\beta_1} \times \lambda_1 = \frac{7.2}{8.1} \times 630 = 560$ nm

Ex. A double slit is illuminated by light of wave length 6000Å. The slit are 0.1 cm apart and the screen is placed one metre away. Calculate :

(i) The angular position of the 10^{th} maximum in radian and

(ii) Separation of the two adjacent minima.

Sol. **(i)** $\lambda = 6000$ Å $= 6\times 10^{-7}$ m, d = 0.1 cm = 1×10^{-3} m, D = 1m, n = 10

Angular position $\theta_n = \frac{n\lambda}{d} = \frac{10\times6\times10^{-7}}{10^{-3}} = 6\times10^{-3}$ rad.

(ii) Separation between two adjacent minima = fringe width β

$$\beta = \frac{\lambda D}{d} = \frac{6\times10^{-7}\times1}{1\times10^{-3}} = 6\times10^{-4}\text{m} = 0.6\text{ mm}$$

Ex. In Young's double slit experiment the fringes are formed at a distance of 1m from double slit of separation 0.12 mm. Calculate

(i) The distance of 3rd dark band from the centre of the screen.

(ii) The distance of 3rd bright band from the centre of the screen, given $\lambda = 6000$Å

Sol. **(i)** For m^{th} dark fringe $x'_m = (2m-1)\frac{D\lambda}{2d}$ given, D = 1m = 100 cm, d = 0.12 mm = 0.012 cm

$$x'_3 = \frac{(2\times3-1)\times100\times6\times10^{-7}}{2\times0.012} = 1.25\text{ cm}\ [\because\ m=3 \text{ and } \lambda = 6\times10^{-7}\text{ m}]$$

(ii) For n^{th} bright fringe $x_n = \frac{nD\lambda}{d} \Rightarrow x_3 = \frac{3\times100\times6\times10^{-7}}{0.012} = 1.5\times10^{-2}\text{m} = 1.5\text{ cm}\ [\because n=3]$

Ex. In Young's double slit experiment the two slits are illuminated by light of wavelength 5890Å and the distance between the fringes obtained on the screen is 0.2°. The whole apparatus is immersed in water, then find out angular fringe width, (refractive index of water = $\frac{4}{3}$).

Sol. $\alpha_{air} = \frac{\lambda}{d} \Rightarrow \alpha_{air} = 0.2° \Rightarrow \alpha \propto \lambda \Rightarrow \frac{\alpha_w}{\alpha_{air}} = \frac{\lambda_w}{\lambda_{air}} \Rightarrow \lambda_w = \frac{\lambda_{air}}{\mu} \Rightarrow \alpha_w = \frac{\alpha_{air}\lambda}{\mu.\lambda} = \frac{0.2\times3}{4} = 0.15$

Ex. The path difference between two interfering waves at a point on screen is 171.5 times the wavelength. If the path difference is 0.01029 cm. Find the wavelength.

Sol. Path difference = $171.5\,\lambda = \frac{343}{2}\lambda$ = odd multiple of half wavelength . It means dark fringe is observed

According to question $0.01029 = \frac{343}{2}\lambda \Rightarrow \lambda = \frac{0.01029\times2}{343} = 6\times10^{-5}$ cm $\Rightarrow \lambda = 6000$ Å

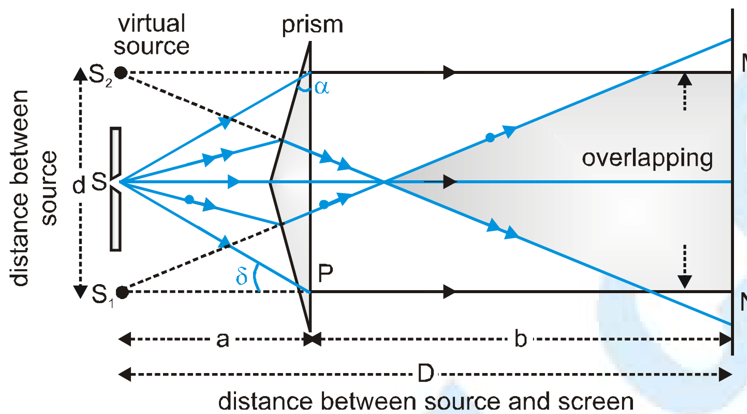
Ex. In young's double slit interference experiment, the distance between two sources is 0.1/π mm. The distance of the screen from the source is 25 cm. Wavelength of light used is 5000Å. Then what is the angular position of the first dark fringe ?

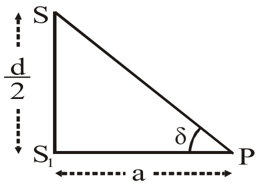
Sol. The angular position $\theta = \frac{\beta}{D} = \frac{\lambda}{d}$ $(\because\ \beta = \frac{\lambda D}{d})$ The first dark fringe will be at half the fringe width from the mid point of central maximum. Thus the angular position of first dark fringe will be-

$$\alpha = \frac{\theta}{2} = \frac{1}{2}\left[\frac{\lambda}{d}\right] = \frac{1}{2}\left[\frac{5000\times\pi}{.1\times10^{-3}}\times10^{-10}\right]\frac{180}{\pi} = 0.45°.$$

FRESNEL'S BIPRISM

It is an optical device to obtain two coherent sources by refraction of light. It is prepared by rubbing an optically pure glass plate slightly on two sides so that each angle of prism is generally $\frac{1^\circ}{2}$ or 1°. The fringes of equal width are observed in the limited region MN due to superposition.





Distance between source and biprism = a

Distance between biprism and eye piece (screen) = b

The distance between source and screen D = a + b

Refracting angle = α , refractive index of the material of prism = μ

The distance between two coherent source = d

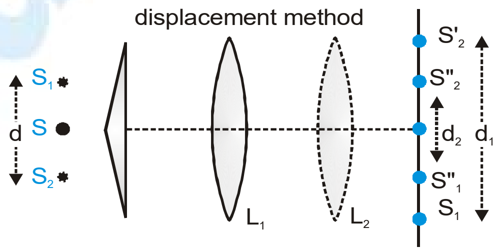
From ΔSS_1P $\tan\delta = \frac{d/2}{a}$ for very-very small δ hence $\tan\delta = \delta$ so $\delta = \frac{d}{2a} \Rightarrow d = 2a\delta$

For prism $\delta = (\mu-1)\alpha$ $\therefore$ $d = 2a(\mu-1)\alpha$, Fringe width $\beta = \frac{\lambda D}{d}$ $\therefore$ $\beta = \frac{(a+b)\lambda}{2a(\mu-1)\alpha}$

- **To calculate the value of d by displacement method**

In this method a convex lens is placed between prism and screen. The lens is adjusted in two position L_1 and L_2 and image is obtained on screen. Let d_1 an d_2 be the real image in these two cases.

The distance d between the virtual source $d = \sqrt{d_1 d_2}$ Fringe width $\beta = \frac{(a+b)\lambda}{\sqrt{d_1 d_2}}$



ETOOS KEY POINTS

(i) If the fresnel biprism experiment is performed in water instead of air then

(i) Fringe width in water increases $\left[\beta_w = \frac{\mu_g - 1}{\mu_g - \mu_w}\beta_{air}\right]$ $\beta_w = 3\ \beta_{air}\left[\because \mu_g = \frac{3}{2}, \mu_w = \frac{4}{3}\right]$

(ii) Separation between the two virtual sources decreases.

(but in Young's double slit experiment it does not change.)

$d_{air} = 2\,a\,(\mu_g - 1)\,\alpha \;\therefore\; d_w = 2\,a\,({}_w\mu_g - 1)\,\alpha \Rightarrow d_w = 2\,a\left[\frac{\mu_g}{\mu_w} - 1\right]\alpha$

$$\therefore \frac{d_w}{d_{air}} = \frac{\frac{\mu_g}{\mu_w} - 1}{\mu_g - 1} = \frac{\frac{3/2}{4/3} - 1}{3/2 - 1} = \frac{1}{4} \Rightarrow d_w = \frac{1}{4}d_{air}$$

(ii) If we use white light instead of monochromatic light then coloured fringes of different width are obtained. Central fringe is white.

(iii) With the help of this experiment the wavelength of monochromatic light, thickness of thin films and their refractive index and distance between apparent coherent sources can be determined.

Ex. Fringes are obtained with the help of a biprism in the focal plane of an eyepiece distant 1m from the slit. A convex lens produces images of the slit in two position between biprism and eyepiece. The distances between two images of the slit in two positions are 4.05×10^{-3} m and 2.9×10^{-3} m respectively. Calculate the distance between the slits.

Sol. $d = \sqrt{d_1 d_2} = \sqrt{4.05 \times 10^{-3} \times 2.9 \times 10^{-3}} = 3.43 \times 10^{-3}$ m

Ex. In fresnel's biprism experiment a mica sheet of refractive index 1.5 and thickness 6×10^{-6} m is placed in the path of one of interfering beams as a result of which the central fringe gets shifted through five fringe widths. Then calculate the wavelength of light.

Sol. $x = \frac{(\mu - 1)t\beta}{\lambda} = \frac{(1.5 - 1)t\beta}{\lambda}$ but, $t = 5\beta$ $\quad\therefore 5\beta = \frac{0.5\ t\beta}{\lambda} \Rightarrow \lambda = \frac{t}{10} = \frac{6 \times 10^{-6}}{10} = 6000$Å

Ex. A whole biprism experiment is immersed in water. If the fringe width in air is β_a and refractive index of biprism material and water are 1.5 and 1.33 respectively. Find the value of the fringe width.

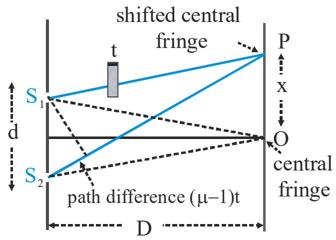
Sol. $$\beta_w = \frac{\mu_g - 1}{\mu_g - \mu_w}\beta_a = \frac{\frac{3}{2} - 1}{\frac{3}{2} - \frac{4}{2}} = 3\beta_a$$

Ex. In fresnel's biprism experiment the distance between the source and the screen is 1m and that between the source and biprism is 10 cm. The wavelength of light used is 6000Å. The fringe width obtained is 0.03 cm and the refracting angle of biprism is 1°. Then calculate the refractive index of the material of biprism.

Sol. $\beta = \frac{D\lambda}{2a(\mu - 1)\alpha} \quad \therefore (\mu - 1) = \frac{D\lambda}{2a\beta_\alpha} = \frac{1 \times 6 \times 10^{-7} \times 180}{2 \times 0.1 \times 3 \times 10^{-4} \times 3.14} \Rightarrow (\mu - 1) = 0.573 \Rightarrow \mu = 1.573$

THICKNESS OF THIN FILMS

When a glass plate of thickness t and refractive index μ is placed in front of the slit in YDSE then the central fringe shifts towards that side in which glass plate is placed because extra path difference is introduced by the glass plate. In the path S_1P distance travelled by wave in air = $S_1P - t$



Distance travelled by wave in the sheet = t

Time taken by light to reach up to point P will be same from S_1 and S_2

$$\frac{S_2P}{c} = \frac{S_1P - t}{c} + \frac{t}{c/\mu} \Rightarrow \frac{S_2P}{c} = \frac{S_1P + (\mu-1)t}{c} \Rightarrow S_2P = S_1P + (\mu-1)t \Rightarrow S_2P - S_1P = (\mu-1)t$$

Path difference = $(\mu-1)t \Rightarrow$ Phase difference $\phi = \frac{2\pi}{\lambda}(\mu-1)t$

Distance of shifted fringe from central fringe $x = \frac{D(\mu-1)t}{d}$ $\left[\because \frac{xd}{D} = (\mu-1)t\right]$

$\therefore\ x = \frac{\beta(\mu-1)t}{\lambda}$ and $\beta = \frac{D\lambda}{d}$ Number of fringes displaced = $\frac{(\mu-1)t}{\lambda}$

Ex. When a mica sheet of thickness 7 microns and μ = 1.6 is placed in the path of one of interfering beams in the biprism experiment then the central fringe gets at the position of seventh bright fringe. What is the wavelength of light used ?

Sol. $\lambda = \frac{(\mu-1)t}{n} = \frac{(1.6-1)\,7\times10^{-6}}{7} = 6 \times 10^{-7}$ meter

ETOOS KEY POINTS

(i) If a glass plate of refractive index μ_1 and μ_2 having same thickness t is placed in the path of ray coming from S_1 and S_2 then path difference $x = \frac{D}{d}(\mu_1 - \mu_2)t$

(ii) Distance of displaced fringe from central fringe $x = \frac{\beta(\mu_1-\mu_2)t}{\lambda}$ ✈ $\frac{\beta}{\lambda} = \frac{D}{d}$

Colours In Thin Films

When white light is made incident on a thin film (like oil film on the surface of water or a soap bubble) Then interference takes place between the waves reflected from its two surfaces and waves refracted through it. The intensity becomes maximum and minimum as a result of interference and colours are seen.

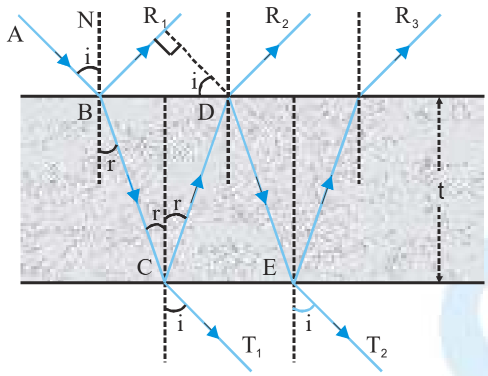
(i) The source of light must be an extended source

(ii) The colours obtained in reflected and transmitted light are mutually complementary.

(iii) The colours obtains in thin films are due to interference whereas those obtained in prism are due to dispersion.

INTERFERENCE DUE TO THIN FILMS

Consider a thin transparent film of thickness t and refractive index μ. Let a ray of light AB incident on the film at B. At B, a part of light is reflected along BR_1, and a part of light refracted along BC. At C a part of light is reflected along CD and a part of light transmitted along CT_1. At D, a part of light is refracted along DR_2 and a part of light is reflected along DE. Thus interference in this film takes place due to reflected light in between BR_1 and DR_2 also in transmitted light in between CT_1 and ET_2.



(i) Reflected System

The path difference between BR_1 and DR_2 is x = 2μt cos r due to reflection from the surface of denser medium involves an additional phase difference of π or path difference $\lambda/2$. Therefore the exact path difference between BR_1 and DR_2 is. $\Rightarrow x' = 2\mu t \cos r - \lambda/2$ maximum or constructive Interference occurs when path difference between the light waves is $n\lambda$.$2\mu t \cos r - \lambda/2 = n\lambda \Rightarrow 2\mu t \cos r = n\lambda + \lambda/2$

So the film will appear bright if $2\mu t \cos r = (2n+1)\lambda/2$ (n = 0, 1, 2, 3 .....)

(ii) For minima or destructive interference

When path difference is odd multiple of $\frac{\lambda}{2} \Rightarrow 2\mu t \cos r - \frac{\lambda}{2} = (2n-1)\frac{\lambda}{2}$

So the film will appear dark if $2\mu t \cos r = n\lambda$

(iii) For transmitted system

Since No additional path difference between transmitted rays CT_1 and ET_2.

So the net path difference between them is $x = 2\mu t \cos r$

For maxima $2\mu t \cos r = n\lambda$, n = 0, 1, 2.............

Minima $2\mu t \cos r = (2n+1)\frac{\lambda}{2}$, n = 0, 1, 2............s

USES OF INTERFERENCE EFFECT

Thin layer of oil on water and soap bubbles show different colours due to interference of waves reflected from two surfaces of their films. Similarly when a lens of large radius of curvature is placed on a plane glass plate, an air film exist between the plate and the lens. If sodium light is put on this film, concentric bright and dark interference rings are formed. These rings are called as Newton's rings.

Uses

(i) Used to determine the wavelength of light precisely.

(ii) Used to determine refractive index or thickness of transparent sheet.

(iii) Used to test the flatness of plane surfaces. These surfaces are knows as optically plane surfaces.

(iv) Used to calibrate meters in terms of wavelength of light.

(v) Used to design optical filter which allows a narrow band of wavelength to pass through it.

(vi) Used in holography to produce 3-D images.

Ex. Light of wavelength 6000Å is incident on a thin glass plate of refractive index 1.5 such that angle of refraction into the plate is 60°. Calculate the smallest thickness of plate which will make it appear dark by reflection.

Sol. $2\mu t \cos r = n\lambda \Rightarrow t = \dfrac{n\lambda}{2\mu \cos r} = \dfrac{1 \times 6 \times 10^{-7}}{2 \times 1.5 \times \cos 60} = \dfrac{6 \times 10^{-7}}{1.5} = 4 \times 10^{-7}\,m$

Ex. Light is incident on a glass plate ($\mu = 1.5$) such that angle of refraction is 60°. Dark band is observed corresponding to the wavelength of 6000Å . If the thickness of glass plate is 1.2×10^{-3} mm. calculate the order of the interference band.

Sol. $\mu = 1.5, r = 60°, \lambda = 6000Å = 6 \times 10^{-7} m \Rightarrow t = 1.2 \times 10^{-3} = 1.2 \times 10^{-6} m$

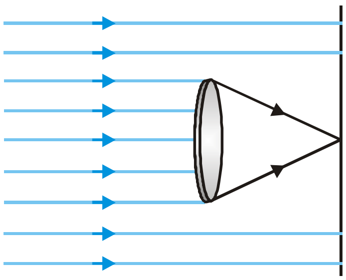
For dark band in the reflected light $2\mu t \cos r = n\lambda$

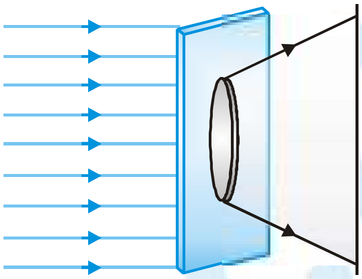
$$n = \frac{2\mu t \cos r}{\lambda} = \frac{2 \times 1.5 \times 1.2 \times 10^{-6} \times \cos 60^\circ}{6 \times 10^{-7}} = \frac{2 \times 1.5 \times 1.2 \times 10^{-6} \times \frac{1}{2}}{6 \times 10^{-7}} = 3$$

Thus third dark band is observed.

DIFFRACTION OF LIGHT

Bending of light rays from sharp edges of an opaque obstacle or aperture and its spreading in the eometricle shaddow region is defined as diffraction of light or deviation of light from its rectilinear propogation tendency is defined as diffraction of light.


diffraction from obstacle


diffraction from aperture

(i) Diffraction was discovered by Grimaldi

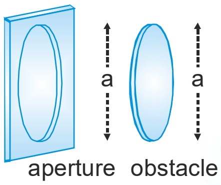
(ii) Theoritically explained by Fresnel

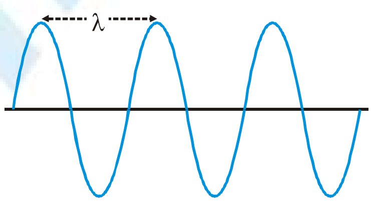
(iii) Diffraction is possible in all type of waves means in mechanical or electromagnetic waves shows diffraction.

Diffraction depends on two factors :

(a) Size of obstacles or aperture

(b) Wave length of the wave





Condition of diffraction — Size of obstacle or aperture should be nearly equal to the wave length of light

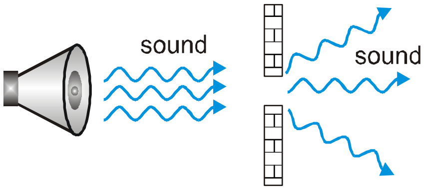
$\lambda \simeq a$ $\qquad \frac{a}{\lambda} \simeq 1$

If size of obstacle is much greater than wave length of light, then rectilinear motion of light is observed.

(iv) It is practically observed when size of aperture or obstacle is greater than 50 λ then obstacle or aperature does not shows diffraction.

(v) Wave length of light is in the order 10^{-7} m. In general obstacle of this wave length is not present so light rays does not show diffraction and it appears to travel in straight line Sound wave shows more diffraction as compare to light rays because wavelength of sound is high (16 mm to 16m). So it is generally diffracted by the objects in our daily life.

(vi) Diffraction of ultrasonic wave is also not observed as easily as sound wave because their wavelength is of the order of about 1 cm. Diffraction of radio waves is very conveniently observed because of its very large wavelength (2.5 m to 250 m). X-ray can be diffracted easily by crystel. It was discovered by Lave.



diffraction of sound from a window

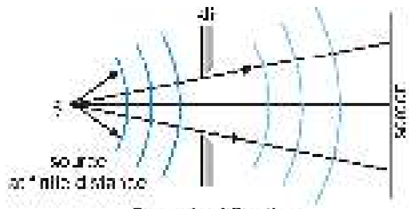
Types of Diffraction

There are two type of diffraction of light : (a) Fresnel's diffraction. (b) Fraunhofer's diffraction.

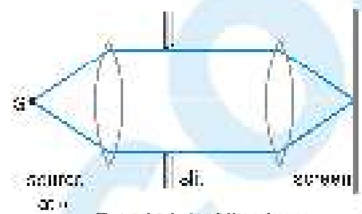
(a) Fresnel diffraction

If either source or screen or both are at finite distance from the diffracting device (obstacle or aperture), the diffraction is called Fresnel diffraction and the pattern is the shadow of the diffracting device modified by diffraction effect.

Ex. Diffraction at a straight edge, small opaque disc, narrow wire are examples of Fresnel diffraction.



Fresnel's diffraction



Fraunhofer's diffraction

(b) Fraunhofer diffraction

Fraunhofer diffraction is a particular limiting case of Fresnel diffraction. In this case, both source and screen are effectively at infinite distance from the diffracting device and pattern is the image of source modified by diffraction effects.

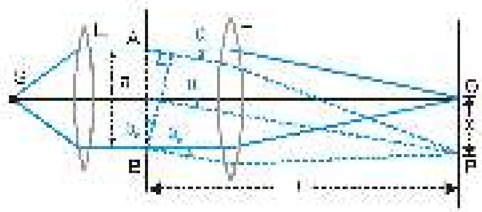
Ex. Diffraction at single slit, double slit and diffraction grating are the examples of fraunhofer diffraction.

Comparison between Fresnel and Fraunhofer diffraction

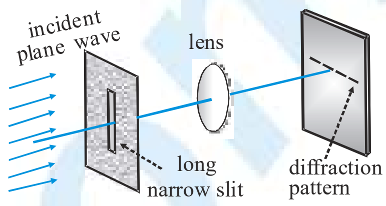
	Fresnel Diffraction	Fraunhofer Diffraction
(a)	Source and screen both are at finite distance from the diffractor.	Source and screen both are at infinite distance from the diffractor.
(b)	Incident and diffracted wave fronts are spherical or cylinderical.	Incident and diffracted wavefronts are plane due to infinite distance from source.
(c)	Mirror or lenses are not used for obtaining the diffraction pattern.	Lens are used in this diffraction pattern.
(d)	Centre of diffraction pattern is sometime bright and sometime dark depending on size of diffractor and distance of observation point.	Centre of diffraction is always bright.
(e)	Amplitude of wave coming from different half period zones are different due to difference of obliquity.	Amplitude of waves coming from different half period zones are same due to same obliquity.

Fraunhofer diffraction due to single slit

AB is single slit of width a. Plane wavefront is incident on a slit AB. Secondary wavelets coming from every part of AB reach the axial point P in same phase forming the central maxima. The intensity of central maxima is maximum in this diffraction. Where θ_1 represents direction of 1st minima Path difference BB' = a sin θ_1



for n^{th} minima $a \sin\theta_n = n\lambda$ $\quad\therefore\quad$ $\sin\theta_n \approx \theta_n = \dfrac{n\lambda}{a}$ (if θ_n is small)



(i) When path difference between the secondary wavelets coming from A and B is $n\lambda$ or $2n\left[\dfrac{\lambda}{2}\right]$ or even multiple of $\dfrac{\lambda}{2}$ then minima occurs

For minima $\quad a\sin\theta_n = 2n\left[\dfrac{\lambda}{2}\right]$ where $n = 1, 2, 3 \ldots$

(ii) When path difference between the secondary wavelets coming from A and B is $(2n+1)\dfrac{\lambda}{2}$ or odd multiple of $\dfrac{\lambda}{2}$ then maxima occurs

For maxima $\quad a\sin\theta_n = (2n+1)\dfrac{\lambda}{2}$ where $n = 1, 2, 3 \ldots$

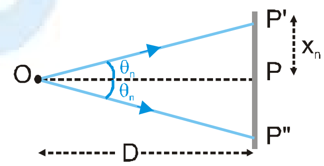
$n = 1 \rightarrow$ first maxima and $n = 2 \rightarrow$ second maxima

(iii) In alternate order minima and maxima occurs on both sides of central maxima.

For n^{th} minima

If distance of n^{th} minima from central maxima $= x_n$

distance of slit from screen = D , width of slit = a



Path difference $\delta = a\sin\theta_n = \dfrac{2n\lambda}{2} \Rightarrow \sin\theta_n = \dfrac{n\lambda}{a}$

In Δ POP' $\tan\theta_n = \dfrac{x_n}{D}$ $\quad$ If θ_n is small $\Rightarrow \sin\theta_n \approx \tan\theta_n \approx \theta_n$

$x_n = \dfrac{n\lambda D}{a} \Rightarrow \theta_n = \dfrac{x_n}{D} = \dfrac{n\lambda}{a}$ First minima occurs both sides on central maxima.

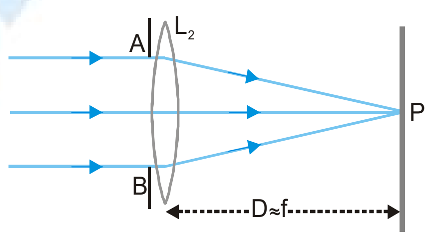
For first minima $x = \dfrac{D\lambda}{a}$ and $\theta = \dfrac{x}{D} = \dfrac{\lambda}{a}$

(iv) Linear width of central maxima $\quad w_x = 2x \Rightarrow w_x = \dfrac{2D\lambda}{a}$

(v) Angular width of central maxima $\quad w_\theta = 2\theta = \dfrac{2\lambda}{a}$

Special case

Lens L_2 is shifted very near to slit AB. In this case distance between slit and screen will be nearly equal to the focal length of lense L_2 (i.e. $D \approx f$) $\theta_n = \dfrac{x_n}{f} = \dfrac{n\lambda}{a} \Rightarrow x_n = \dfrac{n\lambda f}{a}$



$w_x = \dfrac{2\lambda f}{a}$ and angular width of central maxima $w_B = \dfrac{2x}{f} = \dfrac{2\lambda}{a}$

Fringe width : Distance between two consecutive maxima (bright fringe) or minima (dark fringe) is known as fringe width. Fringe width of central maxima is doubled then the width of other maximas i.e.,

$$\beta = x_{n+1} - x_n = (n+1)\frac{\lambda D}{a} - \frac{n\lambda D}{a} = \frac{\lambda D}{a}$$

Intensity curve of Fraunhofer's diffraction

Intensity of maxima in Fraunhofer's diffrection is determined by

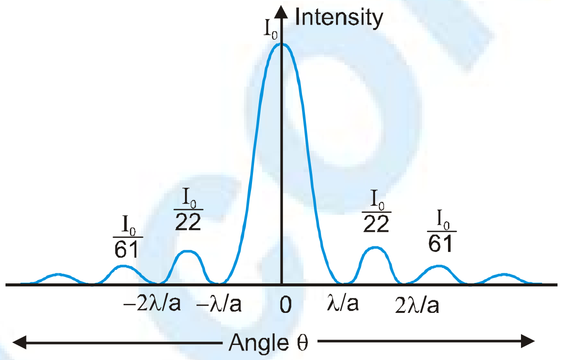
$$I = \left[\frac{2}{(2n+1)\pi}\right]^2 I_0$$

I_0 =intensity of central maxima

n = order of maxima

intensity of first maxima $I_1 = \frac{4}{9\pi^2} I_0 \approx \frac{I_0}{22}$

intensity of second maxima $I_2 = \frac{4}{25\pi^2} I_0 \approx \frac{I_0}{61}$



- Diffraction occurs in slit is always fraunhofer diffraction as diffraction pattern obtained from the cracks between the fingers, when viewed a distant tubelight and in YDSE experiment are fraunhofer diffraction.

ETOOS KEY POINTS

(i) The width of central maxima ∝ λ, that is, more for red colour and less for blue.

i.e., $w_x \propto \lambda$ as $\lambda_{blue} < \lambda_{red}$ ⇒ $w_{blue} < w_{red}$

(ii) For obtaining the fraunhofer diffraction, focal length of second lens (L_2) is used.

$w_x \propto \lambda \propto f \propto 1/a$ width will be more for narrow slit

(iii) By decreasing linear width of slit, the width of central maxima increase.

Resolving Power (R.P.)

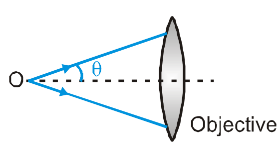
A large number of images are formed as a consequence of light diffraction from a source. If two sources are separated such that their central maxima do not overlap, their images can be distinguished and are said to be resolved R.P. of an optical instrument is its ability to distinguish two neighbouring points.

Linear R.P. = d/λD here D = Observed distance

Angular R.P. = d/λ d = Distance between two points

(1) Microscope : In reference to a microscope, the minimum distance between two lines at which they are just distinct is called Resolving limit (RL) and it's reciprocal is called Resolving power (RP)

$$\text{R.L.} = \frac{\lambda}{2\mu\sin\theta} \text{ and } \text{R.P.} = \frac{2\mu\sin\theta}{\lambda} \Rightarrow \text{R.P.} \propto \frac{1}{\lambda}$$



λ = Wavelength of light used to illuminate the object

μ = Refractive index of the medium between object and objective,

θ = Half angle of the cone of light from the point object, μsinθ = Numerical aperture.

(2) **Telescope :** Smallest angular separations ($d\theta$) between two distant object, whose images are separated in the telescope is called resolving limit. So resolving limit $d\theta = \frac{1.22\lambda}{a}$ and resolving power

$$(RP) = \frac{1}{d\theta} = \frac{a}{1.22\lambda} \Rightarrow R.P. \propto \frac{1}{\lambda}$$ where a = aperture of objective.

Ex. Light of wavelength 6000Å is incident normally on a slit of width 24×10^{-5} cm. Find out the angular position of second minimum from central maximum ?

Sol. $a \sin\theta = 2\lambda$ given $\lambda = 6 \times 10^{-7}$ m, $a = 24 \times 10^{-5} \times 10^{-2}$ m

$$\sin\theta = \frac{2\lambda}{a} = \frac{2 \times 6 \times 10^{-7}}{24 \times 10^{-7}} = \frac{1}{2} \qquad \therefore \theta = 30°$$

Ex. Light of wavelength 6328Å is incident normally on a slit of width 0.2 mm. Calculate the angular width of central maximum on a screen distance 9 m ?

Sol. given $\lambda = 6.328 \times 10^{-7}$ m, $a = 0.2 \times 10^{-3}$ m

$$w_\theta = \frac{2\lambda}{a} = \frac{2 \times 6.328 \times 10^{-7}}{2 \times 10^{-4}} \text{ radian} = \frac{6.328 \times 10^{-3} \times 180}{3.14} = 0.36°$$

Ex. Light of wavelength 5000Å is incident on a slit of width 0.1 mm. Find out the width of the central bright line on a screen distance 2m from the slit ?

Sol. $$w_x = \frac{2f\lambda}{a} = \frac{2 \times 2 \times 5 \times 10^{-7}}{10^{-4}} = 20 \text{ mm}$$

Ex. The fraunhofer diffraction pattern of a single slit is formed at the focal plane of a lens of focal length 1m. The width of the slit is 0.3 mm. If the third minimum is formed at a distance of 5 mm from the central maximum then calculate the wavelength of light.

Sol. $$x_n = \frac{nf\lambda}{a} \Rightarrow \lambda = \frac{ax_n}{fn} = \frac{3 \times 10^{-4} \times 5 \times 10^{-3}}{3 \times 1} = 5000Å$$ • ✈ n=3]

Ex. Find the half angular width of the central bright maximum in the Fraunhofer diffraction pattern of a slit of width 12×10^{-5} cm when the slit is illuminated by monochromatic light of wavelength 6000 Å.

Sol. ✈ $\sin\theta = \frac{\lambda}{a}$ θ = half angular width of the central maximum.

$a = 12 \times 10^{-5}$ cm, $\lambda = 6000$ Å $= 6 \times 10^{-5}$ cm $\therefore$ $\sin\theta = \frac{\lambda}{a} = \frac{6 \times 10^{-5}}{12 \times 10^{-5}} = 0.50 \Rightarrow \theta = 30°$

Ex. Light of wavelength 6000 Å is incident on a slit of width 0.30 mm. The screen is placed 2 m from the slit. Find **(a)** the position of the first dark fringe and **(b)** the width of the central bright fringe.

Sol. The first fringe is on either side of the central bright fringe.

here $n = \pm 1$, $D = 2$ m, $\lambda = 6000$ Å $= 6 \times 10^{-7}$ m

$\therefore$ $\sin\theta = \frac{x}{D}$ $\Rightarrow$ $a = 0.30$ mm $= 3 \times 10^{-4}$ m $\Rightarrow$ $a\sin\theta = n\lambda$ $\Rightarrow$ $\frac{ax}{D} = n\lambda$

(a) $$x = \frac{n\lambda D}{a} \Rightarrow x = \pm\left[\frac{1 \times 6 \times 10^{-7} \times 2}{3 \times 10^{-4}}\right] = \pm 4 \times 10^{-3} \text{ m}$$

The positive and negative signs corresponds to the dark fringes on either side of the central bright fringe.

(b) The width of the central bright fringe $y = 2x = 2 \times 4 \times 10^{-3} = 8 \times 10^{-3}$ m $= 8$ mm

Difference between interference and diffraction :

	Interference		Diffraction
1	It is the phenomenon of superposition of two waves coming from two different coherent sources.	1	It is the phenomenon of superposition of two waves coming from two different parts of the same wave front.
2	In interference pattern, all bright lines are equally bright and equally spaced.	2	All bright lines are not equally bright and equally wide. Brightness and width goes on decreasing with the angle of diffraction.
3	All dark lines are totally dark	3	Dark lines are not perfectly dark. Their on decreasing with angle of diffraction.
4	In interference bands are large in number	4	In diffraction bands are a few in number.

Ex. A Slit of width a is illuminated by monochromatic light of wavelength 650nm at normal incidence. Calculate the value of a when -

(a) the first minimum falls at an angle of diffraction of 30°

(b) the first maximum falls at an angle of diffraction of 30°.

Sol. **(a)** For first minimum $\sin\theta_1 = \frac{\lambda}{a}$ $\therefore$ $a = \frac{\lambda}{\sin\theta_1} = \frac{650\times10^{-9}}{\sin 30^\circ} = \frac{650\times10^{-9}}{0.5} = 1.3\times10^{-6}\,m$

(b) For first maximum $\sin\theta_1 = \frac{3\lambda}{2a}$ $\therefore$ $a = \frac{3\lambda}{2\sin\theta} = \frac{3\times650\times10^{-9}}{2\times0.5} = 1.95\times10^{-6}\,m$

Ex. Red light of wavelength 6500Å from a distant source falls on a slit 0.50 mm wide. What is the distance between the first two dark bands on each side of the central bright of the diffraction pattern observed on a screen placed 1.8 m from the slit.

Sol. Given λ = 6500Å = 65×10^{-8} m, a = 0.5 mm = 0.5×10^{-3} m, D = 1.8 m.

Required distance between first two dark bands will be equal to width of central maxima.

$$W_x = \frac{2\lambda D}{a} = \frac{2\times6500\times10^{-10}\times1.8}{0.5\times10^{-3}} = 468\times10^{-5}\,m = 4.68\,mm$$

Ex. In a single slit diffraction experiment first minimum for λ_1 = 660 nm coincides with first maxima for wavelength λ_2. Calculate λ_2.

Sol. For minima in diffraction pattern $d\sin\theta = n\lambda$

For first minima $d\sin\theta = (1)\lambda_1 \rightarrow \sin\theta = \frac{\lambda_1}{d}$

For first maxima $d\sin\theta_2 = \frac{3}{2}\lambda_2 \rightarrow \sin\theta_2 = \frac{3\lambda_2}{2d}$

The two will coincide if $\theta_1 = \theta_2$ or $\sin\theta_1 = \sin\theta_2$

$\therefore$ $\frac{\lambda_1}{d} = \frac{3\lambda_2}{2d} \rightarrow \lambda_2 = \frac{2}{3}\lambda_1 = \frac{2}{3}\,660\,nm = 440\,nm.$

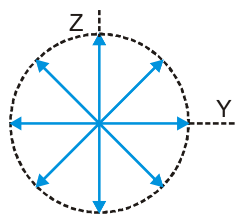
POLARISATION

Experiments on interference and diffraction have shown that light is a form of wave motion. These effects do not tell us about the type of wave motion i.e., whether the light waves are longitudinal or transverse.

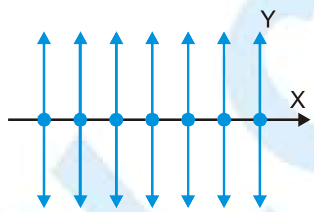
The phenomenon of polarization has helped to establish beyond doubt that light waves are transverse waves.

Unpolarised light

An ordinary beam of light consists of a large number of waves emitted by the atoms of the light source. Each atom produces a wave with its own orientation of electric vector $\vec{E}$ so all direction of vibration of $\vec{E}$ are equally probable.



unpolarised light propagating along . X-axis



unpolarised light

The resultant electromagnetic wave is a super position of waves produced by the individual atomic sources and it is called unpolarised light. In ordinary or unpolarised light, the vibrations of the electric vector occur symmetrically in all possible directions in a plane perpendicular to the direction of propagation of light.

Polarisation

The phenomenon of restricting the vibration of light (electric vector) in a particular direction perpendicular to the direction of propagation of wave is called polarisation of light. In polarised light, the vibration of the electric vector occur in a plane perpendicular to the direction of propagation of light and are confined to a single direction in the plane (do not occur symmetrically in all possible directions). After polarisation the vibrations become asymmetrical about the direction of propagation of light.

Polariser

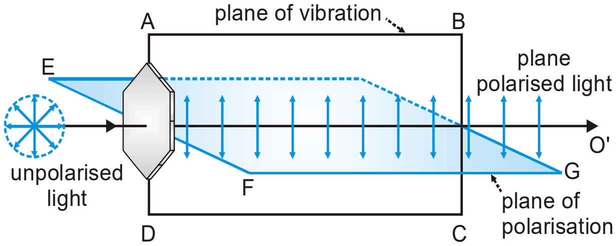
Tourmaline crystal : When light is passed through a tourmaline crystal cut parallel to its optic axis, the vibrations of the light carrying out of the tourmaline crystal are confined only to one direction in a plane perpendicular to the direction of propagation of light. The emergent light from the crystal is said to be plane polarised light.

Nicol Prism : A nicol prism is an optical device which can be used for the production and detection of plane polarised light. It was invented by William Nicol in 1828.

Polaroid : A polaroid is a thin commercial sheet in the form of circular disc which makes use of the property of selective absorption to produce an intense beam of plane polarised light.

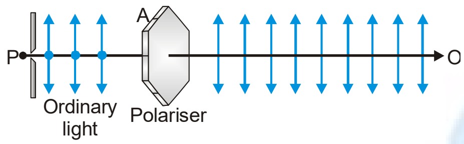
Plane of polarisation and Plane of vibration :

The plane in which vibrations of light vector and the direction of propogation lie is known as plane of vibration A plane normal to the plane of vibration and in which no vibration takes place is known as plane of polarisation



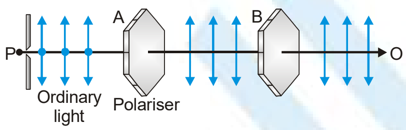
Experimental demonstration of polarisation of light

Take two tourmaline crystals cut parallel to their crystallographic axis (optic axis).

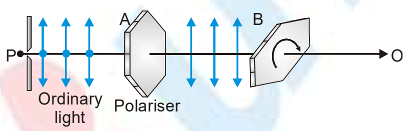


First hold the crystal A normally to the path of a beam of colour light. The emergent beam will be slightly coloured. Rotate the crystal A about PO. No change in the intensity or the colour of the emergent beam of light.

Take another crystal B and hold it in the path of the emergent beam of so that its axis is parallel to the axis of the crystal A. The beam of light passes through both the crystals and outcoming light appears coloured.



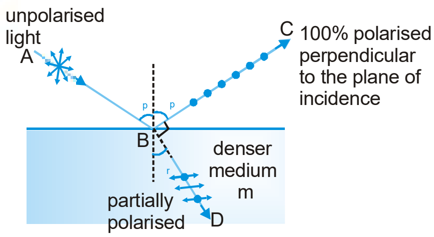
Now, rotate the crystal B about the axis PO. It will be seen that the intensity of the emergent beam decreases and when the axes of both the crystals are at right angles to each other no light comes out of the crystal B.



If the crystal B is further rotated light reappears and intensity becomes maximum again when their axes are parallel. This occurs after a further rotation of B through 90°. This experiment confirms that the light waves are transverse in nature. The vibrations in light waves are perpendicular to the direction of propagation of the wave. First crystal A polarises the light so it is called polariser. Second crystal B, analyses the light whether it is polarised or not, so it is called analyser.

Methods of obtaining plane polarised light

(i) **Polarisation by reflection** : The simplest method to produce plane polarised light is by reflection. This method was discovered by Malus in 1808. When a beam of ordinary light is reflected from a surface, the reflected light is partially polarised. The degree of polarisation of the polarised light in the reflected beam is greatest when it is incident at an angle called polarising angle or Brewster's angle.



(ii) **Polarising angle** : Polarising angle is that angle of incidence at which the reflected light is completely plane polarisation.

(iii) **Brewster's Law** : When unpolarised light strikes at polarising angle θ_p on an interface separating a rare medium from a denser medium of refractive index μ , such that $\mu = \tan\theta_p$ then the reflected light (light in rare medium) is completely polarised. Also reflected and refracted rays are normal to each other. This relation is known as Brewster's law. The law state that the tangent of the polarising angle of incidence of a transparent medium is equal to its refractive index $\mu = \tan\theta_p$

In case of polarisation by reflection :

(i) For $i = \theta_p$ refracted light is partially polarised.

(ii) For $i = \theta_p$ reflected and refracted rays are perpendicular to each other.

(iii) For $i < \theta_p$ or $i > \theta_p$ both reflected and refracted light become partially polarised.

According to snell's law $\mu = \dfrac{\sin\theta_p}{\sin\theta_r}$ ....**(i)**

But according to Brewster's law $\mu = \tan\theta_p = \dfrac{\sin\theta_p}{\cos\theta_p}$ ....**(ii)**

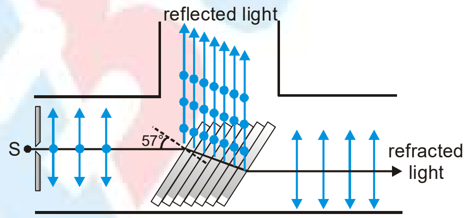
From equation **(i)** and **(ii)** $\dfrac{\sin\theta_p}{\sin\theta_r} = \dfrac{\sin\theta_p}{\cos\theta_p} \Rightarrow \sin\theta_r = \cos\theta_p$

$\therefore \sin\theta_r = \sin(90° - \theta_p) \Rightarrow \theta_r = 90° - \theta_p$ or $\theta_p + \theta_r = 90°$

Thus reflected and refracted rays are mutually perpendicular

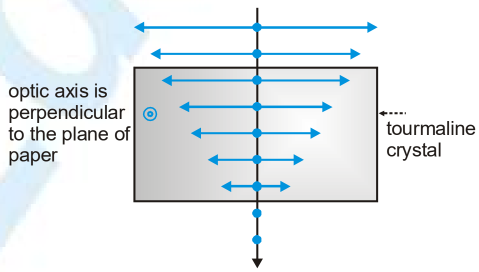
By Refraction

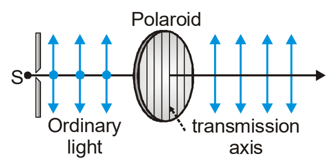
In this method, a pile of glass plates is formed by taking 20 to 30 microscope slides and light is made to be incident at polarising angle 57°. According Brewster law, the reflected light will be plane polarised with vibrations perpendicular to the plane of incidence and the transmitted light will be partially polarised. Since in one reflection about 15% of the light with vibration perpendicular to plane of paper is reflected therefore after passing through a number of plates emerging light will become plane polarised with vibrations in the plane of paper.



By Dichroism

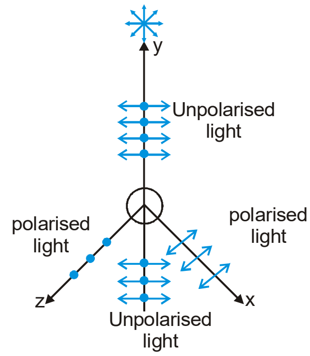
Some crystals such as tourmaline and sheets of iodosulphate of quinone have the property of strongly absorbing the light with vibrations perpendicular of a specific direction (called transmission axis) and transmitting the light with vibration parallel to it. This selective absorption of light is called dichroism. So if unpolarised light passes through proper thickness of these, the transmitted light will plane polarised with vibrations parallel to transmission axis. Polaroids work on this principle.





By scattering

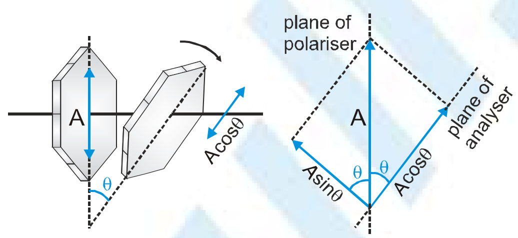
When light is incident on small particles of dust, air molecule etc. (having smaller size as compared to the wavelength of light), it is absorbed by the electrons and is re-radiated in all directions. The phenomenon is called as scattering. Light scattered in a direction at right angles to the incident light is always plane-polarised.



Law of Malus

When a completely plane polarised light beam light beam is incident analyser, then intensity intensity of emergent light varies as the square of cosine of the angle between the planes of transmission of the analyser and the polarizer.

$I \propto \cos^2\theta \;\Rightarrow I = I_0 \cos^2\theta$



(i) If $\theta = 0°$ then $I = I_0$ maximum value (Parallel arrangement)

(ii) If $\theta = 90°$ then $I = 0$ minimum value (Crossed arrangement)

If plane polarised light of intensity $I_0(= KA^2)$ is incident on a polaroid and its vibrations of amplitude A make angle θ with transmission axis, then the component of vibrations parallel to transmission axis will be $A\cos\theta$ while perpendicular to it will be $A\sin\theta$.

Polaroid will pass only those vibrations which are parallel to transmission axis i.e. $A\cos\theta$, ✈ $I_0 \propto A^2$

So the intensity of emergent light $I = K(A\cos\theta)^2 = KA^2\cos^2\theta$

If an unpolarised light is converted into plane polarised light its intensity becomes half.

If light of intensity I_1, emerging from one polaroid called polariser is incident on a second polaroid (called analyser) the intensity of light emerging from the second polaroid is $I_2 = I_1 \cos^2\theta$ θ = angle between the transmission axis of the two polaroids.

Optical Activity

When plane polarised light passes through certain substances, the plane of polarisation of the emergent light is rotated about the direction of propagation of light through a certain angle. This phenomenon is optical rotation.

The substance which rotate the plane of polarision rotates the plane of polarisation is known as optical active substance. Ex. Sugar solution, sugar crystal, sodium chlorate etc.

Optical activity of a substance is measured with the help of polarimeter in terms of specific rotation which is defined as the rotation produced by a solution of length 10 cm (1dm) and of unit concentration (1g/cc) for a given wave length of light at a given temp.

specific rotation $[\alpha]_{t^\circ C}^{\lambda} = \dfrac{\theta}{L \times C}$ θ = rotation in length L at concentration

Types of optically active substances

(a) Dextro rotatory substances

Those substances which rotate the plane of polarisation in clockwise direction are called dextro rotatory of right handed substances.

(b) Laveo rotatory substances

These substances which rotate the plane of polarisation in the anticlockwise direction are called laveo rotatory or left handed substances.

The amount of optical rotation depends upon the thickness and density of the crystal or concentration in case of solutions, the temperature and the wavelength of light used.

Rotation varies inversely as the square of the wavelength of light.

Applications and uses of polarisation

(i) By determining the polarising angle and using Brewster's Law $\mu = \tan\theta_P$ refractive index of dark transparent substance can be determined.

(ii) In calculators and watches, numbers and letters are formed by liquid crystals through polarisation of light called liquid crystal display (L.C.D.)

(iii) In CD player polarised laser beam acts as needle for producing sound from compact disc.

(iv) It has also been used in recording and reproducing three dimensional pictures.

(v) Polarised light is used in optical stress analysis known as photoelasticity.

(vi) Polarisation is also used to study asymmetries in molecules and crystals through the phenomenon of optical activity.

Ex. Two polaroids are crossed to each other. When one of them is rotated through 60°, then what percentage of the incident unpolarised light will be transmitted by the polaroids ?

Sol. Initially the polaroids are crossed to each other, that is the angle between their polarising directions is 90°. When one is rotated through 60°, then the angle between their polarising directions will become 30°.

Let the intensity of the incident unpolarised light = I_0

Then the intensity of light emerging from the first polaroid is $I_1 = \frac{1}{2} I_0$

This light is plane polarised and passes through the second polaroid.

The intensity of light emerging from the second polaroid is $I_2 = I_1 \cos^2\theta$

θ = the angle between the polarising directions of the two polaroids.

$I_1 = \frac{1}{2} I_0$ and $\theta = 30°$ So $I_2 = I_1 \cos^2 30° = \frac{1}{2} I_0 \cos^2 30°$ $\Rightarrow$ $\frac{I_2}{I_0} = \frac{3}{8}$

$\therefore$ transmission percentage $= \frac{I_2}{I_0} \times 100 = \frac{3}{8} \times 100 = 37.5\%$

Ex. At what angle of incidence will the light reflected from water ($\mu = 1.3$) be completely polarised ?

Sol. $\mu = 1.3$,

From Brewster's law $\tan \theta_p = \mu = 1.3 \quad \Rightarrow \quad \theta = \tan^{-1} 1.3 = 53°$

Ex. If light beam is incident at polarising angle (56.3°) on air-glass interface, then what is the angle of refraction in glass?

Sol. ✈ $i_p + r_p = 90°$ $\quad \therefore \; r_p = 90° - i_p = 90° - 56.3° = 33.7°$

Ex. A polariser and an analyser are oriented so that maximum light is transmitted, what will be the intensity of outcoming light when analyser is rotated through 60°.

Sol. According to Malus Law $I = I_0 \cos^2 \theta = I_0 \cos^2 60° = I_0 \left[\frac{1}{2}\right]^2 = \frac{I_0}{4}$

Ex. A 300 mm long tube containing 60 cm^3 of sugar solution produces an optical rotations of 10° when placed in a saccharimeter. If specific rotation of sugar is 60°, calculate the quantity of sugar contained in the tube solution.

Sol. ● = 300 mm = 30 cm = 3 decimetre, $\theta = 10°$, $[\alpha]_\lambda^T = 60°$, volume of solution = 60 cm^3

$$\theta = [\alpha]_\lambda^T lC \Rightarrow C = \frac{\theta}{[\alpha]_\lambda^T l} = \frac{10°}{60° \times 3} = \frac{1}{18} \text{ g cm}^{-3}$$

Quantity of sugar contained $= \frac{1}{18} \times 60 = 3.33$ g

• Etoos Tips & Formulas •

1. Huygen`s Wave Theory :

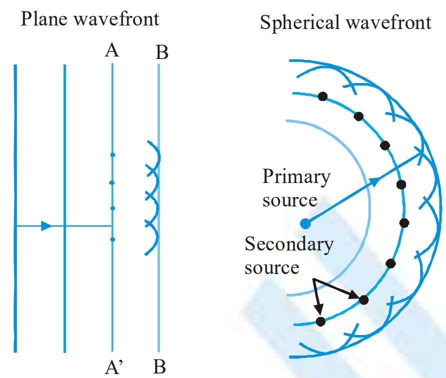
Huygen`s in 1678 assumed that a body emits light in the form of waves.

(a) Each point source of light is a centre of disturbance from which waves spread in all directions. The locus of all the particles of the medium vibrating in the same phase at a given instant is called a wavefront.

(b) Each point on a wave front is a source of new disturbance, called secondary wavelets. These wavelets are spherical and travel with speed of light in that medium.

(c) The forward envelope of the secondary wavelets at any instant gives the new wavefront.

(d) In homogeneous medium, the wavefront is always perpendicular to the direction of wave propagation.



2. Coherent Sources :

Two sources will be coherent if and only if they produce waves of same frequency (and hence wavelength) and have a constant initial phase difference.

3. Incoherent Sources :

The sources are said to be incoherent if they have different frequency and initial phase difference is not constant w.r.t. time.

4. Interference : YDSE

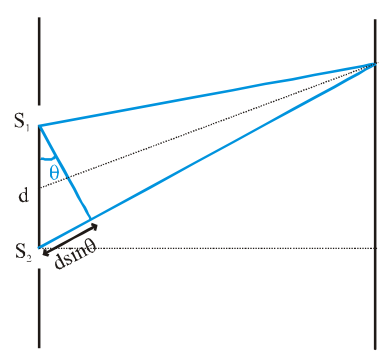
5. Resultant intensity for coherent sources $I = I_1 + I_2 + \sqrt{I_1 I_2}\cos\phi_0$

6. Resultant intensity for incoherent sources $I = I_1 + I_2$

7. Intensity $\propto$ width of slit $\propto$ amplitude

$$\Rightarrow \frac{I_1}{I_2} = \frac{W_1}{W_2} = \frac{a_1^2}{a_2^2} \Rightarrow \frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2}{\left(\sqrt{I_1}-\sqrt{I_2}\right)^2} = \left(\frac{a_1+a_2}{a_1-a_2}\right)^2$$

8. Distance of n^{th} bright fringe $X_n = \dfrac{n\lambda D}{d}$



Path difference $= n\lambda$

where $n = 0, 1, 2, 3, ...$

Distance of m^{th} dark fringe

$$X_m = \frac{(2m+1)\lambda D}{2d}$$

Path difference $= (2m+1)\dfrac{\lambda}{2}$ where $m = 0, 1, 2, 3, ...$

10. Fringe width $\beta = \frac{\lambda D}{d}$

11. Angular fringe width $= \frac{\beta}{D} = \frac{\lambda}{d}$

12. Fringe visibility $= \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \times 100\%$

13. If a transparent sheets of refractive index μ and thickness t is introduced in one of the paths of interfering waves, optical path will becomes 'μt' instead of 't'. Entire fringe pattern is displaced by $\frac{D[(\mu-1)t]}{d} = \frac{\beta}{\lambda}(\mu-1)t$ towards the side in which the thin sheet is introduced without any change in fringe width.

14. The law of conservation of energy holds good in the phenomenon of interference.

15. Fringes are neither image now shadow of slit but locus of a point which moves such a way that its path difference from the two sources remains constant.

16. The interference fringes for two coherent point sources are hyperboloids with axis S_1S_2.

17. If the interference experiment is repeated with bichromatic light, the fringes of two wavelengths will be coincident for the first time when

$$n(\beta)_{longer} = (n+1)(\beta)_{shorter}$$

18. No interference pattern is detected when two coherent sources are infinitely close to each other, because $\beta \propto \frac{1}{d}$

19. If maximum number of maximas/minimas are asked in the question, use the fact that value of sinθ/cosθ can`t be greater than 1.

DIFFRACTION

20. In Fraunhofer diffraction

(a) For minima $a \sin\theta_n = n\lambda$

(b) For maxima $a \sin\theta_n = (2n+1)\frac{\lambda}{2}$

(c) Linear width of central maxima $W_x = \frac{2\lambda D}{a}$

(d) Angular width of central maxima $W_\theta = \frac{2\lambda}{a}$

(e) Intensity of maxima

where I_0 = Intensity of central maxima

$$I = I_0\left[\frac{\sin\left(\beta/2\right)}{\beta/2}\right]^2 \text{ and } \beta = \frac{2\pi}{\lambda} a \sin\theta$$

21. Polarization :

(a) Brewsters' law :

$\mu = \tan\theta_p \rightarrow$

$\theta \rightarrow$ polarization of Brewster's angle

(b) Here reflecting and refracting rays are perpendicular to each other.

(c) Malus law :

$I = I_0 \cos^2\theta$

$I_0 \rightarrow$ Maximum intensity of polarized light.