ELECTROSTATICS

INTRODUCTION

Electromagnetism is, almost unarguably, the most important basic technology in the world today. Almost every modern device, from cars to kitchen appliances to computers, is dependent upon it. Life, for msot of us, would be almost unimaginable without electromagetism. In fact, electromagnetism cuts such a wide path through modern life that the teaching of electromagnetism has developed into several different specialities. Initially electricity and magnetism were classified as independent phenomena, but different specialities. Initially electricity and magnetism were classified as independent phenomena, but after some experiments (we will discuss later) it was found they are interrelated so we use the name **Electromagnetism.** In electromagnetism we have to study basic properties of electromagnetic force and field (the term field will be introduced in later section). The electromagnetic force between charged particles is one of the fundamental force of nature. WE being this chapter by describing some of the basic properties of one manifestation of the electromagnetic force, the electrostatic force between charges (the force between two charges when they are at rest) under the heading **electrostatics.**

Specific Properties of Charge

(i) Charge is a scalar quantity : It represents excess or deficiency of electrons.

(ii) Charge is transferable : If a charged body is put in contact with an another body, then charge can be transferred to another body.

(iii) Charge is always associated with mass

Charge cannot exist without mass though mass can exist without charge.

(a) So the presence of charge itself is a convincing proof of existence of mass.

(b) In charging, the mass of a body changes.

(c) When body is given positive charge, its mass decreases.

(d) When body is given negative charge, its mass increases.

(iv) Charge is quantised

The quantization of electric charge is the property by virtue of which all free charges are integral multiple of a basic unit of charge represented by e. Thus charge q of a body is always given by

$q = ne$ $\quad\quad$ n = positive integer or negative integer

The quantum of charge is the charge that an electron or proton carries.

Note : Charge on a proton = (–) charge on an electron = 1.6×10^{-19} C

(v) Charge is conserved

In an isolated system, total charge does not change with time, though individual charge may change i.e. charge can neither be created nor destroyed. Conservation of charge is also found to hold good in all types of reactions either chemical (atomic) or nuclear. No exceptions to the rule have ever been found.

(vi) Charge is invariant

Charge is independent of frame of reference. i.e. charge on a body does not change whatever be its speed.

(vii) Accelerated charge radiates energy

v = 0 (i.e. at rest)	v = constant	v ≠ constant (i.e. time varying)
⊕ Q produces only $\vec{E}$ (electric field)	⊕→ Q produces both $\vec{E}$ and $\vec{B}$ (magnetic field) but no radiation	⊕→ Q produces $\vec{E}$, $\vec{B}$ and radiates energy

(viii) Attraction – Repulsion

Similar charges repel each other while dissimilar attract

Methods of Charging

(i) Friction

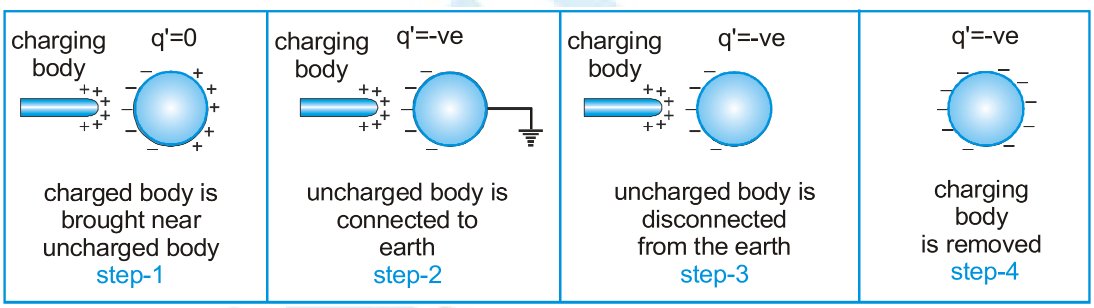
If we rub one body with other body, electrons are transferred from one body to the other.
Transfer of electrons takes places from lower work function body to higher work function body.

Positive charge	Negative charge
Glass rod	Silk cloth
Woollen cloth	Rubber shoes, Amber, Plastic objects
Dry hair	Comb
Flannel or cat skin	Ebonite rod
Note : Clouds become charged by friction	

(ii) Electrostatic induction

If a charged body is brought near a metallic neutral body, the charged body will attract opposite charge and repel similar charge present in the neutral body. As a result of this one side of the neutral body becomes negative while the other positive, this process is called "Electrostatic Induction".

Charging a body by induction (in four successive steps)



Some important facts associated with induction-

(i) Inducing body neither gains nor loses charge

(ii) The nature of induced charge is always opposite to that of inducing charge

(iii) Induction takes place only in bodies (either conducting or non conducting) and not in particles.

(iii) Conduction

The process of transfer of charge by contact of two bodies is known as conduction. If a charged body is put in contact with uncharged body, the uncharged body becomes charged due to transfer of electrons from one body to the other.

(a) The charged body loses some of its charge (which is equal to the charge gained by the uncharged body)

(b) The charge gained by the uncharged body is always lesser than initial charge present on the charged body.

(c) Flow of charge depends upon the potential difference of both bodies. [No potential difference ⇒ No conduction]. Positive charge flows from higher potential to lower potential, while negative charge flows from lower to higher potential.

(iv) Charge differs from mass in the following sense.

(a) In SI units, charge is a derived physical quantity while mass is fundamental quantity.

(b) Charge is always conserved but mass is not (Note : Mass can be converted into energy $E=mc^2$

(c) The quanta of charge is electronic charge while that of mass it is yet not clear.

(d) For a moving charged body mass increases while charge remains constant.

(v) True test of electrification is repulsion and not attraction as attraction may also take place between a charged and an uncharged body and also between two similarly charged bodies.

(vi) For a non relativistic (i.e. $v << c$) charged particle, specific charge $\frac{q}{m}$ =constant

(vii) For a relativistic charged particle $\frac{q}{m}$ decreases as v increases, where v is speed of charged body.

Ex. When a piece of polythene is rubbed with wool, a charge of -2×10^{-7} C is developed on polythene. What is the amount of mass, which is transferred to polythene.

Sol. From Q = ne, So, the number of electrons transferred $n = \frac{Q}{e} = \frac{2 \times 10^{-7}}{1.6 \times 10^{-19}} = 1.25 \times 10^{12}$

Now mass of transferred electrons = n × mass of one electron = $1.25 \times 10^{12} \times 9.1 \times 10^{-31} = 11.38 \times 10^{-19}$ kg

Ex. 10^{12} α – particles (Nuclei of helium) per second falls on a neutral sphere, calculate time in which sphere gets charged by 2μC.

Sol. Number of α – particles falling in t second = $10^{12}t$

Charge on α – particle = +2e , So charge incident in time t = $(10^{12}t).(2e)$

✈ Given charge is 2 μC $\therefore$ $2 \times 10^{-6} = (10^{12}t).(2e)$ $\Rightarrow$ $t = \frac{10^{-18}}{1.6 \times 10^{-19}} = 6.25\,s$

Coulomb's Law

The electrostatic force of interaction between two static point electric charges is directly proportional to the product of the charges, inversely proportional to the square of the distance between them and acts along the straight line joining the two charges.

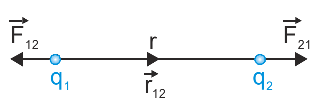
If two points charges q_1 and q_2 separated by a distance r. Let F be the electrostatic force between these two charges. According to Coulomb's law.

$F \propto q_1 q_2$ and $F \propto \frac{1}{r^2}$

$F_e = \frac{kq_1 q_2}{r^2}$ where $\left[k = \frac{1}{4\pi \epsilon_0} = 9 \times 10^9 \frac{Nm^2}{C^2}\right]$ k = coulomb's constant or electrostatic force constant

Coulomb's law in vector form

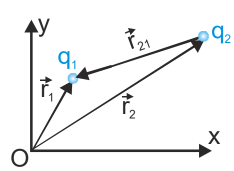
$\vec{F}_{12}$ = force on q_1 due to $q_2 = \frac{kq_1 q_2}{r^2}\hat{r}_{21}$



$\vec{F}_{21} = \frac{kq_1 q_2}{r^2}\hat{r}_{12}$ (here $\hat{r}_{12}$ is unit vector from q_1 to q_2)

Coulomb's law in terms of position vector

$$\vec{F}_{12} = \frac{kq_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3}(\vec{r}_1 - \vec{r}_2)$$



Principle of superposition

The force is a two body interaction, i.e., electrical force between two point charges is independent of presence or absence of other charges and so the principle of superposition is valid, i.e., force on a charged particle due to number of point charges is the resultant of forces due to individual point charges, i.e., $\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + ...$

Note : Nuclear force is many body interaction, so principle of superposition is not valid in case of nuclear force.

When a number of charges are interacting, the total force on a given charge is vector sum of the forces exerted on it by all other charges individually

$$F = \frac{kq_0q_1}{r_1^2} + \frac{kq_0q_2}{r_2^2} + \cdots + \frac{kq_0q_i}{r_i^2} + \ldots \frac{kq_0q_n}{r_n^2} \text{ in vector form } \vec{F} = kq_0 \sum_{i=1}^{n} \frac{q_i}{r_i^2}\hat{r}_i$$

Some important points regarding Coulomb's law and electric force

(i) The law is based on physical observations and is not logically derivable from any other concept. Experiments till today reveal its universal nature.

The law is analogous to Newton's law of gravitation : $F = G\,\frac{m_1m_2}{r^2}$ with the difference that :

(a) Electric force between charged particles is much stronger than gravitational force, i.e., $F_E >> F_G$. This is why when both F_E and F_G are present, we neglect F_G.

(b) Electric force can be attractive or repulsive while gravitational force is always attractive.

(c) Electric force depends on the nature of medium between the charges while gravitational force does not.

(d) The force is an action–reaction pair, i.e., the force which one charge exerts on the other is equal and opposite to the force which the other charge exerts on the first.

(ii) The force is conservative, i.e., work done in moving a point charge once round a closed path under the action of Coulomb's force is zero.

(iii) The net Coulomb's force on two charged particles in free space and in a medium filled upto infinity are

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2} \text{ and } F' = \frac{1}{4\pi\varepsilon}\frac{q_1q_2}{r^2}. \text{ So } \frac{F}{F'} = \frac{\varepsilon}{\varepsilon_0} = K,$$

(iv) Dielectric constant (K) of a medium is numerically equal to the ratio of the force on two point charges in free space to that in the medium filled upto infinity.

(v) The law expresses the force between two point charges at rest. In applying it to the case of extended bodies of finite size care should be taken in assuming the whole charge of a body to be concentrated at its 'centre' as this is true only for spherical charged body, that too for external point.

Although net electric force on both particles change in the presence of dielectric but force due to one charge particle on another charge particle does not depend on the medium between them.

(vi) Electric force between two charges does not depend on neighbouring charges.

Ex. If the distance between two equal point charges is doubled and their individual charges are also doubled, what would happen to the force between them?

Sol. $F = \frac{1}{4\pi\varepsilon_0}\frac{q\times q}{r^2}$ ....**(i)** Again, $F' = \frac{1}{4\pi\varepsilon_0}\frac{(2q)(2q)}{(2r)^2}$ or $F' = \frac{1}{4\pi\varepsilon_0}\frac{4q^2}{4r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} = F$

So, the force will remain the same.

Ex. A particle of mass m carrying charge '$+q_1$' is revolving around a fixed charge '$-q_2$' in a circular path of radius r. Calculate the period of revolution.

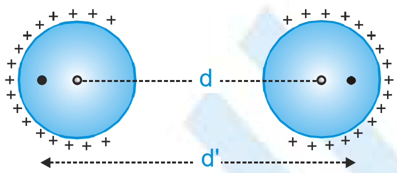
Sol. $$\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2} = mr\omega^2 = \frac{4\pi^2 mr}{T^2}$$

$$T^2 = \frac{(4\pi\varepsilon_0)r^2(4\pi^2 mr)}{q_1q_2} \quad \text{or} \quad T = 4\pi r\sqrt{\frac{\pi\varepsilon_0 mr}{q_1q_2}}$$

where $\vec{r}$ is the vector drawn from source charge is test charge.

Ex. The force of repulsion between two point charges is F, when these are at a distance of 1 m. Now the point charges are replaced by spheres of radii 25 cm having the charge same as that of point charges. The distance between their centres is 1 m, then compare the force of repulsion in two cases.

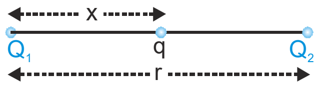
Sol. In 2nd case due to mutual repulsion, the effective distance between their centre of charges will be increased (d' > d) so force of repulsion decreases as $F \propto \frac{1}{d^2}$



Equilibrium of charged particles

In equilibrium net electric force on every charged particle is zero. The equilibrium of a charged particle, under the action of Colombian forces alone can never be stable.

(1) Equilibrium of three point charges



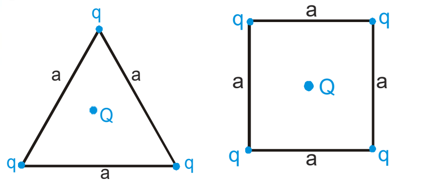
(a) Two charges must be of like nature as $F_q = \frac{KQ_1q}{x^2} + \frac{KQ_2q}{(r-x)^2} = 0$

(b) Third charge should be of unlike nature as $F_{Q_1} = \frac{KQ_1Q_2}{r^2} + \frac{KQ_1q}{x^2} = 0$

Therefore $x = \frac{\sqrt{Q_1}}{\sqrt{Q_1}+\sqrt{Q_2}}r$ and $q = \frac{-Q_1Q_2}{(\sqrt{Q_1}+\sqrt{Q_2})^2}$

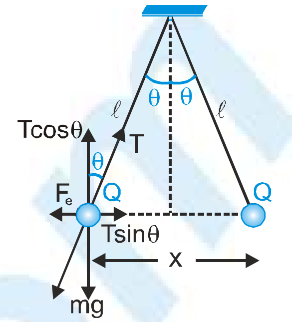
(2) Equilibrium of symmetric geometrical point charged system

Value of Q at centre for which system to be in state of equilibrium



(i) For equilateral triangle $Q = \frac{-q}{\sqrt{3}}$ **(ii)** For square $Q = \frac{-q(2\sqrt{2}+1)}{4}$

(3) Equilibrium of Suspended Point Charge System



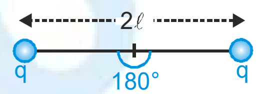
For equilibrium position

$T\cos\theta = mg$ and $T\sin\theta = F_e = \dfrac{kQ^2}{x^2} \Rightarrow \tan\theta = \dfrac{F_e}{mg} = \dfrac{kQ^2}{x^2 mg}$

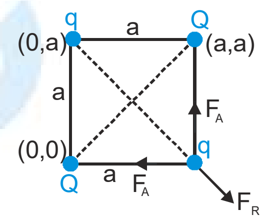
(a) If θ is small then tan

$$\theta \approx \sin\theta = \frac{x}{2\lambda} \Rightarrow \frac{x}{2l} = \frac{kQ^2}{x^2 mg} \Rightarrow x^3 = \frac{2kQ^2\lambda}{mg} \Rightarrow x = \left[\frac{Q^2 l}{2\pi \epsilon_0 \, mg}\right]^{\frac{1}{3}}$$

(b) If whole set up is taken into an artificial satellite ($g_{eff} \simeq 0$) then $T = F_e = \dfrac{kq^2}{4l^2}$



Ex. For the system shown in figure find Q for which resultant force on q is zero.

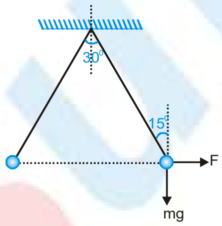


Sol. For force on q to be zero, charges q and Q must be of opposite of nature.

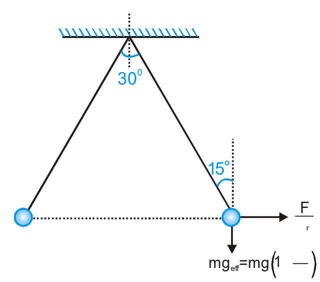
Net attraction force on q due to charges Q = Repulsion force on q due to q

$$\sqrt{2}\,F_A = F_R \Rightarrow \sqrt{2}\,\frac{kQq}{a^2} = \frac{kq^2}{(\sqrt{2}a)^2} \Rightarrow q = 2\sqrt{2}\,Q \text{ Hence } q = -2\sqrt{2}\,Q$$

Ex. Two identically charged spheres are suspended by strings of equal length. The strings make an angle of 30^0 with each other. When suspended in a liquid of density 0.8 g/cc the angle remains same. What is the dielectric constant of liquid. Density of sphere = 1.6 g/cc.



Sol. When set up shown in figure is in air, we have $\tan 15^0 = \dfrac{F}{mg}$ When set up is immersed in the medium as shown in figure, the electric force experienced by the ball will reduce and will be equal to $\dfrac{F}{\varepsilon_r}$ and the effective gravitational force will

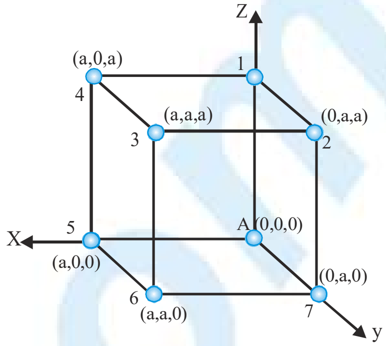


become $mg\left(1 - \dfrac{\rho_l}{\rho_s}\right)$ Thus we have $\tan 15^0 = \dfrac{F}{mg \in_r \left(1 - \dfrac{\rho_l}{\rho_s}\right)} = \dfrac{F}{mg} \Rightarrow \in_r = \dfrac{1}{1 - \dfrac{\rho_l}{\rho_s}} = 2$

Ex. Given a cube with point charges q on each of its vertices. Calculate the force exerted on any of the charges due to rest of the 7 charges.

Sol. The net force on particle A can be given by vector sum of force experienced by this particle due to all the other charges on vertices of the cube. For this we use vector form of coulomb's law $\vec{F} = \dfrac{kq_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3}(\vec{r}_1 - \vec{r}_2)$

From the figure the different forces acting on A are given as $\vec{F}_{A_1} = \dfrac{kq^2(-a\hat{k})}{a^3}$



$$\vec{F}_{A_2} = \frac{kq^2(-a\hat{j}-a\hat{k})}{(\sqrt{2}a)^3},\ \vec{F}_{A_3} = \frac{kq^2(-a\hat{i}-a\hat{j}-a\hat{k})}{(\sqrt{3}a)^3};\ \vec{F}_{A_4} = \frac{kq^2(-a\hat{i}-a\hat{k})}{(\sqrt{2}a)^3}$$

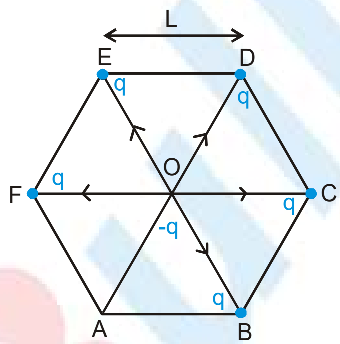
$$\vec{F}_{A_5} = \frac{kq^2(-a\hat{i})}{a^3},\ \vec{F}_{A_6} = \frac{kq^2(-a\hat{i}-a\hat{j})}{(\sqrt{2}a)^3},\ \vec{F}_{A_7} = \frac{kq^2(-a\hat{j})}{a^3}$$

The net force experienced by A can be given as

$$\vec{F}_{net} = \vec{F}_{A_1} + \vec{F}_{A_2} + \vec{F}_{A_3} + \vec{F}_{A_4} + \vec{F}_{A_5} + \vec{F}_{A_6} + \vec{F}_{A_7} = \frac{-kq^2}{a^2}\left[\left(\frac{1}{3\sqrt{3}} + \frac{1}{\sqrt{2}} + 1\right)(\hat{i}+\hat{j}+\hat{k})\right]$$

Ex. Five point charges, each of value +q are placed on five vertices of a regular hexagon of side Lm. What is the magnitude of the force on a point charge of value –q coulomb placed at the centre of the hexagon?

Sol. If there had been a sixth charge +q at the remaining vertex of hexagon force due to all the six charges on –q at O will be zero (as the forces due to individual charges will balance each other).



Now if $\vec{f}$ is the force due to sixth charge and $\vec{F}$ due to remaining five charges.

$$\vec{F} + \vec{f} = 0 \Rightarrow \vec{F} = -\vec{f} \Rightarrow F = f = \frac{1}{4\pi\varepsilon_0}\frac{q \times q}{L^2} = \frac{q^2}{4\pi\varepsilon_0 L^2}$$

Electric Field

Electric field is the region around charged particle or charged body in which if another charge is placed, it experiences electrostatic force.

Electric field intensity $\vec{E}$: Electric field intensity at a point is equal to the electrostatic force experienced by a unit positive point charge both in magnitude and direction.

If a test charge q_0 is placed at a point in an electric field and experiences a force $\vec{F}$ due to some charges (called source charges), the electric field intensity at that point due to source charges is given by $\vec{E} = \dfrac{\vec{F}}{q_0}$

If the $\vec{E}$ is to be determined practically then the test charge q_0 should be small otherwise it will affect the charge distribution on the source which is producing the electric field and hence modify the quantity which is measured.

Ex. A positively charged ball hangs from a long silk thread. We wish to measure E at a point P in the same horizontal plane as that of the hanging charge. To do so, we put a positive test charge q_0 at the point and measure F/q_0. Will F/q_0 be less than, equal to, or greater than E at the point in question?

Sol. When we try to measure the electric field at point P then after placing the test charge at P, it repels the source charge (suspended charge) and the measured

Properties of electric field intensity $\vec{E}$:

(i) It is a vector quantity. Its direction is the same as the force experienced by positive charge.

(ii) Direction of electric field due to positive charge is always away from it while due to negative charge, always towards it.

(iii) Its S.I. unit is Newton/Coulomb.

(iv) Its dimensional formula is $[MLT^{-3}A^{-1}]$

(v) Electric force on a charge q placed in a region of electric field at a point where the electric field intensity is $\vec{E}$ is given by $\vec{F} = q\vec{E}$.

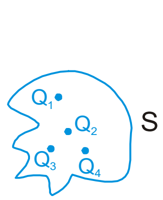
Electric force on point charge is in the same direction of electric field on positive charge and in opposite direction on a negative charge.

(vi) It obeys the superposition principle, that is, the field intensity at a point due to a system of charges is vector sum of the field intensities due to individual point charges.

i.e. $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + ...$

(vii) It is produced by source charges. The electric field will be a fixed value at a point unless we change the distribution of source charges.

Ex. Electrostatic force experienced by $-3\mu C$ charge placed at point 'P' due to a system 'S' of fixed point charges as shown in figure is $\vec{F} = (21\hat{i} + 9\hat{j})\ \mu N$.



(i) Find out electric field intensity at point P due to S.

(ii) If now, $2\mu C$ charge is placed and $-3\ \mu C$ is removed at point P then force experienced by it will be.

Sol. **(i)** $\vec{F} = q\vec{E} \quad \Rightarrow \quad (21\hat{i} + 9\hat{j})\mu N = -3\mu C\,(\vec{E}) \quad \Rightarrow \quad \vec{E} = -7\hat{i} - 3\hat{j}\ \frac{N}{C}$

(ii) Since the source charges are not disturbed the electric field intensity at 'P' will remain same.

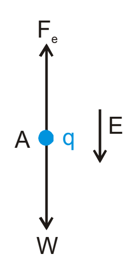
$\vec{F}_{2\mu C} = +2(\vec{E}) = 2(-7\hat{i} - 3\hat{j}) = (-14\hat{i} - 6\hat{j})\ \mu N$

Ex. Calculate the electric field intensity which would be just sufficient to balance the weight of a particle of charge $-10\ \mu c$ and mass 10 mg. (take $g = 10\ ms^2$)

Sol. As force on a charge q in an electric field $\vec{E}$ is

$\vec{F}_q = q\vec{E}$

So, according to given problem:

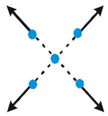
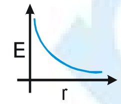
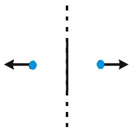
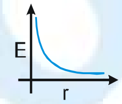
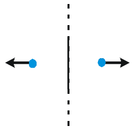
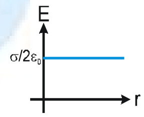
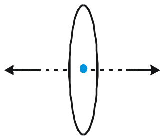
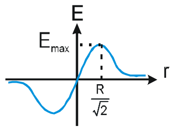
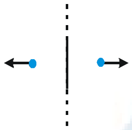
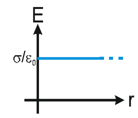
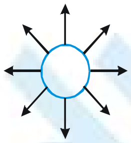
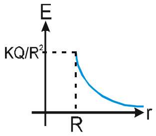
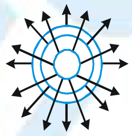
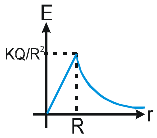


[W = weight of particle]

$|\vec{F}_q| = |\vec{W}|$ i.e., $|q|E = mg$

i.e., $E = \frac{mg}{|q|} = 10$ N/C., in downward direction.

List of formula for Electric Field Intensity due to various types of charge distribution :

Name / Type	Formula	Note	Graph
Point charge 	$\vec{E} = \frac{Kq}{\lvert \vec{r} \rvert^2} \cdot \hat{r}$	* q is source charge. * $\vec{r}$ is vector drawn from source charge to the test point. outwards due to +charges & inwards due to –charges.	
Infinitely long line charge 	$\frac{\lambda}{2\pi\varepsilon_0 r}\hat{r} = \frac{2K\lambda\hat{r}}{r}$	* q is linear charge density (assumed uniform) * r is perpendicular distance of point from line charge. * r is radial unit vector drawn from the charge to test point.	
Infinite non-conducting thin sheet 	$\frac{\sigma}{2\varepsilon_0}\hat{n}$	* is surface charge density. (assumed uniform) * n is unit normal vector. * x = distance of point on the axis from centre of the ring. * electric field is always along the axis.	
Uniformly charged ring 	$E = \frac{KQx}{\left(R^2 + x^2\right)^{3/2}}$ $E_{centre} = 0$	* Q is total charge of the ring * x = distance of point on the axis from centre of the ring. * electric field is always along the axis.	
Infinitely large charged conducting sheet 	$\frac{\sigma}{\varepsilon_0}\hat{n}$	* is the surface charge. density (assumed uniform) * $\hat{n}$ is the unit vector perpendicular to the surface.	
Uniformly charged hollow conducting/ nonconducting /solid conducting sphere 	(i) for r R $\vec{E} = \frac{kQ}{\lvert \vec{r} \rvert^2}\hat{r}$ (ii) for r < R E = 0	* R is radius of the sphere. * $\vec{r}$ is vector drawn from centre of sphere to the point. * Sphere acts like a point charge. placed at centre for points outside the sphere. * $\vec{E}$ is always along radial direction. * Q is total charge (= 4 R^2). (= surface charge density)	
Uniformly charged solid nonconducting sphere (insulating material) 	(i) for r R $\vec{E} = \frac{kQ}{\lvert \vec{r} \rvert^2}\hat{r}$ (ii) for r R $\vec{E} = \frac{kQ}{R^3}\vec{r}$	* $\vec{r}$ is vector drawn from centre of sphere to the point * Sphere acts like a point charge placed at the centre for points outside the sphere $\vec{E}$ is always along radial dir^n * Q is total charge ($\rho . \frac{4}{3}\pi R^3$). (= volume charge density) * Inside the sphere E r. * Outside the sphere E 1/r^2.	

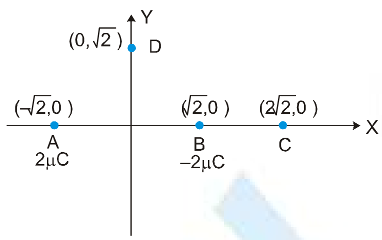
Ex. Find out electric field intensity at point A (0, 1m, 2m) due to a point charge –20μC situated at point B($\sqrt{2}$ m, 0, 1m).

Sol. $E = \frac{KQ\,\vec{r}}{|\vec{r}|^3} = \frac{KQ}{|\vec{r}|^2}\hat{r}$ $\Rightarrow$ $\vec{r}$ = P.V. of A – P.V. of B (P.V. = Position vector)

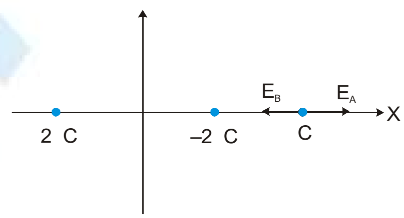
$= (-\sqrt{2}\ \hat{i} + \hat{j} + \hat{k})$ $\quad |\vec{r}| = \sqrt{(\sqrt{2})^2 + (1)^2 + (1)^2} = 2$

$$E = \frac{9\times10^9 \times (-20\times10^{-6})}{8}(-\sqrt{2}\,\hat{i} + \hat{j} + \hat{k}) = -22.5\times10^3(-\sqrt{2}\ \hat{i} + \hat{j} + \hat{k})\text{ N/C.}$$

Ex. Two point charges 2μc and – 2μc are placed at points A and B as shown in figure. Find out electric field intensity at points C and D. [All the distances are measured in meter].



Sol. Electric field at point C :

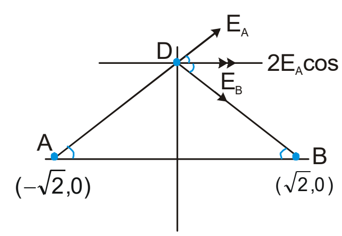


(E_A, E_B are magnitudes only and arrows represent directions)
Electric field due to positive charge is away from it while due to negative charge, it is towards the charge. It is clear that $E_B > E_A$.

$\therefore$ $E_{Net} = (E_B - E_A)$ towards negative X-axis

$$= \frac{K(2\mu c)}{(\sqrt{2})^2} - \frac{K(2\mu c)}{(3\sqrt{2})^2}\text{ towards negative X-axis } = 8000\,(-\hat{i})\text{ N/C}$$

Electric field at point D :



Since magnitude of charges are same and also AD = BD

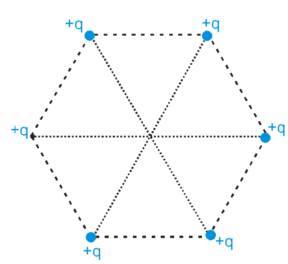
So, $E_A = E_B$

Vertical components of $\vec{E}_A$ and $\vec{E}_B$ cancel each other while horizontal components are in the same direction.

So, $E_{net} = 2E_A\cos\theta = \frac{2.K(2\mu c)}{2^2}\cos45^0$

$$= \frac{K\times10^{-6}}{\sqrt{2}} = \frac{9000}{\sqrt{2}}\hat{i}\text{ N/C.}$$

Ex. Six equal point charges are placed at the corners of a regular hexagon of side 'a'. Calculate electric field intensity at the centre of hexagon ?

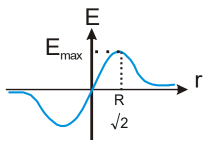


Ans Zero (By symmetry)

The y-component of electric field due to all the elements will be cancelled out to each other. So net electric field intensity at the point will be only due to X-component of each element.

$$E_{net} = \int dE_x = \int dE \cos\theta = \int_0^Q \frac{K(dq)}{R^2+x^2} \times \frac{x}{\sqrt{R^2+X^2}} = \frac{k\,x}{(R^2+x^2)^{3/2}} \int_0^Q dq$$

$$E_{net} = \frac{KQx}{[R^2+x^2]^{3/2}}$$



Graph for variation of E with r.

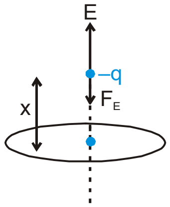
E will be max when $\frac{dE}{dx} = 0$, that is at x= $\frac{R}{\sqrt{2}}$ and $E_{max} = \frac{2KQ}{3\sqrt{3}\,R^2}$

Case (i) : if x>>R, $E = \frac{KQ}{x^2}$ Hence the ring will act like a point charge

Case (ii) : if x<<R, $E = \frac{KQ\,x}{R^3}$,

Ex. Positive charge Q is distributed uniformly over a circular ring of radius a. A point particle having a mass m and a negative charge –q, is placed on its axis at a distance y from the centre. Find the force on the particle. Assuming y << a, find the time period of oscillation of the particle if it is released from there. (Neglect gravity)

Sol. When the negative charge is shifted at a distance x from the centre of the ring along its axis then force acting on the point charge due to the ring:



$F_E = qE$ (towards centre)

$$= q\left[\frac{KQy}{(a^2+y^2)^{3/2}}\right]$$

If a >> y then

$a^2 + y^2 \simeq a^2$

$$\therefore \quad F_E = \frac{1}{4\pi\varepsilon_0}\,\frac{Qqy}{a^3} \quad \text{(Towards centre)}$$

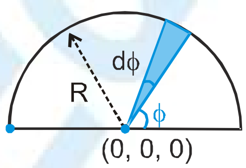
Since, restoring force $F_E \propto y$, therefore motion of charge the particle will be S.H.M.

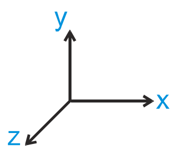
Time period of SHM:

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{\left(\frac{Qq}{4\pi\varepsilon_0 a^3}\right)}} = \left[\frac{16\pi^3\varepsilon_0 m a^3}{Qq}\right]^{1/2}$$

Ex. Calculate electric field intensity at a point on the axis which is at distance x from the centre of half ring, having total charge Q distributed uniformly on it. The radius of half ring is R.

Sol.





Consider an element of small angle $d\phi$ at an angle ϕ as shown.
Coordinates of element : $(R\cos\phi, R\sin\phi, 0)$
Coordinates of point : $(0, 0, x)$
Now electric field due to element :-

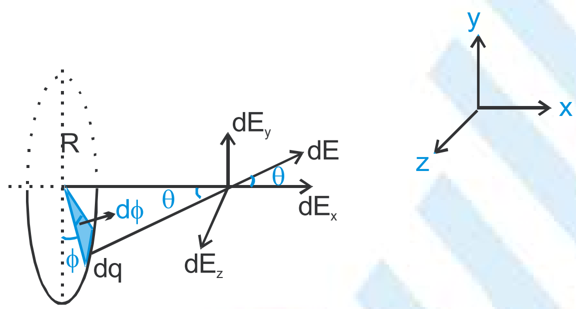
$$\vec{dE} = \frac{K(\lambda R d\varphi)\cdot[-R\cos\varphi\hat{i} - R\sin\varphi\hat{j} + x\hat{k}]}{(R^2\cos^2\varphi + R^2\sin^2\varphi + x^2)^{3/2}} \Rightarrow E_x = \Sigma dE_x = -\int_0^{\pi}\frac{K\lambda R^2\cos\varphi d\varphi}{(R^2+x^2)^{3/2}} = 0$$

$$E_y = \Sigma dE_y = -\int_0^{\pi}\frac{K\lambda R^2\sin\varphi d\varphi}{(R^2+x^2)^{3/2}} = \frac{2K\lambda R^2}{(R^2+x^2)^{3/2}} = \frac{2KQR}{\pi(R^2+x^2)^{3/2}}$$

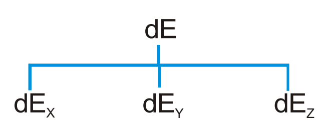
$$E_z = \Sigma dE_z = \int_0^{\pi}\frac{K\lambda R x d\varphi}{(R^2+x^2)^{3/2}} = \frac{KQx}{(R^2+x^2)^{3/2}}$$

$$E_{net} = \sqrt{E_x^2 + E_y^2 + E_z^2} \qquad = \frac{KQ}{(R^2+x^2)^{3/2}}\sqrt{\frac{4R^2}{\pi^2} + x^2}$$

Alternate solution



Consider an element of charge dq at an angle ϕ on circumference of half ring. Due to this element electric field at the point on axis, which is at a distance x from the centre of half ring is dE.
This electric field can be resolved into three component



$$E_Z = \int_{-\pi/2}^{\pi/2} dE\sin\theta\sin\varphi = 0$$

$$E_X = \int_{-\pi/2}^{\pi/2} dE\cos\theta = \frac{KQx}{[R^2+x^2]^{3/2}} \qquad \text{...........(1)}$$

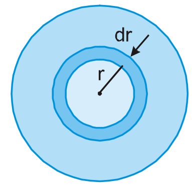
$$E_Y = \int_{-\pi/2}^{\pi/2} dE\sin\theta\cos\varphi = \int\frac{Kdq}{R^2+x^2}\sin\theta.\cos\phi = \frac{2K\lambda R\sin\theta}{R^2+x^2} \qquad \text{...........(2)}$$

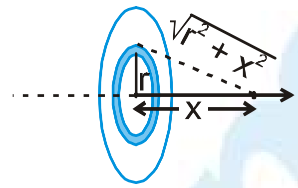
$$\therefore \quad dq = \lambda R d\phi,\ \sin\theta = \frac{R}{\sqrt{R^2+x^2}} \qquad \Rightarrow E_{net} = \sqrt{E_X^2 + E_Y^2}$$

Ex. Derive the expression of electric field intensity at a point 'P' which is situated at a distance x on the axis of uniformly charged disc of radius R and surface charge density σ. Also, derive results for
(i) $x >> R$ **(ii)** $x << R$

Sol. The disc can be considered to be a collection of large number of concentric rings.
Consider an element of the shape of rings of radius r and of width dr. Electric field due to this ring at P is

$$dE = \frac{K.\sigma 2\pi r.dr.x}{(r^2 + x^2)^{3/2}}$$





Put, $r^2 + x^2 = y^2$

$2rdr = 2ydy$

$$\therefore \quad dE = \frac{K.\sigma 2\pi y.dy.x}{y^3} = 2K\sigma\pi.x\,\frac{ydy}{y^3}$$

Electric field at P due to all rings is along the axis :-

$$\therefore \quad E = \int dE \Rightarrow E = 2K\sigma\pi x \int_x^{\sqrt{R^2+x^2}} \frac{1}{y^2}dy = 2K\rho\pi x.\left[-\frac{1}{y}\right]_x^{\sqrt{R^2+x^2}}$$

$$= 2K\sigma\pi x\left[+\frac{1}{x} - \frac{1}{\sqrt{R^2+x^2}}\right] = 2K\sigma\pi\left[1 - \frac{x}{\sqrt{R^2+x^2}}\right]$$

$$= \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{R^2+x^2}}\right], \text{ along the axis}$$

Cases : (i) If $x >> R$

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{x\sqrt{\frac{R^2}{x^2}+1}}\right] = \frac{\sigma}{2\varepsilon_0}\left[1 - \left(1+\frac{R^2}{x^2}\right)^{-1/2}\right]$$

$$= \frac{\sigma}{2\varepsilon_0}\left[1 - 1 + \frac{1}{2}\frac{R^2}{x^2} + \text{higher order terms}\right]$$

$$= \frac{\sigma}{4\varepsilon_0}\frac{R^2}{x^2} = \frac{\sigma\pi R^2}{4\pi\varepsilon_0 x^2} = \frac{Q}{4\pi\varepsilon_0 x^2}$$

i.e. behaviour of the disc is like a point charge.

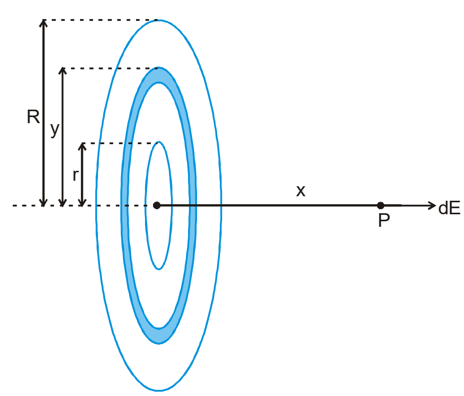
(ii) If $x << R$

$$E = \frac{\sigma}{2\varepsilon_0}[1 - 0] = \frac{\sigma}{2\varepsilon_0}$$

i.e. behaviour of the disc is like infinite sheet.

Ex. Calculate electric field at a point on axis, which at a distance x from centre of uniformly charged disc having surface charge density σ and R which also contains a concentric hole of radius r.

Sol. Consider a ring of radius y (r < y < R) and width dy concentric with disc and in the plane of the disc. Due to this ring, the electric field at the point P :



$$dE = \frac{K(dq)x}{[X^2 + Y^2]^{3/2}}$$

$$E_{net} = \int_r^R \frac{K\,x.\sigma(2\pi y)dy}{[x^2 + y^2]^{3/2}} \qquad [\because dq = \sigma 2\pi ydy]$$

$$E_{net} = \frac{2\pi\sigma kx}{2}\int_{x^2+r^2}^{x^2+R^2}\frac{dt}{t^{3/2}}, \quad \text{put } x^2 + y^2 = t,\ 2y.\,dy = dt$$

$$= \frac{\sigma x}{2\varepsilon_0}\left[\frac{1}{\sqrt{x^2+r^2}}-\frac{1}{\sqrt{x^2+R^2}}\right] \quad \text{away from centre}$$

Alternate method

We can also use superposition principle to solve this problem.

(i) Assume a disc without hole of radius R having surface charge density + σ .

(ii) Also assume a concentric disc of radius r in the same plane of first disc having charge density – σ .

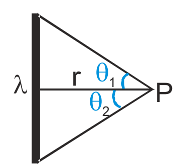
Now using derived formula in last example the net electric field at the centre is :

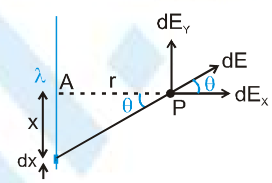
$$\vec{E}_{net} = \vec{E}_R + \vec{E}_r$$

$$= \frac{\sigma x}{2\varepsilon_0}\left[\frac{1}{\sqrt{r^2+x^2}}-\frac{1}{\sqrt{R^2+x^2}}\right] \text{ away from centre.}$$

Electric field due to uniformly charged wire

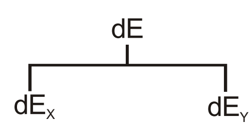
(i) **Line charge of finite length :** Derivation of expression for intensity of electric field at a point due to line charge of finite size of uniform linear charge density λ. The perpendicular distance of the point from the line charge is r and lines joining ends of line charge distribution make angle θ_1 and θ_2 with the perpendicular line.





Consider a small element dx on line charge distribution at distance x from point A (see fig.). The charge of this element will be dq = λdx. Due to this charge (dq), the intensity of electric field at the point P is dE.

Then $dE = \frac{K(dq)}{r^2+x^2} = \frac{K(\lambda dx)}{r^2+x^2}$



There will be two components of this field :

$$E_x = \int dE_x = \int dE\cos\theta = \int \frac{K\lambda dx}{r^2+x^2}.\cos\theta$$

Assuming, $x = r\tan\theta \Rightarrow dx = r\sec^2\theta.\, d\theta$

So $$E_x = \int_{-\theta_2}^{+\theta_1} \frac{K\lambda r\sec^2\theta\cdot\cos\theta\cdot d\theta}{r^2+r^2\tan^2\theta}$$

$$= \frac{K\lambda}{r}\int_{-\theta_2}^{+\theta_1}\cos\theta.d\theta = \frac{K\lambda}{r}[\sin\theta_1+\sin\theta_2] \qquad \text{...........(1)}$$

Similarly y-component.

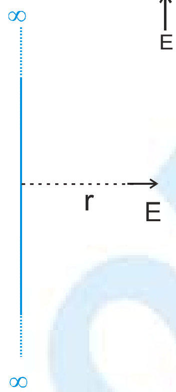
$$E_y = \frac{K\lambda}{r}\int_{-\theta_2}^{+\theta_1}\sin\theta.d\theta = \frac{K\lambda}{r}[\cos\theta_2 - \cos\theta_1]$$

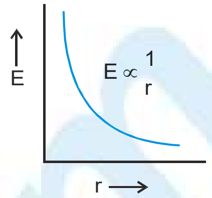
Net electric field at the point:

$$E_{net} = \sqrt{E_x^2+E_y^2}$$

(ii) **We can derive a result for infinitely long line charge:**

In above eq. **(1)** & **(2)**, if we put $\theta_1 = \theta_2 = 90°$, we can get required result.



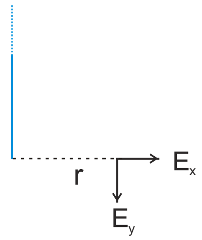


$$E_{net} = E_x = \frac{2K\lambda}{r}$$

(iii) **For Semi- infinite wire :**

$\theta_1 = 90°$ and $\theta_2 = 0°$, so,

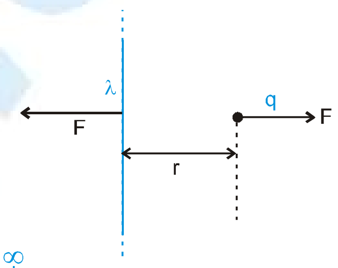
$$E_x = \frac{K\lambda}{r}, \; E_y = \frac{K\lambda}{r}$$



Ex. A point charge q is placed at a distance r from a very long charged thread of uniform linear charge density λ. Find out total electric force experienced by the line charge due to the point charge. (Neglect gravity).

Sol. Force on charge q due to the thread,

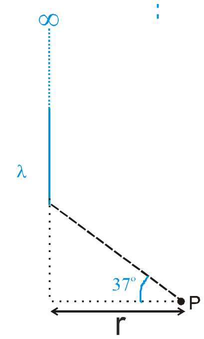
$$F = \left(\frac{2K\lambda}{r}\right).q$$



By Newton's III law, every action has equal and opposite reaction So, force on the thread $= \frac{2K\lambda}{r}.q$

(away from point charge)

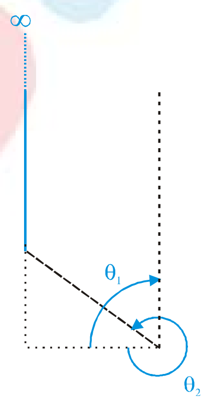
Ex. Figure shows a long wire having uniform charge density λ as shown in figure. Calculate electric field intensity at point P.



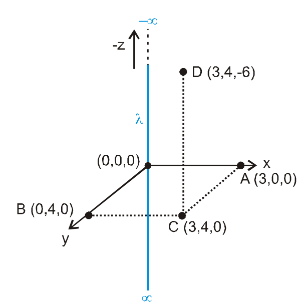
Sol. $\theta_1 = 90°$ and $\theta_2 = 360° - 37°$ So

$$E_x = \frac{K\lambda}{r}[\sin\theta_1 + \sin\theta_2]$$

$$E_y = \frac{K\lambda}{r}[\cos\theta_2 - \cos\theta_1]$$



Ex. Find electric field at point A, B, C, D due to infinitely long uniformly charged wire with linear charge density λ and kept along z-axis (as shown in figure).Assume that all the parameters are in S.I. units.



Sol. $E_A = \frac{2K\lambda}{3}(\hat{i}) \Rightarrow E_B = \frac{2K\lambda}{4}(\hat{j})$

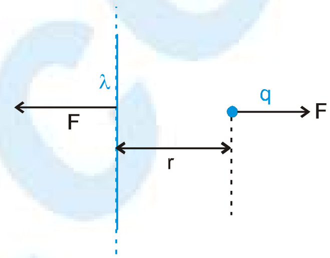
$$E_C = \frac{2K\lambda}{5}\hat{OC} = \frac{2K\lambda}{5}\left(\frac{3\hat{i}+4\hat{j}}{5}\right)$$

$$E_D = \frac{2K\lambda}{5}\left(\frac{3\hat{i}+4\hat{j}}{5}\right) \Rightarrow E_D = E_C$$

Ex. A point charge q is placed at a distance r from a very long charged thread of uniform linear charge density λ. Find out total electric force experienced by the line charge due to the point charge. (Neglect gravity).

Sol. Force on charge q due to the thread,

$$F = \left(\frac{2K\lambda}{r}\right).q$$



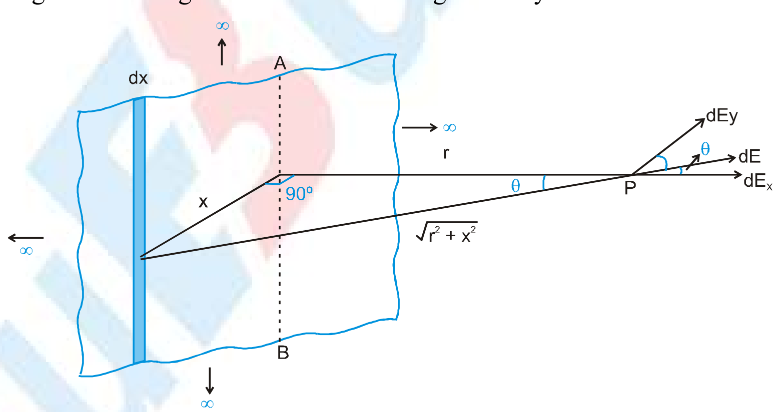
By Newton's III law, every action has equal and opposite reaction, so force on the thread = $\frac{2K\lambda}{r}.q$ (away from point charge)

Electric field due to uniformly charged infinite sheet

$$E_{net} = \frac{\sigma}{2\varepsilon_o} \text{ towards normal direction}$$

Electric field due to An infinitely large, uniformly charged sheet

Derivation of expression for intensity of electric field at a point which is at a perpendicular distance r from the thin sheet of large size having uniform surface charge density σ.



Assume a thin strip of width dx at distance x from line AB (see figure), which can be considered as a infinite line charge of charge density λ = σdx

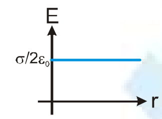
Due to this line charge the electric field intensity at point P will be

$$dE = \frac{\sigma K(dx)}{\sqrt{r^2 + x^2}}$$

Take another element similar to the first element on the other side of AB. Due to symmetry, Y-component of all such elements will be cancelled out.

So net electric field will be given by :

$$E_{net} = \int dE_x = \int dE\cos\theta = \int \frac{2K(\sigma dx)}{\sqrt{r^2 + x^2}} \times \cos\theta$$



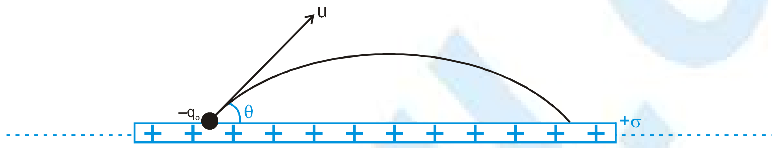
Assume, $x = r\tan\theta \Rightarrow dx = r\sec^2\theta.\, d\theta$

$$\therefore\ E_{net} = 2K\sigma \int_{-\pi/2}^{+\pi/2} \frac{r\sec^2\theta.d\theta.\cos\theta}{\sqrt{r^2 + r^2\tan^2\theta}} = \frac{\sigma}{2\varepsilon_o}$$, away from sheet

Note: **(1)** The direction of electric field is always perpendicular to the sheet.

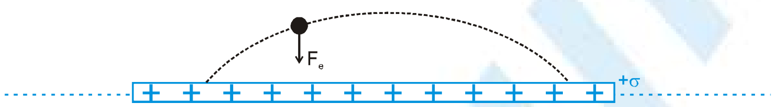
(2) The magnitude of electric field is independent of distance from sheet.

Ex. An infinitely large plate of surface charge density $+\sigma$ is lying in horizontal xy plane. A particle having charge $-q_o$ and mass m is projected from the plate with velocity u making an angle θ with sheet. Find :



(i) The time taken by the particle to return on the plate..
(ii) Maximum height achieved by the particle.
(iii) At what distance will it strike the plate (Neglect gravitational force on the particle)

Sol.



Electric force acting on the particle $F_e = q_oE : F_e = (q_o)\left(\frac{\sigma}{2\varepsilon_o}\right)$ downward

So, acceleration of the particle : $a = \frac{F_e}{m} = \frac{q_o\sigma}{2\varepsilon_o m}$ = uniform

This acceleration will act like 'g' (acceleration due to gravity)
So, the particle will perform projectile motion.

(i) $$T = \frac{2u\sin\theta}{g} = \frac{2u\sin\theta}{\left(\frac{q_o\sigma}{2\varepsilon_o m}\right)}.$$

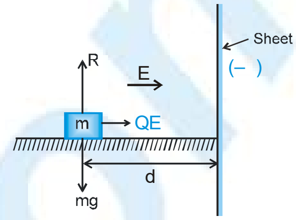
(ii) $$H = \frac{u^2\sin^2\theta}{2g} = \frac{u^2\sin^2\theta}{2\left(\frac{q_o\sigma}{2\varepsilon_o m}\right)}.$$

(iii) $$R = \frac{u^2\sin 2\theta}{g} = \frac{u^2\sin 2\theta}{\left(\frac{q_o\sigma}{2\varepsilon_o m}\right)}.$$

Ex. A block having mass m and charge Q is resting on a frictionless plane at a distance d from fixed large non-conducting infinite sheet of uniform charge density $-\sigma$ as shown in Figure. Assuming that collision of the block with the sheet is perfectly elastic, find the time period of oscillatory motion of the block. Is it SHM ?

Sol. The situation is shown in Figure. Electric force produced by sheet will accelerate the block towards the sheet producing an acceleration. Acceleration will be uniform because electric field E due to the sheet is uniform.

$$a = \frac{F}{m} = \frac{QE}{m}, \text{ where } E = \sigma/2\varepsilon_0$$



As initially the block is at rest and acceleration is constant, from second equation of motion, time taken by the block to reach the wall

$$d = \frac{1}{2}at^2 \qquad \text{i.e.,} \qquad t = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2md}{QE}} = \sqrt{\frac{4md\varepsilon_0}{Q\sigma}}$$

As collision with the wall is perfectly elastic, the block will rebound with same speed and as now its motion is opposite to the acceleration, it will come to rest after travelling same distance d in same time t. After stopping, it will again be accelerated towards the wall and so the block will execute oscillatory motion with 'span' d and time period.

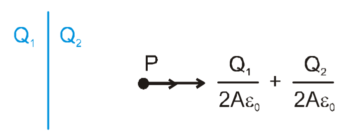
$$T = 2t = 2\sqrt{\frac{2md}{QE}} = 2\sqrt{\frac{4md\varepsilon_0}{Q\sigma}}$$

However, as the restoring force F = QE is constant and not proportional to displacement x, the motion is not simple harmonic.

Ex. If an isolated infinite sheet contains charge Q_1 on its one surface and charge Q_2 on its other surface then prove that electric field intensity at a point in front of sheet will be $\frac{Q}{2A\varepsilon_0}$, where $Q = Q_1 + Q_2$

Sol. Electric field at point P :

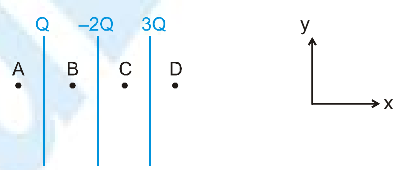
$$\vec{E} = \vec{E}_{Q_1} + \vec{E}_{Q_2}$$



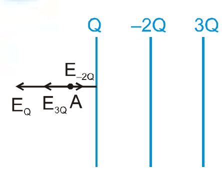
$$= \frac{Q_1}{2A\varepsilon_0}\hat{n} + \frac{Q_2}{2A\varepsilon_0}\hat{n} \qquad = \frac{Q_1 + Q_2}{2A\varepsilon_0}\hat{n} \qquad = \frac{Q}{2A\varepsilon_0}\hat{n}$$

[This shows that the resultant field due to a sheet depends only on the total charge of the sheet and not on the distribution of charge on individual surfaces].

Ex. Three large conducting parallel sheets are placed at a finite distance from each other as shown in figure. Find out electric field intensity at points A, B, C & D.

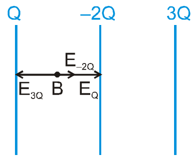


Sol. **For point A**



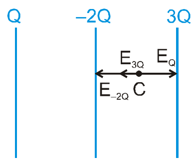
$$\vec{E}_{net} = \vec{E}_Q + \vec{E}_{3Q} + \vec{E}_{-2Q} = -\frac{Q}{2A\varepsilon_0}\hat{i} - \frac{3Q}{2A\varepsilon_0}\hat{i} + \frac{2Q}{2A\varepsilon_0}\hat{i} = -\frac{Q}{A\varepsilon_0}\hat{i}$$

For point B:



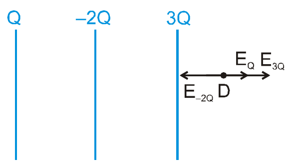
$$\vec{E}_{net} = \vec{E}_{3Q} + \vec{E}_{-2Q} + \vec{E}_{Q} = -\frac{3Q}{2A\varepsilon_0}\hat{i} + \frac{2Q}{2A\varepsilon_0}\hat{i} + \frac{Q}{2A\varepsilon_0}\hat{i} = 0$$

For point C :



$$\vec{E}_{net} = \vec{E}_{Q} + \vec{E}_{3Q} + \vec{E}_{-2Q} = +\frac{Q}{2A\varepsilon_0}\hat{i} - \frac{3Q}{2A\varepsilon_0}\hat{i} - \frac{2Q}{2A\varepsilon_0}\hat{i} = -\frac{2Q}{A\varepsilon_0}\hat{i}$$

For point D :

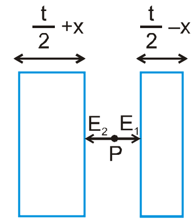


$$\vec{E}_{net} = \vec{E}_{Q} + \vec{E}_{3Q} + \vec{E}_{-2Q} = +\frac{Q}{2A\varepsilon_0}\hat{i} + \frac{3Q}{2A\varepsilon_0}\hat{i} - \frac{2Q}{2A\varepsilon_0}\hat{i} = \frac{Q}{A\varepsilon_0}\hat{i}$$

Ex. Determine and draw the graph of electric field due to infinitely large non-conducting sheet of thickness t and uniform volume charge density ρ as a function of distance x from its symmetry plane.

(a) $x \le \frac{t}{2}$ **(b)** $x \ge \frac{t}{2}$

Sol. We can consider two sheets of thickness $\left(\frac{t}{2} - x\right)$ and $\left(\frac{t}{2} + x\right)$.



Where the point P lies inside the sheet.

Now, net electric field at point P :

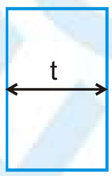
$E = E_1 - E_2$

$$= \frac{Q_1}{2A\varepsilon_0} - \frac{Q_2}{2A\varepsilon_0}$$ [Q_1 : charge of left sheet;

Q_2 : charge of right sheet.]

$$= \frac{A\rho\left(\frac{t}{2} + x\right) - \rho A\left(\frac{t}{2} - x\right)}{2A\varepsilon_0} = \frac{\rho x}{\varepsilon_0}$$

For point which lies outside the sheet we can consider a complete sheet of thickness t



$$\longrightarrow \frac{Q}{2A\varepsilon_0} \qquad E = \frac{\sigma t A}{2A\varepsilon_0} = \frac{\sigma t}{2\varepsilon_0}$$

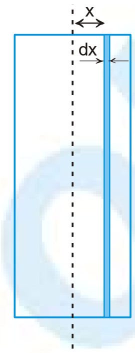
Alternate

We can assume thick sheet to be made of large number of uniformly charged thin sheets. Consider an elementary thin sheet of width dx at a distance x from symmetry plane.

Charge in sheet = $\rho A dx$

(A : assumed area of sheet)

Surface charge density,



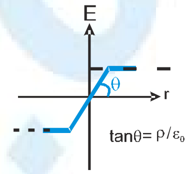
$$s = \frac{\rho A dx}{A}$$

so, electric field intensity due to elementary sheet :

$$dE = \frac{\rho dx}{2\varepsilon_0}$$

(a) When $x < \frac{t}{2}$ $\Rightarrow$ $E_{Net} = \int_{-t/2}^{x} \frac{\rho dx}{2\varepsilon_0} - \int_{x}^{t/2} \frac{\rho dx}{2\varepsilon_0} = \frac{\rho x}{\varepsilon_0}$

(b) When $x > \frac{t}{2}$

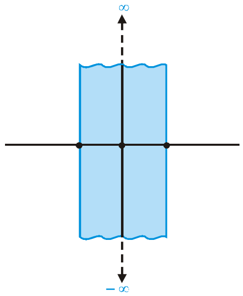


$$E_{Net} = \int_{-t/2}^{t/2} \frac{\rho dx}{2\varepsilon_0} = \frac{\rho t}{2\varepsilon_0}$$

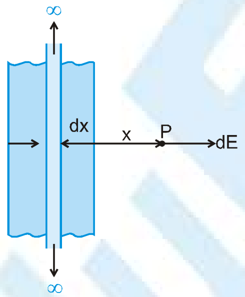
Ex. Thin infinite sheet of width w contains uniform charge distribution σ. Find out electric field intensity at following points :

(a) A point which lies in the same plane at a distance d from one of its edge.

(b) A point which is on the symmetry plane of sheet at a perpendicular distance d from it.



Sol. **(a)**



Consider a thin strip of width dx.

Linear charge density of strip :

$$\lambda = \sigma dx$$

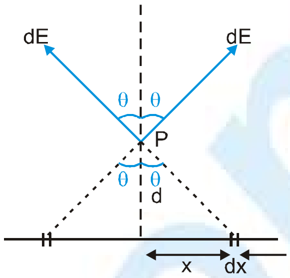
So, electric field due to this strip at point P $dE = \frac{2k\sigma dx}{x}$

$$E_{net} = \int_{d}^{d+w} \frac{\sigma}{2\pi\varepsilon_0} \frac{dx}{x} = \frac{\sigma}{2\pi\varepsilon_0} \ln\left(\frac{d+w}{d}\right)$$

(b) Consider a thin strip of width dx.

Linear charge density of strip :

$$\lambda = \sigma dx$$

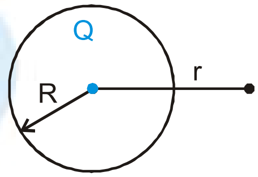


$\therefore$ $E_p = \int 2dE\cos\theta$

or $E_p = 2.\int_0^{w/2} \frac{\sigma dx}{2\pi\varepsilon_0\sqrt{d^2+x^2}} \cdot \frac{d}{\sqrt{d^2+x^2}}$

$$= \frac{\sigma d}{\pi\varepsilon_0}\int_0^{w/2}\frac{dx}{d^2+x^2} = \frac{\sigma}{\pi\varepsilon_0}\tan^{-1}\frac{w}{2d}$$

Electric field due to uniformly charged spherical shell

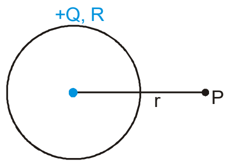


$E = \frac{KQ}{r^2}$ $r \geq R$ $\Rightarrow$ For the outside points & point on the surface the uniformly

$E = 0$ $r < R$ charged spherical shell behaves as a point charge placed at the centre

Electric field due to spherical shell outside it is always along the radial direction.

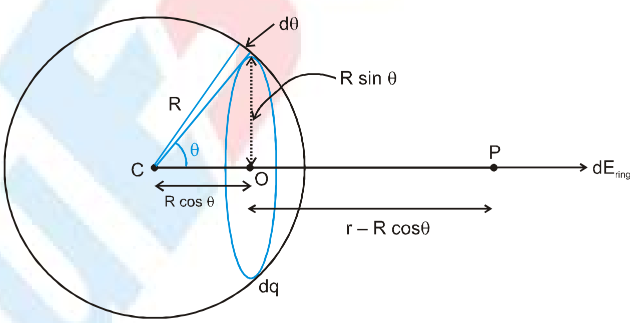
Finding electric field due to a uniformly charged spherical shell

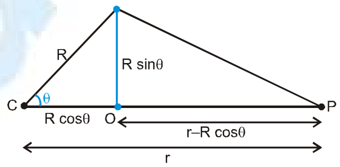


Suppose we have a spherical shell of radius R and charge +Q uniformly distributed on its surface. We have to find electric field at a point P, which is at a distance 'r' from the centre of the sphere.

For this, we can divide the shell into thin rings. Let's consider a ring making an angle θ with the axis and subtending a small angle dθ. Its width will be Rdθ.(arc = radius × angle = Rdθ).

For the points outside the sphere :





Electric field due to this small ring element :

$$dE = \frac{K\,dq\,x}{[(\text{ring radius})^2 + x^2]^{3/2}} \qquad .....(1)$$

So, total electric field $E_{net} = \int \frac{K\,dq\,x}{[(\text{ring radius})^2 + x^2]^{3/2}}$

Here, radius of the ring element = R sinθ

& x = axial distance of point P from the ring = r – R cos θ

Area of the ring element = (length) (width) = (2π (radius of the ring)) Rdθ = (2πR sinθ) Rdθ

dq = charge of the small ring element. We can find dq by unitary method.

In $4\pi R^2$ Area, charge is Q

In unit Area, charge is $\frac{Q}{4\pi R^2}$.

In (2πR sinθ) Rdθ Area, charge = $\frac{Q}{4\pi R^2} \times (2\pi R \sin\theta)\,R d\theta = dq$

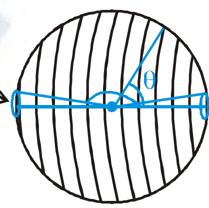
Putting values of r and dq in equation .(1)

We get $$E_{out} = \int_{\theta=0}^{\theta=\pi} \frac{K\left(\frac{Q}{4\pi R^2} \times 2\pi (R\sin\theta) R d\theta\right)(r - R\cos\theta)}{\left[(R\sin\theta)^2 + (r - R\cos\theta)^2\right]^{3/2}}$$

(The first ring will make angle θ = 0 and the last ring will make θ = 180°.So, limit will be from θ = 0 to θ = 180°)

Steps of integration : From above integral :

$$E_{out} = \frac{KQ}{2}\int_0^{\pi} \frac{(r - R\cos\theta)\sin\theta\, d\theta}{(R^2 + r^2 - 2Rr\cos\theta)^{3/2}}$$



Now, let $z^2 = R^2 + r^2 - 2Rr\cos\theta$

⇒ $2\,zdz = 0 + 0 - 2Rr(-\sin\theta)\,d\theta$

∴ $zdz = Rr\sin\theta\, d\theta$

& $\cos\theta = \frac{R^2 + r^2 - z^2}{2Rr}$

Now, when θ = 0 → z = (r – R)

θ = π → z = r + R

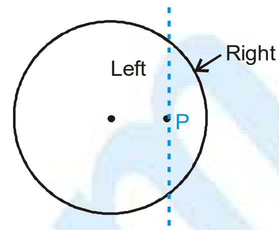
$$\therefore \quad E_{out} = \frac{KQ}{2}\int_{r-R}^{r+R} \frac{\left[r - R\frac{(R^2 + r^2 - z^2)}{2Rr}\right]\frac{zdz}{Rr}}{z^3} = \frac{KQ}{2}\int_{r-R}^{r+R} \frac{(2Rr^2 - R^3 - Rr^2 + Rz^2)\,dz}{2R^2r^2z^2}$$

$$= \frac{KQ}{4R^2r^2}\left[\int_{r-R}^{r+R} \frac{Rr^2dz}{z^2} - \int_{r-R}^{r+R} \frac{R^3dz}{z^2} + \int_{r-R}^{r+R} Rdz\right] = \frac{KQ}{4R^2r^2}\left[-\frac{Rr^2}{z} + \frac{R^3}{z} + Rz\right]_{r-R}^{r+R}$$

On solving above, we will get :

$$E_{out} = \frac{KQ}{r^2} \quad \text{if } r > R$$

For The Points Inside The Sphere :



Now lets derive the electric field due to uniformly charged solid sphere at a point 'P' inside it. The sphere is divided into two parts, the rings on the left part of point 'P' will produce electric field towards right and the rings on right part will produce electric field towards left and $E_{net} = E_{right} - E_{left}$. For this, limit of integration is divided into two parts.

$$E_{net} = \int_{\theta=0}^{\theta=\cos^{-1}\left(\frac{r}{R}\right)} \begin{pmatrix}\text{Electric} \\ \text{field due} \\ \text{to rings} \\ \text{of right} \\ \text{part}\end{pmatrix} - \int_{\theta=\cos^{-1}\left(\frac{r}{R}\right)}^{\theta=\pi} \begin{pmatrix}\text{Electric} \\ \text{field due} \\ \text{to rings} \\ \text{of left} \\ \text{part}\end{pmatrix}$$

As $\quad z^2 = R^2 + r^2 - 2rR\cos\theta$

When $\quad \theta = \cos^{-1}\left(\frac{r}{R}\right) \quad \Rightarrow \quad z = \sqrt{R^2 - r^2}$

When $\quad \theta = 0 \quad \Rightarrow \quad z = R - r$

When $\quad \theta = \pi \quad \Rightarrow \quad z = R + r$

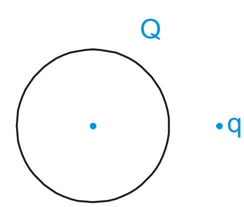
From the result of previous case and just by changing limits we can write

$$E_{in} = \left[-\frac{Rr^2}{z} + \frac{R^3}{z} + Rz\right]_{R-r}^{\sqrt{R^2-r^2}} - \left[-\frac{Rr^2}{z} + \frac{R^3}{z} + Rz\right]_{\sqrt{R^2-r^2}}^{R+r}$$

On solving this expression, we will get E and E = 0 if r < R.

Finding electric field due to shell by integration is very lengthy, so we will not use this method. The given hand-out was just for knowledge. The best method to find E due to shell is by Gauss theorem which we will study later.

Ex. Figure shows a uniformly charged sphere of radius R and total charge Q. A point charge q is situated outside the sphere at a distance r from centre of sphere. Find out the following :



(i) Force acting on the point charge q due to the sphere.
(ii) Force acting on the sphere due to the point charge.

Sol. **(i)** Electric field at the position of point charge

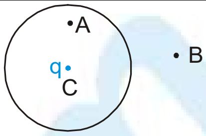
$$\vec{E} = \frac{KQ}{r^2}\hat{r}$$

So, $\quad \vec{F} = \frac{KqQ}{r^2}\hat{r} \quad \Rightarrow \quad |\vec{F}| = \frac{KqQ}{r^2}$

(ii) Since we know that every action has equal and opposite reaction so

$$\vec{F}_{sphere} = -\frac{KqQ}{r^2}\hat{r} \quad \Rightarrow \quad |\vec{F}_{sphere}| = \frac{KqQ}{r^2}.$$

Ex. Figure shows a uniformly charged thin sphere of total charge Q and radius R. A point charge q is also situated at the centre of the sphere.



Find out the following :

(i) Force on charge q

(ii) Electric field intensity at A.

(iii) Electric field intensity at B.

Sol. **(i)** Electric field at the centre of the uniformly charged hollow sphere = 0

So force on charge q = 0

(ii) Electric field at A

$$\vec{E}_A = \vec{E}_{Sphere} + \vec{E}_q = 0 + \frac{Kq}{r^2} \quad ; \; r = CA$$

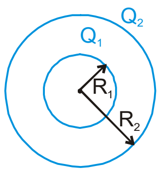
E due to sphere = 0 , because point lies inside the charged hollow sphere.

(iii) Electric field $\vec{E}_B$ at point B = $\vec{E}_{Sphere} + \vec{E}_q$

$$= \frac{KQ}{r^2}.\hat{r} + \frac{Kq}{r^2}.\hat{r} = \frac{K(Q+q)}{r^2}.\hat{r} \; ; \; r = CB$$

Note : Here, we can also assume that the total charge of sphere is concentrated at the centre, for calculation of electric field at B.

Ex. Two concentric uniformly charged spherical shells of radius R_1 and R_2 ($R_2 > R_1$) have total charges Q_1 and Q_2 respectively. Derive an expression of electric field as a function of r for following positions.



(i) $r < R_1$ **(ii)** $R_1 \le r < R_2$ **(iii)** $r \ge R_2$

Sol. **(i)** For $r < R_1$,

therefore, point lies inside both the spheres

$$E_{net} = E_{Inner} + E_{outer} = 0 + 0$$

(ii) For $R_1 \le r < R_2$,

point lies outside inner sphere but inside outer sphere:

$$\therefore \quad E_{net} = E_{inner} + E_{outer}$$

$$= \frac{KQ_1}{r^2}\hat{r} + 0 = \frac{KQ_1}{r^2}\hat{r}$$

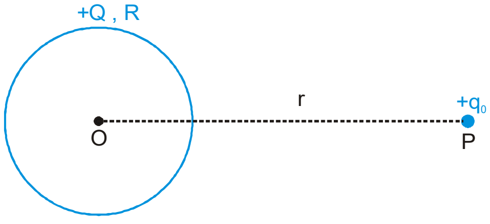
(iii) For $r \ge R_2$

point lies outside inner as well as outer sphere.

Therefore, $E_{Net} = E_{inner} + E_{outer}$

$$= \frac{KQ_1}{r^2}\hat{r} + \frac{KQ_2}{r^2}\hat{r} = \frac{K(Q_1+Q_2)}{r^2}\hat{r}$$

Ex. A spherical shell having charge +Q (uniformly distributed) and a point charge + q_0 are placed as shown. Find the force between shell and the point charge(r>>R).



(i) Force on the point charge + q_0 due to the

$$\text{shell} = q_0\vec{E}_{shell} = (q_0)\left(\frac{KQ}{r^2}\right)\hat{r} = \frac{KQq_0}{r^2}\hat{r}$$

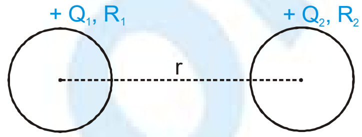
where $\hat{r}$, is unit vector along OP.

From action - reaction principle, force on the shell due to the point charge will

also be : $F_{shell} = \frac{KQq_0}{r^2}(-\hat{r})$

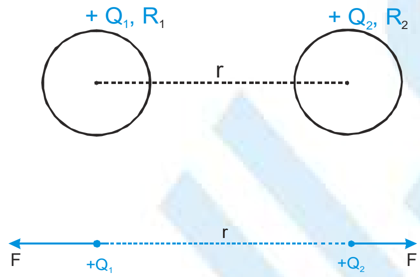
Conclusion :- To find the force on a hollow sphere due to outside charges , we can replace the sphere by a point charge kept at centre.

Ex. Find force acting between two shells of radius R_1 and R_2 which have uniformly distributed charges Q_1 and Q_2 respectively and distance between their centres is r.



Sol. The shells can be replaced by point charges kept at centre, so force between them

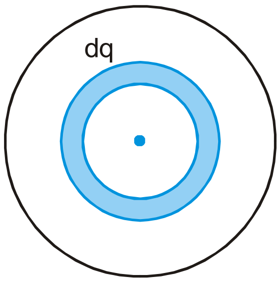
$$F = \frac{KQ_1Q_2}{r^2};$$



Electric field due to uniformly charged solid sphere

Derive an expression for electric field due to solid sphere of radius R and total charge Q which is uniformly distributed in the volume, at a point which is at a distance r from centre for given two cases.

(i) $r \geq R$ **(ii)** $r \leq R$



Assume an elementary concentric shell of charge dq. Due to this shell, the electric field at the point (r > R) will be:

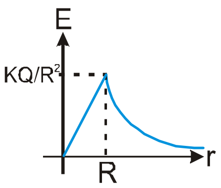
$$dE = \frac{Kdq}{r^2} \quad \text{[from above result of hollow sphere]}$$

$$E_{net} = \int dE = \frac{KQ}{r^2}$$

For r < R, there will be no electric field due to shell of radius greater than r, so electric field at the point will be present only due to shells having radius less than r.

$$E'_{net} = \frac{KQ'}{r^2}$$

Here, $Q' = \frac{Q}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi r^3 = \frac{Qr^3}{R^3}$



$\therefore$ $E'_{net} = \frac{KQ'}{r^2} = \frac{KQr}{R^3}$; away from the centre.

NOTE : The electric field inside and outside the sphere is always in radial direction.

Ex. A solid non conducting sphere of radius R and uniform volume charge density ρ has its centre at origin. Find out electric field intensity in vector form at following positions :

(i) (R/2, 0, 0) **(ii)** $\left(\frac{R}{\sqrt{2}},\frac{R}{\sqrt{2}},0\right)$ **(iii)** (R, R, 0)

Sol. **(i)** At (R/2, 0, 0) : Distance of point from centre = $\sqrt{(R/2)^2+0^2+0^2}$ = R/2 < R, so point lies inside the sphere, so

$$\vec{E} = \frac{\rho\vec{r}}{3\varepsilon_0} = \frac{\rho}{3\varepsilon_0}\left[\frac{R}{2}\hat{i}\right]$$

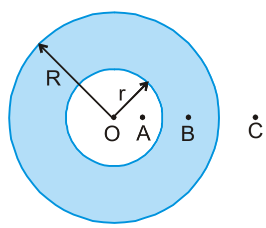
(ii) At $\left(\frac{R}{\sqrt{2}},\frac{R}{\sqrt{2}},0\right)$; distance of point from centre = $\sqrt{(R/\sqrt{2})^2+(R/\sqrt{2})^2+0^2}$ = R = R, so point lies at the surface of sphere, therefore

$$\vec{E} = \frac{KQ\vec{r}}{R^3} = \frac{K\frac{4}{3}\pi R^3\rho}{R^3}\left[\frac{R}{\sqrt{2}}\hat{i}+\frac{R}{\sqrt{2}}\hat{j}\right] = \frac{\rho}{3\varepsilon_0}\left[\frac{R}{\sqrt{2}}\hat{i}+\frac{R}{\sqrt{2}}\hat{j}\right]$$

(iii) The point is outside the sphere

So, $$\vec{E} = \frac{KQ\vec{r}}{r^3} = \frac{K\frac{4}{3}\pi R^3\rho}{(\sqrt{2}R)^3}\left[R\hat{i}+R\hat{j}\right] = \frac{\rho}{6\sqrt{2}\varepsilon_0}\left[R\hat{i}+R\hat{j}\right]$$

Ex. A Uniformly charged solid non-conducting sphere of uniform volume charge density ρ and radius R is having a concentric spherical cavity of radius r. Find out electric field intensity at following points, as shown in the figure :



(i) Point A **(ii)** Point B

(iii) Point C **(iv)** Centre of the sphere

Sol. **Method I :**

(i) For point A :

We can consider the solid part of sphere to be made of large number of spherical shells which have uniformly distributed charge on its surface. Now, since point A lies inside all spherical shells so electric field intensity due to all shells will be zero.

$$\vec{E}_A = 0$$

(ii) For point B :

All the spherical shells for which point B lies inside will make electric field zero at point B. So electric field will be due to charge present from radius r to OB.

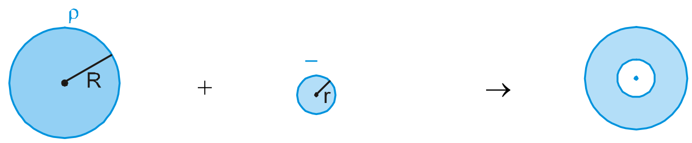
So, $$\vec{E}_B = \frac{K\frac{4}{3}\pi(OB^3-r^3)\rho}{OB^3}\overrightarrow{OB} = \frac{\rho}{3\varepsilon_0}\frac{[OB^3-r^3]}{OB^3}\overrightarrow{OB}$$

(iii) For point C, similarly we can say that for all the shell points C lies outside the shell

So, $$\vec{E}_C = \frac{K[\frac{4}{3}\pi(R^3-r^3)]}{[OC]^3}\overrightarrow{OC} = \frac{\rho}{3\varepsilon_0}\frac{R^3-r^3}{[OC]^3}\overrightarrow{OC}$$

Method : II

We can consider that the spherical cavity is filled with charge density ρ and also –ρ, thereby making net charge density zero after combining. We can consider two concentric solid spheres: One of radius R and charge density ρ and other of radius r and charge density –ρ. Applying superposition principle :



(i) $\vec{E}_A = \vec{E}_\rho + \vec{E}_{-\rho} = \dfrac{\rho(\vec{OA})}{3\varepsilon_0} + \dfrac{[-\rho(\vec{OA})]}{3\varepsilon_0} = 0$

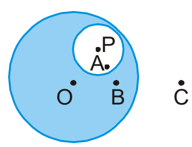
(ii) $\vec{E}_B = \vec{E}_\rho + \vec{E}_{-\rho} = \dfrac{\rho(\vec{OB})}{3\varepsilon_0} + \dfrac{K\left[\frac{4}{3}\pi r^3(-\rho)\right]}{(OB)^3}\vec{OB} = \left[\dfrac{\rho}{3\varepsilon_0} - \dfrac{r^3\rho}{3\varepsilon_0(OB)^3}\right]\vec{OB} = \dfrac{\rho}{3\varepsilon_0}\left[1 - \dfrac{r^3}{OB^3}\right]\vec{OB}$

(iii) $\vec{E}_C = \vec{E}_\rho + \vec{E}_{-\rho} = \dfrac{K\left(\frac{4}{3}\pi R^3\rho\right)}{OC^3}\vec{OC} + \dfrac{K\left(\frac{4}{3}\pi r^3(-\rho)\right)}{OC^3}\vec{OC} = \dfrac{\rho}{3\varepsilon_0(OC)^3}[R^3 - r^3]\vec{OC}$

(iv) $\vec{E}_O = \vec{E}_\rho + \vec{E}_{-\rho} = 0 + 0 = 0$

Ex. In above question, if cavity is not concentric and centered at point P then repeat all the steps.

Sol. Again assume ρ and –ρ in the cavity, (similar to the previous example) :



(i) $\vec{E}_A = \vec{E}_\rho + \vec{E}_{-\rho} = \dfrac{\rho[\vec{OA}]}{3\varepsilon_0} + \dfrac{(-\rho)\vec{PA}}{3\varepsilon_0}$

$= \dfrac{\rho}{3\varepsilon_0}[\vec{OA} - \vec{PA}] = \dfrac{\rho}{3\varepsilon_0}[\vec{OP}]$

Note : Here, we can see that the electric field intensity at point A is independent of position of point A inside the cavity. Also the electric field is along the line joining the centres of the sphere and the spherical cavity.

(ii) $\vec{E}_B = \vec{E}_\rho + \vec{E}_{-\rho} = \dfrac{\rho(\vec{OB})}{3\varepsilon_0} + \dfrac{K[\frac{4}{3}\pi r^3(-\rho)]}{[PB]^3}\vec{PB}$

(iii) $\vec{E}_C = \vec{E}_\rho + \vec{E}_{-\rho} = \dfrac{K[\frac{4}{3}\pi R^3\rho]}{[OC]^3}\vec{OC} + \dfrac{K[\frac{4}{3}\pi r^3(-\rho)]}{[PC]^3}\vec{PC}$

(iv) $\vec{E}_O = \vec{E}_\rho + \vec{E}_{-\rho} = 0 + \dfrac{K[\frac{4}{3}\pi r^3(-\rho)]}{[PO]^3}\vec{PO}$

Ex. A non-conducting solid sphere has volume charge density that varies as $\rho = \rho_0 r$, where ρ_0 is a constant and r is distance from centre. Find out electric field intensities at following positions.

(i) $r < R$ **(ii)** $r \geq R$

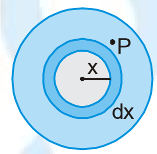
Sol. **Method I :**

(i) For $r < R$:

The sphere can be considered to be made of large number of spherical shells. Each shell has uniform charge density on its surface. So. the previous results of the spherical shell can be used. Consider a shell of radius x and thickness dx as an element. Charge on shell $dq = (4\pi x^2 dx)\rho_0 x$

$\therefore$ Electric field intensity at point P due to shell,

$$dE = \frac{Kdq}{x^2}$$



Since all the shell will have electric field in same direction

$$\therefore \quad E = \int_0^R dE = \int_0^r dE + \int_r^R dE$$

Due to shells which lie between region $r < x \leq R$, electric field at point P will be zero.

$$\therefore \quad |\vec{E}| = \int_0^r \frac{Kdq}{r^2} + 0 = \int_0^r \frac{K.4\pi x^2 dx \rho_0 x}{r^2} = \frac{4\pi K \rho_0}{r^2}\left[\frac{x^4}{4}\right]_0^r = \frac{\rho_0 r^2}{4\varepsilon_0}\hat{r}$$

(ii) For $r \geq R$, $$E = \int_0^R dE = \int_0^R \frac{K.4\pi x^2 dx \rho_0 x}{r^2} = \frac{\rho_0 R^4}{4\varepsilon_0 r^2}\hat{r}$$

Method II :

(i) The sphere can be considered to be made of large number of spherical shells. Each shell has uniform charge density on its surface. So the previous results of the spherical shell can be used. we can say that all the shells for which point lies inside will make electric field zero at that point,

$$\text{So} \quad \vec{E}_{(r<R)} = \frac{K\int_0^r (4\pi x^2 dx)\rho_0 x}{r^2} = \frac{\rho_0 r^2}{4\varepsilon_0}\hat{r}$$

(ii) Similarly, for $r \geq R$, all the shells will contribute in electric field, Therefore :

$$\vec{E}_{(r<R)} = \frac{K\int_0^R (4\pi x^2 dx)\rho_0 x}{r^2} = \frac{\rho_0 R^4}{4\varepsilon_0 r^2}\hat{r}$$

Electric Potential

In electrostatic field, the electric potential (due to some source charges) at a point P is defined as the work done by external agent in taking a unit point positive charge from a reference point (generally taken at infinity) to that point P without changing its kinetic energy..

Mathematical representation

If $(W_{\infty \to P})_{ext}$ is the work required in moving a point charge q from infinity to a point P, the electric potential of the point P is

$$V_p = \left.\frac{W_{\infty\to p})_{ext}}{q}\right]_{\Delta K=0} = \frac{(-W_{elc})_{\infty\to p}}{q}$$

ETOOS KEY POINTS

(i) $(W_{\infty \to P})_{ext}$ can also be called as the work done by external agent against the electric force on a unit positive charge due to the source charge.

(ii) Write both W and q with proper sign.

Properties

(i) Potential is a scalar quantity, its value may be positive, negative or zero.

(ii) S.I. Unit of potential is volt = $\frac{\text{joule}}{\text{coulmb}}$ and its dimensional formula is $[M^1L^2T^{-3}I^{-1}]$.

(iii) Electric potential at a point is also equal to the negative of the work done by the electric field in taking

the point charge from reference point (i.e. infinity) to that point.

(iv) Electric potential due to a positive charge is always positive and due to negative charge it is always negative except at infinity. (taking $V_\infty = 0$).

(v) Potential decreases in the direction of electric field.

(vi) $V = V_1 + V_2 + V_3 + ...$

USE OF POTENTIAL

If we know the potential at some point (in terms of numerical value or in terms of formula) then we can find out the work done by electric force when charge moves from point 'P' to ∞ by the formula

$$W_{ep})_{p \to \infty} = qV_p$$

Ex. A charge 2μC is taken from infinity to a point in an electric field, without changing its velocity. If work done against electrostatic forces is –40μJ, then find the potential at that point.

Sol. $V = \frac{W_{ext}}{q} = \frac{-40\mu J}{2\mu C} = -20\ V$

Ex. When charge 10 μC is shifted from infinity to a point in an electric field, it is found that work done by electrostatic forces is –10 μJ. If the charge is doubled and taken again from infinity to the same point without accelerating it, then find the amount of work done by electric field and against electric field.

Sol. $W_{ext})_{\infty \to p} = -w_{el})_{\infty \to p} = w_{el})_{p \to \infty} = 10\ \mu J$

because $\Delta KE = 0$

$\therefore \quad V_P = \frac{(W_{ext})_{\infty \to p}}{20\mu C} = \frac{10\mu J}{10\mu C} = 1V$

So, if now the charge is doubled and taken from infinity then

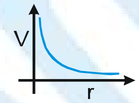
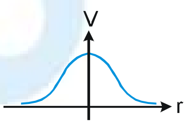
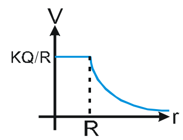
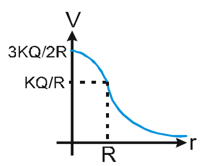
$1 = \frac{W_{ext})_{\infty \to p}}{20\mu C} \Rightarrow$ or $W_{ext})_{\infty \to P} = 20\ \mu J \Rightarrow \quad W_{el})_{\infty \to P} = -20\ \mu J$

Ex. A charge 3μC is released from rest from a point P where electric potential is 20 V then its kinetic energy when it reaches infinity is :

Sol. $W_{el} = \Delta K = K_f - 0$

$\therefore \quad W_{el})_{P \to \infty} = qV_P = 60\ \mu J \qquad$ So, $\quad K_f = 60\ \mu J$

ELECTRIC POTENTIAL DUE TO VARIOUS CHARGE DISTRIBUTIONS

Name / Type	Formula	Note	Graph
Point charge	$\frac{Kq}{r}$	* q is source charge. * r is the distance of the point from the point charge.	
Ring (uniform/nonuniform charge distribution)	at centre: $\frac{KQ}{R}$ at the axis: $\frac{KQ}{\sqrt{R^2+x^2}}$	* Q is source chage. * x is the distance of the point on the axis from centre f ring	
Uniformly charged hollow conducting/nonconducting /solid conducting sphere	for $r \geq R$, $V = \frac{kQ}{r}$ for $r \leq R$, $V = \frac{kQ}{R}$	* R is radius of sphere * r is the distance from centre of sphere to the point * Q is total charge $= \sigma 4\pi R^2$.	
Uniformly charged solid nonconducting sphere .	For $r \geq R$, $V = \frac{kQ}{r}$ For $r \leq R$ $V = \frac{KQ(3R^2 - r^2)}{2R^3}$ $= \frac{\rho}{6\varepsilon_0}(3R^2 - r^2)$	* R is radius of sphere * r is distance from centre to the point * $V_{centre} = \frac{3}{2} V_{surface}$. * Q is total charge $= \rho \frac{4}{3}\pi R^3$. * Inside the sphere potential varies parabolically * Outside the sphere potential varies hyperbolically.	
Infinite line charge	Not defined	* Absolute potential is not defined. * Potential difference between two points is given by formula : $v_B - v_A = -2K\lambda \ \ln(r_B / r_A)$	
Infinite nonconducting thin sheet	Not defined	* Absolute potential is not defined. * Potential difference between two points is given by formula $V_B - V_A = -\frac{\sigma}{\varepsilon_0}(r_B - r_A)$	
Infinite charged conducting thin sheet	Not defined	* Absolute potential is not defined. * Potential difference between two points is given by formula $V_B - V_A = -\frac{\sigma}{\varepsilon_0}(r_B - r_A)$	

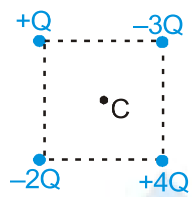
(1) Potential due to a point charge

Derivation of expression for potential due to point charge Q, at a point which is at a distance r from the point charge.

From definition of potential

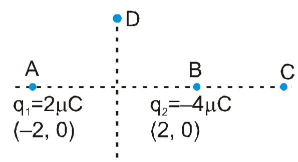
$$V = \frac{W_{ext(\infty \to p)}}{q_o} = \frac{-\int_{\infty}^{r} (q_0\vec{E})\cdot d\vec{r}}{q_o} = -\int_{\infty}^{r} \vec{E}\cdot d\vec{r} \quad \Rightarrow \quad V = -\int_{\infty}^{r} \frac{KQ}{r^2}(-dr)\cos 180^o = \frac{KQ}{r}$$

Ex. Four point charges are placed at the corners of a square of side ● Calculate potential at the centre of square.



Sol. V = 0 at 'C'. [Use $V = \frac{Kq}{r}$]

Ex. Two point charges 2μC and – 4μC are situated at points (–2m, 0m) and (2 m, 0 m) respectively. Find out potential at point C (4 m, 0 m) and D (0 m, $\sqrt{5}$ m).



Sol. Potential at point C

$$V_C = V_{q_1} + V_{q_2} = \frac{K(2\mu C)}{6} + \frac{K(-4\mu C)}{2} = \frac{9\times10^9 \times 2\times10^{-6}}{6} - \frac{9\times10^9\times4\times10^{-6}}{2} = -15000\ V.$$

Similarly, $V_D = V_{q_1} + V_{q_2} = \frac{K(2\mu C)}{\sqrt{(\sqrt{5})^2+2^2}} + \frac{K(-4\mu C)}{\sqrt{(\sqrt{5})^2+2^2}} = \frac{K(2\mu C)}{3} + \frac{K(-4\mu C)}{3} = -6000\ V.$

Finding potential due to continuous charges

If formula of E is tough, then we take a small element and integrate

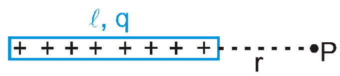
$V = \int dv$

If formula of E is easy then, we use

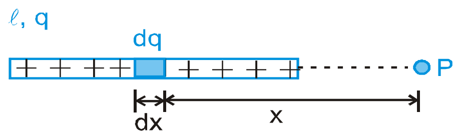
$V = -\int_{r\to 8}^{r=r} \vec{E}.d\vec{r}$

(i.e. for sphere, plate, infinite wire etc.)

Ex. A rod of length ● is uniformly charged with charge q Calculate potential at point P.



Sol. Take a small element of length dx, at a distance x from left end. Potential due to this small element



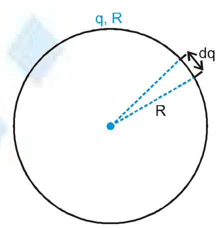
$dV = \frac{K(dq)}{x}$ $\therefore$ Total potential $\Rightarrow$ $V = \int_{x=0}^{x=l} \frac{k\,dq}{x}$

Where $dq = \frac{q}{l}dx$ $\Rightarrow$ $V = \int_{x=r}^{x=r+l} \frac{K\left(\frac{q}{l}dx\right)}{x} = \frac{Kq}{l}\log_e\left(\frac{l+r}{r}\right)$

(2) Potential due to a ring

(i) Potential at the centre of uniformly charged ring :

Potential due to the small element dq

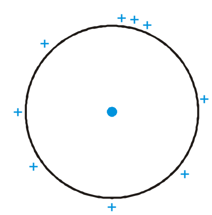


$$dV = \frac{Kdq}{R}$$

$\therefore$ Net potential: $V = \int \frac{K\,dq}{R}$

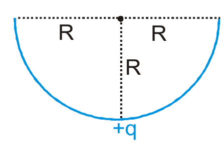
$\therefore$ $V = \frac{K}{R}\int dq = \frac{Kq}{R}$

(ii) For non-uniformly charged ring potential at the center is



$$V = \frac{Kq_{total}}{R}$$

(iii) Potential due to half ring at center is :



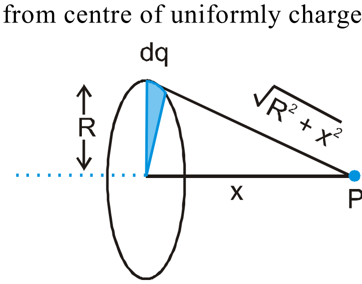
$$V = \frac{Kq}{R}$$

(iv) Potential at the axis of a ring:

Calculation of potential at a point on the axis which is a distance x from centre of uniformly charged (total charge Q) ring of radius R.

Consider an element of charge dq on the ring.

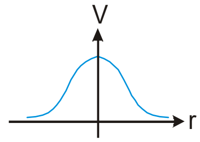
Potential at point P due to charge dq will be



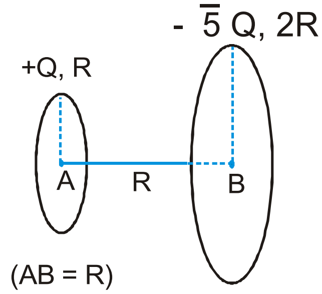
$$dV = \frac{K(dq)}{\sqrt{R^2+x^2}}$$

$\therefore$ Net potential at point P due to all such element will be:

$$V = \int dv = \frac{KQ}{\sqrt{R^2 + x^2}}$$



Ex. Figure shows two rings having charges Q and $-\sqrt{5}$ Q. Find Potential difference between A and B i.e. $(V_A - V_B)$.



Sol. $V_A = \frac{KQ}{R} + \frac{K(-\sqrt{5}\,Q)}{\sqrt{(2R)^2 + (R)^2}}$; $V_B = \frac{K(-\sqrt{5}\,Q)}{2\,R} + \frac{K(\,Q)}{\sqrt{(R)^2 + (R)^2}}$

From above, we can easily find $V_A - V_B$.

Ex. A point charge q_0 is placed at the centre of uniformly charged ring of total charge Q and radius R. If the point charge is slightly displaced with negligible force along axis of the ring then find out its speed when it reaches a large distance.

Sol. Only electric force is acting on q_0

$\therefore \quad W_{el} = \Delta K = \frac{1}{2}mv^2 - 0 \quad \Rightarrow \quad \text{Now } W_{el})_{c\to\infty} = q_0 V_c = q_0 \cdot \frac{KQ}{R}$

$\therefore \quad \frac{Kq_0Q}{R} = \frac{1}{2}mv^2 \quad \Rightarrow \quad v = \sqrt{\frac{2Kq_0Q}{mR}}$

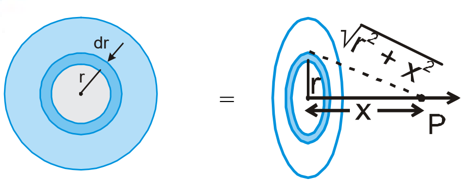
(3) Potential due to uniformly charged disc

$V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2 + x^2} - x\right)$, where σ is the charge density and x is the distance of the point on the axis from the center of the disc & R is the radius of disc.

Finding potential due to a uniformly charged disc

A disc of radius 'R' has surface charge density (charge/area) = σ. We have to find potential at its axis, at point 'P' which is at a distance x from the centre.

For this, we can divide the disc into thin rings and let's consider a thin ring of radius r and thickness dr. Suppose charge on the small ring element = dq. Potential due to this ring at point 'P' is:



$$dV = \frac{Kdq}{\sqrt{r^2 + x^2}}$$

So, net potential : $V_{net} = \int \frac{Kdq}{\sqrt{r^2 + x^2}}$

Here, σ = charge/area = $\frac{dq}{d\,(area)}$

So, $dq = \sigma \times d\,(area) = \sigma\ (2\pi r\ dr)$

(Here, d (area) = area of the small ring element =(length of ring) × (width of the ring) = $(2\pi r)$. (dr)

So, $V_{net} = \int_{r=0}^{r=R} \frac{K\sigma(2\pi r\ dr)}{\sqrt{r^2 + x^2}}$

To integrate it, let $r^2 + x^2 = y^2$

$2r\,dr = 2y\,dy$. Substituting we will get:

$$V_{net} = \int_{r=0}^{r=R} \frac{1}{4\pi\varepsilon_0} \frac{\sigma(2\pi)\,y\,dy}{y} \quad \Rightarrow \quad V_{net} = \frac{\sigma}{2\varepsilon_0}[\,y\,]_{r=0}^{r=R}$$

$$V_{net} = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{r^2 + x^2}\right)_{r=0}^{r=R} \quad \Rightarrow \quad V_{net} = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2 + x^2} - x\right)$$

If a test charge q_0 is placed at point P, then potential energy of this charge q_0 due to the disc = U = q_0V

$$\Rightarrow \quad U = q_0\left[\frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2 + x^2} - x\right)\right]$$

Graph of V v/s x :

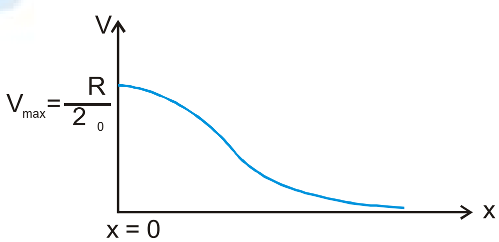
$$V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2 + x^2} - x\right)$$

at $x = 0\ V = \frac{\sigma R}{2\varepsilon_0}$

to check whether V will increase with x or decrease, lets multiply and divide by conjugate.

$$V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2 + x^2} - x\right) \times \frac{\left(\sqrt{R^2 + x^2} + x\right)}{\left(\sqrt{R^2 + x^2} + x\right)} \quad \Rightarrow \quad V = \frac{\sigma R^2}{2\varepsilon_0}\left(\frac{1}{\left(\sqrt{R^2 + x^2} + x\right)}\right)$$

Now, we can say that as x ↑ ⇒ V↓ so curve will be



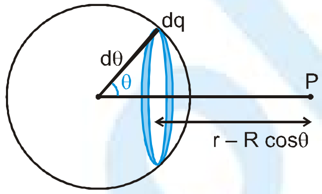
(4) Potential Due To Uniformly Charged Spherical shell

Derivation of expression for potential due to uniformly charged hollow sphere of radius R and total charge Q, at a point which is at a distance r from centre for the following situation

(i) $r > R$ (ii) $r < R$

Assume a ring of width $Rd\theta$ at angle θ from X axis (as shown in figure).

Potential due to the ring at the point P will be



$$dV = \frac{K(dq)}{\sqrt{(r - R\cos\theta)^2 + (R\sin\theta)^2}}$$

Where $dq = 2\pi R \sin\theta \,(Rd\theta)\sigma$

where $Q, = 4\pi R^2\sigma$

then, net potential

$$V = \int dV = \frac{KQ}{2}\int_0^{\pi}\frac{\sin\theta . d\theta}{\sqrt{(r - R\cos\theta)^2 + (R\sin\theta)^2}}$$

Solving this eq.we find

$$V = \frac{KQ}{r} \quad (\text{for } r > R)$$

& $$V = \frac{KQ}{R} \quad \text{for } (r < R)$$

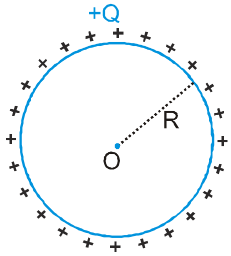
Alternate Method :

As the formula of E is easy , we use $V = -\int_{r\to\infty}^{r=r} \vec{E}\cdot d\vec{r}$

(i) **At outside point ($r > R$):**

$$V_{out} = -\int_{r\to\infty}^{r=r}\left(\frac{K\,Q}{r^2}\right)dr \quad \Rightarrow \quad V_{out} = \frac{KQ}{r} = \frac{KQ}{(\text{Distance from centre})}$$

For outside point, the hollow sphere acts like a point charge.



(ii) **Potential at the centre of the sphere (r=0) :**

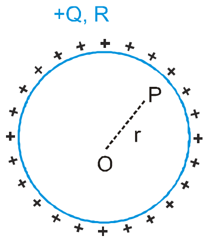
As all the charges are at a distance R from the centre ,

So, $$V_{centre} = \frac{KQ}{R} = \frac{KQ}{(\text{Radius of the sphere})}$$

(iii) **Potential at inside point ($r < R$) :**

Suppose we want to find potential at point P , inside the sphere.

$\therefore$ Potential difference between Point P and O :



$$V_P - V_O = -\int_O^P \vec{E}_{in}\cdot d\vec{r} \quad \text{Where, } E_{in} = 0$$

So $$V_P - V_O = 0$$

$$\Rightarrow \quad V_P = V_O = \frac{KQ}{R}$$

$$\Rightarrow \quad V_{in} = \frac{KQ}{R} = \frac{KQ}{(\text{Radius of the sphere})}$$

(5) POTENTIAL DUE TO UNIFORMLY CHARGED SOLID SPHERE

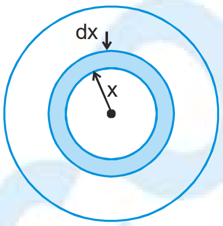
Derivation of expression for potential due to uniformly charged solid sphere of radius R and total charge Q (distributed in volume), at a point which is at a distance r from centre for the following situations.

(i) $r \geq R$ **(ii)** $r \leq R$

Consider an elementary shell of radius x and width dx

(i) For $r \geq R$:

$$V = \int_0^R \frac{K \cdot 4\pi x^2 dx \rho}{r} = \frac{KQ}{r}$$



(ii) for $r \leq R$

$$V = \int_0^r \frac{K \cdot 4\pi x^2 dx\rho}{r} + \int_r^R \frac{K 4\pi x^2 dx \rho}{x}$$

$$= \frac{KQ}{2R^3}(3R^2 - r^2) \quad \Rightarrow \quad \left(\rho = \frac{Q}{\frac{4}{3}\pi R^3}\right)$$

From definition of potential

(i) For $r \geq R$:

$$V = -\int_\infty^r \frac{KQ}{r^2}\hat{r} \cdot dr = \frac{KQ}{r}$$

(ii) For $r \leq R$:

$$V = -\int_\infty^R \frac{KQ}{r^2}.dr - \int_R^r \frac{KQr}{R^3} dr$$

$$V = \frac{KQ}{R} - \frac{KQ}{2R^3}[r^2 - R^2] = \frac{KQ}{2R^3}[2R^2 - r^2 + R^2] = \frac{KQ}{2R^3}(3R^2 - r^2)$$

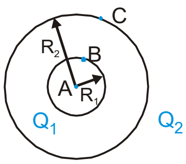
Ex. Two concentric spherical shells of radius R_1 and R_2 ($R_2 > R_1$) are having uniformly distributed charges Q_1 and Q_2 respectively. Find out potential

(i) at point A

(ii) at surface of smaller shell (i.e. at point B)

(iii) at surface of larger shell (i.e. at point C)

(iv) at $r \leq R_1$ **(v)** at $R_1 \leq r \leq R_2$ **(vi)** at $r \geq R_2$



Sol. Using the results of hollow sphere as given in the table 7.4.

(i) $V_A = \frac{KQ_1}{R_1} + \frac{KQ_2}{R_2}$ **(ii)** $V_B = \frac{KQ_1}{R_1} + \frac{KQ_2}{R_2}$

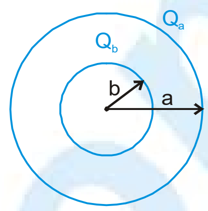
(iii) $V_C = \frac{KQ_1}{R_2} + \frac{KQ_2}{R_2}$ **(iv)** for $r \leq R_1$, $V = \frac{KQ_1}{R_1} + \frac{KQ_2}{R_2}$

(v) for $R_1 \leq r \leq R_2$, $V = \frac{KQ_1}{r} + \frac{KQ_2}{R_2}$

(vi) for $r \geq R_2$, $V = \frac{KQ_1}{r} + \frac{KQ_2}{r}$

Ex. Two hollow concentric non-conducting spheres of radius the a and b (a > b) contain charges Q_a and Q_b respectively. Prove that potential difference between the two spheres is independent of charge on outer sphere. If outer sphere is given an extra charge, is there any change in potential difference?

Sol. $V_{\text{inner sphere}} = \frac{KQ_b}{b} + \frac{KQ_a}{a}$



$V_{\text{outer sphere}} = \frac{KQ_b}{a} + \frac{KQ_a}{a}$

$V_{\text{inner sphere}} - V_{\text{outer sphere}} = \frac{KQ_b}{b} - \frac{KQ_b}{a}$

$\therefore \Delta V = KQ_b\left[\frac{1}{b} - \frac{1}{a}\right]$

Which is independent of charge on outer sphere.

If outer sphere in given any extra charge, then there will be no change in potential difference.

Potential difference

The potential difference between two points A and B is work done by external agent against electric field in taking a unit positive charge from A to B with no change in kinetic energy between initial and final points ie. $\Delta K = 0$ or $K_i = K_f$

(a) Mathematical representation

If $(W_{A\to B})_{ext}$ = Work done by external agent against electric field in taking the unit charge from A to B

Then, $V_B - V_A = \left(\frac{(W_{A\to B})_{ext}}{q}\right)_{\Delta K=0} = \frac{-(W_{A\to B})_{electric}}{q} = \frac{U_B - U_A}{q} = \frac{-\int_A^B \vec{F}_e.d\vec{r}}{q} = -\int_A^B \vec{E}.d\vec{r}$

Note : Take W and q both with sign

(b) Properties

(i) The difference of potential between two points is called potential difference. It is also called voltage.

(ii) Potential difference is a scalar quantity. Its S.I. unit is also volt.

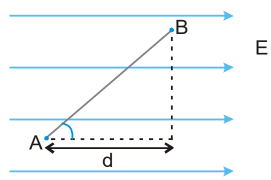
(iii) If V_A and V_B be the potential of two points A and B, then work done by an external agent in taking the charge q from A to B is

$$(W_{ext})_{AB} = q\,(V_B - V_A) \text{ or } (W_{el})_{AB} = q\,(V_A - V_B).$$

(iv) Potential difference between two points is independent of reference point.

(1) Potential difference in a uniform electric field

$V_B - V_A = -\vec{E}\cdot\overrightarrow{AB}$



$\Rightarrow \quad V_B - V_A = -|E|\,|AB| \cos\theta$

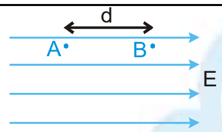
$= -|E|\,d$

$= -Ed$

where d = effective distance between A and B along electric field.

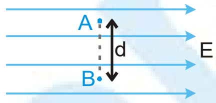
or we can also say that $E = \frac{\Delta V}{\Delta d}$

Special Cases :



Case 1. Line AB is parallel to electric field.

$\therefore \quad V_A - V_B = Ed$



Case 2. Line AB is perpendicular to electric field.

$\therefore \quad V_A - V_B = 0 \quad \Rightarrow \quad V_A = V_B$

Note : In the direction of electric field potential always decreases.

Ex. 1μC charge is shifted from A to B and it is found that work done by an external force is 40μJ in doing so against electrostatic forces, then find potential difference $V_A - V_B$

Sol. $(W_{AB})_{ext} = q(V_B - V_A) \quad \Rightarrow \quad 40\ \mu J = 1\mu C\,(V_B - V_A) \quad \Rightarrow \quad V_A - V_B = -40\ V$

Ex. A uniform electric field is present in the positive x-direction. If the intensity of the field is 5N/C then find the potential difference $(V_B - V_A)$ between two points A (0m, 2 m) and B (5 m ,3 m)

Sol. $V_B - V_A = -\vec{E}\cdot\vec{AB} \qquad = -\hat{i}(5)\cdot(5\hat{i}+\hat{j}) = -25V.$

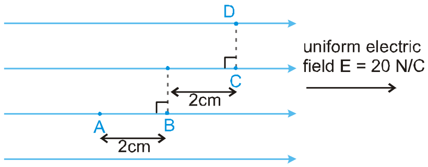
The electric field intensity in uniform electric field, $E = \dfrac{\Delta V}{\Delta d}$

Where ΔV = potential difference between two points.

Δd = effective distance between the two points.

(projection of the displacement along the direction of electric field.)

Ex. Find out following



(i) $V_A - V_B$
(ii) $V_B - V_C$
(iii) $V_C - V_A$
(iv) $V_D - V_C$
(v) $V_A - V_D$
(vi) Arrange the order of potential for points A, B, C and D.

Sol. **(i)** $|\Delta V_{AB}| = Ed = 20 \times 2 \times 10^{-2} = 0.4$

So, $V_A - V_B = 0.4\ V$

because **In the direction of electric field potential always decreases.**

(ii) $|\Delta V_{BC}| = Ed = 20 \times 2 \times 10^{-2} = 0.4$ so, $V_B - V_C = 0.4\ V$

(iii) $|\Delta V_{CA}| = Ed = 20 \times 4 \times 10^{-2} = 0.8$ so, $V_C - V_A = -0.8\ V$

because **In the direction of electric field potential always decreases.**

(iv) $|\Delta V_{DC}| = Ed = 20 \times 0 = 0$ so, $V_D - V_C = 0$

because the effective distance between D and C is zero.

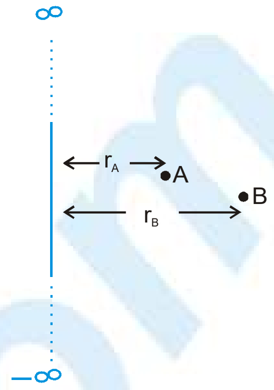
(v) $|\Delta V_{AD}| = Ed = 20 \times 4 \times 10^{-2} = 0.8$ so, $V_A - V_D = 0.8\ V$

because **In the direction of electric field potential always decreases.**

(vi) The order of potential is :

$V_A > V_B > V_C = V_D$.

Potential difference due to infinitely long wire



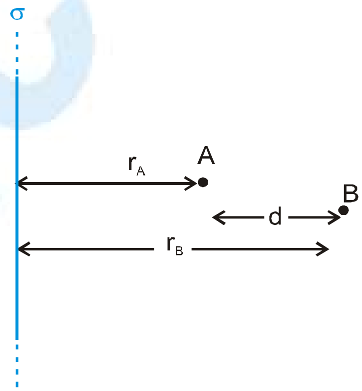
Derivation of expression for potential difference between two points, which have perpendicular distance r_A and r_B from infinitely long line charge of uniform linear charge density λ. :

From definition of potential difference :

$$V_{AB} = V_B - V_A = -\int_{r_A}^{r_B} \vec{E} \cdot \vec{dr} = -\int_{r_A}^{r_B} \frac{2K\lambda}{r} \hat{r} \cdot \vec{dr}$$

$$\therefore \; V_{AB} = -2K\lambda \ln\left(\frac{r_B}{r_A}\right)$$

Potential difference due to infinitely long thin sheet



Derivation of expression for potential difference between two points, having separation d in the direction perpendicularly to a very large uniformly charged thin sheet of uniform surface charge density σ.

Let the points A and B have perpendicular distance r_A and r_B respectively then from definition of potential difference.

$$V_{AB} = V_B - V_A = -\int_{r_A}^{r_B} \vec{E} \cdot \vec{dr} = -\int_{r_A}^{r_B} \frac{\sigma}{2\varepsilon_0} \hat{r} \cdot \vec{dr}$$

$$\Rightarrow \quad V_{AB} = -\frac{\sigma}{2\varepsilon_o}(r_B - r_A) = -\frac{\sigma d}{2\varepsilon_o}$$

Equipotential Surface

Definition : If potential of a surface (imaginary or physically existing) is same throughout, then such surface is known as an equipotential surface.

Properties of equipotential surfaces

(i) When a charge is shifted from one point to another point on an equipotential surface, then work done against electrostatic forces is zero.

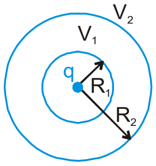
(ii) Electric field is always perpendicular to equipotential surfaces.

(iii) Two equipotential surfaces do not cross each other.

Examples of equipotential surfaces

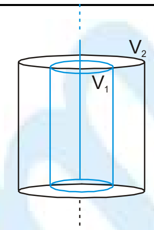
(i) Point charge :

Equipotential surfaces are concentric and spherical as shown in figure. In figure, we can see that sphere of radius R_1 has potential V_1 throughout its surface and similarly for other concentric sphere potential is same.



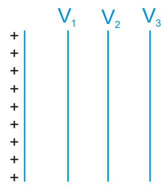
(ii) Line charge :

Equipotential surfaces have curved surfaces as that of coaxial cylinders of different radii.



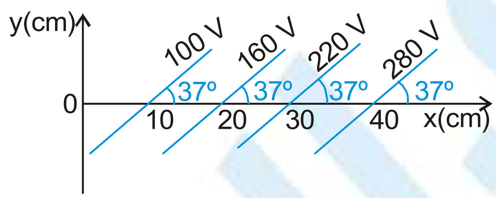
(iii) Uniformly charged large conducting / non conducting sheets:

Equipotential surfaces are parallel planes.



Note : In uniform electric field equipotential surfaces are always parallel planes.

Ex. Some equipotential surfaces are shown in figure. What can you say about the magnitude and the direction of the electric field ?



Sol. Here, we can say that the electric will be perpendicular to equipotential surfaces.

Also, $|\vec{E}| = \dfrac{\Delta V}{\Delta d}$

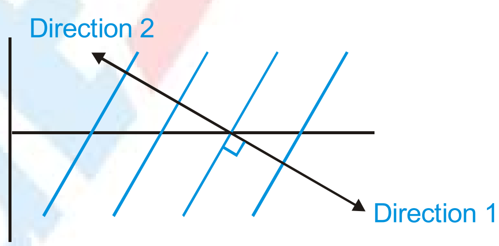
Where, ΔV = potential difference between two equipotential surfaces.

Δd = perpendicular distance between two equipotential surfaces.

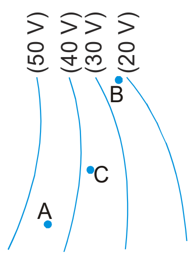
So $|\vec{E}| = \dfrac{60}{(10\sin 37^\circ)\times 10^{-2}} = 1000$ V/m

Now there are two perpendicular directions: either direction 1 or direction 2 as shown in figure, but since we know that in the direction of electric field, electric potential decreases, so the correct direction is direction 2.

Hence E = 1000 V/m, making an angle 127° with the x-axis



Ex. Figure shows some equipotential surfaces produce by some charges. At which point, the value of electric field is greatest?

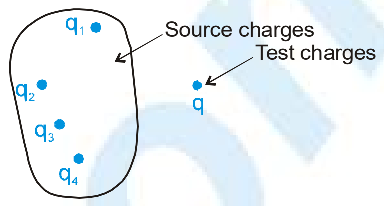


Sol. E is larger where equipotential surfaces are closer. ELOF are ⊥ to equipotential surfaces. In the figure, we can see that for point B, they are closer so E at point B is maximum.

ELECTROSTATIC POTENTIAL ENERGY

Electrostatic potential energy of a point charge due to many charges

The electrostatic potential energy of a point charge at a point in electric field is the work done in taking the charge from reference point (generally at infinity) to that point without change in kinetic energy



Its Mathematical formula is

$$U = W_{\infty\to P)ext}\big|_{\Delta K = 0} = qV = -\,W_{P\to\infty)ele}$$

Here, q is the charge whose potential energy is being calculated and V is the potential at its position due to the source charges.

Note : Always put q and V with sign.

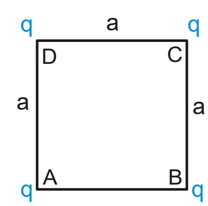
Properties

(i) Electric potential energy is a scalar quantity but may be positive, negative or zero.

(ii) Its unit is same as unit of work or energy i.e joule (in S.I. system).
Some times energy is also given in electron-volts.
Where, $1eV = 1.6 \times 10^{-19}$ J

(iii) Electric potential energy depends on reference point. (Generally Potential Energy at r= ∞ is taken zero)

Ex. The four identical charges (q each) are placed at the corners of a square of side a. Find the potential energy of one of the charges due to the remaining charges.



Sol. The electric potential of point A due to the charges placed at B, C and D is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{a} + \frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{2}a} + \frac{1}{4\pi\varepsilon_0}\frac{q}{a} = \frac{1}{4\pi\varepsilon_0}\left(2+\frac{1}{\sqrt{2}}\right)\frac{q}{a}$$

∴ Potential energy of the charge at A is $= qV = \frac{1}{4\pi\varepsilon_0}\left(2+\frac{1}{\sqrt{2}}\right)\frac{q^2}{a}$.

Ex. A particle of mass 40 mg and carrying a charge 5×10^{-9} C is moving directly towards a fixed positive point charge of magnitude 10^{-8} C. When it is at a distance of 10 cm from the fixed point charge it has speed of 50 cm/s. At what distance from the fixed point charge will the particle come momentarily to rest? Is the acceleration constant during the motion?

Sol. If the particle comes to rest momentarily at a distance r from the fixed charge, then from conservation of energy, we have ?

$$\frac{1}{2}mu^2 + \frac{1}{4\pi\varepsilon_0}\frac{Qq}{a} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r}$$

Substituting the given data, we get:

$$\frac{1}{2}\times 40 \times 10^{-6} \times \frac{1}{2}\times\frac{1}{2} = 9\times 10^{9}\times 5\times 10^{-8}\times 10^{-9}\left[\frac{1}{r}-10\right]$$

or, $\frac{1}{r} - 10 = \frac{5\times10^{-6}}{9\times5\times10^{-8}} = \frac{100}{9} \Rightarrow \frac{1}{r} = \frac{190}{9} \quad \Rightarrow r = \frac{9}{190}$ m

or, i.e., $r = 4.7 \times 10^{-2}$ m

As here, $F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$ So, acc. $= \frac{F}{m} \propto \frac{1}{r^2}$

i.e., Acceleration is not constant during the motion.

Ex. A proton moves from a large distance with a speed u m/s directly towards a free proton originally at rest. Find the distance of closest approach for the two protons in terms of mass of proton m and its charge e.

Sol. As here the particle at rest is free to move, when one particle approaches the other, due to electrostatic repulsion other will also start moving and so the velocity of first particle will decrease while of other will increase and at closest approach, both will move with same velocity. So, if v is the common velocity of each particle at closest approach, then by 'conservation of momentum' of the two proton system.

$$mu = mv + mv \quad \text{i.e.,} \quad v = \frac{1}{2}u$$

And by conservation of energy,

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

$$\Rightarrow \quad \frac{1}{2}mu^2 - m\left(\frac{u}{2}\right)^2 = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r} \quad [\text{as } v = \frac{u}{2}] \quad \Rightarrow \quad \frac{1}{4}mu^2 = \frac{e^2}{4\pi\varepsilon_0 r}$$

$$\Rightarrow \quad r = \frac{e^2}{\pi m\varepsilon_0 u^2}$$

Electrostatic potential energy of a system of charges

(This concept is useful when more than one charges move.)

It is the work done by an external agent against the internal electric field required to make a system of charges in a particular configuration from infinite separation without change in their kinetic energies.

Types of system of charges

(i) Point charge system

(ii) Continuous charge system.

Derivation for a system of point charges

(i) Keep all the charges at infinity. Now bring the charges one by one to its corresponding position and find work required. PE of the system is algebraic sum of all the works.

Let W_1 = Work done in bringing first charge.

W_2 = Work done in bringing second charge against force due to 1st charge.

W_3 = Work done in bringing third charge against force due to 1st and 2nd charge.

$PE = W_1 + W_2 + W_3 + \ldots\ldots$ (This will contain $\frac{n(n-1)}{2} = {}^nC_2$ terms)

(ii) Method of calculation (to be used in problems) :

U = sum of the interaction energies of the charges.

$= (U_{12} + U_{13} + ... + U_{1n}) + (U_{23} + U_{24} + ... + U_{2n}) + (U_{34} + U_{35} + ... + U_{3n}) ....$.

(iii) Method of calculation useful for symmetrical point charge systems.

Find PE of each charge due to rest of the charges.

If U_1 = PE of first charge due to all other charges.

$= (U_{12} + U_{13} + ... + U_{1n})$

U_2 = PE of second charge due to all other charges.

$= (U_{21} + U_{23} + ... + U_{2n})$ then U = PE of the system $= \dfrac{U_1 + U_2 + ...U_n}{2}$

Ex. Find out potential energy of the two point charge system having charges q_1 and q_2 separated by distance r.

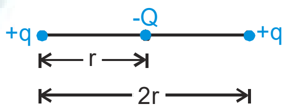
Sol. Let both the charges be placed at a very large separation initially.

Let, W_1 = work done in bringing charge q_1 in absence of $q_2 = q_1(V_f - V_i) = 0$

W_2 = work done in bringing charge q_2 in presence of $q_1 = q_2(V_f - V_i) = q_2(Kq_1/r - 0)$

$\therefore$ $PE = W_1 + W_2 = 0 + Kq_1q_2 / r = Kq_1q_2 / r$

Ex. Figure shows an arrangement of three point charges. The total potential energy of this arrangement is zero. Calculate the ratio $\dfrac{q}{Q}$.

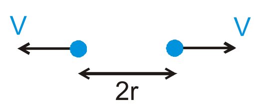


Sol. $$U_{sys} = \frac{1}{4\pi\varepsilon_0}\left[\frac{-qQ}{r} + \frac{(+q)(+q)}{2r} + \frac{Q(-q)}{r}\right] = 0$$

$$-Q + \frac{q}{2} - Q = 0 \quad \text{or} \quad 2Q = \frac{q}{2} \quad \text{or} \quad \frac{q}{Q} = \frac{4}{1}.$$

Ex. Two point charges, each of mass m and charge q are released when they are at a distance r from each other. What is the speed of each charged particle when they are at a distance 2r ?

Sol. According to momentum conservation, both the charge particles will move with same speed. Now applying energy conservation : →



$$0 + 0 + \frac{Kq^2}{r} = 2\,\frac{1}{2}\,mv^2 + \frac{Kq^2}{2r} \quad \Rightarrow \quad v = \sqrt{\frac{Kq^2}{2rm}}$$

Ex. Two charged particles each having equal charges 2×10^{-5} C are brought from infinity to within a separation of10 cm. Calculate the increase in potential energy during the process and the work required for this purpose.

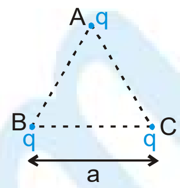
Sol. $\Delta U = U_f - U_i = U_f - 0 = U_f$

We have to simply calculate the electrostatic potential energy of the given system of charges

$$\therefore \quad \Delta U = U_f = \frac{1}{4\pi\varepsilon_0}\,\frac{q_1 q_2}{r} = \frac{9\times10^9 \times 2\times10^{-5} \times 2\times10^{-5} \times 100}{10}\ \text{J} = 36\ \text{J}$$

$\therefore$ Work required = 36 J = equal to change in potential energy of system

Ex. Three equal charges q each are placed at the corners of an equilateral triangle of side a.

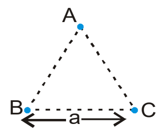


(i) Find out potential energy of charge system.

(ii) Calculate work required to decrease the side of triangle to a/2.

(iii) If the charges are released from the shown position and each of them has same mass m then find the speed of each particle when they lie on triangle of side 2a.

Sol. **(i) Method I** (Derivation)



Assume all the charges are at infinity initially.

Work done in putting charge q at corner A

$\Rightarrow \quad W_1 = q\,(v_f - v_i) = q\,(0 - 0)$

Since potential at A is zero in absence of charges, work done in putting q at corner B in presence of charge at A :

$$\Rightarrow \quad W_2 = \left(\frac{Kq}{a} - 0\right)q = \frac{Kq^2}{a}$$

Similarly work done in putting charge q at corner C in presence of charge at A and B.

$$\Rightarrow \quad W_3 = q(v_f - v_i) \quad = q\left[\left(\frac{Kq}{a} + \frac{Kq}{a}\right) - 0\right] \quad = \frac{2\,Kq^2}{a}$$

So, net potential energy PE = $W_1 + W_2 + W_3$

$$= 0 + \frac{Kq^2}{a} + \frac{2Kq^2}{a} \quad = \frac{3Kq^2}{a}$$

Method II (using direct formula):

$$U = U_{12} + U_{13} + U_{23} \quad = \frac{Kq^2}{a} + \frac{Kq^2}{a} + \frac{Kq^2}{a} = \frac{3Kq^2}{a}$$

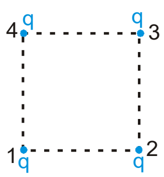
(ii) Work required to decrease the sides

$$W = U_f - U_i \quad = \frac{3Kq^2}{a/2} - \frac{3Kq^2}{a} = \frac{3Kq^2}{a} \text{ Joules}$$

(iii) Work done by electrostatic forces = Change is kinetic energy of particles.

$$U_i - U_f = K_f - K_i \;\Rightarrow\; \frac{3Kq^2}{a} - \frac{3Kq^2}{2a} = 3\left(\frac{1}{2}mv^2\right) - 0 \;\Rightarrow\; v = \sqrt{\frac{Kq^2}{am}}$$

Ex. Four identical point charges q each are placed at four corners of a square of side a. Find out potential energy of the charge system



Sol. **Method 1 (using direct formula) :**

$$U = U_{12} + U_{13} + U_{14} + U_{23} + U_{24} + U_{34}$$

$$= \frac{Kq^2}{a} + \frac{Kq^2}{a\sqrt{2}} + \frac{Kq^2}{a} + \frac{Kq^2}{a} + \frac{Kq^2}{a\sqrt{2}} + \frac{Kq^2}{a}$$

$$= \left[\frac{4Kq^2}{a} + \frac{2Kq^2}{a\sqrt{2}}\right] \quad = \frac{2Kq^2}{a}\left[2 + \frac{1}{\sqrt{2}}\right]$$

Method 2 [Using, $U = \frac{1}{2}(U_1 + U_2 + ...)$] **:**

U_1 = total P.E. of charge at corner 1 due to all other charges.

U_2 = total P.E. of charge at corner 2 due to all other charges.

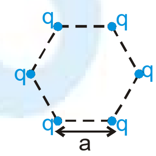
U_3 = total P.E. of charge at corner 3 due to all other charges.

U_4 = total P.E. of charge at corner 4 due to all other charges.

Since, due to symmetry, $U_1 = U_2 = U_3 = U_4$

$$U_{Net} = \frac{U_1 + U_2 + U_3 + U_4}{2} = 2U_1 = 2\left[\frac{Kq^2}{a} + \frac{Kq^2}{a} + \frac{Kq^2}{\sqrt{2}a}\right] = \frac{2Kq^2}{a}\left[2 + \frac{1}{\sqrt{2}}\right]$$

Ex. Six equal point charges q each are placed at six corners of a hexagon of side a. Find out potential energy of charge system



Sol. $$U_{Net} = \frac{U_1 + U_2 + U_3 + U_4 + U_5 + U_6}{2}$$

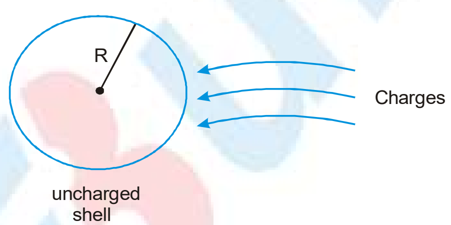
Due to symmetry $U_1 = U_2 = U_3 = U_4 = U_5 = U_6$ So $U_{net} = 3U_1 = \frac{3Kq^2}{a}\left[2 + \frac{2}{\sqrt{3}} + \frac{1}{2}\right]$

Derivation of electric potential energy for continuous charge system

This energy is also known as self energy.

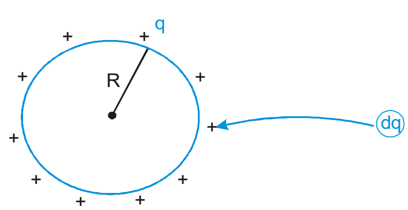
(i) Finding P.E., (Self Energy) of a uniformly Charged spherical shell

For this, lets use method 1 : Take an uncharged shell. Now bring charges one by one from infinity to the surface fo the shell. The work required in this process will be stored as potential Energy.



Suppose, we have given charge q to the sphere and now we are giving extra charge dq to it.

Work required to bring dq charge from infinity to the shell is



$$dW = (dq)\,(V_f - V_i)$$

$$\Rightarrow \quad dW = (dq)\left(\frac{Kq}{R} - 0\right) = \frac{Kq}{R}\,dq$$

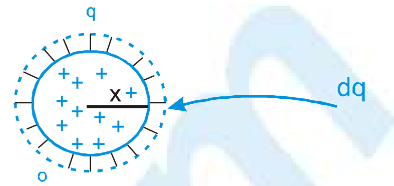
$\Rightarrow$ Total work required to give charge Q is $W = \int_{q=o}^{q=Q} \frac{Kq}{R}\,dq = \frac{KQ^2}{2R}$

This work will be stored in the form of P.E. (self energy)

So, P.E. of a charged spherical shell:

$$U = \frac{KQ^2}{2R}$$

(ii) Self energy of uniformly charged solid sphere



In this case we have to assemble a solid charged sphere. So we bring the charges one-by-one from infinity to the sphere so that the size of the sphere increases.

Suppose we have given charge q to the sphere, and its radius becomes 'x' .Now we are giving extra charge dq to it, which will increase its radius by 'dx'

∴ Work required to bring charge dq from infinity to the sphere

$$= dq\,(V_f - V_i) = (dq)\left(\frac{Kq}{x} - o\right) = \frac{Kqdq}{x}$$

∴ Total work required to give charge Q to the sphere

$$W = \int \frac{Kqdq}{x}, \quad \text{where} \quad q = \rho\left(\frac{4}{3}\pi x^3\right)$$

& $$dq = \rho\,(4\,\pi x^2\,dx) \Rightarrow W = \int_{x=o}^{x=R} K\frac{\rho\left(\frac{4}{3}\pi x^3\right)\rho\,(4\pi x^2 dx)}{x}$$

Solving, well get:

$$W = \frac{3}{5}\frac{KQ^2}{R} = U_{self}\ \text{(for a solid sphere)}$$

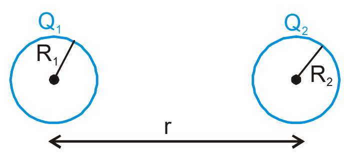
Ex. A spherical shell of radius R with uniform charge q is expanded to a radius 2R. Find the work performed by the electric forces and external agent against electric forces in this process.

Sol. $$W_{ext} = U_f - U_i = \frac{q^2}{16\pi\varepsilon_0 R} - \frac{q^2}{8\pi\varepsilon_0 R} = -\frac{q^2}{16\pi\varepsilon_0 R}$$

$$W_{elec} = U_i - U_f = \frac{q^2}{8\pi\varepsilon_0 R} - \frac{q^2}{16\pi\varepsilon_0 R} = \frac{q^2}{16\pi\varepsilon_0 R}$$

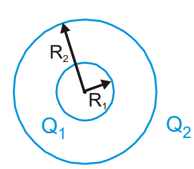
Ex. Two non-conducting hollow uniformly charged spheres of radii R_1 and R_2 with charge Q_1 and Q_2 respectively are placed at a distance r.

Find out total energy of the system.



Sol. $$U_{total} = U_{self} + U_{Interaction} = \frac{Q_1^2}{8\pi\varepsilon_0 R_1} + \frac{Q_2^2}{8\pi\varepsilon_0 R_2} + \frac{Q_1Q_2}{4\pi\varepsilon_0 r}$$

Ex. Two concentric spherical shells of radius R_1 and R_2 ($R_2 > R_1$) are having uniformly distributed charges Q_1 and Q_2 respectively. Find out total energy of the system.



Sol. $$U_{total} = U_{self\,1} + U_{self\,2} + U_{Interaction} = \frac{Q_1^2}{8\pi\varepsilon_0 R_1} + \frac{Q_2^2}{8\pi\varepsilon_0 R_2} + \frac{Q_1Q_2}{4\pi\varepsilon_0 R_2}$$

Energy density

Def : Energy density is defined as energy stored in unit volume in any electric field. Its mathematical formula is given as following :

$$\text{Energy density} = \frac{1}{2}\varepsilon E^2$$

where E = electric field intensity at that point

. $\varepsilon = \varepsilon_0\varepsilon_r$ electric permittivity of medium

Ex. Find out energy stored in an imaginary cubical volume of side a in front of a infinitely large non-conducting sheet of uniform charge density σ.

Sol. Energy stored :

$$U = \int \frac{1}{2}\varepsilon_0 E^2\, dV \text{ ; where dV is small volume}$$

$\therefore$ $U = \frac{1}{2}\varepsilon_0 E^2 \int dV$

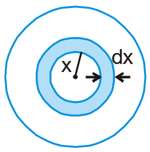
✈ E is constant .

$\therefore$ $U = \frac{1}{2}\varepsilon_0 \frac{\sigma^2}{4\varepsilon_0^2} \,.\, a^3 .= \frac{\sigma^2 a^3}{8\varepsilon_0}$

Ex. Find out energy stored in the electric field of uniformly charged thin spherical shell of total charge Q and radius R.

Sol. We know that electric field inside the shell is zero, so the energy is stored only outside the shell, which can be calculated by using energy density formula.

$$U_{self} = \int_{x=R}^{x\to\infty} \frac{1}{2}\varepsilon_0 E^2 dV$$



Consider an elementary shell of thickness dx and radius x (x > R).

Volume of the shell = $(4\pi x^2 dx) = dV$

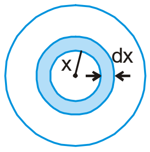
$$U = \int_R^{\infty} \frac{1}{2}\varepsilon_0 \left[\frac{KQ}{x^2}\right]^2 .\, 4\pi x^2\, dx = \frac{1}{2}\varepsilon_0\, K^2Q^2\, 4\pi \int_R^{\infty} \frac{1}{x^2}\, dx$$

$$= \frac{4\pi\varepsilon_0}{2} \frac{Q^2}{(4\pi\varepsilon_0)^2} \,.\left(\frac{1}{R}\right) = \frac{Q^2}{8\pi\varepsilon_0 R} = \frac{KQ^2}{2R}.$$

Ex. Find out energy stored inside a solid non-conducting sphere of total charge Q and radius R. [Assume charge is uniformly distributed in its volume.]

Sol. We can consider solid sphere to be made of large number of concentric spherical shells. Also electric field intensity at the location of any particular shell is constant.

$$\therefore \quad U_{inside} = \int_0^R \frac{1}{2}\varepsilon_0 E^2 dV$$



Consider an elementary shell of thickness dx and radius x.

Volume of the shell = $(4\pi x^2 dx)$

$$U_{inside} = \int_0^R \frac{1}{2}\varepsilon_0 \left[\frac{KQx}{R^3}\right]^2 .\, 4\pi x^2\, dx = \frac{1}{2}\varepsilon_0 \frac{K^2Q^2 4\pi}{R^6} \int_0^R x^4 dx$$

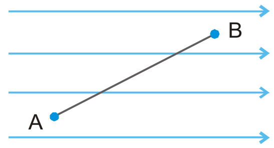
$$= \frac{4\pi\varepsilon_0}{2R^6} \frac{Q^2}{(4\pi\varepsilon_0)^2} \,.\, \frac{R^5}{5} = \frac{Q^2}{40\pi\varepsilon_0 R} = \frac{KQ^2}{10R}.$$

Relation between Electric Field Intensity and electric Potential

(i) For uniform electric field

Potential difference between two points A and B

$V_B - V_A = -\vec{E} \cdot \vec{AB}$



(ii) Non uniform electric field

$$E_x = -\frac{\partial V}{\partial x}, E_y = -\frac{\partial V}{\partial y}, E_z = -\frac{\partial V}{\partial z}$$

$$\Rightarrow \quad \vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$$

$$= -\left[\hat{i}\frac{\partial}{\partial x}V + \hat{j}\frac{\partial}{\partial y}V + \hat{k}\frac{\partial}{\partial z}V\right] = -\left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right]V = -\nabla V = -\text{grad } V$$

Where, $\frac{\partial V}{\partial x}$ = derivative of V with respect to x (keeping y and z constant)

$\frac{\partial V}{\partial y}$ = derivative of V with respect to y (keeping z and x constant)

$\frac{\partial V}{\partial z}$ = derivative of V with respect to z (keeping x and y constant)

If electric potential and electric field depends only on one coordinate, say r

(i) $\vec{E} = -\frac{\partial V}{\partial r}\hat{r}$

where, $\hat{r}$ is a unit vector along increasing r.

(ii) $\int dV = -\int \vec{E} \cdot \vec{dr}$

$$\Rightarrow \quad V_B - V_A = -\int_{r_A}^{r_B} \vec{E} \cdot \vec{dr}$$

$\vec{dr}$ is along the increasing direction of r.

(iii) The potential of a point

$$V = -\int_{\infty}^{r} \vec{E} \cdot \vec{dr}$$

Ex. A uniform electric field is along x – axis . The potential difference $V_A - V_B = 10$ V is between two points A (2m , 3m) and B (4m, 8m). Find the electric field intensity.

Sol. $E = \frac{\Delta V}{\Delta d} = \frac{10}{2} = 5\ V/m.$ (It is along + ve x-axis)

Ex. $V = x^2 + y$, Find $\vec{E}$.

Sol. $\frac{\partial V}{\partial x} = 2x$, $\frac{\partial V}{\partial y} = 1$ and $\frac{\partial V}{\partial z} = 0$

$\vec{E} = -\left(\hat{i}\frac{\partial V}{\partial x} + \hat{j}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right) = -(2x\,\hat{i} + \hat{j})$ $\therefore$ Electric field is non-uniform.

Ex. For given $\vec{E} = 2x\hat{i} + 3y\hat{j}$, find the potential at (x, y) if V at origin is 5 volts.

Sol. $\int_5^v dV = -\int \vec{E}\cdot\overline{dr} = -\int_0^x E_x dx - \int_0^y E_y dy$

$\Rightarrow \quad V - 5 = -\frac{2x^2}{2} - \frac{3y^2}{2} \quad \Rightarrow \quad V = -\frac{2x^2}{2} - \frac{3y^2}{2} + 5.$

Electric Dipole

If two point charges, equal in magnitude 'q' and opposite in sign separated by a distance 'a' such that the distance of field point r>>a, the system is called a dipole. The electric dipole moment is defined as a vector quantity having magnitude p = (q × a) and direction from negative charge to positive charge.

ETOOS KEY POINTS

[In chemistry, the direction of dipole moment is assumed to be from positive to negative charge.] The C.G.S unit of electric dipole moment is **debye** which is defined as the dipole moment of two equal and opposite point charges each having charge 10^{-10} Franklin and separation of 1 Å, i.e.,

$1 \text{ debye (D)} = 10^{-10} \times 10^{-8} = 10^{-18}\ \text{Fr} \times \text{cm}$

or $\quad 1\ D = 10^{-18} \times \frac{C}{3\times10^9} \times 10^{-2}\ m = 3.3 \times 10^{-30}\ C \times m.$

S.I. Unit is coulomb × metre = C . m

Ex. A system has two charges $q_A = 2.5 \times 10^{-7}$ C and $q_B = -2.5 \times 10^{-7}$ C located at points A : (0, 0, – 0.15 m) and B ; (0, 0, + 0.15 m) respectively. What is the net charge and electric dipole moment of the system ?

Sol. Net charge = $2.5 \times 10^{-7} - 2.5 \times 10^{-7} = 0$

Electric dipole moment,

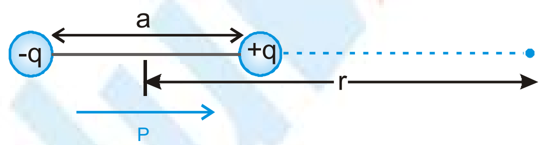
P = (Magnitude of charge) × (Separation between charges)

$= 2.5 \times 10^{-7}[0.15 + 0.15]$ C m $\quad = 7.5 \times 10^{-8}$ C m

The direction of dipole moment is from B to A.

ELECTRIC FIELD INTENSITY DUE TO DIPOLE

(i) At the axial point :-



$$\vec{E} = \frac{Kq}{\left(r-\frac{a}{2}\right)^2} - \frac{Kq}{\left(r+\frac{a}{2}\right)^2} \text{ (along the } \vec{P}) = \frac{Kq(2ra)}{\left(r^2-\frac{a^2}{4}\right)^2}\hat{P}$$

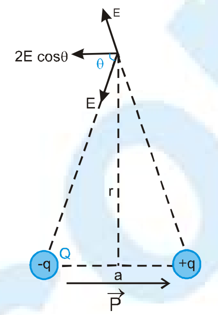
If r >> a then,

$$\vec{E} = \frac{Kq\,2ra}{r^4}\hat{P} = \frac{2K\vec{P}}{r^3},$$

As the direction of electric field at axial position is along the dipole moment ($\vec{P}$)

So, $$\vec{E}_{axial} = \frac{2K\vec{P}}{r^3}$$

(ii) Electric field at perpendicular Bisector (Equatorial Position)



$E_{net} = 2\,E\cos\theta$ (along $-\hat{P}$)

$$\vec{E}_{net} = 2\left(\frac{Kq}{\left(\sqrt{r^2+\left(\frac{a}{2}\right)^2}\right)^2}\right)\frac{\frac{a}{2}}{\sqrt{r^2+\left(\frac{a}{2}\right)^2}}(-\hat{P}) = \frac{Kqa}{\left(r^2+\left(\frac{a}{2}\right)^2\right)^{3/2}}(-\hat{P})$$

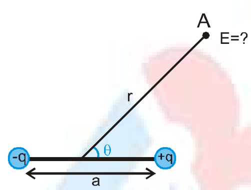
If r >> a then

$$\vec{E}_{net} = \frac{KP}{r^3}(-\hat{P})$$

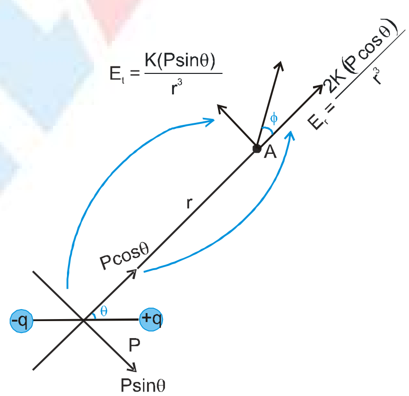
As the direction of $\vec{E}$ at equatorial position is opposite of $\vec{P}$ so we can write in vector form:

$$\vec{E}_{eqt} = -\frac{K\vec{P}}{r^3}$$

(iii) Electric field at general point (r, θ) :



For this, let's resolve the dipole moment $\vec{P}$ into components.



One component is along radial line (=P cosθ) and other component is $\perp_r$ to the radial line (=Psinθ)

From the given figure $E_{net} = \sqrt{E_r^2 + E_t^2} = \sqrt{\left(\frac{2KP\cos\theta}{r^3}\right)^2 + \left(\frac{KP\sin\theta}{r^3}\right)^2} = \frac{KP}{r^3}\sqrt{1+3\cos^2\theta}$

$$\tan\phi = \frac{E_t}{E_r} = \frac{\frac{KP\sin\theta}{r^3}}{\frac{2KP\cos\theta}{r^3}} = \frac{\tan\theta}{2}$$

$$E_{net} = \frac{KP}{r^3}\sqrt{1+3\cos^2\theta} \quad ; \quad \tan\phi = \frac{\tan\theta}{2}$$

Ex. The electric field due to a short dipole at a distance r, on the axial line, from its mid point is the same as that of electric field at a distance r', on the equatorial line, from its mid-point. Determine the ratio $\frac{r}{r'}$.

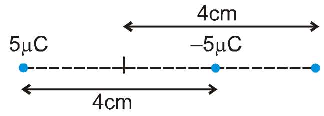
Sol. $\frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r'^3}$ or $\frac{2}{r^3} = \frac{1}{r'^3}$ or $\frac{r^3}{r'^3} = 2$ or, $\frac{r}{r'} = 2^{1/3}$

Ex. Two charges, each of 5 µC but opposite in sign, are placed 4 cm apart. Calculate the electric field intensity of a point that is at a distance 4 cm from the mid point on the axial line of the dipole.

Sol. We cannot use formula of short dipole here because distance of the point is comparable to the distance between the two point charges.

$q = 5 \times 10^{-6}$ C, $a = 4 \times 10^{-2}$ m, $r = 4 \times 10^{-2}$ m

$$E_{res} = E_+ + E_- = \frac{K(5\mu C)}{(2cm)^2} - \frac{K(5\mu C)}{(6cm)^2} = \frac{144}{144\times10^{-8}}\ NC^{-1} = 10^8\ N\,C^{-1}$$



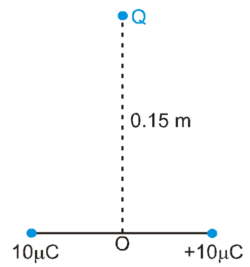
Ex. Two charges ± 10 µC are placed 5×10^{-3} m apart as shown in figure. Determine the electric field at a point Q which is 0.15 m away from O, on the equatorial line.

Sol. In the given problem, r >> a

$\therefore$ $$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3} = \frac{1}{4\pi\varepsilon_0}\frac{q(a)}{r^3}$$

or $$E = 9 \times 10^9 \times \frac{10\times10^{-6}\times5\times10^{-3}}{0.15\times0.15\times0.15}\ NC^{-1}$$

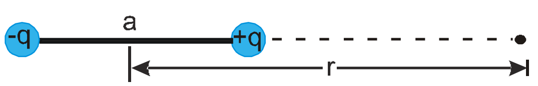
$= 1.33 \times 10^5\ NC^{-1}$



ELECTRIC POTENTIAL DUE TO A SMALL DIPOLE

(i) Potential at axial position :

$$V = \frac{Kq}{\left(r-\frac{a}{2}\right)} + \frac{K(-q)}{\left(r+\frac{a}{2}\right)}$$



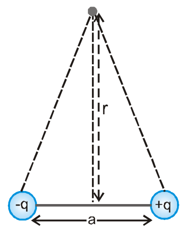
$$V = \frac{Kqa}{\left(r^2 - \left(\frac{a}{2}\right)^2\right)}$$

If r >> a then

$V = \frac{Kqa}{r^2}$; where, qa = p $\therefore$ $V_{axial} = \frac{KP}{r^2}$

(ii) Potential at equatorial position :

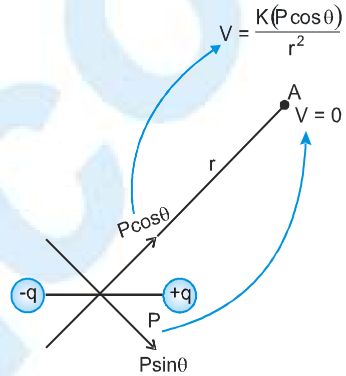
$$V = \frac{Kq}{\sqrt{r^2 + \left(\frac{a}{2}\right)^2}} + \frac{K(-q)}{\sqrt{r^2 + \left(\frac{a}{2}\right)^2}} = 0$$



$$V_{eqt} = 0$$

(iii) Potential at general point (r,θ) :

Lets resolve the dipole moment $\vec{P}$ into components: Pcosθ along radial line and Psinθ $\perp_r$ to the radial line



For the Pcosθ component, the point A is an axial point,

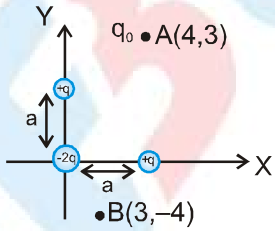
So, potential at A due to Pcosθ $= \frac{K(P\cos\theta)}{r^2}$

And for Psinθ component, the point A is an equatorial point,
So potential at A due to Psinθ = 0

$$V_{net} = \frac{K(P\cos\theta)}{r^2}$$

$$\therefore \quad V = \frac{K\left(\vec{P}\cdot\vec{r}\right)}{r^3}$$

Ex. **(i)** Find potential at point A and B due to the small charge - system fixed near origin.(Distance between the charges is negligible).
(ii) Find work done to bring a test charge q_0 from point A to point B, slowly. All parameters are in S.I. units.



Sol. **(i)** Dipole moment of the system is

$$\vec{P} = (qa)\,\hat{i} + (qa)\,\hat{j}$$

Potential at point A due to the dipole

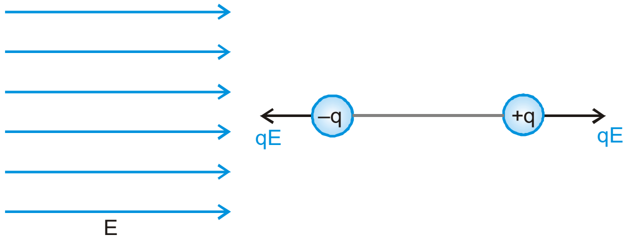
$$V_A = K\frac{(\vec{P}\cdot\vec{r})}{r^3} = \frac{K[(qa)\hat{i} + (qa)\hat{j}]\cdot(4\hat{i} + 3\hat{j})}{5^3} = \frac{k(qa)}{125}(7)$$

$$\Rightarrow \quad V_B = \frac{K\left[(qa)\hat{i} + (qa)\hat{j}\right]\cdot\left(3\hat{i} - 4\hat{j}\right)}{(5)^3} = \frac{-K(qa)}{125}$$

(ii) $W_{A\to B} = U_B - U_A = q_0(V_B - V_A) = q_0\left[-\frac{K(qa)}{125} - \left(\frac{K(qa)(7)}{125}\right)\right] \Rightarrow W_{A\to B} = \frac{-K\,q\,q_0 a}{125}(8)$

DIPOLE IN UNIFORM ELECTRIC FIELD

(i) Dipole is placed along electric field :



In this case, $F_{net} = 0$, $\tau_{net} = 0$, so it is an equilibrium state. And it is a stable equilibrium position.

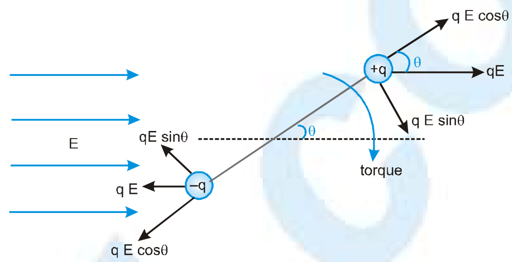
(ii) If the dipole is placed at angle θ from $\vec{E}$:-

In this case $F_{net} = 0$ but

Net torque $\tau = (qE\sin\theta)$ (a)

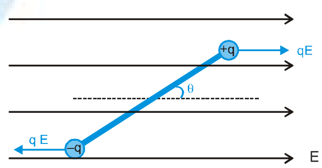
Here qa = P $\Rightarrow$ $\tau = PE\sin\theta$

in vector form : $\vec{\tau} = \vec{P} \times \vec{E}$



Ex. A dipole is formed by two point charge –q and +q, each of mass m, and both the point charges are connected by a rod of length ● and mass m. This dipole is placed in uniform electric field $\vec{E}$. If the dipole is disturbed by a small angle θ from stable equilibrium position, prove that its motion will be almost SHM. Also find its time period.

Sol.



If the dipole is disturbed by θ angle,

$\tau_{net} = -PE\sin\theta$ (Here – ve sign indicates that direction of torque is opposite to θ)

If θ is very small, $\sin\theta \approx \theta$

$\therefore \quad \tau_{net} = -(PE)\theta$

$\tau_{net} \propto (-\theta)$ so motion will be almost SHM & C = PE (where, P = q●)

$\Rightarrow \quad T = 2\pi\sqrt{\dfrac{I}{C}}$

$$\therefore \quad T = 2\pi\sqrt{\frac{\frac{ml}{12} + 2m\left(\frac{1}{2}\right)^2}{P.E}} = 2\pi\sqrt{\frac{\frac{ml}{12} + \frac{ml^2}{2}}{qlE}} = 2\pi\sqrt{\frac{7ml^2}{12qlE}} = 2\pi\sqrt{\frac{7ml}{12qE}} = T = \pi\sqrt{\frac{7ml}{3qE}}$$

(iii) Potential energy of a dipole placed in uniform electric field :

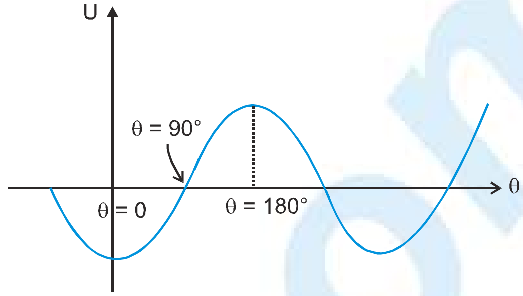
$$U_B - U_A = -\int_A^B \vec{F}.\vec{dr} \qquad \text{(for translational motion)}$$

Here, $$U_B - U_A = -\int_A^B \vec{\tau}.\vec{d\theta} \qquad \text{(for rotational motion)}$$

In the case of dipole, at θ = 90° , P.E. is assumed to be zero.

$U_\theta - U_{90^\circ} = -\int_{\theta=90^\circ}^{\theta=\theta} (-PE\sin\theta)(d\theta)$ (As the direction of torque is opposite of θ)

$U_\theta - 0 = -PE\cos\theta$



θ = 90° is chosen as reference,
so that the lower limit comes out to be zero.

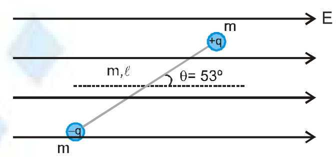
$$U_\theta = -\vec{P}\cdot\vec{E}$$

From the potential energy curve, we can conclude :

(i) At θ = 0, there is minimum of P.E. so it is a stable equilibrium position.

(ii) At θ = 180° , there is maxima of P.E. so it is a position of unstable equilibrium.

Ex. Two point masses of mass m and equal and opposite charge of magnitude q are attached on the corners of a non-conducting uniform rod of mass m and the system is released from rest in uniform electric field E as shown in figure from θ = 53°



(i) Find angular acceleration of the rod just after releasing

(ii) What will be angular velocity of the rod when it passes through stable equilibrium.

(iii) Find work required to rotate the system by 180°.

Sol. **(i)** $\tau_{net} = PE\sin 53^\circ = I\alpha$

$$\therefore \quad \alpha = \frac{(ql)E\left(\frac{4}{5}\right)}{\frac{ml^2}{12} + m\left(\frac{l}{2}\right)^2 + m\left(\frac{l}{2}\right)^2} = \frac{48qE}{35\ ml}$$

(ii) From energy conservation :

$K_i + U_i = K_f + U_f$

$$\therefore \quad 0 + (-PE\cos 53^\circ) = \frac{1}{2}I\omega^2 + (-PE\cos 0^\circ)$$

where $I = \frac{ml^2}{12} + m\left(\frac{l}{2}\right)^2 + m\left(\frac{l}{2}\right)^2 = \frac{7ml^2}{12}$ $\quad\therefore\quad \frac{1}{2}I\omega^2 = PE(1-3/5) = \frac{2}{5}PE$

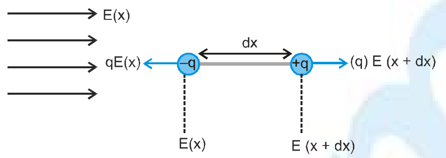
$$\therefore \quad \frac{1}{2}\times\frac{7ml^2}{12}\times\omega^2 = \frac{2}{5}q\bullet E \text{ or } \qquad \omega = \sqrt{\frac{48qE}{35\ ml}}$$

(iii) ✈ $W_{ext} = U_f - U_i$

$\therefore \quad W_{ext} = (-PE\cos(180^\circ + 53^\circ)) - (-PE\cos 53^\circ)$

or $\quad W_{ext} = (q\bullet)E\left(\frac{3}{5}\right) + (q\bullet)E\left(\frac{3}{5}\right) \quad\Rightarrow\quad W_{ext} = \left(\frac{6}{5}\right)q\bullet E$

Dipole in Non-uniform Electric Field



If the dipole is placed along $\vec{E}$, (shown in figure)

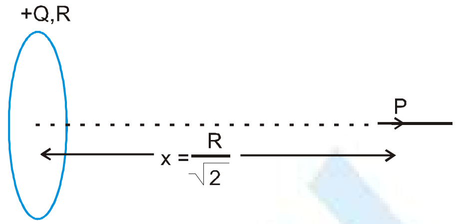
Then, Net force on the dipole :

$$F_{net} = q\,E(x+dx) - q\,E(\mathbf{x})$$

$$F_{net} = q\,\frac{E(x+dx)-E(x)}{dx}\,(dx) \quad ; \quad \text{here } (q\,(dx) = P)$$

$$\therefore \quad F_{net} = P\left(\frac{dE}{dx}\right)$$

Ex. A short dipole is placed on the axis of a uniformly charged ring (total charge –Q, radius R) at a distance $\frac{R}{\sqrt{2}}$ from centre of ring as shown in figure. Find the Force on the dipole due to the ring



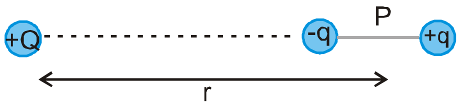
Sol. ✈ $F = P\left(\frac{dE}{dx}\right)$

$$\therefore \quad F = P\,\frac{d}{dx}\left(\frac{KQx}{(R^2+x^2)^{3/2}}\right) \quad ; \quad \left(\text{at } x = \frac{R}{\sqrt{2}}\right)$$

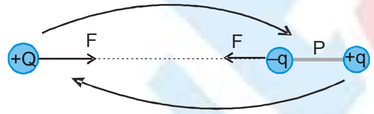
Solving we get, $F = 0$

Force between a dipole and a point charge

Ex. A short dipole of dipole moment P is placed near a point charge Q as shown in figure. Find force on the dipole due to the point charge



Sol.



Force on the point charge due to the dipole

$$F = (Q)\,E_{dipole}$$

$$F = (Q)\left(\frac{2KP}{r^3}\right) \text{ (towards right)}$$

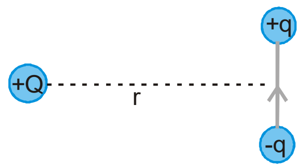
From action reaction concept, force on the dipole due to point charge will be equal to the force on charge due to dipole

$$F = \frac{2KPQ}{r^3} \text{ (towards left)}$$

is force on dipole due to point Charge.

FORCE BETWEEN TWO DIPOLES

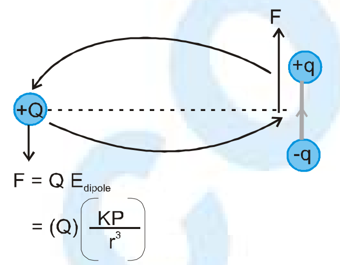
Ex. A short dipole of dipole moment P is placed near a point charge Q as shown in figure. Find force on the dipole due to the point charge.



Sol. Force on the point charge due to dipole

$$F = (Q)\,(E_{dipole})$$

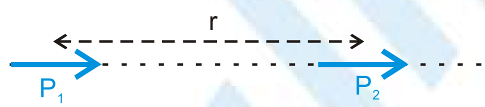
$$F = (Q)\left(\frac{KP}{r^3}\right)(\downarrow)$$



So force on the dipole due to the point charge will also be

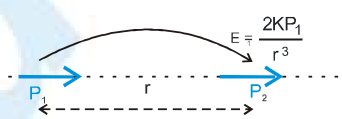
$$F = \left(\frac{KPQ}{r^3}\right)(\uparrow)$$ (but in opposite direction) as shown

Ex. Find force on short dipole P_2 due to short dipole P_1 if they are placed at a distance r a part as shown in figure.



Sol. Force on P_2 due to P_1

$$F_2 = (P_2)\left(\frac{dE_1}{dr}\right)$$

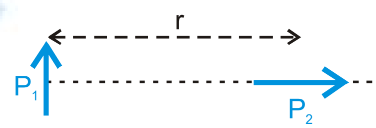


$\therefore \quad F_2 = (P_2)\left(\frac{d}{dr}\left(\frac{2kP_1}{r^3}\right)\right)$

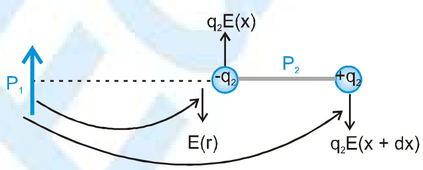
or $\quad F_2 = -\dfrac{6KP_1P_2}{r^4}$

Here – sign indicates that this force will be attractive (opposite to r)

Ex. Find force on short dipole P_2 due to short dipole P_1 if they are placed a distance r apart as shown in figure.



Sol.



$$F_{net} = q_2 E(x + dx) - q_2 E(x)$$

$$F_{net} = q_2 \left(\frac{E(x + dx) - E(x)}{dx} \right) dx \qquad \text{or} \qquad F_{net} = (P_2)\left(\frac{dE}{dx}\right)$$

(Usually this formula is valid when the dipole is placed along $\vec{E}$. However, in this case also, we are getting the same formula)

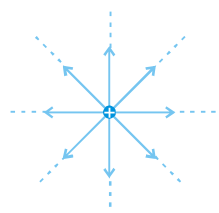
$$\therefore F_{net} = (P_2)\left(\frac{d}{dr}\left(\frac{KP_1}{r^3}\right)\right) \quad \Rightarrow \quad F_{net} = \frac{3KP_1P_2}{r^4} \text{ (in magnitude) \& (direction upwards)}$$

Electric Lines of Force (ELOF)

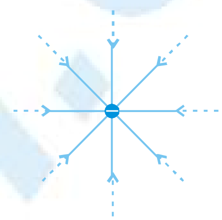
The line of force in an electric field is an imaginary line, the tangent to which at any point on it represents the direction of electric field at the given point.

Properties

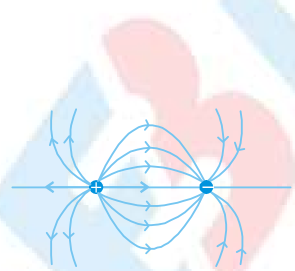
(i) Line of force originates out from a positive charge and terminates on a negative charge. If there is only one positive charge, then lines starts from positive charge and terminates at ∞. If there is only one negative charge, then lines starts from ∞ and terminates at negative charge.



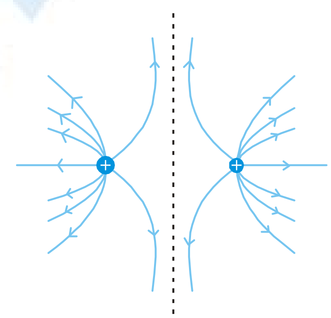
ELOF of Isolated positive charge



ELOF of Isolated negative charge

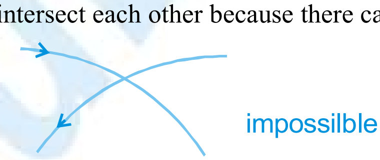


ELOF due to positive and negative charges

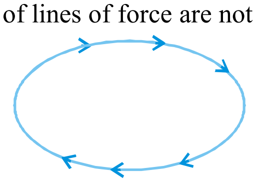


ELOF due to two positive charges

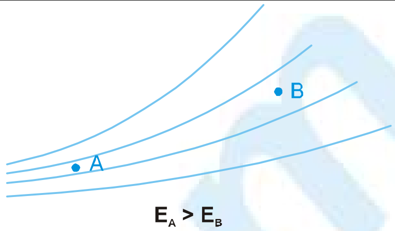
(ii) Two lines of force never intersect each other because there cannot be two directions of $\vec{E}$ at a single Point



(iii) Electric lines of force produced by static charges do not form closed loop.
If lines of force make a closed loop, then work done to move a +q charge along the loop will be non-zero. So it will not be conservative field. So these type of lines of force are not possible in electrostatics.



(iv) The Number of lines per unit area (line density) represents the magnitude of electric field.
If lines are dense $\Rightarrow$ E will be more
If Lines are rare $\Rightarrow$ E will be less
and if E = O, no line of force will be found there



(v) Number of lines originating (terminating) at a charge is proportional to the magnitude of charge

Ex. If number of electric lines of force from charge q are 10, then find out number of electric lines of force from 2q charge.

Sol. No. of ELOF $\propto$ charge

$10 \propto q$

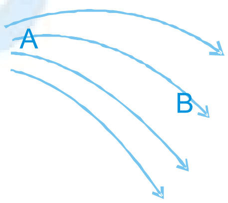
$\Rightarrow \quad 20 \propto 2q$

So, number of ELOF will be 20.

(vi) Electric lines of force end or start perpendicularly on the surface of a conductor.

(vii) Electric lines of force never enter into conductors.

Ex. Some electric lines of force are shown in figure. For points A and B



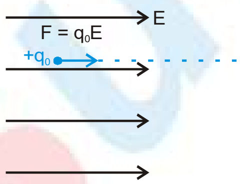
(A) $E_A > E_B$ (B) $E_B > E_A$
(C) $V_A > V_B$ (D) $V_B > V_A$

Sol. Lines are more dense at A, so $E_A > E_B$ In the direction of Electric field, potential decreases so $V_A > V_B$

Ex. If a charge is released in electric field, will it follow lines of force?

Sol. **Case I**

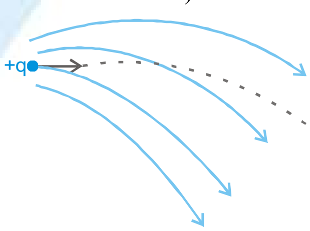
If lines of force are parallel (in uniform electric field) :-



In this type of field, if a charge is released, force on it will be q_oE and its direction will be along $\vec{E}$. So the charge will move in a straight line , along the lines of force.

Case II

If lines of force are curved (in non-uniform electric field) :-



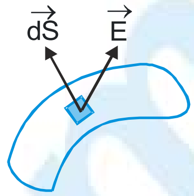
The charge will not follow lines of force

Ex. A charge + Q is fixed at a distance d in front of an infinite metal plate. Draw the lines of force indicating the directions clearly.

Sol. There will be induced charge on two surfaces of conducting plate, so ELOF will start from +Q charge and terminate at conductor and then will again start from other surface of conductor.

Electric Flux

Consider some surface in an electric field $\vec{E}$. Let us select a small area element $\vec{dS}$ on this surface. The electric flux of the field over the area element is given by $d\phi_E = \vec{E}.\vec{dS}$



Direction of $\vec{dS}$ is normal to the surface. It is along $\hat{n}$

or $\quad d\phi_E = EdS\cos\theta$

or $\quad d\phi_E = (E\cos\theta)\,dS$

or $\quad d\phi_E = E_n\,dS$

where E_n is the component of electric field in the direction of $\vec{dS}$.

The electric flux over the whole area is given by $\phi_E = \int_S \vec{E}.\vec{dS} = \int_S E_n dS$

If the electric field is uniform over that area then $\phi_E = \vec{E}\cdot\vec{S}$

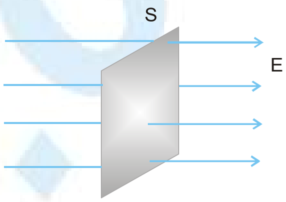
Special Cases

Case I If the electric field is normal to the surface,

then angle of electric field $\vec{E}$ with normal will be zero

So $\phi = ES\cos 0$

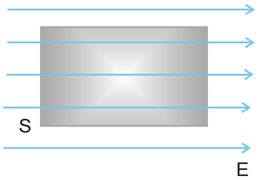
or $\phi = ES$



Case II If electric field is parallel of the surface (grazing),

then angle made by $\vec{E}$ with normal = 90º

So $\phi = ES\cos 90º = 0$



Physical Meaning

The electric flux through a surface inside an electric field represents the total number of electric lines of force crossing the surface. It is a property of electric field

Unit

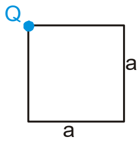
(i) The SI unit of electric flux is $Nm^2\,C^{-1}$ (Gauss) or $J\,m\,C^{-1}$.

(ii) Electric flux is a scalar quantity. (It can be positive, negative or zero)

Ex. The electric field in a region is given by $\vec{E} = \frac{3}{5}E_0\hat{i} + \frac{4}{5}E_0\hat{j}$ with $E_0 = 2.0 \times 10^3$ N/C. Find the flux of this field through a rectangular surface of area $0.2m^2$ parallel to the Y–Z plane.

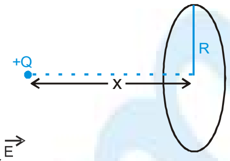
Sol. $\phi_E = \vec{E}\cdot\vec{S} = \left(\frac{3}{5}E_0\hat{i} + \frac{4}{5}E_0\hat{j}\right).\left(0.2\hat{i}\right) = 240\frac{N-m^2}{C}$

Ex. A point charge Q is placed at the corner of a square of side a, then find the flux through the square.

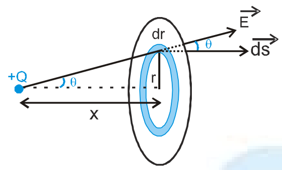


Sol. The electric field due to Q at any point of the square will be along the plane of square and the electric field lines are perpendicular to square ; so $\phi = 0$.
In other words we can say that no line is crossing the square so, flux = 0.

Ex. Find the electric flux due to a point charge 'Q' through the circular region of radius R if the charge is placed on the axis of ring at a distance x.



Sol.

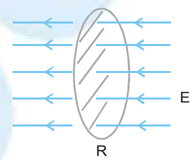


We can divide the circular region into small rings.
Lets take a ring of radius r and width dr.
Flux through this small element
$d\phi = E\, ds \cos\theta$

$$\therefore \quad \phi_{net} = \int E\, ds \cos\theta = \int_{r=o}^{r=R} \frac{KQ}{(x^2+r^2)} (2\pi r\, dr) \left(\frac{x}{\sqrt{x^2+r^2}}\right) = \frac{Q}{2\epsilon_0}\left[1 - \frac{x}{\sqrt{x^2+R^2}}\right]$$

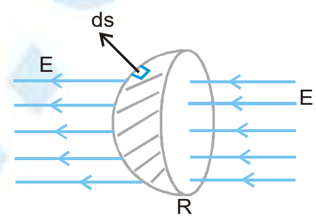
Case-III

Curved surface in uniform electric field
Suppose a circular surface of radius R is placed in a uniform electric field as shown.



Flux passing through the surface $\phi = E(\pi R^2)$

(ii) Now suppose, a hemispherical surface, is placed in the electric field. Flux through hemispherical surface:

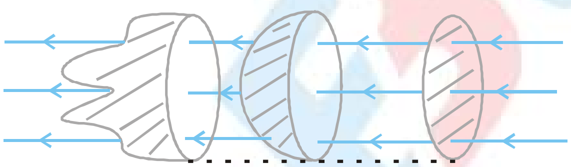


$\phi = \int E ds \cos\theta$
$\phi = E \int ds \cos\theta$
where, $\int ds \cos\theta$ is
projection of the spherical surface Area on base.

& $\int ds \cos\theta = \pi R^2$

So, $\phi = E(\pi R^2)$ = same Ans. as in previous case
So, we can conclude that

If the number of electric field lines passing through two surfaces are same, then flux passing through these surfaces will also be same, irrespective of the shape of surface

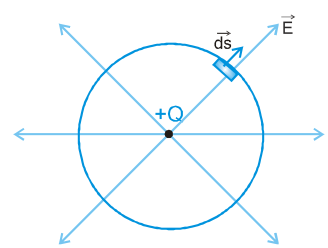


$$\phi_1 = \phi_2 = \phi_3 = E(\pi R^2)$$

Case IV

Flux through a closed surface

Suppose there is a spherical surface and a charge 'q' is placed at centre.
$\therefore$ Flux through the spherical surface



$$\phi = \int \vec{E} \cdot \vec{ds} = \int E\, ds \quad (\text{as } \vec{E} \text{ is along } \vec{ds} \text{ (normal)})$$

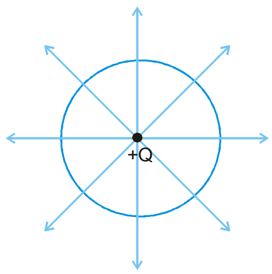
$$\therefore \quad \phi = \frac{1}{4\pi\varepsilon_o}\frac{Q}{R^2}\int ds \qquad \text{where, } \int ds = 4\pi R^2$$

$$\phi = \left(\frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}\right)(4\pi R^2) \Rightarrow \quad \phi = \frac{Q}{\varepsilon_o}$$

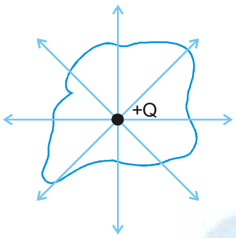
Now if the charge Q is enclosed by any other closed surface, still same no of lines of force will pass through the surface.

So, here also flux will be $\phi = \frac{Q}{\varepsilon_o}$. That's what Gauss Theorem is.

$\phi = \frac{Q}{\varepsilon_o}$



$\phi = \frac{Q}{\varepsilon_o}$



Gauss's Law in Electrostatics or Gauss's theorem

This law was stated by a mathematician Karl F Gauss. This law gives the relation between the electric field at a point on a closed surface and the net charge enclosed by that surface. This surface is called Gaussian surface. It is a closed hypothetical surface. Its validity is shown by experiments. It is used to determine the electric field due to some symmetric charge distributions.

Statement and Details

Gauss's law is stated as given below :

The surface integral of the electric field intensity over any closed hypothetical surface (called Gaussian surface) in free space is equal to $\frac{1}{\varepsilon_0}$ times the total charge enclosed within the surface. Here, ε_0 is the permittivity of free space.

If S is the Gaussian surface and $\sum_{i=1}^{n} q_i$ is the total charge enclosed by the Gaussian surface, then according to Gauss's law,

$$\phi_E = \oint \vec{E} \cdot d\vec{S} = \frac{1}{\varepsilon_0} \sum_{i=1}^{n} q_i .$$

The circle on the sign of integration indicates that the integration is to be carried out over the closed surface.

ETOOS KEY POINTS

(i) Flux through Gaussian surface is independent of its shape.

(ii) Flux through Gaussian surface depends only on total charge present inside Gaussian surface.

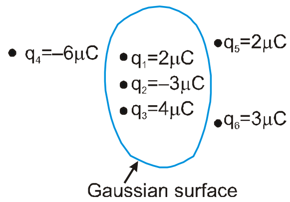
(iii) Flux through Gaussian surface is independent of position of charges inside Gaussian surface.

(iv) Electric field intensity at the Gaussian surface is due to all the charges present inside as well as outside the Gaussian surface.

(v) In a closed surface incoming flux is taken negative, while outgoing flux is taken positive, because $\hat{n}$ is taken positive in outward direction.

(vi) In a Gaussian surface, $\phi = 0$ does not imply $E = 0$ at every point of the surface but $E = 0$ at every point implies $\phi = 0$.

Ex. Find out flux through the given Gaussian surface.



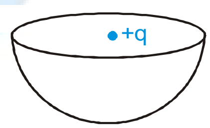
Sol. $\phi = \frac{Q_{in}}{\varepsilon_0} = \frac{2\mu C - 3\mu C + 4\mu C}{\varepsilon_0} = \frac{3 \times 10^{-6}}{\varepsilon_0}$ Nm²/C

Ex. If a point charge q is placed at the centre of a cube, then find out flux through any one face of cube.

Sol. Flux through all 6 faces = $\frac{q}{\varepsilon_0}$. Since, all the surfaces are symmetrical

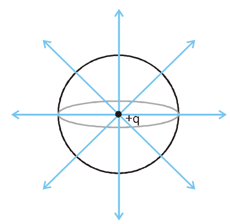
So, flux through one face = $\frac{1}{6}\frac{q}{\varepsilon_0}$

Flux through open surfaces using Gauss's Theorem



Ex. A point charge +q is placed at the centre of curvature of a hemisphere. Find flux through the hemispherical surface.

Sol. Lets put an upper half hemisphere.

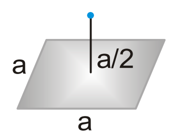


Now, flux passing through the entire sphere = $\frac{q}{\varepsilon_0}$

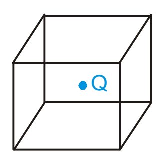
As the charge q is symmetrical to the upper half and lower half hemispheres , so half-half flux will emit from both the surfaces.

Flux emitting from lower half surface = $\frac{q}{2_{\,0}}$

Flux emitting from upper half surface = $\frac{q}{2_{\,0}}$

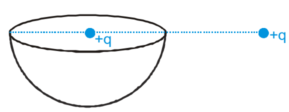


Ex. A charge Q is placed at a distance a/2 above the centre of a horizontal, square surface of edge a as shown in figure. Find the flux of the electric field through the square surface.



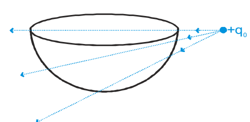
Sol. We can consider imaginary faces of cube such that the charge lies at the centre of the cube. Due to symmetry, we can say that flux through the given area (which is one face of cube), $\phi = \frac{Q}{6\varepsilon_0}$

Ex. Find flux through the hemispherical surface



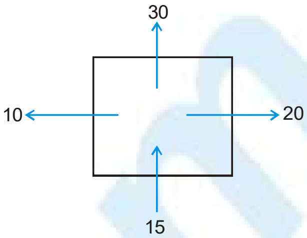
Sol. **(i)** Flux through the hemispherical surface due to +q = $\frac{q}{2\varepsilon_0}$ (we have seen in previous examples)

(ii) Flux through the hemispherical surface due to $+q_0$ is 0, because due to $+q_0$, field lines entering the surface = field lines coming out of the surface.



Finding qin from flux

Ex. Flux (in S.I.units) coming out and entering a closed surface is shown in the figure . Find charge enclosed by the closed surface.



Sol. Net flux through the closed surface = + 20 + 30 + 10 -15 = 45 N.m²/c

From Gauss`s theorem :

$$\phi_{net} = \frac{q_{in}}{\varepsilon_0} \quad \text{or} \quad 45 = \frac{q_{in}}{\varepsilon_0}$$

$\therefore \quad q_{in} = (45)\varepsilon_0$

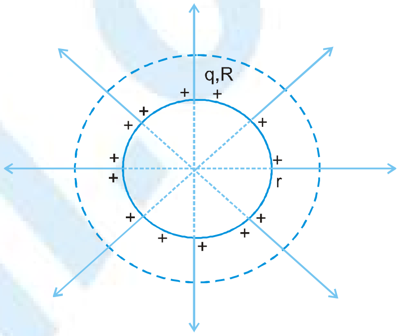
Finding electric field from Gauss`s Theorem

From Gauss`s theorem, we can say

$$\int \vec{E}.\vec{ds} = \phi_{net} = \frac{q_{in}}{\varepsilon_0}$$

Finding E due to a spherical shell

(a) Electric field outside the Sphere



Since, electric field due to a shell will be radially outwards. So lets choose a spherical Gaussian surface Applying Gauss`s theorem for this spherical Gaussian surface,

$$\int \vec{E}\,\vec{ds} = \phi_{net} = \frac{q_{in}}{\varepsilon_o} = \frac{q}{\varepsilon_o}$$

$\downarrow$

$\int |\vec{E}||\vec{ds}|\cos 0$ (because $\vec{E}$ is normal to the surface)

$\downarrow$

$E\int ds$ (because value of E is constant at the surface)

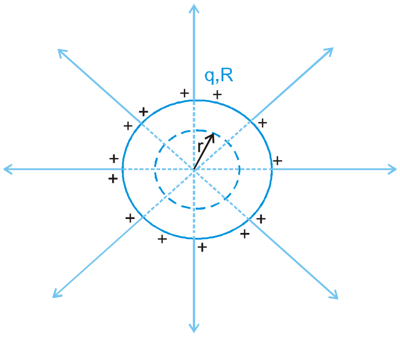
$E(4\pi r^2)$ ($\therefore \int ds$ total area of the spherical surface $= 4\pi r^2$)

$$\Rightarrow \quad E(4\pi r^2) = \frac{q_{in}}{\varepsilon_0} \quad \Rightarrow \quad \therefore E_{out} = \frac{q}{4\pi\varepsilon_o r^2}$$

(b) Electric field inside a spherical shell

Lets choose a spherical Gaussian surface inside the shell.

Applying Gauss`s theorem for this surface



$$\int \vec{E}\,\vec{ds} = \phi_{net} = \frac{q_{in}}{\varepsilon_0} = 0$$

$\downarrow$

$\int |\vec{E}||\vec{ds}|\cos 0$

$\downarrow$

$E\int ds$

$\downarrow$

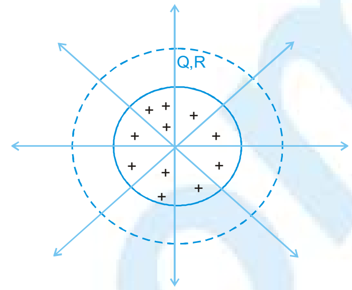
$E(4\pi r^2)$

$\Rightarrow \quad E(4\pi r^2) = 0 \qquad \therefore \quad E_{in} = 0$

Electric field due to solid sphere (having uniformly distributed charge Q and radius R)

(a) Electric field outside the sphere

Direction of electric field is radially outwards, so we will choose a spherical Gaussian surface Applying Gauss`s theorem



$$\int \vec{E}\,\vec{ds} = \phi_{net} = \frac{q_{in}}{\varepsilon_o} = \frac{Q}{\varepsilon_o}$$

$\downarrow$

$\int |\vec{E}||\vec{ds}|\cos 0$

$\downarrow$

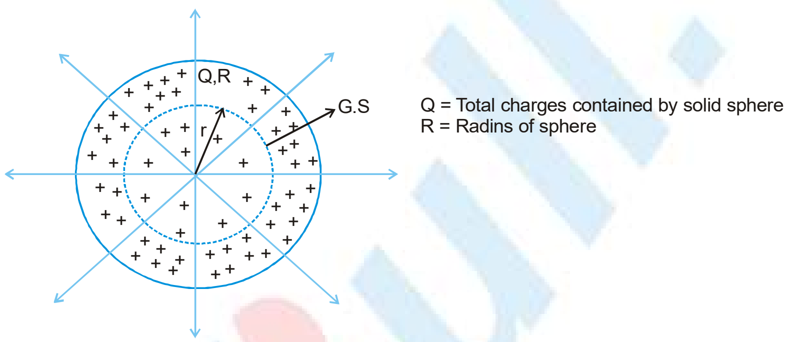
$E\int ds$

$\downarrow$

$E\,(4\pi r^2)$

$\Rightarrow \quad E\,(4\pi r^2) = \dfrac{Q}{\varepsilon_o} \quad \Rightarrow \quad$ or $\quad E_{out} = \dfrac{Q}{4\pi\varepsilon_o r^2}$

(b) Electric field inside a solid sphere :



For this choose a spherical Gaussian surface inside the solid sphere Applying Gauss`s theorem for this surface

$$\int \vec{E}\,\vec{ds} = \phi_{net} = \frac{q_{in}}{\varepsilon_o} = \frac{\dfrac{Q}{\frac{4}{3}\pi R^3}\times\frac{4}{3}\pi r^3}{\varepsilon_o} = \frac{Qr^3}{\varepsilon_o R^3}$$

$\downarrow$

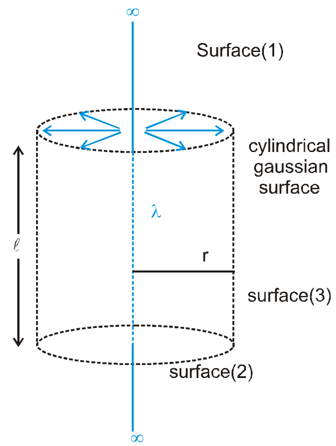
$\int E\,ds$

$\downarrow$

$E\,(4\pi r^2) \quad \Rightarrow \quad E(4\pi r^2) = \dfrac{Qr^3}{\varepsilon_o R^3}$

$E = \dfrac{Q\ r}{4\pi\varepsilon_o R^3} \quad \Rightarrow \quad \therefore E_{in} = \dfrac{kQ}{R^3}\ r$

Electric field due to infinite line charge (having uniformly distributed charged of charge density l)



Electric field due to infinitely wire is radial so we will choose cylindrical Gaussian surface as shown is figure:

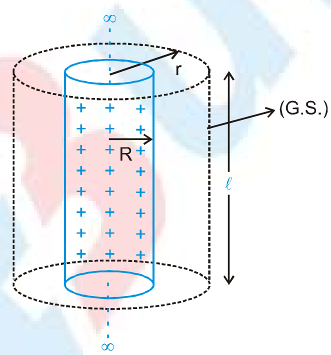
ϕ_{net} → $\phi_1 = 0$, $\phi_2 = 0$, $\phi_3 \neq 0$

$$= \frac{q_{in}}{\varepsilon_o} = \frac{\lambda l}{\varepsilon_o}$$

$$\phi_3 = \int \vec{E}.\vec{ds} = \int E\ ds = E\int ds = E\,(2\pi r\bullet)$$

$$✈ E\,(2\pi r\bullet) = \frac{\lambda l}{\varepsilon_o} \quad \Rightarrow \quad \therefore \quad E = \frac{\lambda}{2\pi\varepsilon_o r} = \frac{2k\lambda}{r}$$

Electric field due to infinitely long charged tube (having uniform surface charge density s and radius R))



(i) **E outside the tube :-** Lets choose a cylindrical Gaussian surface of length ● :

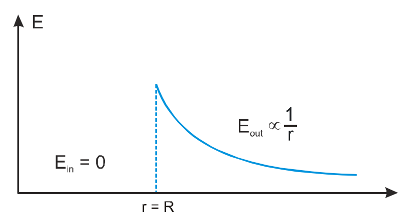
$$\therefore \phi_{net} = \frac{q_{in}}{\varepsilon_o} = \frac{\sigma 2\pi R l}{\varepsilon_o} \quad \Rightarrow \quad E_{out} \times 2\pi r\bullet = \frac{\sigma 2\pi R l}{\varepsilon_o} \quad \therefore \quad E = \frac{\sigma R}{r\ \varepsilon_0}$$

(ii) **E inside the tube :**

Lets choose a cylindrical Gaussian surface inside the tube.

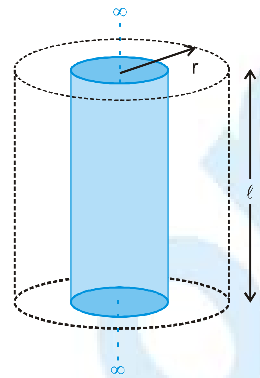
$$\phi_{net} = \frac{q_{in}}{\varepsilon_o} = 0$$

So $E_{in} = 0$



E due to infinitely long solid cylinder of radius R (having uniformly distributed charge in volume (volume charge density ρ)) :

(i) **E at outside point :-**



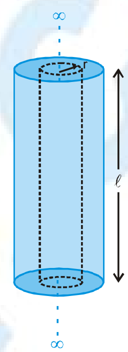
Lets choose a cylindrical Gaussian surface.
Applying Gauss`s theorem :

$$E \times 2\pi r \ell = \frac{q_{in}}{\varepsilon_o} = \frac{\rho \times \pi R^2 l}{\varepsilon_o}$$

$$E_{out} = \frac{\rho R^2}{2r\,\varepsilon_0}$$

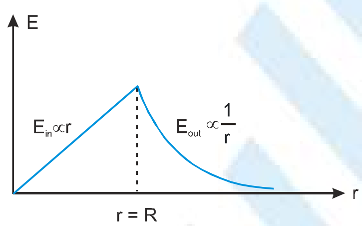
(ii) **E at inside point :**

Lets choose a cylindrical Gaussian surface inside the solid cylinder.
Applying Gauss`s theorem :



$$E \times 2\pi r \ell = \frac{q_{in}}{\varepsilon_o} = \frac{\rho \times \pi r^2 l}{\varepsilon_o}$$

$$E_{in} = \frac{\rho r}{2\varepsilon_0}$$



Conductor and it's properties [For electrostatic condition]

(i) Conductors are materials which contain large number of free electrons which can move freely inside the conductor.

(ii) In electrostatics conductors are always equipotential surfaces.

(iii) Charge always resides on outer surface of conductor.

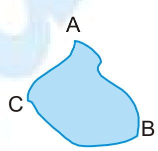
(iv) If there is a cavity inside the conductor having no charge then charge will always reside only on outer surface of conductor.

(v) Electric field is always perpendicular to conducting surface.

(vi) Electric lines of force never enter into conductors.

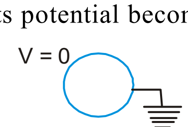
(vii) Electric field intensity near the conducting surface is given by formula

$$\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$$



$$\vec{E}_A = \frac{\sigma_A}{\varepsilon_0}\hat{n} \;;\; \vec{E}_B = \frac{\sigma_B}{\varepsilon_0}\hat{n} \text{ and } \vec{E}_C = \frac{\sigma_C}{\varepsilon_0}\hat{n}$$

(viii) When a conductor is grounded its potential becomes zero.



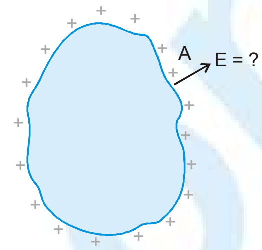
(ix) When an isolated conductor is grounded then its charge becomes zero.

(x) When two conductors are connected there will be charge flow till their potentials become equal.

(xi) Electric pressure : Electric pressure at the surface of a conductor is given by formula

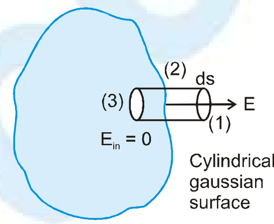
$$P = \frac{\sigma^2}{2\varepsilon_0}$$, where σ is the local surface charge density.

Finding field due to a conductor



Suppose we have a conductor and at any 'A', local surface charge density = σ. We have to find electric field just outside the conductor surface.

For this, let's consider a small cylindrical Gaussian surface, which is partly inside and partly outside the conductor surface, as shown in figure.



It has a small cross section area ds and negligible height.

Applying Gauss's theorem for this surface :

$$\phi_{net} = \frac{q_{in}}{\varepsilon_0} = \frac{\sigma\, ds}{\varepsilon_0}$$

- flux through surface (1): $\phi_1 = Eds$ (because $\vec{E}$ is normal to the surface of conductor)
- flux through surface (2): $\phi_2 = 0$ ($\vec{E}$ is normal to curved Gaussian surface)
- flux through surface (3): $\phi_3 = 0$ (as E inside the conductor = 0)

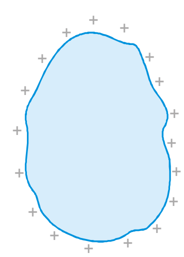
So, $\quad Eds = \frac{\sigma ds}{\varepsilon_0} \quad \Rightarrow \quad E = \frac{\sigma}{\varepsilon_0}$

Electric field just outside the surface of conductor :

$$E = \frac{\sigma}{\varepsilon_0}$$ (direction will be normal to the surface)

in vector form: $\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$ (Here, $\hat{n}$ = unit vector normal to the conductor surface)

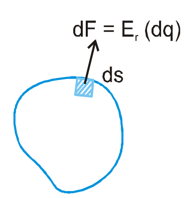
Electrostatic pressure at the surface of the conductor



uppose a conductor is given some charge. Due to repulsion, all the charges will reach the surface of the conductor. But the charges will still repel each other. So an outward force will be felt by each charge due to others. Due to this force, there will be some pressure at the surface, which is called electrostatic pressure.

To find the electrostatic pressure, lets take a small surface element having Area 'ds'.

Force on this element due to the remaining charges :



$$dF = \left(\begin{array}{c}\text{electric field at} \\ \text{that place due to} \\ \text{remainig charges}\end{array}\right)\left(\begin{array}{c}\text{charge of} \\ \text{the small} \\ \text{element}\end{array}\right)$$

Let electric field at that point due to the remaining charges = E_r
and charge of the small element = $dq = \sigma ds$

$\Rightarrow$ $dF = (E_r)(dq) = (E_r)(\sigma ds)$

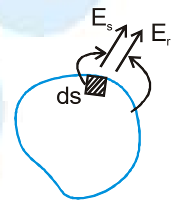
So, pressure on this small element

$$P = \frac{dF}{ds} = \frac{(E_r)(\sigma ds)}{ds}$$

$\Rightarrow$ $P = (E_r)(\sigma)$ ....(1)

Now to find pressure, we have to find E_r (electric field at that position due to the remaining charges)
Suppose,
Electric field due to the small element near the surface = E_s

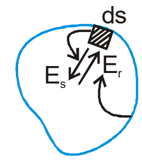
Electric field due to the remaining part near the surface = E_r



At a point just outside the surface, electric field due to the small element (E_s) will be normally out wards, and electric field due to the remaining part (E_r) will also be normally out wards.
So Net electric field just outside the surface $= E_s + E_r$ and we have proved that

electric field just outside the conductor surface = $\frac{\sigma}{\varepsilon_o}$

$\Rightarrow$ $E_s + E_r = \frac{\sigma}{\varepsilon_o}$ ....(2)



Now, lets see the electric field just inside the metal surface. Here, electric field due to the remaining charges (E_r) will be in the same direction (normally outward), but the electric field due to the small element will be in opposite direction (normally inward)
So net electric field just inside the metal surface $= E_r - E_s$ and we know that electric field inside a conductor $= 0$

So, $E_r - E_s = 0$ $\Rightarrow$ $E_r = E_s$ ....(3)

from eqn. (2) and eqn. (3), we can say that :

$$2E_r = \frac{\sigma}{\varepsilon_o} \quad \Rightarrow \quad E_r = \frac{\sigma}{2\varepsilon_o}$$

Now, we can easily find the pressure from eqn. (1)

$$P = (E_r)(\sigma) = \frac{\sigma}{2\varepsilon_o}(\sigma) = \frac{\sigma^2}{2\varepsilon_o}$$

So, electrostatic pressure at the surface of the conductor $P = \frac{\sigma^2}{2\varepsilon_o}$

where, σ = local surface charge density.

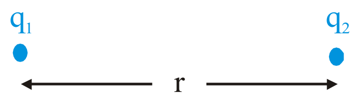
• Etoos Tips & Formulas •

1. Electric Charge

Charge of a material body is that property due to which it interacts with other charges. There are two kinds of charges-positive and negative. S.I. unit is coulomb. Charge is quantized, conserved, and additive.

2. Coulomb`s law

Force between two charges $\vec{F} = \dfrac{1}{4\pi \in_0 \in_r} \dfrac{q_1 q_2}{r^2} \hat{r}$ $\in_r$ = dielectric constant



Note : The Law is applicable only for static and point charges. Moving charges may result in magnetic interaction. And if charges are extended, induction may change the charge distribution.

3. Principle Of Superposition

Force on a point charge due to many charges is given by

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ...$$

Note : The force due to one charge is not affected by the presence of other charges.

4. Electric Field or Electric Intensity or Electric Field Strength (Vector Quantity)

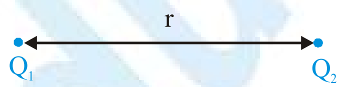
In the surrounding region of a charge there exist a physical property due to which other charge experiences a force. The direction of electric field is direction of force experienced by a positively charged particle and the magnitude of the field (electric field intensity) is the force experienced by a unit charge.

$$\vec{E} = \frac{\vec{F}}{q} \text{ unit is N/C or V/m.}$$

5. Electric field intensity due to charge Q

$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0} = \frac{1}{4\pi \in_0} \frac{Q}{r^2} \hat{r}$$

6. Null point for two charges



If $|Q_1| > |Q_2|$

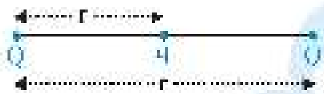
$\Rightarrow$ Null point near Q2

$$x = \frac{\sqrt{Q_1}\, r}{\sqrt{Q_1} \pm \sqrt{Q_2}}$$

(+) for like charges

(–) for unlike charges

7. Equilibrium of three point charges

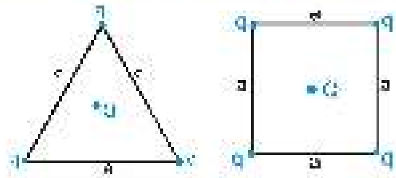


(i) Two charges must be of like nature.

(ii) Third charge should be unlike nature.

$$x = \frac{\sqrt{Q_1}}{\sqrt{Q_1}+\sqrt{Q_2}}\,r \qquad \text{and} \qquad q = \frac{Q_1 Q_2}{\left(\sqrt{Q_1}+\sqrt{Q_2}\right)^2}$$

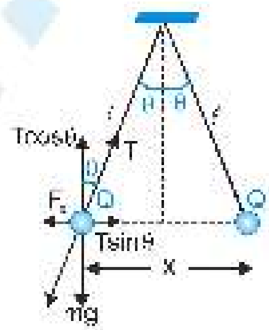
8. Equilibrium of symmetric geometrical point charged system



Value of Q at centre for which system to be in state of equilibrium

(a) For equilateral triangle $Q = \frac{-q}{\sqrt{3}}$ (b) For square $Q = \frac{-q\left(2\sqrt{2}+1\right)}{4}$

9. Equilibrium of suspended point charge system



For equilibrium position

$T\cos\theta = mg$ & $T\sin\theta = F_e = \frac{kQ^2}{x^2} \Rightarrow \tan\theta = \frac{F_e}{mg} = \frac{kQ^2}{x^2 mg}$

If whole set up is taken into an artificial satellite ($g_{eff} \approx 0$)

$T = F_e =$

10. Electric potential difference $= W/q$

11. Electric potential

It is the work done against the field to take a unit positive charge from infinity (reference point) to the given point.

(i) For point charge : $V = K$ (ii) For several point charges : $V = K$

12. Relation between & V

$= -$ grad ; ; ; $V =$

13. Electrical potential energy of two charges :

14. Electric dipole

(a) Electric dipole moment $\vec{p} = q\vec{d}$

(b) Torque on dipole placed in uniform electric field

(c) Work done in rotating dipole placed in uniform electric field

(d) Potential energy of dipole placed in an uniform field U =

(e) At a point which is at a distance r from dipole midpoint and making angle θ with dipole axis.

(1) Potential

(2) Electric field

(3) Direction

(f) Electric field at axial point (or End-on) of dipole

(g) Electric field at equatorial position (Broad-on) of dipole

15. Equipotential Surface And Equipotential Region

In an electric filed the locus of points of equal potential is called an equipotential surface. An equipotential surface and the electric field line meet at right angles. The region where E = 0. Potential of the whole region must remains constant as no work is done in displacement of charge in it. It is called as equipotential region i.e. conducting bodies.

16. Mutual Potential Energy Or Interaction Energy

"The work to be done to integrate the charge system".

For 2 particle system

For 3 particle system

For n particle there will be terms.

Total energy of a system $= U_{self} + U_{mutual}$

17. Electric flux :

(a) For uniform electric field : = E.A cosθ where θ = angle between & area vector . Flux is contributed only due to the component of electric field which is perpendicular to the plane.

(b) If is not uniform throughout the area A, then

18. Gauss's Law : (Applicable only to closed surface)

Net flux emerging out of a closed surface is .

where q_{in} = net charge enclosed by the closed surface does not depend on the

(a) Shape and size of the closed surface

(b) The charges located outside the closed surface.

19. For a conducting sphere

(a) For

(b) For

20. For a non - conducting sphere

(a) For

(b) For

21. **For a conducting/non conducting spherical shell**

(i) For

(ii) For

22. **For a charged circular ring**

Electric field will be maximum at

23. **For a charge long conducting cylinder**

(a) For

(b) For $r < R$: $E = 0$

24. **Electric field intensity at a point near a charged conductor**

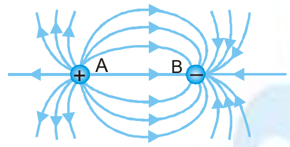
25. **Mechanical pressure on a charged conductor**

26. **For non-conducting long sheet of surface charge density**

27. **For conducting long sheet of surface charge density**

28. **Energy density in electric field**

29. Electric lines of force



Electric lines of electrostatic field following properties

(a) Imaginary

(b) Can never be closed each other

(c) Can never be closed loops

(d) The number of lines originating or terminating on a charge is proportional to the magnitude of charge. In rationalised MKS system $(1/\varepsilon_0)$ electric lines are associated with it (called flux) will be q/ε_0.

(e) Lines of force ends or starts normally at the surface of a conductor.

(f) If there is no electric field there will be no lines of force.

(g) Lines of forceper unit area normal to the area at a point represents magnitude of intensity, crowded lines represent strong field while distant lines weak field.

(h) Tangent to the line of force at a point is an electric field gives the direction of intensity.