ELECTROMAGNETIC INDUCTION

INTRODUCTION

From previous chapters, we know that current produces magnetic field. Is reverse possible i.e. can magnetic field produce electric current ? The answer is 'yes'. It is found that currents were induced in closed coils when subjected to changing magnetic fields.

The phenomenon in which electric current is generated by changing magnetic fields is known as electromagnetic induction. The current so produced is known as induced current.

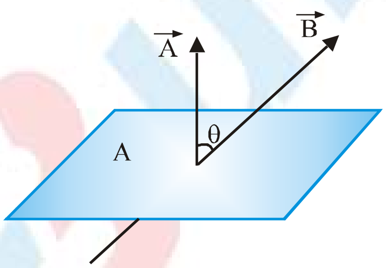
If current is produced in the circuit, this must be due to some emf produced in the circuit. This emf produced in the circuit. This emf produced as a result of change in magnetic field is known as induced emf.

The phenomenon of electromagnetic induction (EMI) is the basis of working of power generators, dynamos, transformers etc.

Magnetic Flux

Magnetic flux though any surface held in a magnetic field is measured as the total number of magnetic field lines crossing the surface. It is a scalar quantity.

Consider a plane area A placed in a magnetic field B as shown in figure. Area vector $\vec{A}$ makes an angle θ with the direction of magnetic field.



Now the magnetic flux passing through the area is defined as

$$\phi = \vec{B}.\vec{A} \qquad \text{[for uniform } \vec{B}\text{]}$$

or $$\phi = BA\cos\theta = (B\cos\theta)A = B_{\perp}A$$

where $B_{\perp} = B\cos\theta$ is the component of the magnetic field B perpendicular to the face of the area. Direction of area vector is normal to the face of the area.

Special Cases

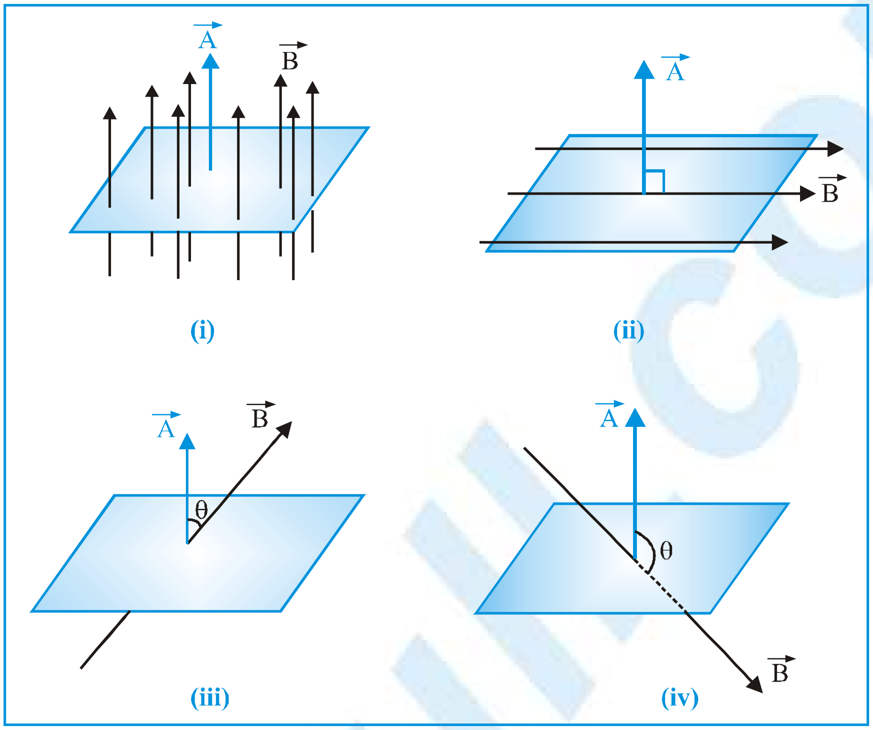
(i) if $\theta = 0°$, then $\phi = BA\cos 0° \Rightarrow \phi = BA$ (maximum flux)

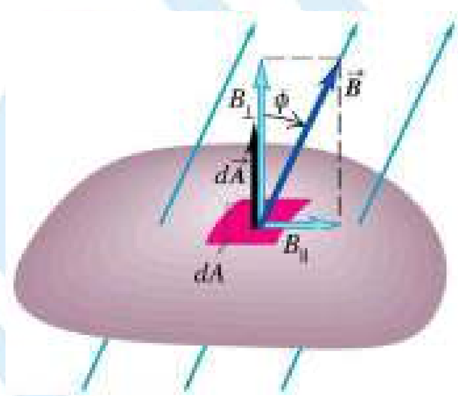
(ii) if $\theta = 90°$, then $\phi = BA\cos 90° \Rightarrow \phi = 0$ (maximum flux)

(iii) if $\theta < 90°$, then $\cos\theta > 0 \Rightarrow \phi > 0$ (positive flux)

(iv) if $\theta > 90°$, then $\cos\theta < 0 \Rightarrow \phi < 0$ (negative flux)

If the surface is not plane, we can divide any surface into elements of area dA (as shown in figure). For each element we determine $B_\perp$, the component of normal to the surface at the position of that element, as shown. from above figure, $B_\perp = \text{B}\cos\phi$, where ϕ is the angle between the direction of B and a line perpendicular to the surface. In general, this component varies from point to point on the surface.





We define magnetic flux $d\Phi_B$ through the area element dA as

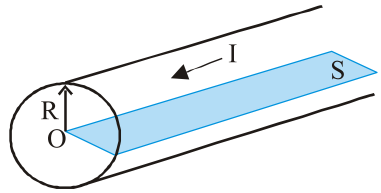
$$d\Phi = B_\perp dA$$

The total magnetic flux through the surface is the sum of the contributions from the individual area elements :

$$\phi = \int B_\perp dA = \int B\cos\phi\, dA = \int \vec{B}.d\vec{A} \quad \text{(magnetic flux through a surface)}$$

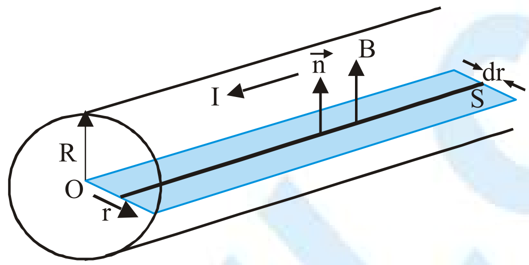
The unit of magnetic flux is tesla-metre^2 which is called weber (Wb) in honour of Wilhelm Weber. 1 Wb = 1 Tm^2. Clearly, B can be measured in Wb m^{-2} = 1 T. Sometimes it is referred to as flux density.

Ex. A long copper wire carries a current of I ampere. Calculate the magnetic flux per metre of wire for a plane surface S inside the wire as shown in figure.



Sol. Consider an element of width *dr* at a distance *r* from the axis of the wire. The field due to the current I in the wire at the position of element will be $B = \frac{\mu_0 I_r}{2\pi R^2}$

and as its direction is perpendicular to the plane surface, flux linked with the element is given by



$$d\phi = B\ ds\cos\theta = \frac{\mu_0}{2\pi}\frac{I}{R^2}r(1dr)\cos 0$$

So, the flux linked with the plane surface is given by

$$\phi = \frac{\mu_0}{2\pi}\frac{I}{R^2}\int_0^R r = \frac{\mu_0 I}{4\pi}$$

Faraday's Law of Electromagnetic Induction

On the basis of several experimental observations, Michael Faraday came to the following conclusions.

1. Whenever there is a relative motion between a magnet (source of magnetic field) and a closed conducting loop, electric current appears in the loop. It happens because of the change in magnetic flux associated with the loop.
2. Since emf causes current in a circuit, when loop and magnet are brought in relative motion current flows in the loop. This implies that an emf is set up in the loop. This emf is known as induced emf and its magnitude is directly proportional to the rate of change of magnetic flux through the loop with time.

Now, we come to the Faraday's law of electromagnetic induction. In mathematical form induced emf can be given by the expression $\varepsilon = -\frac{d\varphi_B}{dt}$

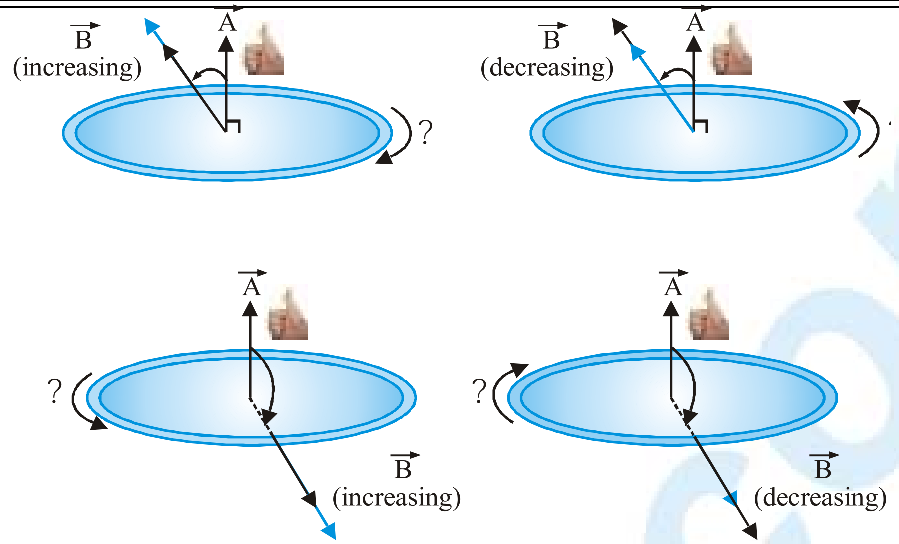
As such Faraday's law in itself is complete to tell the magnitude and polarity of induced emf but Lenz's rule is commonly used to determine the polarity of induced emf or direction of induced current.

Direction of Induced emf

We can find the direction of an induced emf or current $\varepsilon = -d\Phi_B / dt$ together with some simple sign rules. Here is the procedure :

(1) Define a positive direction for the area vector $\vec{A}$.

(2) From the directions of $\vec{A}$ and the magnetic field $\vec{B}$, determine the sign of the magnetic flux Φ_B and its rate of change $d\Phi_B / dt$. Figure shows several examples.



(i) $\vec{A}$ is upward to anticlockwise direction is positive.

(ii) $\phi = BA\cos\theta$

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{d}{dt}(BA\cos\theta) \Rightarrow \varepsilon = -A\cos\theta\frac{dB}{dt} \qquad ..... \text{(i)}$$

Here θ is acute, so cos θ is positive. B is increasing, so dB/dt is positive. Then we find that ε is negative. So ε is clockwise as shown.

For Fig. (b)

Here B is decreasing, so dB/dt is negative. Then from eq. **(i)**, ε is positive. So ε is anticlockwise as shown.

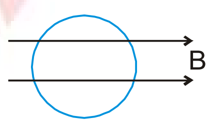
For Fig (c)

Here θ is obtuse, so cos θ is negative. B is increasing, so dB/dt is positive. Then from eq. **(i)** ε is positive, so ε is anticlockwise as shown.

For. Fig. (d)

Here θ is obtuse, so cos θ is negative. B is decreasing, so dB/dt is negative. Then from eq. **(i)** ε is negative, so ε is clockwise as shown.

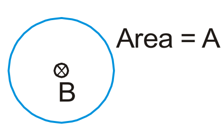
Ex. 1. A coil is placed in a constant magnetic field .The magnetic field is parallel to the plane of the coil as shown in figure. Find the emf induced in the coil .



Sol. : $\phi = 0$ (always) since area is perpendicular to magnetic field.

$\therefore$ emf $= 0$

Ex. 2. Find the emf induced in the coil shown in figure. The magnetic field is perpendicular to the plane of the coil and is constant.

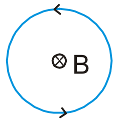


Sol. : $\phi = BA$ (always)

$= $ const.

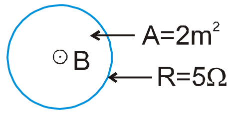
$\therefore$ emf $= 0$

Ex. 3. Find the direction of induced current in the coil shown in figure. Magnetic field is perpendicular to the plane of coil and it is increasing with time.



Sol. : Inward flux is increasing with time. To opposite it outward magnetic field should be induced. Hence current will flow anticlockwise.

Ex. 4. Figure shows a coil placed in decreasing magnetic field applied perpendicular to the plane of coil .The magnetic field is decreasing at a rate of 10T/s. Find out current in magnitude and direction

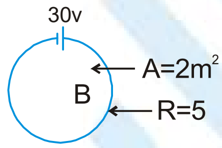


Sol. : $\phi = B.A$

$$emf = A.\frac{dB}{dt} = 2 \times 10 = 20\ v$$

$\therefore$ i = 20/ 5 = 4 amp. From Lenz's law direction of current will be anticlockwise.

Ex. 5. Figure shows a coil placed in a magnetic field decreasing at a rate of 10T/s. There is also a source of emf 30 V in the coil. Find the magnitude and direction of the current in the coil.

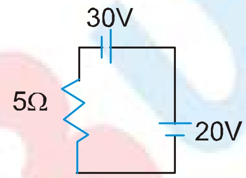


Sol. :

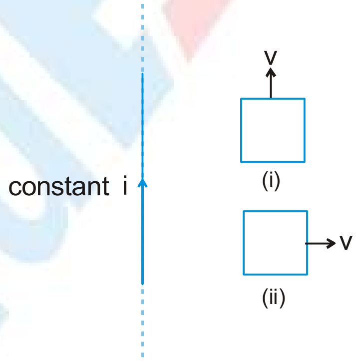
Induce emf = 20V

equivalent

i = 2A clockwise



Ex. 6. Figure shows a long current carrying wire and two rectangular loops moving with velocity v. Find the direction of current in each loop.



Sol. : In loop **(i)** no emf will be induced because there is no flux change.

In loop **(ii)** emf will be induced because the coil is moving in a region of decreasing magnetic field inward in direction. Therefore to oppose the flux decrease in inward direction, current will be induced such that its magnetic field will be inwards. For this direction of current should be clockwise.

Lenz's Law

Direction of the Induced Current in a Circuit

Lenz's law states that 'when the magnetic flux through a loop changes, a current is induced in the loop such that the magnetic field due to the induced current opposes the change in the magnetic flux through the loop.'

The above rule can be systematically applied as follows to determine the direction of the induced currents.

(i) Identify the loop in which the induced current is to be determined.

(ii) Determine the direction of the magnetic field in this loop (i.e in or out of the loop).

(iii) The direction of flux is the same as the direction of the magnetic field. Determine if the flux through the loop is increasing or decreasing (because of change in area or change in B).

Choose the appropriate current in the loop that will oppose the change in flux.

(i) If the flux is into the paper and increasing them the flux due to the induced current should be out of the paper.

(ii) If the flux is into the paper and decreasing, the flux due to the induced current should be into the paper.

(iii) If the flux is out of the paper and increasing, the flux due to the induced current should be into the paper.

(iv) If the flux is out of the paper and decreasing, the flux due to the induced current should be out of the paper.

The above description is the physical interpretation of Lenz's law. We can determine the direction of the induced current mathematically by simply applying Lenz's law $\xi_{ind} = \frac{d\Phi_B}{dt}$ with the appropriate conventions.

The right hand sign convention used is as follows :

(i) For counterclockwise current, emf is positive.

(ii) For clockwise current, emf is negative.

(iii) Magnetic flux out of the paper is positive.

(iv) Magnetic flux into the plane of the paper is negative.

(v) The rate of change of an increasing positive flux is positive.

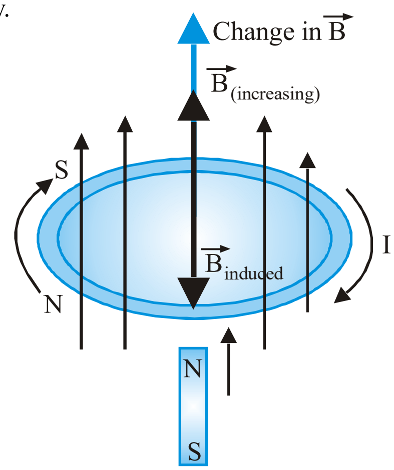
(vi) The rate of change of an decreasing positive flux is negative.

(vii) The rate of change of an increasing negative flux is negative.

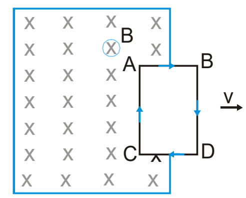
(viii) The rate of change of an decreasing negative flux is positive.

Let us consider some of the cases regarding application of Lenz's law.

(ix) Suppose north pole of a bar magnet is moved towards a loop as shown in figure. Because of change in magnetic flux associated with the loop current is induced in it. Due to induced current, magnetic field is induced in such a way that it opposes the motion of the bar magnet. As the north pole is moving towards the loop, hence to oppose the motion of the bar magnet only the north pole will be induced on that face of the loop which faces the magnet. The induced current due to the change in $\vec{B}$ is clockwise, as seen from above the loop. The added field $\vec{B}_{induced}$ that it causes is downward, opposing the change in the upward field $\vec{B}$.



(x) Consider figure. A rectangular loop ABCD is being pulled out of the magnetic field directed into the plane of the paper. As the loop is dragged out of the field, the flux associated with the loop which is directed into the plane of the paper decreases. The induced current will flow in the loop in the sense to oppose the decrease of this flux. For this to happen, magnetic field due to the induced current in the loop must be directed into the plane of the paper. Hence current in the loop must flow in the clockwise sense.



Ex. A closed loop with the geometry shown in figure (a) is placed in a uniform magnetic field directed into the plane of the paper. If the magnetic field decreases with time, determine the direction of the induced e.m.f. in this loop.

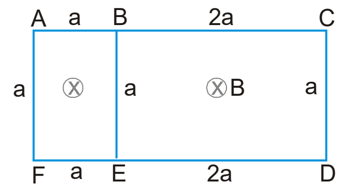


Figure (a)

Sol. There are two loops that are immersed in the magnetic field, namely, ABEFA and BCDEB.

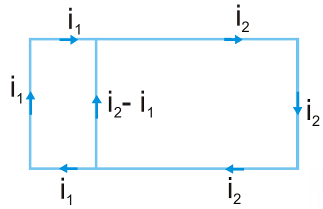


Figure (b)

Consider loop ABEFA. magnetic flux is into the plane of the paper. For loop BCDEB too, the magnetic flux is into the plane of the paper.

In both loops, the magnetic flux is decreasing with time.

Therefore, the induced current in loop ABEFA, i_1, will be in a direction so as to induce a flux into the paper. The direction of i_1 is clockwise.

Like wise in loop BCDEB the current i_2 will be in a direction so as to induce a flux into the paper. The direction of i_2 is also clockwise. The final induced current in all the arms of the loop are shown Fig. (b).

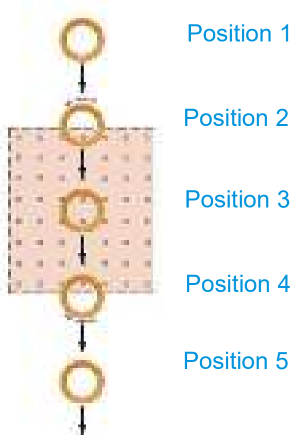
Lenz's Law (Deciding direction of induced e.m.f.)

The induced emf resulting from a changing magnetic flux has a polarity that leads to an induced current whose direction is such that the induced magnetic field opposes the original flux change. We shall discuss later on how it is in accordance with conservation of energy.

Ex. In figure there is a constant magnetic field in a rectangular region of space. This field is directed perpendicularly into the page. Outside this region there is no magnetic field. A copper ring moves through the region from position 1 to position 5. Find the induced current in the ring at it passes through positions.

(a) 1 **(b)** 2 **(c)** 3 **(d)** 4 **(e)** 5

Sol.



(a) Since the field is zero outside the rectangular region, no flux passes through the region in position 1, there is no change in the flux through the ring, and there is no induced emf or current in the ring.

(b) In position 2 the flux increases. According to Lenz's law, the induced current must create an induced magnetic field that op poses the increase. To oppose the increase, the induced field must point opposite to the external field and, therefore, must point out of the page, for which the induced current must be counter clockwise.

(c) Here the field is not zero. Hence nonzero flux passes through the ring in position 3 but the flux through the ring is constant, and there is no induced emf or current in the ring.

(d) In position 4 the flux decreases. According to Lenz's law, the induced current must create an induced magnetic field that opposes the decrease. To oppose the decrease, the induced field must point in the direction of external field and, therefore, must point into the page. For which the induced current must be clock-wise.

(e) Since the field is zero outside the rectangular region, no flux passes through the ring in position 5, there is no change in the flux through the ring, and there is no induced emf or current in the ring.

Note down the following points regarding the Faraday's law :

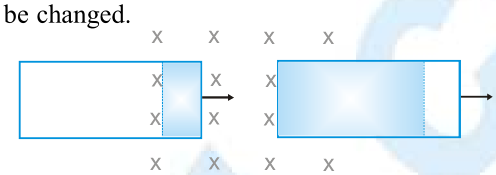
1. As we have seen, induced emf is produced only when there is a change in magnetic flux passing through a loop. The flux passing through the loop is given by ϕ = BA cos θ

Thus, flux can be changed in several ways "

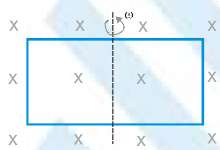
(i) The magnitude of $\vec{B}$ can change with time. In the problems if magnetic field is given as a function of time, it implies that the magnetic field is changing. Thus, B = B(t).

(ii) The area enclosed by the loop can change with time.

This can be done by pulling a loop inside (or outside) a magnetic field. By doing so, the area enclosed by the loop (hatched area) can be changed.



(iii) The angle θ between $\vec{B}$ and the normal to the loop can change with time. This can be done by rotating a loop in a magnetic field.



(iv) Any combination of the above can occur.

2. When the magnetic field passing through a loop is changed, an induced emf and hence an induced current is produced in the circuit. If R is the resistance of the circuit, then induced current is given by

$$i = \frac{e}{R} = \frac{1}{R}\left(\frac{-d\phi_B}{dt}\right)$$

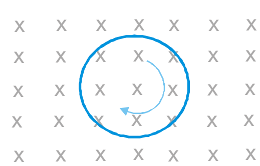
Current starts flowing in the circuit, i.e., flow of charge takes place. Charge flown in the circuit in time dt will be given by

$$dq = i.dt = \frac{1}{R}(-d\phi_B)$$

Thus, for a time interval Δt we can write $e = -\frac{\Delta\phi_B}{\Delta t}, i = \frac{1}{R}\left(\frac{-\Delta\phi_B}{dt}\right)$ and $\Delta q = i\Delta t = \frac{1}{R}(-\Delta\phi_B)$

From these equations we can see that e and i are inversely proportional to Δt and Δq is independent of Δt. It depends on the magnitude of change in flux, not the time taken by it.

Ex. A circular loop of radius a having n turns is kept in a horizontal plane. A uniform magnetic field B exists in a vertical direction shown in figure. Find the emf induced in the loop if the loop is rotated with a uniform angular velocity ω about

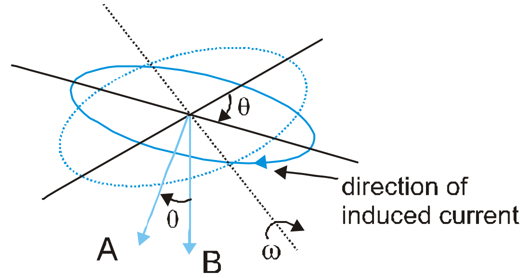


(a) an exist passing through the centre and perpendicular to the plane of the loop.

(b) the diameter.

Sol. **(a)** The emf induces when there is change of flux. As in this case(figure) there is no change of flux, hence no emf will be induced in the coil.

(b) If the loop is rotated about the diameter there will be change of flux with time. In this case e.m.f. will be induced in the coil. The area of the loop is A – πa^2. If the normal of the loop makes an angle θ = 0 with the magnetic field at t = 0, this angle will become $\theta = \omega t$ at time t. The flux of the magnetic field at this time is

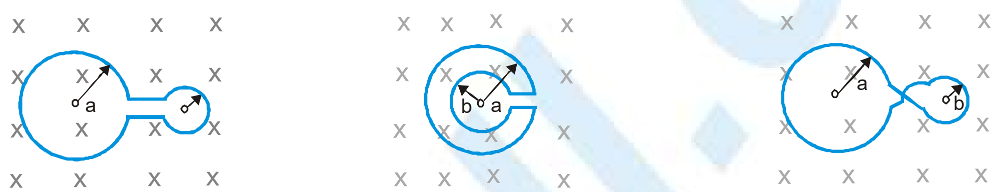


$$\phi = nB\pi a^2 \cos\theta = nB\pi a^2 \cos\omega t$$

The induced e.m.f. is $\varepsilon = -\frac{d\phi}{dt} = \pi n a^2 B\omega \sin\omega t$

e is coming out to be positive, so direction of induced emf will be as shown in figure. Because here this sense is positive if we look at direction of area vector.

Ex. Figure **(a)** shows two circular rings of radii a and b (a > b) joined together with wires of negligible resistance. Figure **(b)** shows the pattern obtained by folding the small loop in the plane of the large loop.



(a) (b) (c)

The pattern shown in figure **(c)** is obtained by twisting the small loop of Fig. **(a)** through 180°.

All the three arrangements are placed in a uniform time varying magnetic field $\frac{dB}{dt} = k$, perpendicular to the plane of the loops. If the resistance per unit length of the wire is λ, then determine the induced current in each case.

Sol. **(a)** $\phi_B = \pi(a^2 + b^2)B$

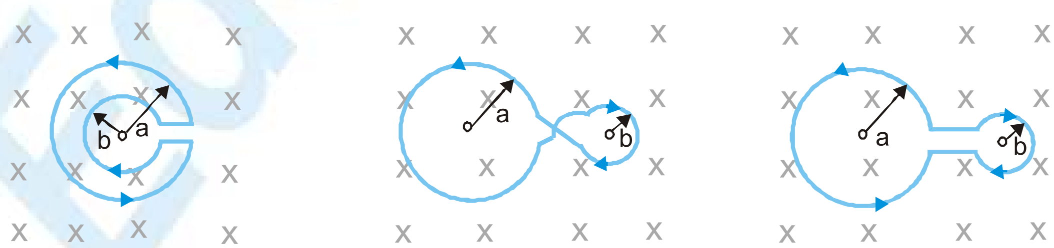
$$\varepsilon = \frac{d\phi_B}{dt} = \pi(a^2 + b^2) \times \frac{dB}{dt} = \pi(a^2 + b^2)k$$

Induced current, $I = \frac{\varepsilon}{R} = \frac{\pi(a^2 + b^2)k}{\lambda[2\pi(a+b)]} = \frac{k(a^2 + b^2)}{2\lambda(a+b)}$

(b) $\phi_B = \pi a^2 B - \pi b^2 B = \pi B(a^2 - b^2)$

$$\varepsilon = \frac{d\phi_B}{dt} = \frac{\pi dB}{dt} \times (a^2 - b^2) = \pi k(a^2 - b^2)$$

Induced current, $I = \frac{\varepsilon}{R} = \frac{\pi k(a^2 - b^2)}{2\pi\lambda(a+b)} = \frac{k(a-b)}{2\lambda}$



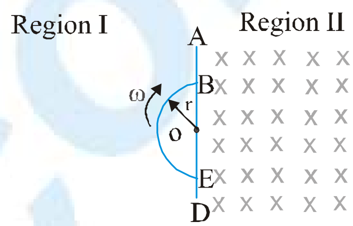
(c) Here induced emf in both loops will oppose each other, hence effective flux is given by

$$\Phi_B = \pi a^2 B - \pi b^2 B = \pi B(a^2 - b^2)$$

$$\Rightarrow \qquad \varepsilon = \frac{d\Phi_B}{dt} = \pi k(a^2 - b^2)$$

Induced current, $I = \dfrac{\varepsilon}{R} = \dfrac{k(a-b)}{2\lambda}$

Ex. Space is divided by the line AD into two refions. Region I is field free and region II has a uniform magnetic field B directed into the plane of the paper. BCE is a semicircular conducting loop of radius r with centre at O, the plane of the loop being in the plane of the paper. The loop is now made to rotate with a constant angular velocity ω about an axis passing through O and the perpendicular to the plane of the paper. The effective resistance of the loop is R.

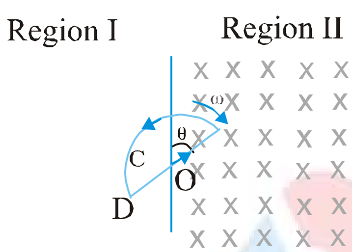


(i) Obtain an expression for the magnitude of the induced current in the loop.

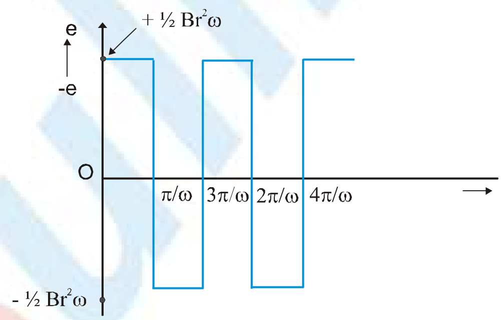
(ii) Show the direction of the current when the loop is entering into Region II.

(iii) Plot a graph between the induced emf and the time of rotation for the two periods of the rotation.

Sol. **(i)** When the loop is rotated about an axis passing centre O and perpendicular to the plane of the paper, the angle between magnetic field vector B and area A is always 0°. When the loop is region I, the magnetic flux linked with loop = BA cos 0 = 0 the (since B = 0 in region I).



(a)



(b)

When the loop enters the magnetic field in region II, the magnetic flux linked with it is given by ϕ = BA where $A = \frac{1}{2}r^2\theta$. Therefore, emf induced

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(BA) = -B\frac{dA}{dt} = \frac{-Br^2}{2}\frac{d\theta}{dt}$$

$$\Rightarrow \qquad e = -\frac{-Br^2}{2}\omega$$

A resistance of the loop is R, the current induced is given by

$$i = \frac{e}{R} = \frac{1}{2}\frac{Br^2\omega}{R}$$

This is the required expression for current induced in the loop.

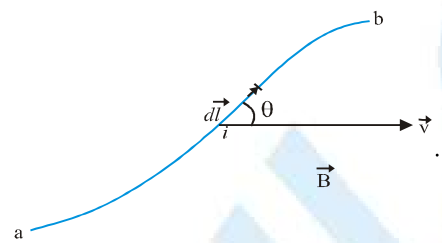
(ii) According to Lenz's law, the direction of current induced is to oppose the change in magnetic flux. So, when entering into region II the field produced by the current induced must be upward. For this, the current in the loop must be anticlockwise as shown in Fig. (a).

(iii) When the loop enters the magnetic field the magnetic flux linked with it increases and the emf $i = \frac{1}{2}Br^2\omega$ is induced in one direction. When the loop comes out of the field, the flux decreases and emf is induced in opposite sense. The graph for representing the emf induced versus time for two periods (T = 2π/ω) is shown in Fig. (b). Here we have taken anticlockwise direction as positive.

MOTIONAL ELECTROMOTIVE FORCE

When a conductor moves in a magnetic field, then charges inside conductor experience magnetic force given by $\vec{F} = q\vec{v} \times \vec{B}$, where $\vec{v}$ is the velocity of conductor (here we assume that velocity of charges inside conductor is same as the velocity of conductor and random motion of charges inside the conductor is neglected). Due to this force charges start moving in a particular direction in the conductor or we say that an emf is induced in the conductor. This emf is known as motional emf.

Consider a conductor of arbitrary shape as shown in figure moving in some magnetic field $\vec{B}$



The different parts of the conductor may have different velocity. Consider an element $\vec{dl}$ have velocity $\vec{v}$.
Force due to magnetic field on a charge in conductor.

$$\vec{F} = q\vec{v} \times \vec{B}$$

work done by this force on charge q in passing through $\vec{dl}$

$$dW = \vec{F}.\vec{dl} = q(\vec{v} \times \vec{B}).\vec{dl}$$

So emf induced within $\vec{dl}$: $d\varepsilon = \frac{dW}{q} = (\vec{v} \times \vec{B}).\vec{dl}$

Integrate this to find net emf : $\varepsilon = \int_a^b (\vec{v} \times \vec{B}).\vec{dl}$

This emf is directed from a to b.

Other Approach :

Due to the magnetic force, charges star moving in the conductor an electric field is set up in the conductor. In equilibrium force of electric field balances the force of magnetic field. Let electric field induced is $\vec{E}$, then

$$q\vec{v} \times \vec{B} + q\vec{E} = 0 \quad \Rightarrow \quad \vec{E} = -\vec{v} \times \vec{B}$$

Now potential difference developed across the ends of the conductor :

$$V_b - V_a = -\int_a^b \vec{E}.\vec{dl} = \int_a^b (\vec{v} \times \vec{B}).\vec{dl}$$

If circuit is incomplete, then this terminal potential difference will be equal to emf induced. Hence emf induced :

$$\varepsilon = \int_a^b (\vec{v} \times \vec{B}).\vec{dl}$$

Note : We can also write $\varepsilon = \int_a^b \vec{B}.(\vec{dl} \times \vec{v})$

As $\vec{dl} \times \vec{v}$ is the area swept per unit time by length $\vec{dl}$ and hence $\vec{B}.(\vec{dl} \times \vec{v})$ is the flux of induction through the area.

Therefore, the motional e.m.f. is equal to the flux of induction cut by the conductor per unit time.

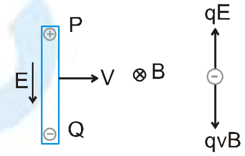
* We can write the expression for induced emf in various forms :

$$\varepsilon = \int_a^b (\vec{v} \times \vec{B}).\vec{dl} = \int_a^b (\vec{B} \times \vec{dl}).\vec{v} = \int_a^b (\vec{dl} \times \vec{v}).\vec{B}$$

if any two out of $\vec{v}$, $\vec{B}$ and $\vec{dl}$ become parallel or antiparallel, ε will become zero.

Explanation of emf induced in rod on the basis of magnetic force:

If a rod is moving with velocity v in a magnetic field B ,as shown , the free electrons in a rod will experience a magnetic force in downward direction and hence free electrons will accumulate at the lower end and there will be a deficiency of free electrons and hence a surplus of positive charge at the upper end. These charges at the ends will produce an electric field in downward direction which will exert an upward force on electron. If the rod has been moving for quite some time enough charges will accumulate at the ends so that the two forces qE and qvB will balance each other. Thus E = v B.



$V_P - V_Q = VB\ell$

The moving rod is equivalent to the following diagram, electrically.

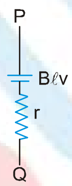
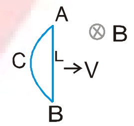
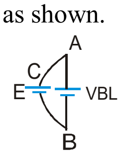


Figure shows a closed coil ABCA moving in a uniform magnetic field B with a velocity v. The flux passing through the coil is a constant and therefore the induced emf is zero.



Now consider rod AB , which is a part of the coil. Emf induced in the rod =B L v

Suppose the emf induced in part ACB is E , as shown.



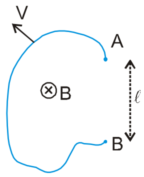
Since the emf in the coil is zero, Emf (in ACB) + Emf (in BA) = 0

or $-E + vBL = 0$

or $E = vBL$

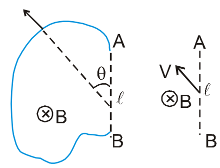
Thus emf induced in any path joining A and B is same, provided the magnetic field is uniform. Also the equivalent emf between A and B is BLv (here the two emf's are in parallel)

Ex. Figure shows an irregular shaped wire AB moving with velocity v, as shown.



Find the emf induced in the wire.

Sol. The same emf will be induced in the straight imaginary wire joining A and B , which is Bv● sin θ



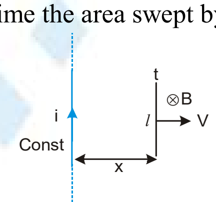
Ex. A rod of length l is kept parallel to a long wire carrying constant current i. It is moving away from the wire with a velocity v. Find the emf induced in the wire when its distance from the long wire is x.

Sol. $E = Bl V = \frac{\mu_0 i l v}{2\pi x}$

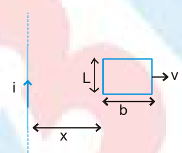
OR

Emf is equal to the rate with which magnetic field lines are cut . In dt time the area swept by the rod is l v dt. The magnetic field lines cut in dt time $= B\, l\, vdt = \frac{\mu_0 i l vdt}{2\pi x}$.

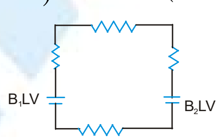
$\therefore$ The rate with which magnetic field lines are cut $= \frac{\mu_0 i l v}{2\pi x}$



Ex. A rectangular loop ,as shown in the figure ,moves away from an infinitely long wire carrying a current i. Find the emf induced in the rectangular loop.



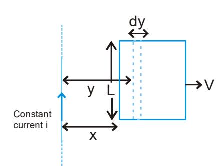
Sol. $E = B_1 LV - B_2 Lv = \frac{\mu_0 i}{2\pi x} Lv - \frac{\mu_0 i}{2\pi(x+b)} Lv = \frac{\mu_0 iLbv}{2\pi x(x+b)}$



Aliter:

Consider a small segment of width dy at a distance y from the wire.

Let flux through the segment be $d\phi$.



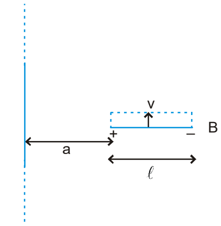
$\therefore \quad d\phi = \frac{\mu_0 i}{2\pi y} L\, dy$

$\therefore \quad \phi = \frac{\mu_0 iL}{2\pi} \int_x^{x+b} \frac{dy}{y} = \frac{\mu_0 iL}{2\pi} (\lambda n(x+b) - \lambda n x)$

Now $\frac{d\phi}{dt} = \frac{\mu_0 iL}{2\pi}\left[\frac{1}{x+b}\frac{dx}{dt} - \frac{1}{x}\frac{dx}{dt}\right] = \frac{\mu_0 iL}{2\pi}\left[\frac{(-b)}{x(x+b)}\right] v = \frac{-\mu_0 ibLv}{2\pi x(x+b)}$

$\therefore$ induced emf $= \frac{\mu_0 ibLv}{2\pi x(x+b)}$

Ex. A rod of length l is placed perpendicular to a long wire carrying current i. The rod is moved parallel to the wire with a velocity v. Find the emf induced in the rod , if its nearest end is at a distance 'a' from the wire.

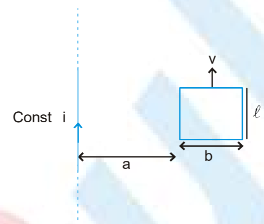


Sol. Consider a segment of rod of length dx , at a distance x from the wire. Emf induced in the segment

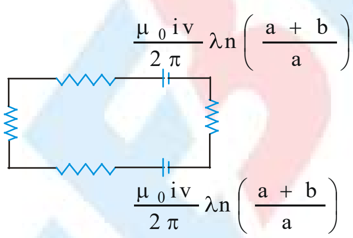
$$dE = \frac{\mu_0 i}{2\pi x} dx.v$$

$$\therefore \quad E = \int_a^{a+\lambda} \frac{\mu_0 ivdx}{2\pi x} = \frac{\mu_0 iv}{2\pi}\lambda n\left(\frac{\lambda + a}{a}\right)$$

Ex. A rectangular loop is moving parallel to a long wire carrying current i with a velocity v. Find the emf induced in the loop , if its nearest end is at a distance 'a' from the wire. Draw equivalent electrical diagram.



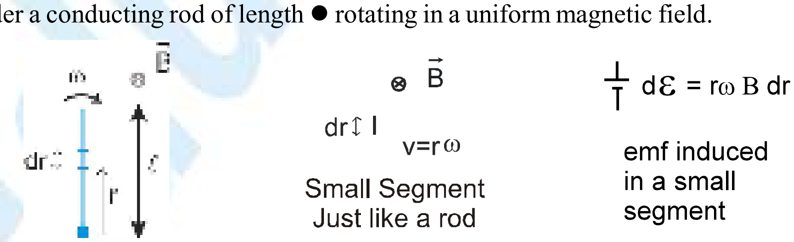
Sol. emf = 0 ;



Induced emf due to rotation

Rotation of the Rod

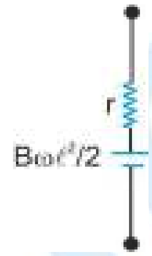
Consider a conducting rod of length ● rotating in a uniform magnetic field.



Emf induced in a small segment of length dr, of the rod = v B dr = rω B dr

$\therefore$ emf induced in the rod $= \omega B\int_0^{\lambda} r dr = \frac{1}{2}B\omega\lambda^2$

equivalent of this rod is as following

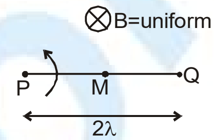


or $\varepsilon = \frac{d\Phi}{dt}$

$$\varepsilon = \frac{d\Phi}{dt} = \frac{\text{flux through the area swept by the rod in time dt}}{dt}$$

$$= \frac{B\frac{1}{2}\lambda^2\omega dt}{dt} \quad = \frac{1}{2}B\omega\lambda^2$$

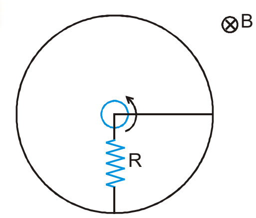
Ex. A rod PQ of length 2l is rotating about one end P in a uniform magnetic field B which is perpendicular to the plane of rotation of the rod. Point M is the mid point of the rod. Find the induced emf between M & Q if that between P & Q = 100V .



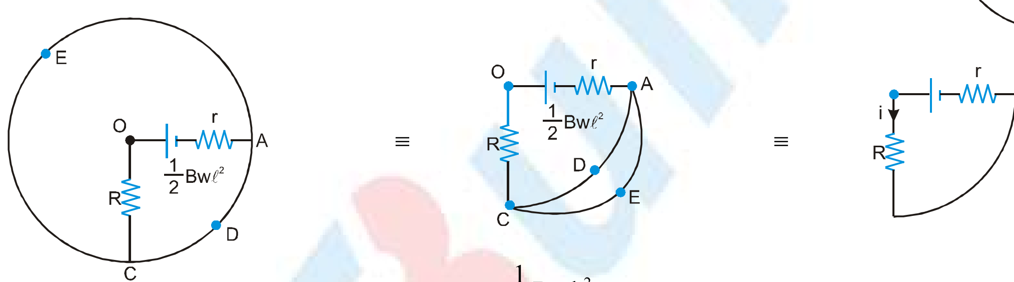
Sol. $E_{MQ} + E_{PM} = E_{PQ} \quad \Rightarrow \quad E_{PM} = \frac{Bw\lambda^2}{2} = 100$

$$E_{MQ} + \frac{B\omega\left(\frac{\lambda}{2}\right)^2}{2} = \frac{B\omega\lambda^2}{2} \Rightarrow \quad E_{MQ} = \frac{3}{8}B\omega●^2 = \frac{3}{4} \times 100\text{ V} = 75\text{ V}$$

Ex. A rod of length ● and resistance r rotates about one end as shown in figure. Its other end touches a conducting ring a of negligible resistance. A resistance R is connected between centre and periphery. Draw the electrical equivalence and find the current in the resistance R. There is a uniform magnetic field B directed as shown.

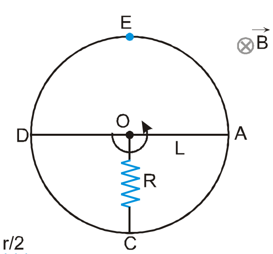


Sol.

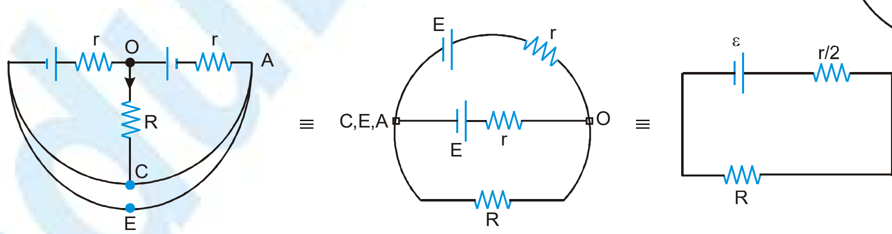


$$\text{current } i = \frac{\frac{1}{2}B\omega l^2}{R + r}$$

Ex. Solve the above question if the length of rod is 2L and resistance 2r and it is rotating about its centre. Both ends of the rod now touch the conducting ring

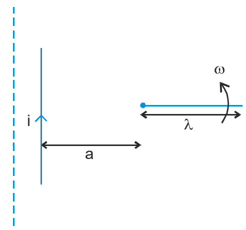


Sol.

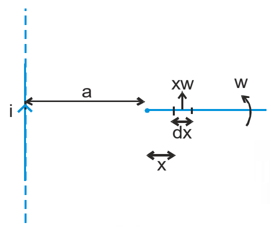


$$i = \frac{\varepsilon}{R + \frac{r}{2}} = \frac{\frac{1}{2}B\omega L^2}{R + \frac{r}{2}}$$

Ex. A rod of length l is rotating with an angular speed ω about its one end which is at a distance 'a' from an infinitely long wire carrying current i. Find the emf induced in the rod at the instant shown in the figure.

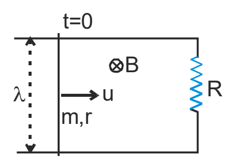


Sol. Consider a small segment of rod of length dx , at a distance x from one end of the rod. Emf induced in the segment

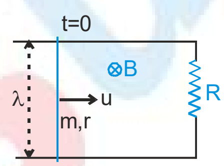


$$dE = \frac{\mu_0 i}{2\pi(x+a)}(x\omega)dx \qquad \therefore \qquad E = \int_0^{\lambda} \frac{\mu_0 i}{2\pi(x+a)}(x\omega)dx = \frac{\mu_0 i\omega}{2\pi}\left[\lambda - a.\lambda n\left(\frac{\lambda+a}{a}\right)\right].$$

Ex. A rod of mass m and resistance r is placed on fixed, resistanceless, smooth conducting rails (closed by a resistance R) and it is projected with an initial velocity u .Find its velocity as a function of time.

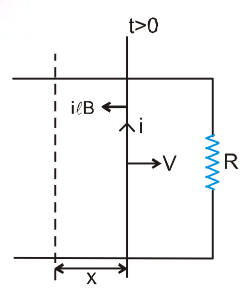


Sol. Let at an instant the velocity of the rod be v .The emf induced in the rod will be vBl. The electrically equivalent circuit is shown in the following diagram.



$\therefore$ Current in the circuit $i = \frac{B\lambda v}{R+r}$

At time t



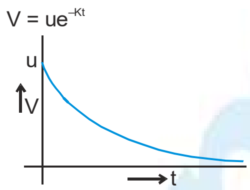
Magnetic force acting on the rod is F = i ● B, opposite to the motion of the rod.

$$i \bullet B = -m\frac{dV}{dt} \qquad \text{...(1)}$$

$$i = \frac{B\lambda v}{R+r} \qquad \text{...(2)}$$

Now solving these two equation

$$\frac{B^2\lambda^2 V}{R+r} = -m\,.\,\frac{dV}{dt} \quad \Rightarrow \quad -\frac{B^2\lambda^2}{(R+r)m}\,.\,dt = \frac{dV}{V}$$



let $$\frac{B^2\lambda^2}{(R+r)m} = k \quad \Rightarrow \quad -K\,.\,dt = \frac{dV}{V}$$

$$\int_u^v \frac{dV}{V} = \int_0^t -K.dt \quad \Rightarrow \quad \ln\left(\frac{v}{u}\right) = -Kt \quad \Rightarrow \quad V = ue^{-Kt}$$

Ex. In the above question find the force required to move the rod with constant velocity v, and also find the power delivered by the external agent .

Sol. The force needed to keep the velocity constant

$$F_{ext} = i\bullet B = \frac{B^2\lambda^2 v}{R+r}$$

Power due to external force $= \dfrac{B^2\lambda^2 v^2}{R+r} = \dfrac{\varepsilon^2}{R+r} = i^2(R+r)$

Note that the power delivered by the external agent is converted into joule heating in the circuit. That means magnetic field helps in converting the mechanical energy into joule heating.

Ex. In the above question if a constant force F is applied on the rod. Find the velocity of the rod as a function of time assuming it started with zero initial velocity.

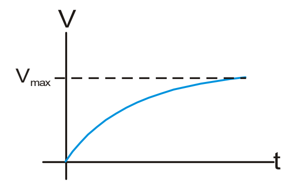
Sol. $$m\frac{dv}{dt} = F - i\bullet B \quad ....(1) \quad \Rightarrow \quad i = \frac{B\lambda v}{R+r} \quad ....(2)$$

$$m\frac{dv}{dt} = F - \frac{B^2\lambda^2 v}{R+r}$$

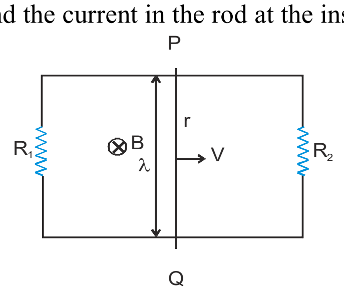
let $$K = \frac{B^2\lambda^2}{R+r} \quad \Rightarrow \quad \int_0^v \frac{dV}{F-Kv} = \int_0^t \frac{dt}{m} \quad \Rightarrow \quad -\frac{1}{K}\left[\lambda n(F-KV)\right]_0^V = \frac{t}{m}$$

$$\bullet n\left(\frac{F-kV}{F}\right) = -\frac{Kt}{m} \quad \Rightarrow \quad F - KV = F\,e^{-kt/m}$$

$$V = \frac{F}{K}\left(1 - e^{-kt/m}\right)$$

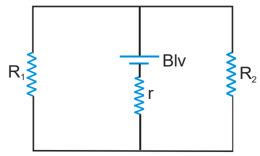


Ex. A rod PQ of mass m and resistance r is moving on two fixed, resistanceless, smooth conducting rails (closed on both sides by resistances R_1 and R_2) .Find the current in the rod at the instant its velocity is v.



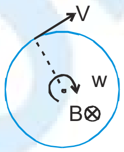
Sol. $i = \dfrac{B\lambda V}{r + \dfrac{R_1 R_2}{R_1 + R_2}}$

this circuit is equivalent to the following diagram.



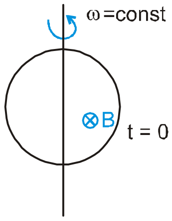
EMF Induced due to rotation of a coil

Ex. A ring rotates with angular velocity ω about an axis perpendicular to the plane of the ring passing through the center of the ring. A constant magnetic field B exists parallel to the axis. Find the emf induced in the ring



Sol. Flux passing through the ring $\phi = B.A$ is a constant here, therefore emf induced in the coil is zero. Every point of this ring is at the same potential, by symmetry.

Ex. A ring rotates with angular velocity ω about an axis in the plane of the ring and which passes through the center of the ring. A constant magnetic field B exists perpendicular to the plane of the ring. Find the emf induced in the ring as a function of time.



Sol. At any time t , $\phi = BA\cos\theta = BA\cos\omega t$

Now induced emf in the loop

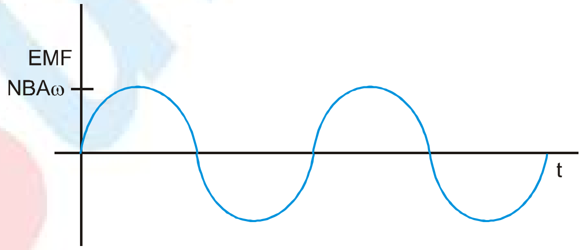
$$e = \frac{-d\phi}{dt} = BA\,\omega\sin\omega t$$

If there are N turns

emf = $BA\omega N\sin\omega t$

$BA\,\omega N$ is the amplitude of the emf

$e = e_m \sin\omega t$

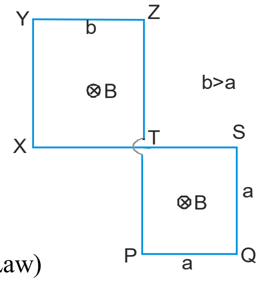


$$i = \frac{e}{R} = \frac{e_m}{R}\sin wt = i_m \sin wt$$

$$i_m = \frac{e_m}{R}$$

The rotating coil thus produces a sinusoidally varying current or alternating current. This is also the principle used in generator.

Ex. Figure shows a wire frame PQSTXYZ placed in a time varying magnetic field given as $B=\beta t$, where β is a positive constant. Resistance per unit length of the wire is λ. Find the current induced in the wire and draw its electrical equivalent diagram.



Sol. Induced emf in part PQST = βa^2 (in anticlockwise direction, from Lenz's Law)

Similarly Induced emf in part TXYZ = βb^2 (in anticlockwise direction, from Lenz's Law)

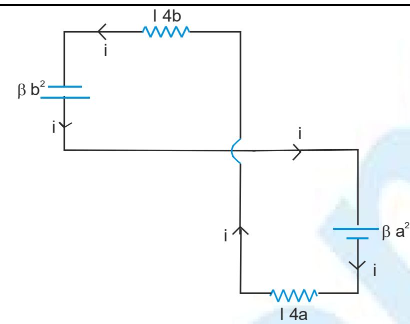
Total resistance of the part PQST = $\lambda 4a$.

Total resistance of the part TXYZ = $\lambda 4b$. The equivalent circuit is as shown in the following diagram.

writing KVL along the current flow

$$\beta b^2 - \beta a^2 - \lambda\, 4ai - \lambda\, 4bi = 0$$

$$i = \frac{\beta}{4\lambda}\,(b - a)$$



EMF Induced in a rotating disc :

Consider a disc of radius r rotating in a magnetic field B.

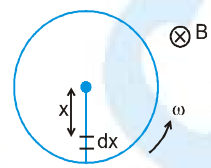
Consider an element dx at a distance x form the centre. This element is moving with speed v = ωx.

∴ Induced emf across dx

$= B(dx)\, v = Bdx\omega x = B\omega x dx$

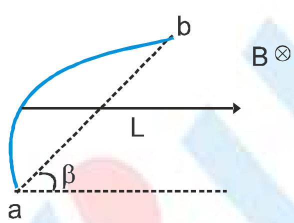
∴ emf between the centre and the edge of disc.

$$= \int_0^r B\omega x dx = \frac{B\omega r^2}{2}$$



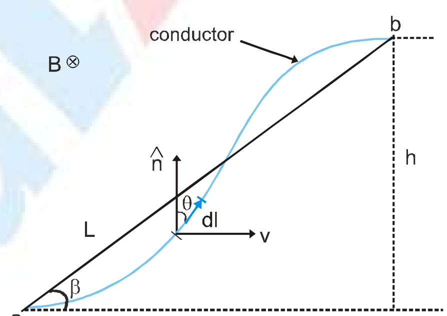
A Conductor of Arbitary Shape

Consider a conductor ab of arbitary shape translating in a uniform magnetic field B.



Then induced emf in this conductor will be same as in a straight conductor connected between a and b, i.e. $\varepsilon = BvL\sin\beta$

Proof : Consider an arbitrary shaped conductor ab



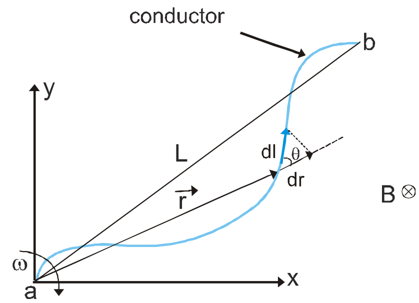
L is shortest length between a and b

Emf induced in the conductor is given by :

$$\varepsilon = \int (\vec{v} \times \vec{B}).\vec{dl} = \int vB\hat{n}.\vec{dl} = vB\int dl\cos\theta = vBh = vBL\sin\beta$$

A Rotating Conductor of Arbitary Shape :

Consider an arbitary shaped conductor ab which is rotating about end a with angular velocity ω as shown. We want to find induced emf between ends a and b.



L is shortest length between a and b

Induced emf developed between ends a and b is given by :

$$\varepsilon = \int (\vec{v} \times \vec{B}).\vec{dl}$$

$$\Rightarrow \quad \varepsilon = \int [(\vec{\omega} \times \vec{r}) \times \vec{B}].\vec{dl}$$

$$= \int [\{-\omega\hat{k} \times (x\hat{i} + y\hat{j})\} \times (-B\hat{k})].\,\vec{dl} \quad = \int [(-\omega x\hat{j} + \omega y\hat{i}) \times (-B\hat{k})].\,\vec{dl}$$

$$= \int [(-\omega xB\hat{i} + \omega By\hat{j}).\vec{dl} = B\omega \int (x\hat{i} + y\hat{j}).\vec{dl} \quad = B\omega \int \vec{r}.\,\vec{dl} = B\omega \int r\, dl \cos\theta = B\omega \int_0^L r dr$$

$$\Rightarrow \quad \varepsilon = \frac{1}{2} B\omega L^2$$

Motional EMF when the magnetic Field is Non-Uniform

In some of the cases motion of a conductor may be in a non-uniform magnetic field. Take the following steps while calculating motional emf.

Step 1 : Determine the magnetic field at all points on the rod.

Step 2 : Consider a small element at some distance from one end of the rod.

Step 3 : Assuming B to be uniform over this element, calculate the potential difference across this element using the procedures outlined earlier.

Step 4 : Integrate over the entire rod to calculate the total induced emf.

Let us learn to calculate the induced emf through the examples given below.

Ex. A copper rod of length 0,19 m is moving with uniform velocity 10 m s^{-1} parallel to a long straight wire carrying a current of 0.5 A. The rod is perpendicular to the wire with its ends at distances 0.01 and 0.2 m from it. Calculate the emf induced in the rod.

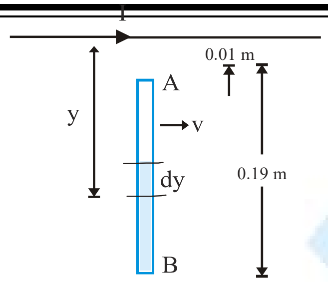
Sol. As shown in figure, consider an element of length dy at a distance y from the wire, then at this position of the element, the magnetic field due to the current-carrying wire PQ will be $B = \frac{\mu_0}{4\pi}\frac{2I}{y}$ into the plane of the paper.

So, the emf induced in the element $d\varepsilon = Bv\,\text{dy} = \frac{\mu_0}{4\pi}\frac{2I}{y} v\, dy$ and hence the emf induced across the ends of the rod due to its motion in the field of the wire.

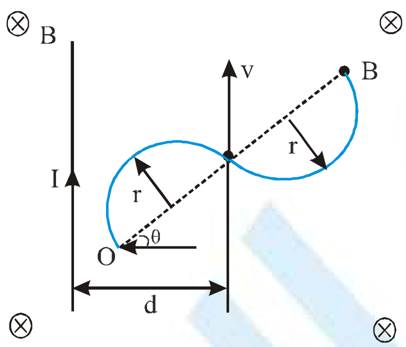
$$\varepsilon=\int_a^b d\varepsilon = \frac{\mu_0}{4\pi}2Iv\int_a^b \frac{dy}{y}, \text{ i.e. } \varepsilon = \frac{\mu_0}{4\pi}2Iv\log_e\left(\frac{b}{a}\right)$$

Substituting the given data with b = (a + ●),

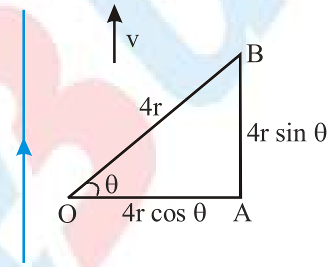
$$\varepsilon = 10^{-7}\times 2\times 5\times 10\log_e\frac{0.20}{0.01} = 10^{-5}\times\log_e 20 = 30\mu V$$



Ex. An infinite wire carries a current I. An 'S' shaped conducting rod of two semicircles each of radius r is placed at an angle q to the wire. The centre of the conductors at a distance d from the wire. If the rod translates parallel to the wire with a velocity v as shown in figure, calculate the emf induced across the ends OB of the rod.



Sol. Join the end point O and B and replace the two semicircles by a straight rod length 4r (in figure).
Induced emf in straight rod OB will be same as in the actual conductor. Now this straight rod can be replaced by a combination of two rods OA and AB. Induced emf in OB will be same as induced emf in OA+ induced emf in AB will be zero because its velocity will be parallel to its length. Hence induced emf in OA will be the net induced emf in actual conductor.



We now have a rod of effective length 4r cosq translating in a non-uniform magnetic field. Consider a small element of the wire of length dx located at a distance x from the wire. The magnetic field at this element is $B = \frac{\mu_0 I}{2\pi x}$

The potential difference across this element is $dV = \frac{\mu_0 I}{2\pi x}v\,dx$.

The potential difference across the ends of the rod can be calculated by intergating over the whole end. Therefore,

$$V = \int dV = \int_{d-2r\cos\theta}^{d+2r\cos\theta}\frac{\mu_0 I}{2\pi x}v\,dx$$

or

$$V = \frac{\mu_0 Iv}{2\pi}\text{In}\left[\frac{d+2r\cos\theta}{d-2r\cos\theta}\right]$$

From the right hand rule we see that electrons will accumulate at end B. Therefore, end O is at a high potential than B.

Induced Electric Field

We have seen that the electric potential difference between two points A and B in an electric field $\vec{E}$ can be written as

$$\Delta V = V_B - V_A = -\int_A^B \vec{E}.d\vec{s}$$

When the electric field is conservative, as is the case of electrostatics, the line integral of it is path-independent, which implies $\oint \vec{E}.d\vec{s} = 0$

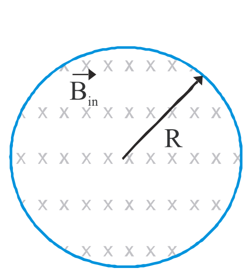
Faraday's law show that as magnetic flux changes with time, an induced current begins to flow. What cause the charges to move ? It is the induced emf which is the work done per unit charge. However, since magnetic field can do no work, as we have shown, the work done on the mobile charges must be electric, and the electric field in this situation cannot be conservative because the line integral of a conservative field must vanish. Therefore, we conclude that there is a non-conservative electric field $\vec{E}_{nc}$ associated with an induced emf $\varepsilon = \oint \vec{E}_{nc}.d\vec{s}$

Combining with Faraday's law then yields :

$$\oint \vec{E}_{nc}.d\vec{s} = -\frac{d\Phi_B}{dt}$$

The above expression implies that a changing magnetic flux will induce a non-conservative electric field which can vary with time. It is important to distinguish between the induced, non-conservative electric field and the conservative electric field which arises from electric charges.

As an example, let's consider a uniform magnetic field which points into the page and is confined to a circular region with radius R, as shown in Figure. Suppose the magnetic of $\vec{B}$ increase with time, i.e., dB/dt>0. let's find the induced electric field everywhere due to the changing magnetic field.

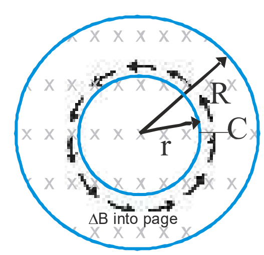


Since the magnetic field is confined to a circular region, from symmetry arguments we choose the integration path to be a circle of radius r. The magnitude of the induced field $\vec{E}_{nc}$ at all points on a circle is the same. According to Lenz's law, the direction $\vec{E}_{nc}$ of must be such that it would drive the induced current to produce a magnetic field opposing the change in magnetic flux. With the area vector $\vec{A}$ pointing out of the page, the magnetic flux is negative or inward. With dB > dt > 0, the inward magnetic flux is increasing. Therefore, to counteract this change the induced current must flow counterclockwise to produce more outward flux. The direction of $\vec{E}_{nc}$ is shown in Figure.

Let's proceed to find the magnitude of $\vec{E}_{nc}$.

In the region r > R

the rate of change of magnetic flux is :



$$\frac{d\Phi_B}{dt} = \frac{d}{dt}(\vec{B}.\vec{A}) = \frac{d}{dt}(-BA) = -\left(\frac{dB}{dt}\right)\pi r^2$$

Using equation, we have

$$\oint \vec{E}_{nc}.d\vec{s} = E_{nc}(2\pi r) = -\frac{d\Phi_B}{dt} = \left(\frac{dB}{dt}\right)\pi r^2$$

which implies

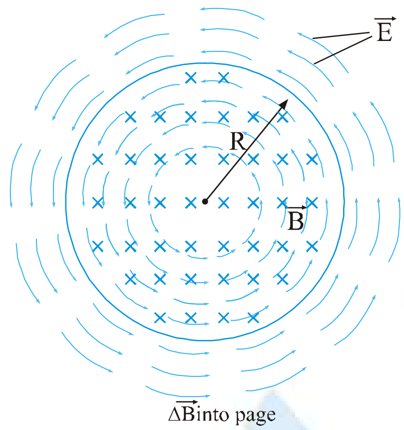
$$E_{nc} = \frac{r}{2}\frac{dB}{dt}$$

Similarly, for r > R

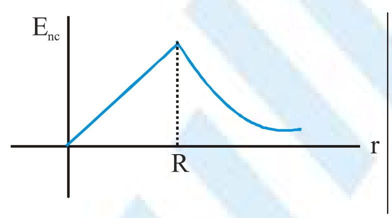
the induced electric field may be obtained as

$$E_{nc}(2\pi r) = -\frac{d\Phi_B}{dt} = \left(\frac{dB}{dt}\right)\pi r^2 \qquad \text{or} \qquad \vec{E}_{nc} = \frac{R^2}{2r}\frac{dB}{dt}$$

The pattern of E_{nc} as a function of r is shown in figure.

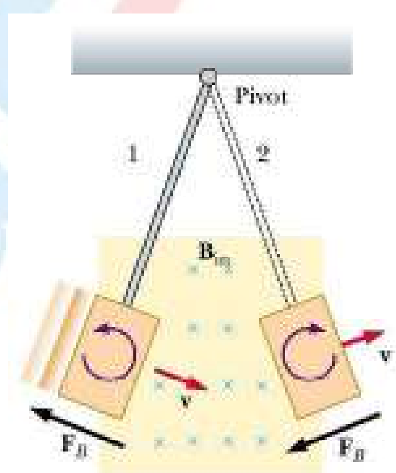


A Plot of E_{nc} as a function of r is shown in figure.

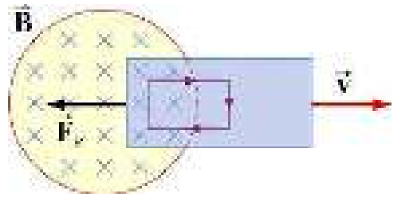


Eddy Currents

We have seen that when a conducting loop moves through a magnetic field, current is induced as the result of changing magnetic flux. If a solid conductor were used instead of a loop, as shown in Figure. current can also be induced along any closed loop in the conductor. The induced current are called an early current.



The induced eddy currents also generate a magnetic force that opposes the motion, making it more difficult to move the conductor across the magnetic field (Figure).



Since the conductor has non-vanashing resistance R, joule heating causes a loss of power by an amount $P = \varepsilon^2 / R$. Therefore by increasing the value of R, power loss can be reduced. One way to increase R is to laminate the conducting slab, or construct the slab by using gluing together thin strips that are insulated from one another (see Fig.- a). Another way is to make cuts in the slab, thereby disrupting the conducting path (Figure - b).

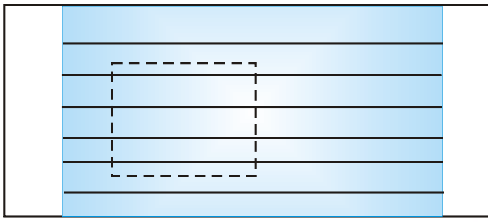


Figure (a)

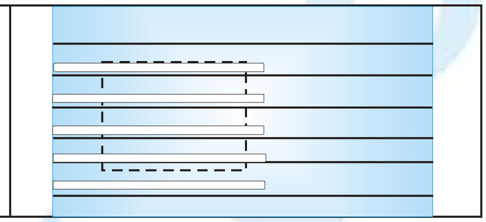
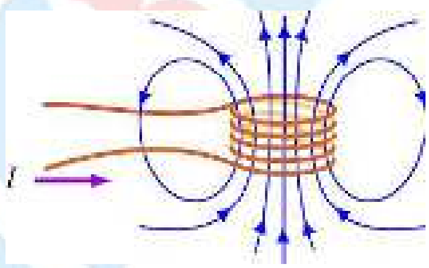


Figure (b)

There are important applications of eddy currents. For example, the currents can be used to suppress unwanted mechanical oscillations. Another application is the magnetic braking system in high-speed transit cars.

Self-Inductance

Consider again a coil consisting of N turns and carrying current I in the counterclockwise direction, as shown in Figure. If the current is already, then the magnetic flux through the loop will remain constant. However, suppose the current I changes with time, then according to faraday's law, an induced emf will arise to oppose the change. According to lenz law the induced emf will be clockwise if $\frac{dI}{dt} > 0$, and counterclockwise if $\frac{dI}{dt} < 0$. The property of the loop in which its own magnetic field opposes any change in current is called :self-inductance", and the emf generated is called the self-induced emf or back emf, which we denote as ε_L. All current-carrying loops exhibit this property. In particular, an inductor is a circuit element (symbol) which has a large self-inductance.



Mathematically, the self-induced emf can be written as

$$\varepsilon_L = -N\frac{d\Phi_B}{dt} = -N\frac{d}{dt}\int \vec{B}.d\vec{A}$$

and is related to the self-inductance L by

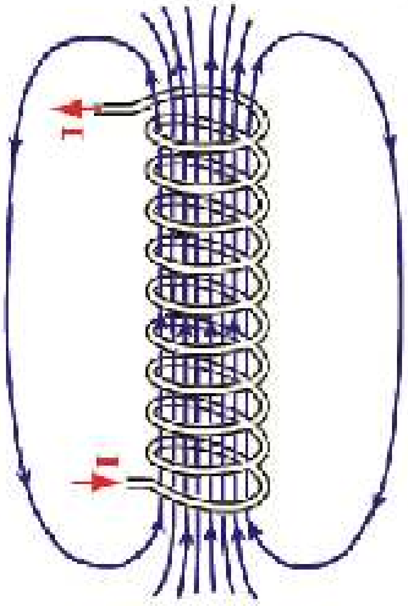
$$\varepsilon_L = -L\frac{dI}{dt}$$

The above two expression can be combined to yield

$$L = \frac{N\Phi_B}{I}$$

Physically, the inductance L is a measure of an inductor's "opposition" to the change of current ; larger the value of L, lower the rate of change of current.

Ex. Compute the self-inductance of a solenoid with N turns, length *l* , and the radius R with a current I flowing through each turn, as shown in Figure. Ignore edge effects



Sol. Ignoring edge effects and applying Ampere's law, the magnetic field inside a solenoid is given by Eq. :

$$\vec{B} = \frac{\mu_0 NI}{l}\hat{k} = \mu_0 nI\,\hat{k}$$

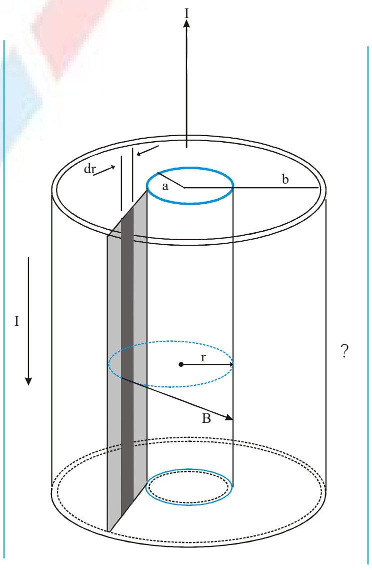
where n = N / ● is the number of turns per unit length. The magnetic flux through each turn is

$$\Phi_B = BA = \mu_0 nI.(\pi R^2) = \mu_0 nI\pi R^2$$

Thus, the self-inductance is $L = \frac{N\Phi_B}{l}\hat{k} = \mu_0 n^2 \pi R^2 l$

Note We see that L depends only on the geometrical factors (n, R and ●) and is independent of the current I.

Ex. A long coaxial cable as consisting of two thin concentric cylindrical conducting shells of radii a and b and length *l*. The conducting shells carry the same current I in opposite directions. Imagine that the inner conductor carries current to a device and that the outer one acts as a return path carrying the current back to the source. Calculate the self-inductance L of this cable.



Sol. Imagine a thin radial slice of the coaxial cable, such as the shaded rectangle in Figure. The magnetic field is perpendicular to the rectangle of length l and width b-a, the cross section of interest. Divide this rectangle into strips of width dr. We see that the area of each strip is Idr and that the flux through each strip is

$$B\,dA = BIdr$$

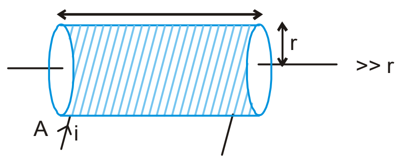
Hence, we find the total flux through the entire cross section by integrating

$$\left|\Phi_B = \int B\,dA = \int_a^b \frac{\mu_0 I}{2\pi r} 1\ dr = \frac{\mu_0 I1}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 I1}{2\pi} In\left(\frac{b}{a}\right)\right|$$

Using this result, we find that the self-inductance of the cable is

$$\left|L = \frac{\Phi_B}{I} = \frac{\mu_0 1}{2\pi} In\left(\frac{b}{a}\right)\right|$$

Self Inductance of Solenoid



Let the volume of the solenoid be V , the number of turns per unit length be n.

Let a current I be flowing in the solenoid. Magnetic field in the solenoid is given as $B=\mu_0 ni$.The magnetic flux through one turn of solenoid $\phi = \mu_0 n\, i\; A$.

The total magnetic flux through the solenoid $= N\,\phi = N\mu_0 ni\, A = \mu_0 n^2 i\, A \bullet$

$\therefore$ $L = \mu_0 n^2 \bullet\ A = \mu_0 n^2 V$

$\phi = \mu_0\, n\, i\, \pi r^2\, (n\bullet)$

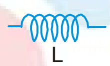
$$L = \frac{\phi}{i} = \mu_0\, n^2\, \pi r^2 \bullet.$$

Inductance per unit volume = $\mu_0 n^2$.

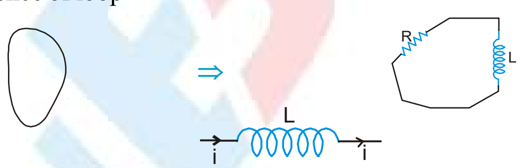
Self inductance is the physical property of the loop due to which it opposes the change in current that means it tries to keep the current constant. Current can not change suddenly in the inductor.

Inductor

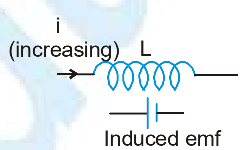
It is represent by

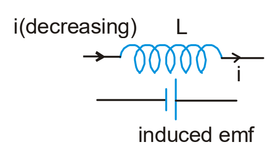


electrical equivalence of loop

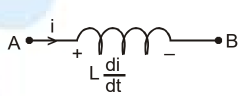


If current i through the inductor is increasing the induced emf will oppose the **increase** in current and hence will be opposite to the current. If current i through the inductor is decreasing the induced emf will oppose the **decrease** in current and hence will be in the direction of the current.



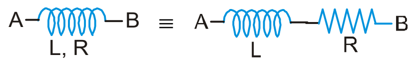


Over all result

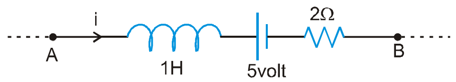


$$V_A - L\frac{di}{dt} = V_B$$

Note : If there is a resistance in the inductor (resistance of the coil of inductor) then :



Ex. A B is a part of circuit. Find the potential difference $v_A - v_B$ if

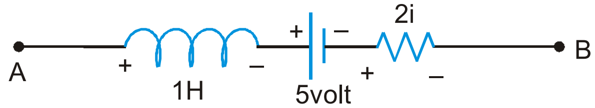


(i) current i = 2A and is constant

(ii) current i = 2A and is increasing at the rate of 1 amp/sec.

(iii) current i = 2A and is decreasing at the rate 1 amp/sec.

Sol.



$$L\frac{di}{dt} = 1\frac{di}{dt}$$

writing KVL from A to B

$$V_A - 1\frac{di}{dt} - 5 - 2i = V_B.$$

(i) Put $i = 2, \frac{di}{dt} = 0$

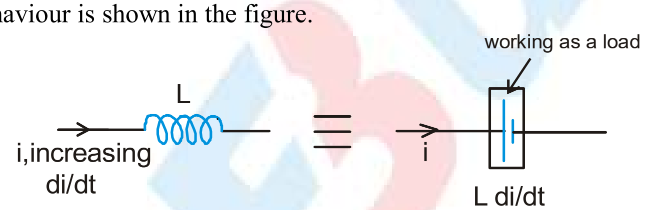
$V_A - 5 - 4 = V_B$ $\therefore V_A - V_B = 9$ volt

(ii) Put $i = 2, \frac{di}{dt} = 1; V_A - 1 - 5 - 4 = V_B$ or $V_A - V_B = 10\,V_0$

(iii) Put $i = 2, \frac{di}{dt} = -1; V_A + 1 - 5 - 2 \times 2 = V_B$ or $V_A = 8$ volt.

Energy Stored in an Inductor

If current in an inductor at an instant is i and is increasing at the rate di/dt,the induced emf will oppose the current . Its behaviour is shown in the figure.



Power consumed by the inductor = $i\,L\frac{di}{dt}$

Energy consumed in dt time = $i\,L\frac{di}{dt}dt$

$\therefore$ total energy consumed as the current increases from 0 to I = $\int_0^I iLdi = \frac{1}{2}LI^2$

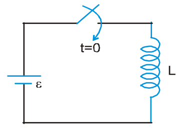
$$= \frac{1}{2}Li^2 \quad \Rightarrow \quad U = \frac{1}{2}LI^2$$

Note : This energy is stored in the magnetic field with energy density

$$\frac{dU}{dV} = \frac{B^2}{2\mu} = \frac{B^2}{2\mu_0\mu_r}$$

Total energy $U = \int \frac{B^2}{2\mu_0\mu_r}dV$

Ex. A circuit contains an ideal cell and an inductor with a switch. Initially the switch is open .It is closed at t=0. Find the current as a function of time.



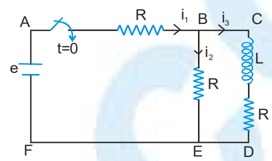
Sol. $\varepsilon = L\dfrac{di}{dt} \Rightarrow \int_0^i \varepsilon dt = \int_0^i L di$

$\varepsilon t = Li \Rightarrow i = \dfrac{\varepsilon t}{L}$

Ex. In the following circuit, the switch is closed at t = 0.

Find the currents i_1, i_2, i_3 and $\dfrac{di_3}{dt}$ at t=0 and at $t = \infty$.

Initially all currents are zero.



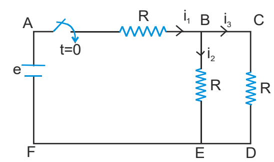
Sol. At **t = 0**

i_3 is zero ,since current cannot suddenly change due to the inductor. $\therefore i_1 = i_2$ (from KCL)

applying KVL in the part ABEF we get $i_1 = i_2 = \dfrac{\varepsilon}{2R}$.

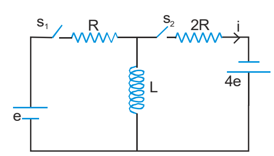
At $t = \infty$

i_3 will become constant and hence potential difference across the inductor will be zero. It is just like a simple wire and the circuit can be solved assuming it to be like shown in the following diagram.



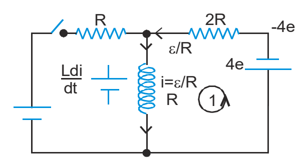
$i_2 = i_3 = \dfrac{\varepsilon}{3R}, \; i_1 = \dfrac{2\varepsilon}{3R}$.

Ex. In the circuit shown in the figure, S_1 remains closed for a long time and S_2 remains open. Now S_2 is closed and S_1 is opened. Find out the di/dt just after that moment.



Sol. Before S_2 is closed and S_1 is opened current in the left part of the circuit $= \dfrac{\varepsilon}{R}$.

Now when S_2 closed S_1 opened, current through the inductor can not change suddenly , current $\dfrac{\varepsilon}{R}$ will continue to move in the inductor.

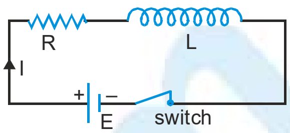


Applying KVL in loop 1.

$$L\frac{di}{dt} + \frac{\varepsilon}{R}(2R) + 4\varepsilon = 0 \Rightarrow \frac{di}{dt} = -\frac{6\varepsilon}{L}$$

R-L DC Circuit

Current Growth



(i) **EMF equation** $E = IR + L\frac{dI}{dt}$

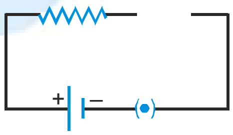
(ii) **Current at any instant**

When key is closed the current in circuit increases exponentially with respect to time. The current in circuit at any instant 't' given by $I = I_0\left[1 - e^{\frac{-t}{\lambda}}\right]$

$t = 0$ (just after the closing of key) $\Rightarrow I = 0$

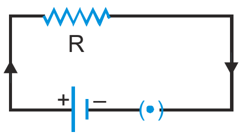
$t = \infty$ (some time after closing of key) $\Rightarrow$ $I \to I_0$

(iii) Just after the closing of the key inductance behaves like open circuit and current in circuit is zero.



Open circuit, $t = 0, I = 0$ Inductor provide infinite resistence

(iv) Some time after closing of the key inductance behaves like simple connecting wire (short circuit) and current in circuit is constant.



Short circuit, $t \to \infty, I \to I_0$, Inductor provide zero resistence $I_0 = \frac{E}{R}$

(Final, steady, maximum or peak value of current) or ultimate current

Note : Peak value of current in circuit does not depends on self inductance of coil.

(v) **Time constant of circuit (λ)**

$\lambda = \frac{L}{R_{sec.}}$ It is a time in which current increases up to 63% or 0.63 times of peak current value.

(vi) **Half life (T)**

It is a time in which current increases upto 50% or 0.50 times of peak current value.

$$I = I_0(1 - e^{-t/\lambda}), t = T, I = \frac{I_0}{2} \Rightarrow \frac{I_0}{2} = I_0(1 - e^{-T/\lambda}) \Rightarrow e^{-T/\lambda} = \frac{1}{2} \Rightarrow e^{T/\lambda} = 2$$

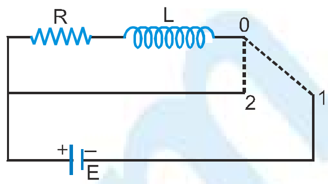
$$\frac{T}{\lambda}\log_e e = \log_e 2 \Rightarrow T = 0.693\,\lambda \Rightarrow T = 0.693\frac{L}{R_{sec.}}$$

(vii) **Rate of growth of current at any instant :–**

$$\left[\frac{dI}{dt}\right] = \frac{E}{L}(e^{-t/\lambda}) \Rightarrow \quad t = 0 \quad \Rightarrow \quad \left[\frac{dI}{dt}\right]_{max} = \frac{E}{L} \quad t = \infty \Rightarrow \left[\frac{dI}{dt}\right] \to 0$$

Note : Maximum or initial value of rate of growth of current does not depends upon resistance of coil.

Current Decay



(i) **Emf equation** $IR + L\frac{dI}{dt} = 0$

(ii) **Current at any instant**

Once current acquires its final max steady value, if suddenly switch is put off then current start decreasing exponentially wrt to time. At switch put off condition $t = 0$, $I = I_0$, source emf E is cut off from circuit $I = I_0(e^{-t/\lambda})$

Just after opening of key $t = 0$ $\Rightarrow I = I_0 = \frac{E}{R}$

Some time after opening of key $t \to \infty$ $\Rightarrow I \to 0$

(iii) **Time constant (λ)**

It is a time in which current decreases up to 37% or 0.37 times of peak current value.

(iv) **Half life (T)**

It is a time in which current decreases upto 50% or 0.50 times of peak current value.

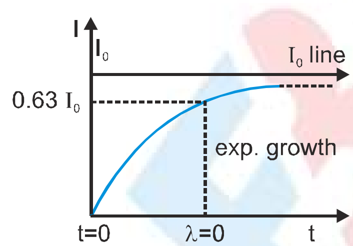
(v) **Rate of decay of current at any instant**

$$\left[-\frac{dI}{dt}\right] = \left[\frac{E}{L}\right]e^{-t/\lambda} \qquad t = 0, \quad \left[-\frac{dI}{dt}\right]_{max.} = \frac{E}{L} \quad t \to \infty \quad \Rightarrow \quad \left[-\frac{dI}{dt}\right] \to 0$$

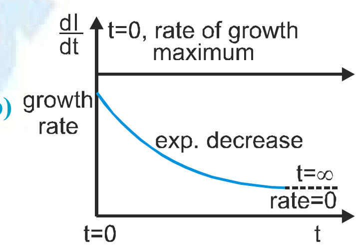
Graph for R–L circuit

Current Growth

(a)

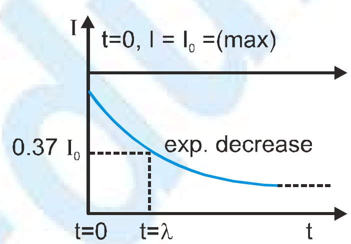


(b)

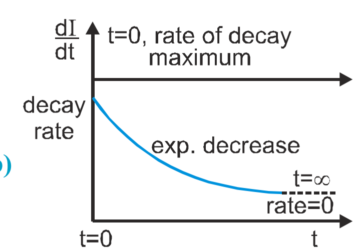


Current decay

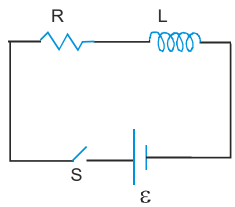
(a)



(b)



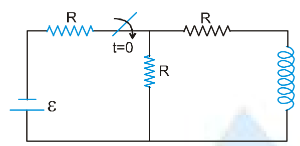
Ex. At t = 0 switch is closed (shown in figure) after a long time suddenly the inductance of the inductor is made η times lesser ($\frac{L}{\eta}$) then its initial value, find out instant current just after the operation.



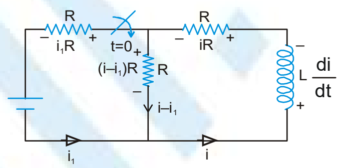
Sol. Using above result (note 4)

$$L_1 i_1 = L_2 i_2 \qquad \Rightarrow \qquad i_2 = \frac{\eta\varepsilon}{R}$$

Ex. In the following circuit the switch is closed at t = 0. Initially there is no current in inductor. Find out current the inductor coil as a function of time.



Sol. : At any time t



$$-\varepsilon + i_1R - (i - i_1)R = 0$$

$$-\varepsilon + 2i_1R - iR = 0$$

$$i_1 = \frac{iR + \varepsilon}{2R}$$

Now, $-\varepsilon + i_1R + iR + L.\frac{di}{dt} = 0$

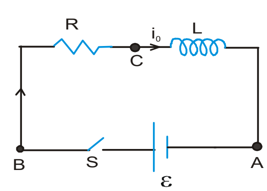
$$-\varepsilon + \left(\frac{iR+\varepsilon}{2}\right) + iR + L.\frac{di}{dt} = 0 \quad \Rightarrow \quad -\frac{\varepsilon}{2} + \frac{3iR}{2} = -L.\frac{di}{dt}$$

$$\left(\frac{-\varepsilon+3iR}{2}\right)dt = -L.di \quad \Rightarrow \quad -\frac{-dt}{2L} = \frac{di}{-\varepsilon+3iR}$$

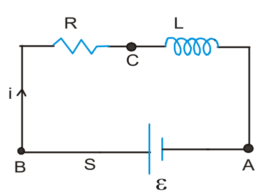
$$-\int_0^t \frac{dt}{2L} = \int_0^i \frac{di}{-\varepsilon+3iR} \quad \Rightarrow \quad -\frac{t}{2L} = \frac{1}{3R}\ln\left(\frac{-\varepsilon+3iR}{-\varepsilon}\right)$$

$$-\ln\left(\frac{-\varepsilon+3iR}{-\varepsilon}\right) = \frac{3Rt}{2L} \quad \Rightarrow \quad i = +\frac{\varepsilon}{3R}\left(1-e^{-\frac{3Rt}{2L}}\right)$$

Ex. Figure shows a circuit consisting of a ideal cell, an inductor L and a resistor R,connected in series. Let the switch S be closed at t = 0. Suppose at t = 0 current in the inductor is i_0 then find out equation of current as a function of time



Sol. Let an instant t current in the circuit is i which is increasing at the rate di/dt.

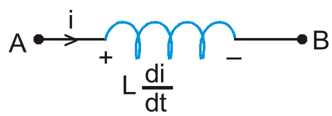


Writing KVL along the circuit , we have $\varepsilon - L\dfrac{di}{dt} - iR = 0$

$$\Rightarrow \quad L\frac{di}{dt} = \varepsilon - iR \qquad \Rightarrow \quad \int_{i_0}^{i}\frac{di}{\varepsilon - iR} = \int_{0}^{t}\frac{dt}{L} \quad \Rightarrow \quad \ln\left(\frac{\varepsilon - iR}{\varepsilon - i_0R}\right) = -\frac{Rt}{L}$$

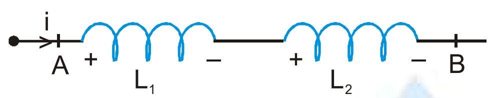
$$\Rightarrow \varepsilon - iR = (\varepsilon - i_0R)\, e^{-Rt/L} \qquad \Rightarrow \quad i = \frac{\varepsilon - (\varepsilon - i_0R)e^{-Rt/L}}{R}$$

Equivalent self inductance



$$L = \frac{V_A - V_B}{di/dt} \qquad ..(1)$$

Series combination

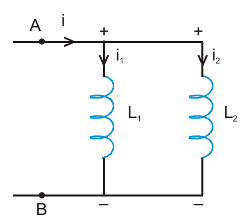


$$V_A - L_1\frac{di}{dt} - L_2\frac{di}{dt} = V_B \qquad ....(2)$$

from **(1)** and **(2)**

$L = L_1 + L_2$ (neglecting mutual inductance)

Parallel Combination



From figure $V_A - V_B = L_1\dfrac{di_1}{dt} = L_2\dfrac{di_2}{dt}$ .....**(3)**

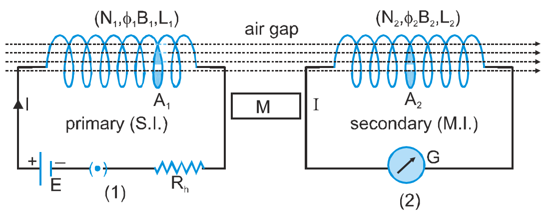
also $i = i_1 + i_2$

or $\dfrac{di}{dt} = \dfrac{di_1}{dt} + \dfrac{di_2}{dt}$

or $\dfrac{V_A - V_B}{L} = \dfrac{V_A - V_B}{L_1} + \dfrac{V_A - V_B}{L_2}$

$\dfrac{1}{L} = \dfrac{1}{L_1} + \dfrac{1}{L_2}$ (neglecting mutual inductance)

Mutual Induction



Whenever the current passing through primary coil or circuit change then magnetic flux neighbouring secondary coil or circuit will also change. Acc. to Lenz for opposition of flux change, so an emf induced in the neighbouring coil or circuit. This phenomenon called as 'Mutual induction'. In case of mutual inductance for two coils situated close to each other, flux linked with the secondary due to current in primary.

Due to Air gap always $\phi_2 < \phi_1$ and $\phi_2 = B_1A_2$ $(\theta = 0°)$.

Case - I When current through primary is constant

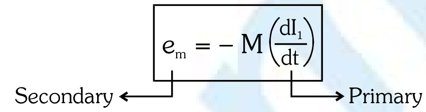
Total flux of secondary is directly proportional to current flow through the primary coil

$N_2\,\phi_2 \propto I_1 \quad \Rightarrow \quad N_2\,\phi_2 = MI_1,\ M = \dfrac{N_2\phi_2}{I_1} = \dfrac{N_2B_1A_2}{I_1} = \dfrac{(\phi_T)_s}{I_p}$ where M : is coefficient of mutual induction.

Case - II When current through primary changes with respect to time

If $\frac{dI_1}{dt} \rightarrow \frac{dB_1}{dt} \rightarrow \frac{d\phi_1}{dt} \rightarrow \frac{d\phi_2}{dt} \Rightarrow$ Static EMI

$N_2\phi_2 = MI_1$ $\qquad -N_2\frac{d\phi_2}{dt} = -M\frac{dI_1}{dt}, \left[-N_2\frac{d\phi}{dt}\right]$



called total mutual induced emf of secondary coil em.

- The units and dimension of M are same as 'L'.
- The mutual inductance does not depends upon current through the primary and it is constant for circuit system.

'M' depends on :

(a) Number of turns (N_1, N_2).

(b) Cofficient of self inductance (L_1, L_2).

(c) Area of cross section.

(d) Magnetic permeability of medium (μr).

(e) Distance between two coils (As $d\downarrow = M\uparrow$).

(f) Orientation between two coils.

(g) Coupling factor 'K' between two coils.

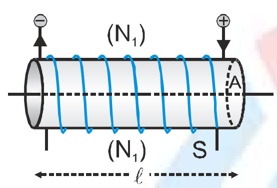
Different Coefficient of Mutual Inductance

(a) In terms of their number of turns

(b) In terms of their coefficient of self inductances

(c) In terms of their nos of turns (N_1, N_2)

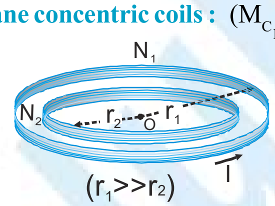
(a) Two co–axial solenoids : ($M_{s_1s_2}$)



Coefficient of mutual inductance between two solenoids

$$M_{s_1s_2} = \frac{N_2B_1A}{I_1} = \frac{N_2}{I_1}\left[\frac{\mu_0 N_1 I_1}{l}\right]A \Rightarrow \quad M_{s_1s_2} = \left[\frac{\mu_0 N_1 N_2 A}{l}\right]$$

(b) Two plane concentric coils : ($M_{c_1c_2}$)

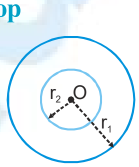


$$M_{c_1c_2} = \frac{N_2B_1A_2}{I_1} \qquad \text{where} \quad B_1 = \frac{\mu_0 N_1 I_1}{2r_1},\ A_2 = \pi r_2^2$$

$$M_{c_1c_2} = \frac{N_2}{I_1}\left[\frac{\mu_0 N_1 I_1}{2r_1}\right](\pi r_2^2) \Rightarrow M_{c_1c_2} = \frac{\mu_0 N_1 N_2 \pi r_2^2}{2r_1}$$

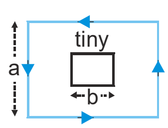
Two concentric loop

$M \propto \frac{r_2^2}{r_1}$ (r1 >> r2)



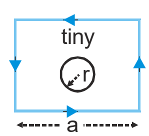
Two concentric square loops

$M \propto \frac{b^2}{a}$



A square and a circular loop

$M \propto \frac{r^2}{a}$

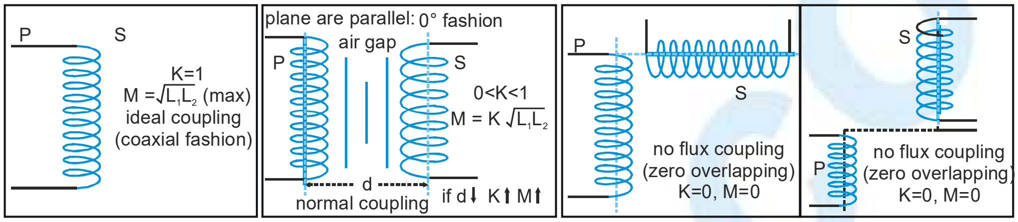


In terms of L_1 and L_2 : For two magnetically coupled coils :-

$M = K\sqrt{L_1 L_2}$ here 'K' is coupling factor between two coils and its range $0 \le K \le 1$

- For ideal coupling Kmax = 1 $\Rightarrow$ $M_{max} = \sqrt{L_1 L_2}$ (where M is geometrical mean of L_1 and L_2)
- For real coupling $(0 < K < 1)$ $M = K\sqrt{L_1 L_2}$
- Value of coupling factor 'K' decided from fashion of coupling.

Different Fashion of Coupling



'K' also defined as $K = \dfrac{\phi_s}{\phi_p} = \dfrac{\text{mag. flux linked with secondary (s)}}{\text{mag. flux linked with primary (p)}}$

Inductance in Series and Parallel

Two coil are connected in series : Coils are lying close together (M)

If M = 0, $L = L_1 + L_2$ If $M \neq 0$ $L = L_1 + L_2 + 2M$

(a) When current in both is in the same direction Then $L = (L_1 + M) + (L_2 + M)$

(b) When current flow in two coils are mutually in opposite directions.

$L = L_1 + L_2 - 2M$

Two coils are connected in parallel :

(a) If M = 0 then $\dfrac{1}{L} = \dfrac{1}{L_1} + \dfrac{1}{L_2}$ or $L = \dfrac{L_1 L_2}{L_1 + L_2}$ **(b)** If $M \neq 0$ then $\dfrac{1}{L} = \dfrac{1}{(L_1 + M)} + \dfrac{1}{(L_2 + M)}$

Ex. On a cylindrical rod two coils are wound one above the other. What is the coefficient of mutual induction if the inductance of each coil is 0.1 H ?

Sol. One coil is wound over the other and coupling is tight,so $K = 1,\ M = \sqrt{L_1 L_2} = \sqrt{0.1 \times 0.1} = 0.1\,H$

Ex. How does the mutual inductance of a pair of coils change when :

(i) the distance between the coils is increased ?

(ii) the number of turns in each coil is decreased ?

(iii) a thin iron rod is placed between the two coils, other factors remaining the same ?

Justify your answer in each case .

Sol. **(i)** The mutual inductance of two coils, decreases when the distance between them is increased. This is because the flux passing from one coil to another decreases.

(ii) Mutual inductance $M = \dfrac{\mu_0 N_1 N_2 A}{l}$ i.e., $M \propto$ N1 N2 Clearly, when the number of turns N1 and N2 in the two coils is decreased, the mutual inductance decreases.

(iii) When an iron rod is placed between th two coils the mutual inductance increases, because $M \propto$ permeability (μ)

Ex. A coil is wound on an iron core and looped back on itself so that the core has two sets of closely would wires in series carrying current in the opposite sense. What do you expect about its self-inductance ? Will it be larger or small ?

Sol. As the two sets of wire carry currents in opposite directions, their induced emf's also act in opposite directions. These induced emf's tend to cancel each other, making the self-inductance of the coil very small.

This situation is similar to two coils connected in series and producing fluxes in opposite directions. Therefore, their equivalent inductance must be Leq = L + L – 2M = L + L – 2L = 0

Ex. A solenoid has 2000 turns wound over a length of 0.3 m. The area of cross-section is 1.2 × 10–3 m2. Around its central section a coil of 300 turns is closely would. If an initial current of 2A is reversed in 0.25 s, find the emf induced in the coil.

Sol. $$M = \frac{\mu_0 N_1 N_2 A}{l} = \frac{4\pi \times 10^{-7} \times 2000 \times 300 \times 1.2 \times 10^{-3}}{0.3} = 3 \times 10^{-3}\,H$$

$$E = -M\frac{dI}{dt} = -3 \times 10^{-3}\left[\frac{-2-2}{0.25}\right] = 48 \times 10^{-3}\,V = 48\,mV$$

Ex. Two insulated wires are wound on the same hollow cylinder, so as to form two solenoids sharing a common air-filled core. Let l be the length of the core, A the cross-sectional area of the core, N_1 the number of times the first wire is wound around the core, and N_2 the number of turns the second wire is wound around the core. Find the mutual inductance of the two solenoids, neglecting the end effects.

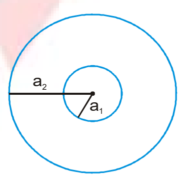
Sol. If a current I_1 flows around the first wire then a uniform axial magnetic field of strength $B_1 = \frac{\mu_0 N_1 I_1}{\lambda}$ is generated in the core. The magnetic field in the region outside the core is of negligible magnitude. The flux linking a single turn of the second wire is B_1A. Thus, the flux linking all N_2 turns of the second wire is

$$\phi_2 = N_2 B_1 A = \frac{\mu_0 N_1 N_2 A I_1}{\lambda} = MI_1 .$$

$$\therefore \quad M = \frac{\mu_0 N_1 N_2 A}{\lambda}$$

As described previously, M is a geometric quantity depending on the dimensions of the core and the manner in which the two wires are wound around the core, but not on the actual currents flowing through the wires.

Ex. Find the mutual inductance of two concentric coils of radii a_1 and a_2 ($a_1 << a_2$) if the planes of coils are same.



Sol. Let a current i flow in coil of radius a_2.

Magnetic field at the centre of coil = $\frac{\mu_0 i}{2a_2}\pi a_1^2$

or $$Mi = \frac{\mu_0 i}{2a_2}\pi a_1^2$$ or $$M = \frac{\mu_0 \pi a_1^2}{2a_2}$$

Ex. Solve the above question, if the planes of coil are perpendicular.

Sol. Let a current i flow in the coil of radius a_1. The magnetic field at the centre of this coil will now be parallel to the plane of smaller coil and hence no flux will pass through it, hence M = 0.

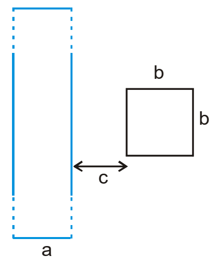
Ex. Solve the above problem if the planes of coils make θ angle with each other.

Sol. If i current flows in the larger coil, magnetic field produced at the centre will be perpendicular to the plane of larger coil. Now the area vector of smaller coil which is perpendicular to the plane of smaller coil will make an angle θ with the magnetic field.

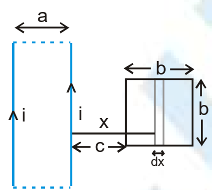
Thus flux = $\vec{B}.\vec{A} = \dfrac{\mu_0 i}{2a_2}.\pi a_1^2.\cos\theta$

or $M = \dfrac{\mu_0 \pi a_1^2 \cos\theta_1}{2a_2}$

Ex. Find the mutual inductance between two rectangular loops, shown in the figure



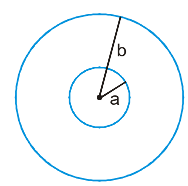
Sol.



Let current i flow in the loop having ∞-by long sides. Consider a segment of width dx at a distance x as shown flux through the regent

$d\phi = \left[\dfrac{\mu_0 i}{2\pi x} - \dfrac{\mu_0 i}{2\pi(x+a)}\right] b\,dx$. $\Rightarrow$ $\phi = \displaystyle\int_c^{c+b} \left[\dfrac{\mu_0 i}{2\pi x} - \dfrac{\mu_0 i}{2\pi(x+a)}\right] b\,dx = \dfrac{\mu_0 i b}{2\pi}\left[\ln\dfrac{c+b}{c} - \ln\dfrac{a+b+c}{a+c}\right]$.

Ex. Figure shows two concentric coplanar coils with radii a and b (a << b). A current i = 2t flows in the smaller loop. Neglecting self inductance of larger loop



(a) Find the mutual inductance of the two coils
(b) Find the emf induced in the larger coil
(c) If the resistance of the larger loop is R find the current in it as a function of time

Sol. **(a)** To find mutual inductance, it does not matter in which coil we consider current and in which flux is calculated (Reciprocity theorem) Let current i be flowing in the larger coil. Magnetic field at the centre = $\dfrac{\mu_0 i}{2b}$.

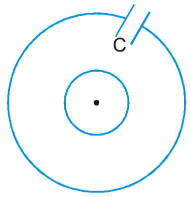
flux through the smaller coil = $\dfrac{\mu_0 i}{2b}\pi a^2$ $\quad\therefore\ M = \dfrac{\mu_0}{2b}\pi a^2$

(b) $|\text{emf induced in larger coil}| = M\left[\left(\dfrac{di}{dt}\right)\text{in smaller coil}\right]$

$= \dfrac{\mu_0}{2b}\pi a^2\ (2)\ = \dfrac{\mu_0 \pi a^2}{b}$

(c) current in the larger coil $= \dfrac{\mu_0 \pi a^2}{bR}$

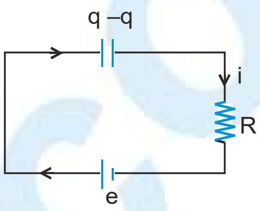
Ex. If the current in the inner loop changes according to $i = 2t^2$ then, find the current in the capacitor as a function of time.



Sol. $M = \frac{\mu_0}{2b}\pi a^2$

$$|\text{emf induced in larger coil}| = M\left[\left(\frac{di}{dt}\right)\text{in smaller coil}\right]$$

$$e = \frac{\mu_0}{2b}\pi a^2\,(4t) \quad = \frac{2\mu_0\pi a^2 t}{b}$$



Applying KVL :-

$$+e - \frac{q}{c} - iR = 0 \qquad \Rightarrow \qquad \frac{2\mu_0\pi a^2 t}{b} - \frac{q}{c} - iR = 0$$

differentiate wrt time :-

$$\frac{2\mu_0\pi a^2}{b} - \frac{i}{c} - \frac{di}{dt}R = 0$$

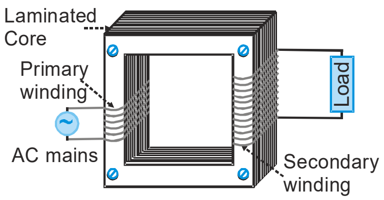
on solving it

$$i = \frac{2\mu_0\pi a^2 C}{b}\left[1 - e^{-t/RC}\right]$$

Transformer

Working principle

Mutual induction



Transformer has basic two section

(a) **Shell :** Consist of primary and secondary coil of copper.
The effective resistance between primary and secondary coil is infinite because electric circuit between two is open ($R_{ps} = \infty$)

(b) **Core :** Which is between two coil and magnetically coupled two coils. Two coils of transformer would on the same core. The alternating current passing through the primary creats a continuously changing flux through the core. This changing flux induces alternating emf in secondary.

Work

It regulates AC voltage and transfer the electrical electrical power without change in freqency of input supply. (The alternating current changes itself.)

Special Points

(i) It can't work with D.C. supply, and if a battery is connected to its primary, then output is across scondary is always zero ie. No working of transformer.

(ii) It can't called 'Amplifier' as it has no power gain like **transistor**.

(iii) It has no moving part, hence there are no mech. losses in transformer.

Types : According to voltage regulation it has two –

(i) Step up transformer : $N_S > N_P$ **(ii)** Step down transformer $N_S < N_P$

Step up transformer : Converts low voltage high current in to High voltage low current

Step down transformer : Converts High voltage low current into low voltage high current.

Power transmission is carried out always at **"High voltage low current"** so that voltage drop and power losses are minimum in transmission line.

voltage drop = $I_L R_L$, I_L = line current R_L = total line resistance,

$I_L = \dfrac{\text{power to be transmission}}{\text{line voltage}}$ power losses = $I_L^2 R_L$

High voltage coil having more number of turns and always **made of thin wire** and **high current coil** having less number of turns and always **made of thick wires.**

Ideal Transformer : (η = 100%)

(a) No flux leakage $\phi_s = \phi_p \Rightarrow \dfrac{-d\phi_s}{dt} = \dfrac{-d\phi_p}{dt}$

$e_s = e_p = e$ induced emf per turn of each coil is also same.

total induced emf for secondary $E_s = N_s e$ total induced emf for primary $E_p = N_p e$

$\dfrac{E_s}{E_p} = \dfrac{N_s}{N_p} = n$ or p where n : turn ratio , p : transformation ratio

(b) No load condition

$V_P = E_P$ and $E_S = V_S$ $\dfrac{V_S}{V_P} = \dfrac{N_S}{N_P}$ from (i) and (ii) $\dfrac{V_S}{V_P} = \dfrac{N_S}{N_P} = n$ or p

(c) No power loss

$P_{out} = P_{in}$ and $V_S I_S = V_P I_P$ $\dfrac{V_S}{V_P} = \dfrac{I_P}{I_S}$ valid only for ideal transformer

from equation (iii) and (iv) $\dfrac{V_S}{V_P} = \dfrac{I_P}{I_S} = \dfrac{N_S}{N_P} = n$ or p

Note : Generally transformers deals in ideal condition i.e. $P_{in} = P_{out}$, if other information are not given.

Real transformer ($\eta \neq$ 100%)

Some power is always lost due to flux leakage, hysteresis, eddy currents, and heating of coils.

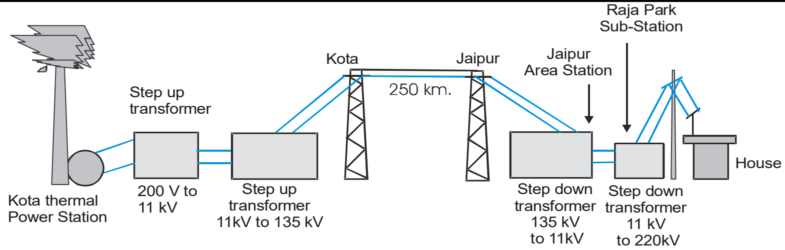
hence $P_{out} < P_{in}$ always. efficiency of transformer $\eta = \dfrac{P_{out}}{P_{in}} = \dfrac{V_S}{V_P}\cdot\dfrac{I_S}{I_P} \times 100$

Applications

The most important application of a transformer is in long distance transmission of electric power from generating station to consumers hundreds of kilometers away through transmission lines at reduced loss of power.

Transmission lines having resistance R and carrying current I have loss of power = I^2R.

This loss is reduced by reducing the current by stepping up the voltage at generating station. This high voltage is transmitted through high-tension transmission lines supported on robust pylons (iron girder pillars). The voltage is stepped down at consumption station. A typical arrangement is shown below :



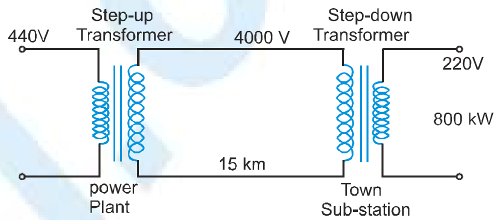
Ex. A small town with a demand of 800 kW of electric power at 220 V is situated 15 km away from an electric power plant generating power at 440 V. The resistance of the two wire line carrying power is 0.5 Ω per km. The town gets from the line through a 4000 – 220 V step down transformer at a sub-station in the town.

(a) Estimate the line power loss in the form of heat.

(b) How much power must be plant supply, assuming there is a negligible power loss due to leakage?

(c) Characterise the step up transformer at the plant.

Sol. The diagram shows the network :



For sub-station, $P = 800\text{ kW} = 800 \times 10^3$ watts

$V = 220\text{ V}$

$$I_S = \frac{P}{V} = \frac{800 \times 10^3}{220} = \frac{40}{11} \times 10^3 \text{ A.}$$

Primary current (I_p) in sub-station transformer will be given by

$$4000 \times I_P = 220 \times I_S,\ I_P = \frac{220 \times 40 \times 10^3}{11 \times 4000} = 200\text{ A}$$

(a) Hence transmission line current = 200 A

transmission line resistance = 2 × 15 × 0.5 = 15 Ω

transmission line power loss = $I^2R = 200 \times 200 \times 15 = 6 \times 10^5$ watt = 600 kW.

(b) power to be supplied by plant = power required at substation + loss of power of transmission

= 800 + 600 = 1400 kW.

(c) Voltage in secondary at power plant has characteristics $= \dfrac{\text{Power}}{\text{Current}} = \dfrac{1400\ \text{kW}}{200\ \text{A}} = \dfrac{1400 \times 1000}{200} = 7000\text{ V}$

Step-up transformer at power plant has characteristics 440 - 7000 V.

Ex. A power transmission line feeds input power at 2300 V to a step down transformer having 4000 turns in its primary. What should be the number of turns in the secondary to get output power at 230 V?

Sol. $E_p = 2300\text{ V}; N_p = 4000, E_S = 230\text{ V}$

$$\frac{E_S}{E_P} = \frac{N_S}{N_P} \quad \therefore N_S = N_P \times \frac{E_S}{E_P} = 4000 \times \frac{230}{2300} = 400$$

Ex. The output voltage of an ideal transformer, connected to a 240 V a.c. mains is 24 V. When this transformer is used to light a bulb with rating 24V, 24W calculate the current in the primary coil of the circuit.

Sol. $E_P = 240\text{ V}, E_S = 24\text{ V}, \qquad E_S I_S = 24\text{ W}$

$$\text{Current in primary coil } I_P = \frac{E_S I_S}{E_P} = \frac{24}{240} = 0.1\text{ A}$$

Losses of Transformer

(a) Copper or joule heating losses

Where : There losses occurs in both coils of shell part

Reason : Due to heating effect of current ($H = I^2Rt$)

Remmady : To minimise these losses, high current coil always made up with thick wire and for removal of produced heat circulation of mineral oil should be used.

(b) Flux leakage losses

Where : There losses occurs in between both the coil of shell part.

Cause : Due to air gap between both the coils.

Remmady : To minimise there losses both coils are tightly wound over a common soft iron core (high magnetic permeability) so a closed path of magnetic field lines formed itself within the core and tries to makes coupling factor $K \rightarrow 1$

(c) Iron losses

Where : There losses occurs in core part.

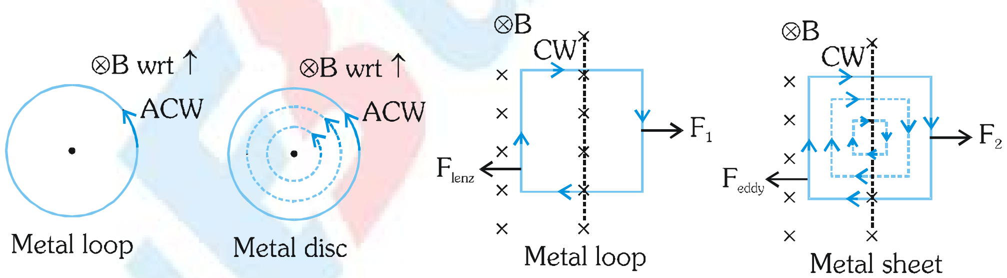
Types : (i) Hysteresis losses (ii) Eddy currents losses

(i) Hysteresis losses

Cause : Transformer core always present in the effect of alternating magnetic field ($B = B_0 \sin\omega t$) so it will magnetised & demagnetised with very high frequeny ($f = 50$ Hz).During its demagnetization a part of magnetic energy left inside core part in form of residual magnetic field. Finally this residual energy waste as heat.

Remmady : To minimise these losses material of transformer core should be such that it can be easily magnetised & demagnetised. For this purpose soft ferromagnetic material should be used. For Ex. soft iron (low retentivity and low coercivity)

Eddy Currents (or Focalt's currents)



($F_1 <<< F_2$)

(i) Eddy currents are basically the induced currents set up inside the body of conductor whenever the magnetic flux linked with it changes.

(ii) Eddy currents tend to follow the path of least resistance inside a conductor. So they from irregularly shaped loops. However, their directions are not random, but guided by Lenz's law.

(iii) Eddy currents have both undesirable effects and practically useful applications.

Applications of eddy currents

(i) Induction furnace
(ii) Electromagnetic damping
(iii) Electric brakes
(iv) Speedometers
(v) Induction motor
(vi) Electromagnetic shielding
(vii) Inductothermy
(viii) Energy meters

ETOOS KEY POINTS

(i) These currents are produced only in closed path within the entire volume and on the surface of metal body. Therefore their measurement is impossible.

(ii) Circulation plane of these currents is always perpendicular to the external field direction.

(iii) Generally resistance of metal bodies is low so magnitude of these currents is very high.

(iv) These currents heat up the metal body and some time body will melt out (Application : Induction furnace)

(v) Due to these induced currents a strong eddy force (or torque) acts on metal body which always apposes the translatory (or rotatory) motion of metal body, according to lenz.

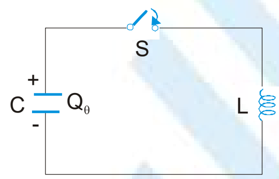
(vi) **Transformer**

Cause : Transformer core is always present in the effect of alternating magnetic field ($B = B_0\sin\omega t$). Due to this eddy currents are produced in its volume, so a part of magnetic energy of core is wasted as heat.

Remmady : To minimise these losses transformer core should be laminated. with the help of lamination process, circulation path of eddy current is greatly reduced & net resistance of system is greatly increased. So these currents become

LC Oscillations

Consider an LC circuit in which a capacitor is connected to an inductor, as shown in Figure.



Suppose the capacitor initially has charge Q_q . When the switch is closed, the capacitor begins to discharge and the electric energy is decreased. On the other hand, the current created from the discharging process generates magnetic energy which then gets stored in the inductor. In the absence of resistance, the total energy is transformed back and forthe between the electric energy in the capacitor and the magnetic energy in the inductor. This phenomenon is called electromagnetic oscillation.

The total energy in the LC circuit at some instant after closing the switch is

$$U = U_C + U_L = \frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}LI^2$$

The fact that U remains constant implies that

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}LI^2\right) = \frac{Q}{C}\frac{dQ}{dt} + LI\frac{dI}{dt} = 0$$

$$\frac{Q}{C} + L\frac{d^2Q}{dt^2} = 0$$

where $$I = -\frac{dQ}{dt}$$

and $$\frac{dI}{dt} = -\frac{d^2Q}{dt^2}$$

Notice the sign convention we have adopted here. The negative sign implies that the current I is equal to the rate of decrease of charge in the capacitor plate immediately after the switch has been closed.

The general solution to equation is

$$Q = Q_0\cos(\omega_0 t + \phi)$$

Where Q_0 is the amplitude of the charge and ϕ is the phase. The angular frequency ω_0 is given by

$$\omega_0 = -\frac{1}{\sqrt{LC}}$$

The corresponding current in the inductor is

$$I = -\frac{dQ}{dt} = \omega_0 Q_0 \sin\ (\omega_0 t + \phi) = I_0 \sin(\omega_0 t + \phi)$$

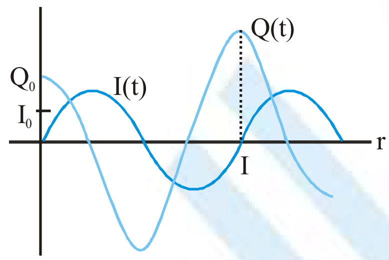
where $\qquad I_0 = \omega_0 Q_0$

From the initial conditions Q (at t = 0) = Q_0 and I (at t = 0) = 0, the phase ϕ can be determined $\phi = 0$. Thus, the solutions for the charge and the current in our LC circuit are

$$Q(t) = Q_0 \cos \omega_0 t$$

and $\qquad I(t) = I_0 \sin \omega_0 t$

The time dependence of Q(t) and I(t) are depicted in figure.

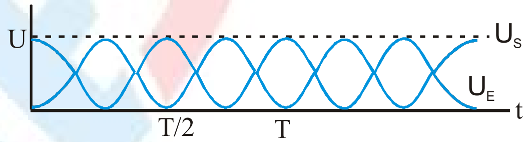


Using eqs., we see that at any instant of time, the electric energy and the magnetic energies are given by

$$U_E = \frac{Q^2(t)}{2C} = \left(\frac{Q_0^2}{2C}\right)\cos^2 \omega_0 t$$

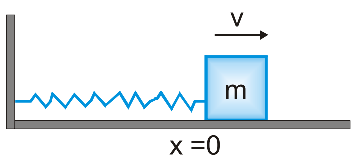
and $$U_B = \frac{1}{2}LI^2(t) = \frac{LI_0^2}{2}\sin^2 \omega t = \frac{L(-\omega_0 Q_0)^2}{2}\sin^2 \omega_0 t = \left(\frac{Q_0^2}{2C}\right)\sin^2 \omega_0 t$$

The electric and magnetic energy oscillation is illustrated in figure.



Mechanical analogy

The mechanical analog of the LC oscillations is the mass-spring system, shown in Figure.



If the mass is moving with a speed v and the spring having a spring constant k is displaced from its equilibrium by x, then the total energy of this mechanical system is

$$U = K + U_{sp} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

where K and U_{sp} are the kinetic energy of the mass and the potential energy of the spring, respectively.
In the absence of friction, U is conserved and we obtain

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2 + \frac{1}{2}kx^2\right) = mv\frac{dv}{dt} + kx\frac{dx}{dt} = 0$$

Using $\quad v = \frac{dx}{dt} \quad$ and $\quad \frac{dv}{dt} = \frac{d^2x}{dt^2}$

the above equation may be rewritten as

$$m\frac{d^2x}{dt^2} + kx = 0$$

The general solution for the displacement is

$$x = x_0 \cos(\omega_0 t + \phi)$$

where $\quad \omega_0 = \sqrt{\frac{k}{m}}$

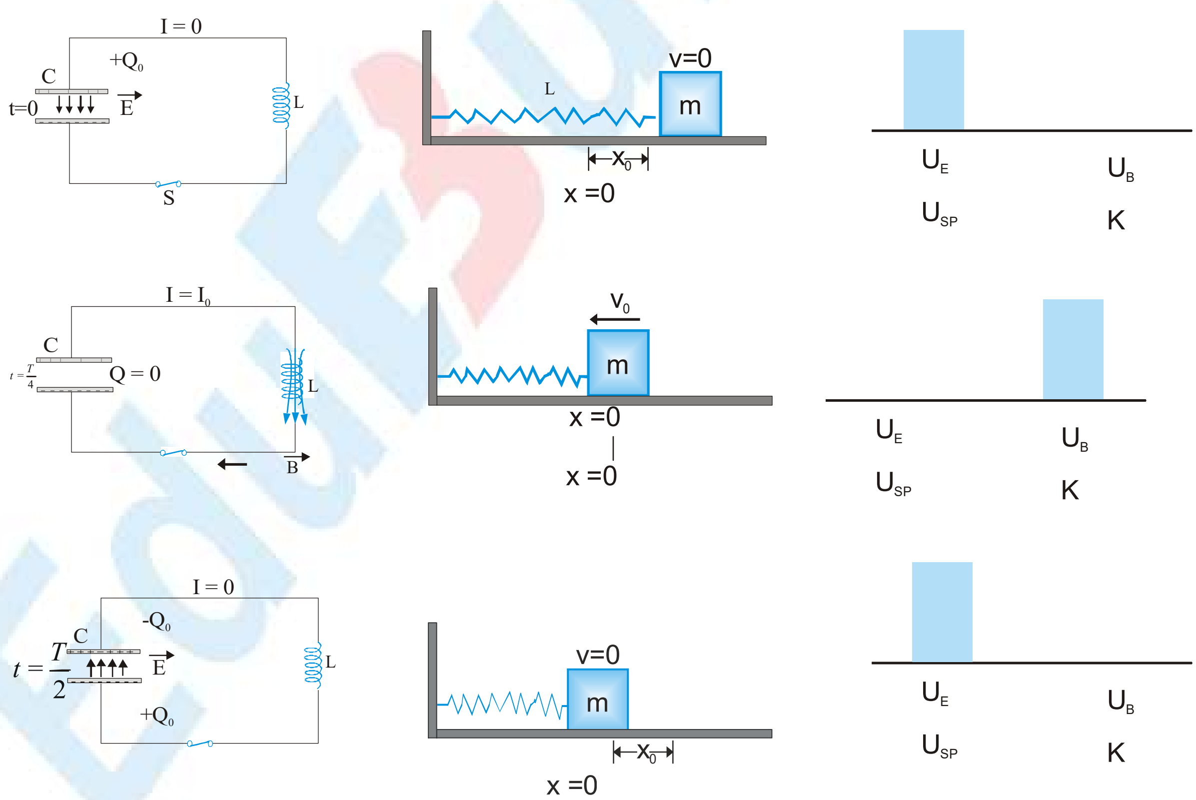
is the angular frequency and x_0 is the amplitude of the oscillations. Thus, at any instant in time, the energy of the system may be written as

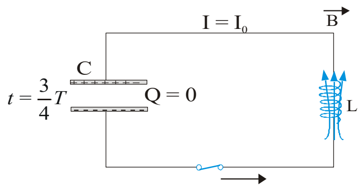
$$U = \frac{1}{2}mx_0^2\omega_0^2 \sin^2(\omega_0 t + \phi) + \frac{1}{2}kx_0^2 \cos^2(\omega_0 t + \phi)$$

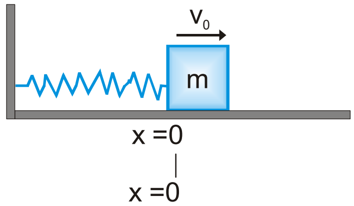
$$= \frac{1}{2}kx_0^2[\sin^2(\omega_0 t + \phi) + \cos^2(\omega_0 t + \phi) = \frac{1}{2}kx_0^2$$

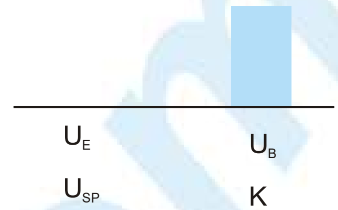
In figure we illustrate the energy oscillations in the LC circuit and the mass spring system (harmonic oscillator).

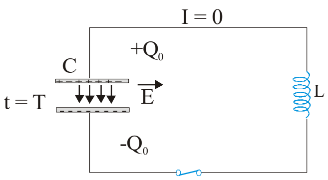
Lc Circuit **Mass spring System** **Energy**

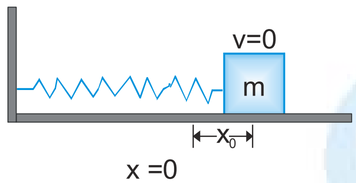


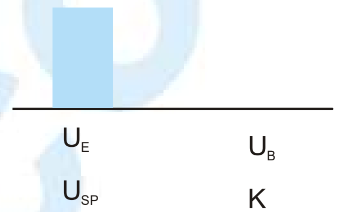












Ex. An inductor of inductance 2mH is connected across a charged capacitor of 5μE. Let q denote the instantaneous charge on the capacitor, and i the current in the circuit. Maximum value of q is Q = 200 μC.

(a) When q = 100 μC, what is the value of |di/dt| ?

(b) When q = 200 μC, what is the value of i ?

Sol. **(a)** Charge stored in the capacitor oscillates simple harmonically as

$$Q = Q_0 \sin(\omega t - \phi) \text{ and}$$

$$U = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(2\times10^{-3}H)(5.0\times10^{-6}F)}} = 10^4 s^{-1}$$

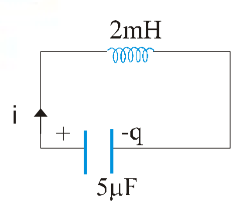
At t = 0, if $Q = Q_0$, then

$$Q(t) = Q_0 \cos \omega t \text{ and if } Q = Q_0/2, \text{ then } \cos \omega t = 1/2$$

$$\frac{di(t)}{dt} = -Q_0\omega^2 \cos(\omega t) = (200\times10^{-6})(10^4 s^{-1})\left(\frac{1}{2}\right) = 10^4 A/s$$

(b) When the energy of the capacitor is maximum, the energy stored in the inductor will be zero.

i.e. $\frac{1}{2}Li^2 = 0 \Rightarrow i = 0$



Ex. A capacitor of capacitance 2 μF is charged to a potential difference of 12 V. It is then connected across an indicator of inductance 0.6 mH. What is the current in the circuit at a time when the potential difference across the capacitor is 6.0 V ?

Sol. As the capacitor is discharged to a potential difference of 12 V, the initial charge on the capacitor is

$q_0 = CV_0 = 2 \times 10^{6} \times 12\ C$ ....**(1)**

At any instant as the capacitor discharges through the inductor (LC circuit), the instantaneous charge on the capacitor is given by

$q = q_0 \cos \omega t$ ....**(2)** (because at t = 0, $q = q_0$)

But $q = CV$ ....**(3)**

where V is the potential difference at the instant 't'.

From **(1)** and **(3)** we obtain $\frac{q}{q_0} = \frac{V}{V_0}$

Putting the value of V and V_0 we obtain

$$\frac{q}{q_0} = \frac{1}{2} \quad \Rightarrow \quad \cos \omega t = \frac{1}{2} \quad \Rightarrow \quad \omega t = \cos^{-1}\left(\frac{1}{2}\right)$$

$\Rightarrow$ ωt = /3 rad ....**(4)**

Here $$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{[0.6 \times 10^{-3} \times 10^{-4}]}$$

$$\Rightarrow \quad \omega = \frac{10^5}{2\sqrt{3}} rad/\sec.$$ ....**(5)**

The current through the circuit at that instant is given by

$$i = \frac{d}{dt}[q_0 \cos \omega t]$$

$\Rightarrow$ $i = -q_0\, \omega \sin\omega t$

Putting the value of q_0 from **(1)**, ω from **(5)** and ωt from **(4)** we obtain

$$|i| = 2 \times 10^7 \times 12 \times \frac{10^3}{2\sqrt{3}} \sin(\pi/3) = 0.6\ A$$

• Etoos Tips & Formulas •

1. Magnetic Flux :

$\phi = \vec{B}.\vec{A} = BA\cos\theta$ for uniform $\vec{B}$.

$\phi = \int \vec{B}.d\vec{A}$ = for non uniform $\vec{B}$.

2. FARADAY'S Laws of Electromagnetic Induction :

(a) An induced emf is setup whenever the magnetic flux linking that circuit changes.

(b) The magnitude of the induced emf in any circuit is proportional to the rate of change of the magnetic flux linking the circuit, $\varepsilon \propto \frac{d\varphi}{dt}$.

3. Lenz's Laws :

The direction of an induced emf is always such as to oppose the cause producing it.

4. Law of EMI : $e = -\frac{d\varphi}{dt}$

The negative sign indicates that the induced emf oppose the change of the flux.

5. EMF Induced in a Straight Conductor in Uniform Magnetic Field :

$E = BLv\sin\theta$

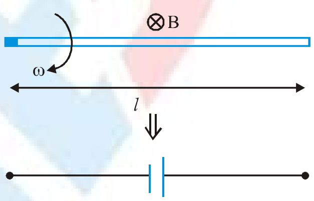
where B = flux density

L = length of the conductor

v = velocity of the conductor

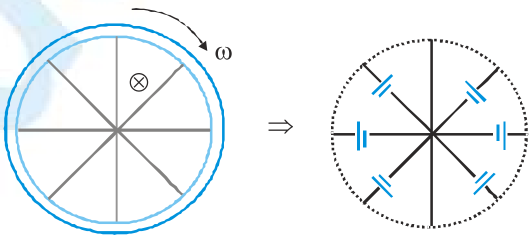
θ = angle between direction of motion of conductor & B.

6. EMF Induced in a Rod Rotating Perpendicular to Magnetic Field



$E = \frac{1}{2}B\omega l^2$

For a wheel rotating in a earth magnetic field effective emf induced between the periphery & centre = $\frac{1}{2}B\omega l^2$



7. Coil Rotation in Magnetic Field such that Axis of Rotation is Perpendicular to the Magnetic Field:

Instantaneous induced emf. $E = NAB\omega \sin \omega t = E_0 \sin \omega t$

where N = number of turns in the coil

A = area of one turn B = magnetic induction

ω = uniform angular velocity of the coil E_0 = maximum induced emf

8. Self Induction & Self Inductance :

When a current flowing through a coil is changed the flux linking flux with the coil its own winding changes & due to the change in linking flux with the coil an emf is induced which is known as self induced emf & this phenomenon is known as self induction. This induced emf opposes the causes of induction. The property of the coil or the circuit due to which it opposes any change of the current coil or the circuit is known as **Self-Inductance.** Its unit is Henry.

Coefficient of self inductance $L = \frac{\varphi_s}{i}$ or $\phi_s = Li$

i = current in the circuit

ϕ_s = magnetic flux linked with the circuit due to the current i.

L depends only on ; **(i)** shape of the loop **(ii)** medium

self induced emf $e_s = \frac{d\varphi_s}{dt} = -\frac{d}{dt}(Li) = -L\frac{di}{dt}$ (if L is constant)

9. Combination of inductors

Series combination $L = L_1 + L_2 + \ldots\ldots$, i same, V in ratio of inductance, U in ratio of inductance, ϕ in ratio of inductance

Parallel combination $\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots\ldots$, V same, i in inverse ratio of inductance, U in inverse ratio of inductance, ϕ same

10. Mutual Induction :

If two electric circuits are such that the magnetic field due to a current in one is partly or wholly linked with the other, the two coils are said to be electromagnetically coupled circuits. Then any change of current in one produces a change of magnetic flux in the other & the later opposes the change by inducing an emf within itself. This phenomenon is called **MUTUAL INDUCTION** & the induced emf in the later circuit due to a change of current in the former is called **MUTUALLY INDUCED EMF.** The circuit in which the current is changed is called the primary & the other circuit in which the emf is induced is called the secondary. The Co-efficient of mutual induction (mutual inductance) between two electromagnetically coupled circuit is the magnetic flux linked with the secondary per unit current in the primary.

Mutual inducatnce = $M = \frac{\varphi_m}{I_p} = \frac{\text{flux linked with secondary}}{\text{current in the primary}}$

mutually induced emf : $E_m = \frac{d\varphi_m}{dt} = -\frac{d}{dt}(MI) = -M\frac{dI}{dt}$ (If M is constant)

M depends on **(1)** geometry of loops **(2)** medium **(3)** orientation & distance of loops.

If two coils of self inductance L_1 and L_2 are wound over each other, the mutual inductance $M = K\sqrt{L_1L_2}$ where K is called coupling constant.

For two coils wound in same direction and connected in series

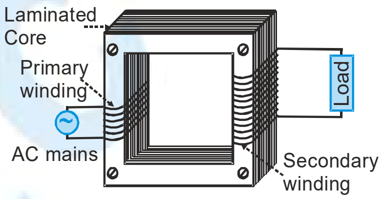
$L = L_1 + L_2 + 2M$

For two coils wound in opposite direction and connected in series

$$L = L_1 + L_2 - 2M$$

For two coils in parallel $L = \dfrac{L_1L_2 - M^2}{L_1 + L_2 \pm 2M}$

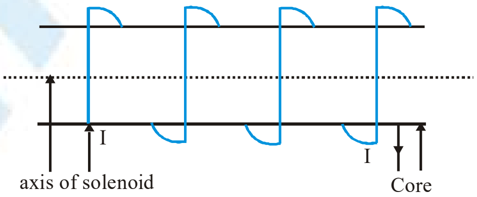
11. Transformer



(a) For ideal transformer $\dfrac{E_2}{E_1} = \dfrac{I_1}{I_2} = \dfrac{N_2}{N_1}$

(b) Efficiency $\eta = \dfrac{P_{out}}{P_{in}} \times 100\ \%$

12. SOLENOID



There is a uniform magnetic field along the axis of the solenoid

$B = \mu ni$ (ideal : length >> diameter)

where m = magnetic permeability of the core material

n = number of turns in the solenoid per unit length

i = current in the solenoid

Self inductance of a solenoid $L = \mu_0 n^2 A l$

A = area of cross section of solenoid.

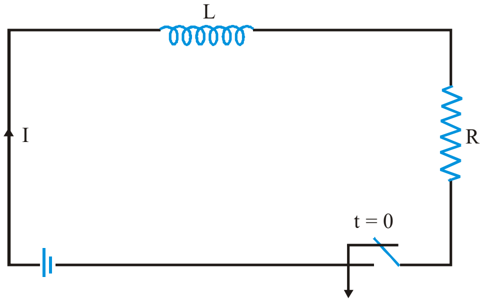
13. Super Conduction Loop in Magnetic Field :

$R = 0$; $\varepsilon = 0$. Therefore ϕ_{total} = constant. Thus in a superconducting loop flux never changes. (or it opposes 100 %)

14. Energy Stored in an Inductor : $W = \dfrac{1}{2}LI^2$

15. Energy of interaction of two loops $U = I_1\phi_2 = I_2\phi_1 = MI_1I_2$ where M is mutual inductance

16. Growth of a Current in an L - R Circuit :



$I = \dfrac{E}{R}\left(1 - e^{-Rt/L}\right)$. [If initial current = 0]

$\dfrac{L}{R}$ = time constant of the circuit.

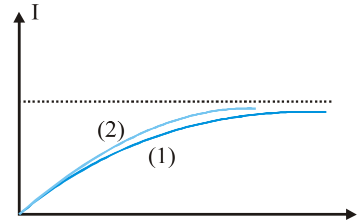
$I_0 = \dfrac{E}{R}$.

(a) L behaves as open circuit at t = 0 [If $i = 0$]

(b) L behaves as short circuit at $t = \infty$ always.

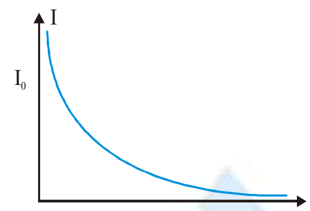
Curve (1) $\rightarrow \frac{L}{R}$ Large

Curve (2) $\rightarrow \frac{L}{R}$ Small



17. Decay of Current :

Initial current through the inductor = I_0 ; Current at any instant $i = I_0e^{-Rt/L}$



18. An emf is induced in a closed loop where magnetic flux is varied. The induced electric field is not conservative field because for induced electric field, the line integral $\oint \vec{E}.d\vec{l}$ around a closed path is non- zero.

19. Acceleration of a magnet falling through a long solenoid decrease because the induced current produced in a circuit always flows in such direction that it opposes the change or the cause that produces it.

20. The mutual inductance of two coils is doubled if the self inductance of the primary and secondary coil is doubled because mutual inductance $M \propto \sqrt{L_1L_2}$.

21. The possibility of an electric bulb fusing is higher at the time of switching ON and OFF because inductive effects produce a surge at the time of switch-off and switch-on.

22. Motional emf : If a conductor is moved in a magnetic field then motional emf will be $E = B\, l_{eff}\, v$

Here $v \perp l_{eff}$ & $v \perp B$ & $B \perp l_{eff}$

$l_{eff} \rightarrow$ effective length between the end points of conductor which is perpendicular to the velocity.