

• CURRENT ELECTRICITY •

INTRODUCTION

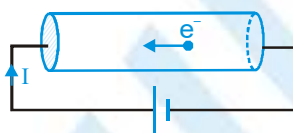
In electrostatics, our discussion of electric phenomena has been focused on charges at rest. In the previous chapter, we treated the concept of electric potential, which is measured in volt. Now we will see that this voltage acts like an “electrical pressure” that can produce a flow of charge or current, which is measured in ampere (or simply, amp and abbreviated as A) and that the resistance that restrains this flow is measured in ohm (Ω).

Example of electric currents about and involve many professions. Meteorologists are concerned with lightning and with the less dramatic slow flow of charge through the atmosphere. Biologists, physiologists, and engineers working in medical technology are concerned with the nerve currents that control muscles and especially with how those currents can be reestablished after spinal cord injuries. Electrical engineers are concerned with countless electrical systems, such as power systems, lightning protection systems, information storage systems, and music systems. Space engineers monitor and study the flow of charged particles from our Sun because that flow can wipe out telecommunication systems in orbit and even power transmission systems on the ground.

In this chapter we discuss the basic physics of electric currents and why they can be established in some materials but not in others. We begin with the meaning of electric current.

ELECTRIC CURRENT

Electric charges in motion constitute an electric current. Any medium having practically free electric charges, free to migrate is a conductor of electricity. The electric charge flows from higher potential energy state to lower potential energy state.



Positive charge flows from higher to lower potential and negative charge flows from lower to higher. Metals such as gold, silver, copper, aluminium etc. are good conductors. When charge flows in a conductor from one place to the other, then the rate of flow of charge is called electric current (**I**). When there is a transfer of charge from one point to other point in a conductor, we say that there is an electric current through the area. If the moving charges are positive, the current is in the direction of motion of charge. If they are negative the current is opposite to the direction of motion. If a charge ΔQ crosses an area in time Δt then the average electric current through the area, during this time as

$$(a) \quad \text{Average current } I_{av} = \frac{\Delta Q}{\Delta t}$$

$$(b) \quad \text{Instantaneous current } I = \lim_{\Delta t \rightarrow 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

CLASSIFICATION OF MATERIALS ACCORDING TO CONDUCTIVITY

(i) Conductor

In some materials, the outer electrons of each atoms or molecules are only weakly bound to it. These electrons are almost free to move throughout the body of the material and are called free electrons. They are also known as conduction electrons. When such a material is placed in an electric field, the free electrons move in a direction opposite to the field. Such materials are called conductors.

(ii) Insulator

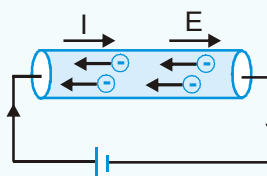
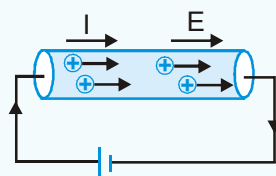
Another class of materials is called insulators in which all the electrons are tightly bound to their respective atoms or molecules. Effectively, there are no free electrons. When such a material is placed in an electric field, the electrons may slightly shift opposite to the field but they can't leave their parent atoms or molecules and hence can't move through long distances. Such materials are also called dielectrics.

(iii) Semiconductor

In semiconductors, the behaviour is like an insulator at low levels of temperature. But at higher temperatures, a small number of electrons are able to free themselves and they respond to the applied electric field. As the number of free electrons in a semiconductor is much smaller than that in a conductor, its behaviour is in between a conductor and an insulator and hence, the name semiconductor. A freed electron in a semiconductor leaves a vacancy in its normal bound position. These vacancies also help in conduction.

ETOOS KEY POINTS

- (i) Current is a fundamental quantity with dimension $[M^0L^0T^0A^1]$
- (ii) Current is a scalar quantity with its SI unit **ampere**.
Ampere : The current through a conductor is said to be one ampere if one coulomb of charge is flowing per second through a cross-section of wire.
- (iii) The conventional direction of current is the direction of flow of positive charge or applied field. It is opposite to direction of flow of negatively charged electrons.



- (iv) The conductor remains uncharged when current flows through it because the charge entering at one end per second is equal to charge leaving the other end per second.
- (v) For a given conductor current does not change with change in its cross-section because current is simply rate of flow of charge.
- (vi) If n particles each having a charge q pass per second per unit area then current associated with cross-sectional area A is $I = \frac{\Delta q}{\Delta t} = nqA$.
- (vii) If there are n particles per unit volume each having a charge q and moving with velocity v then current through cross-sectional area A is $I = \frac{\Delta q}{\Delta t} = nqvA$
- (viii) If a charge q is moving in a circle of radius r with speed v then its time period is $T = 2\pi r/v$. The equivalent current $I = \frac{q}{T} = \frac{qv}{2\pi r}$.

Behavior of conductor in absence of applied potential difference :

In absence of applied potential difference electrons have random motion. The average displacement and average velocity is zero. There is no flow of current due to thermal motion of free electrons in a conductor.

The free electrons present in a conductor gain energy from temperature of surrounding and move randomly in the conductor.

The speed gained by virtue of temperature is called as thermal speed of an electron $\frac{1}{2}mv_{rms}^2 = \frac{3}{2}kT$

So thermal speed $v_{rms} = \sqrt{\frac{3kT}{m}}$ where m is mass of electron

At room temperature $T = 300$ K, $v_{rms} = 10^5$ m/s

(a) **Mean free path λ** : ($\lambda \sim 10\text{\AA}$) $\cdot \lambda = \frac{\text{total distance travelled}}{\text{number of collisions}}$

(b) **Relaxation time** : The time taken by an electron between two successive collisions is called as relaxation

$$\text{time } \tau : (\tau \sim 10^{-14}\text{s}), \text{ Relaxation time : } \tau = \frac{\text{total time taken}}{\text{number of collisions}}$$

Behavior of conductor in presence of applied potential difference :

When two ends of a conductor are joined to a battery then one end is at higher potential and another at lower potential. This produces an electric field inside the conductor from point of higher to lower potential $E = \frac{V}{L}$

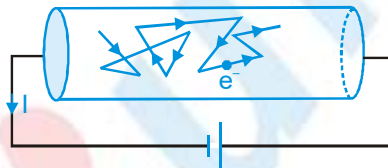
where V = emf of the battery, L = length of the conductor.

The field exerts an electric force on free electrons causing acceleration of each electron.

$$\text{Acceleration of electron } \vec{a} = \frac{\vec{F}}{m} = \frac{-e\vec{E}}{m}$$

DRIFT VELOCITY

Drift velocity is defined as the velocity with which the free electrons get drifted towards the positive terminal under the effect of the applied external electric field. In addition to its thermal velocity, due to acceleration given by applied electric field, the electron acquires a velocity component in a direction opposite to the direction of the electric field. The gain in velocity due to the applied field is very small and is lost in the next collision.



Under the action of electric field :
Random motion of an electron
with superimposed drift

At any given time, an electron has a velocity $\vec{v}_1 = \vec{u}_1 + \vec{a}\tau_1$, where \vec{u}_1 = the thermal velocity and

$\vec{a}\tau_1$ = the velocity acquired by the electron under the influence of the applied electric field.

τ_1 = the time that has elapsed since the last collision. Similarly, the velocities of the other electrons are

$$\vec{v}_2 = \vec{u}_2 + \vec{a}\tau_2, \vec{v}_3 = \vec{u}_3 + \vec{a}\tau_3, \dots, \vec{v}_N = \vec{u}_N + \vec{a}\tau_N.$$

The average velocity of all the free electrons in the conductor is equal to the drift velocity \vec{v}_d of the free electrons

$$\vec{v}_d = \frac{\vec{v}_1 + \vec{v}_2 + \vec{v}_3 + \dots + \vec{v}_N}{N} = \frac{(\vec{u}_1 + \vec{a}\tau_1) + (\vec{u}_2 + \vec{a}\tau_2) + \dots + (\vec{u}_N + \vec{a}\tau_N)}{N} = \frac{(\vec{u}_1 + \vec{u}_2 + \dots + \vec{u}_N)}{N} + \vec{a} \left(\frac{\tau_1 + \tau_2 + \dots + \tau_N}{N} \right)$$

$$\rightarrow \frac{\vec{u}_1 + \vec{u}_2 + \dots + \vec{u}_N}{N} = 0 \therefore \vec{v}_d = \vec{a} \left(\frac{\tau_1 + \tau_2 + \dots + \tau_N}{N} \right) \Rightarrow \vec{v}_d = \vec{a}\tau = -\frac{e\vec{E}}{m}\tau$$

Note : Order of drift velocity is 10^{-4} m/s.

Relation between current and drift velocity :

Let n = number density of free electrons and A = area of cross-section of conductor.

Number of free electrons in conductor of length L = nAL , Total charge on these free electrons $\Delta q = neAL$

Time taken by drifting electrons to cross conductor $\Delta t = \frac{L}{v_d}$ \therefore current $I = \frac{\Delta q}{\Delta t} = neAL \left(\frac{v_d}{L} \right) = neAv_d$

Ex. Find free electrons per unit volume in a metallic wire of density 10^4 kg/m^3 , atomic mass number 100 and number of free electron per atom is one.

Sol. Number of free charge particle per unit volume (n) = $\frac{\text{total free charge particle}}{\text{total volume}}$

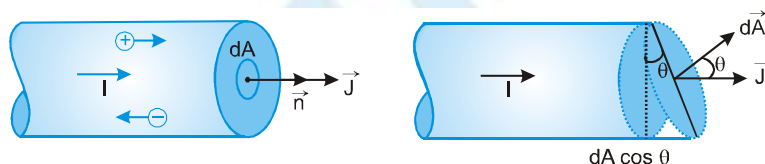
\therefore No. of free electron per atom means total free electrons = total number of atoms = $\frac{N_A}{M_w} \times M$

$$\text{So } n = \frac{\frac{N_A}{M_w} \times M}{V} = \frac{N_A}{M_w} \times d = \frac{6.023 \times 10^{23} \times 10^4}{100 \times 10^{-3}} = 6.023 \times 10^{28}$$

CURRENT DENSITY (J)

Current is a macroscopic quantity and deals with the overall rate of flow of charge through a section. To specify the current with direction in the microscopic level at a point, the term current density is introduced. Current density at any point inside a conductor is defined as a vector having magnitude equal to current per unit area surrounding that point. Remember area is normal to the direction of charge flow (or current passes) through that point.

(a) Current density at point P is given by $\vec{J} = \frac{dI}{dA} \vec{n}$



(b) If the cross-sectional area is not normal to the current, but makes an angle θ with the direction of current then

$$J = \frac{dI}{dA \cos \theta} \Rightarrow dI = J dA \cos \theta = \vec{J} \cdot d\vec{A} \Rightarrow I = \int \vec{J} \cdot d\vec{A}$$

(c) Current density \vec{J} is a vector quantity. Its direction is same as that of \vec{E} . Its S.I. unit is ampere/m² and dimension $[L^{-2}A]$.

Ex. The current density at a point is $\vec{J} = (2 \times 10^4 \hat{j}) \text{ Jm}^{-2}$.

Find the rate of charge flow through a cross sectional area $\vec{S} = (2\hat{i} + 3\hat{j}) \text{ cm}^2$

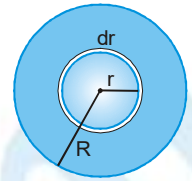
Sol. The rate of flow of charge = current = $I = \int \vec{J} \cdot d\vec{S} \Rightarrow I = \vec{J} \cdot \vec{S} = (2 \times 10^4) [\hat{j} \cdot (2\hat{i} + 3\hat{j})] \times 10^{-4} \text{ A} = 6 \text{ A}$

Ex. A potential difference applied to the ends of a wire made up of an alloy drives a current through it. The current density varies as $J = 3 + 2r$, where r is the distance of the point from the axis. If R be the radius of the wire, then the total current through any cross section of the wire.

Sol. Consider a circular strip of radius r and thickness dr

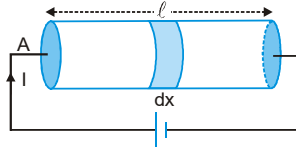
$$dI = \vec{J} \cdot d\vec{S} = (3 + 2r)(2\pi r dr) \cos 0^\circ = 2\pi(3r + 2r^2) dr$$

$$I = \int_0^R 2\pi(3r + 2r^2) dr = 2\pi \left(\frac{3r^2}{2} + \frac{2}{3}r^3 \right)_0^R = 2\pi \left(\frac{3R^2}{2} + \frac{2R^3}{3} \right) \text{ units}$$



Relation between Current density, Conductivity and electric field

Let the number of free electrons per unit volume in a conductor = n



Total number of electrons in dx distance = $n(Adx)$

Total charge $dQ = n(Adx)e$

$$\text{Current } I = \frac{dQ}{dt} = nAe \frac{dx}{dt} = nAe v_d, \text{ Current density } J = \frac{I}{A} = ne v_d$$

$$= ne \left(\frac{eE}{m} \right) \tau \quad \rightarrow v_d = \left(\frac{eE}{m} \right) \tau \Rightarrow J = \left(\frac{ne^2 \tau}{m} \right) E \Rightarrow J = \sigma E, \text{ where conductivity } \sigma = \frac{ne^2 \tau}{m}$$

σ depends only on the material of the conductor and its temperature.

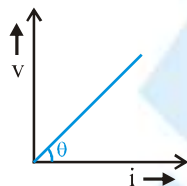
In vector form $\vec{J} = \sigma \vec{E}$ Ohm's law (at microscopic level)

OHM'S LAW

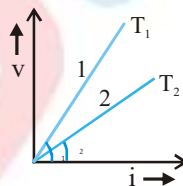
If the physical conditions of the conductor (length, temperature, mechanical strain etc). remains same, then the current flowing through the conductor is directly proportional to the potential difference across its two ends i.e., $i \propto V \Rightarrow V = iR$ where R is a proportionality constant, known as electric resistance.

(i) Ohm's law is not a universal law, the substances, which obey ohm's law are known as ohmic substance,

(ii) Graph between V and i for a metallic conductor is a straight line as shown. At different temperatures V - i curves are different.



(A) slope of the line
 $= \tan \theta = \frac{V}{i} = R$



(B) Here $\tan \theta_1 > \tan \theta_2$
 So $R_1 > R_2$ i.e., $T_1 > T_2$

Resistance

(a) The property of substance by virtue of which it opposes the flow of current through it, is known as the resistance.

(b) **Formula of resistance :** For a conductor if l = length of a conductor A = Area of cross-section of conductor, n = No.

of free electrons per unit volume in conductor, τ = relaxation time then resistance of conductor $R = \rho \frac{l}{A} = \frac{m}{ne^2 \tau} \cdot \frac{l}{A}$

where ρ = resistivity of the material of conductor

(c) **Unit and Dimension :** It's S.I. unit is Volt/amp. or Ohm (Ω). Also ohm = $\frac{1 \text{ volt}}{1 \text{ Amp}} = \frac{10^8 \text{ emu of potential}}{10^{-1} \text{ emu of current}} = 10^9 \text{ emu of resistance}$. It's dimension is $[ML^2T^{-3}A^{-2}]$

PHYSICS FOR JEE MAIN & ADVANCED

Dependence of resistance : Resistance of a conductor depends upon the following factors.

(i) Length of the conductor : Resistance of a conductor is directly proportional to its length i.e $R \propto l$

(ii) Inversely proportional to its area of cross section i.e., $R \propto \frac{1}{A}$

(iii) Nature of material of the conductor $R = \frac{\rho l}{A}$

(iv) Temperature For a conductor

Resistance \propto Temperature.

If R_0 = resistance of conductor at 0°C

R_t = resistance of conductor at $t^\circ \text{C}$

and α, β = temperature co-efficient of resistance then $R_t = R_0(1 + \alpha t + \beta t^2)$ for $t > 300^\circ \text{C}$ and $R_t = R_0$

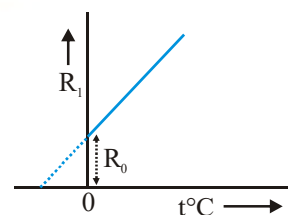
$(1 + \alpha t)$ for $t \leq 300^\circ \text{C}$ or $\alpha = \frac{R_t - R_0}{R_0 \times t}$

If R_1 and R_2 are the resistance at $t_1^\circ \text{C}$ and $t_2^\circ \text{C}$ respectively then $\frac{R_1}{R_2} = \frac{1 + \alpha t_1}{1 + \alpha t_2}$

The value of α is different at different temperature range $t_1^\circ \text{C}$ to $t_2^\circ \text{C}$ is given by $\alpha = \frac{R_2 - R_1}{R_1(t_2 - t_1)}$ which gives

$R_2 = R_1[1 + \alpha(t_2 - t_1)]$. This formula gives an approximate value.

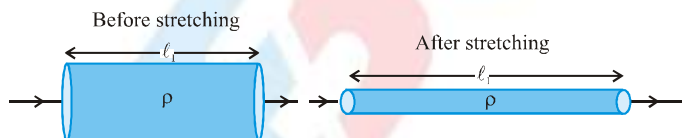
(v) Resistance of the conductor decreases linearly with decrease in temperature and becomes zero at a specific temperature. This temperature is called critical temperature. At this temperature conductor becomes a superconductor.



(vi) **Stretching of wire :** If a conducting wire stretches, its length increases, area of cross-section decreases so resistance increases but volume remains constant.

Suppose for a conducting wire before stretching its length = l_1 , area of cross-section = A_1 radius = r_1

diameter = d_1 , and resistance $R_1 = \rho \frac{l_1}{A_1}$



Volume remains constant i.e., $A_1 l_1 = A_2 l_2$

After stretching length = l_2 area of cross-section = A_2 radius diameter = d_2 and resistance = $R_2 = \rho \frac{l_2}{A_2}$

Ratio of resistance before and after stretching

$$\frac{R_1}{R_2} = \frac{l_1}{l_2} \times \frac{A_2}{A_1} = \left(\frac{l_1}{l_2}\right)^2 = \left(\frac{r_2}{r_1}\right)^4 = \left(\frac{d_2}{d_1}\right)^4$$

(a) If length is given by $R \propto l^2 \Rightarrow \frac{R_1}{R_2} = \left(\frac{l_1}{l_2}\right)^2$

(b) If radius is given then $R \propto \frac{1}{r^4} \Rightarrow \frac{R_1}{R_2} = \left(\frac{r_2}{r_1}\right)^4$



Resistivity (ρ), Conductivity (σ) and Conductance (G)

1. Resistivity : From $R = \rho \frac{l}{A}$; If $l = 1\text{m}$, $A = 1\text{m}^2$ then $R = \rho$ i.e., resistivity is numerically equal to the resistance of a substance having unit area of cross-section and unit length.
- (i) Unit and dimension : It's S.I. unit ohm \times m and dimension is $[ML^3T^{-3}A^{-2}]$
- (ii) It's formula : $\rho = \frac{m}{ne^2\tau}$
- (iii) Resistivity is the intrinsic property of the substance. It is independence of shape and size of the body (i.e., l and A).
- (iv) Resistivity depends on the temperature. For metal $\rho_t = \rho_0(1 + \alpha\Delta t)$ i.e., resistivity increases with temperature.
- (v) Resistivity increases with impurity and mechanical stress.
- (vi) Magnetic field increases the resistivity of all metal except iron, cobalt and nickel.
- (vii) Resistivity of certain substance like selenium, cadmium, sulphide is inversely proportional to intensity of light falling upon them.

2. **Conductivity** : Reciprocal of resistivity is called conductivity i.e., $s = \frac{1}{\rho}$ with unit mho/m and dimensions $[M^{-1}L^3T^{-3}A^2]$

3. **Conductance** : Reciprocal of resistance is known as conductance. $C = \frac{1}{R}$. It's unit is $\frac{1}{\Omega}$ or Ω^{-1} or "siemen".

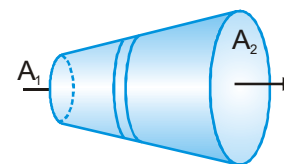
- (i) 1 ampere of current means the flow of 6.25×10^{18} electrons per second through any cross section of conductor.
- (ii) Current is a scalar quantity but current density is a vector quantity.
- (iii) Order of free electron density in conductors = 10^{28} electrons/ m^3

Terms	Thermal speed v_T	Mean free path	Relaxation time	Drift speed v_d
Order	10^5 m/s	10 \AA	10^{-14} s	10^{-4} m/s

- (v) If a steady current flows in a metallic conductor of non uniform cross section.

- (a) Along the wire I is same.
- (b) Current density and drift velocity depends on area

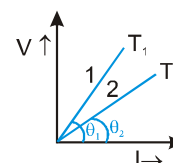
$$I_1 = I_2, A_1 < A_2 \Rightarrow J_1 > J_2, E_1 > E_2, v_{d1} > v_{d2}$$



- (vi) If the temperature of the conductor increases, the amplitude of the vibrations of the positive ions in the conductor also increase. Due to this, the free electrons collide more frequently with the vibrating ions and as a result, the average relaxation time decreases.

- (vii) At different temperatures V - I curves are different.

Here $\tan \theta_1 > \tan \theta_2$ So $R_1 > R_2$ i.e. $T_1 > T_2$



PHYSICS FOR JEE MAIN & ADVANCED

Ex. What will be the number of electron passing through a heater wire in one minute, if it carries a current of 8 A.

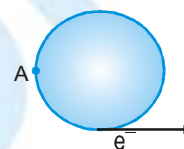
Sol. $I = \frac{Ne}{t} \Rightarrow N = \frac{It}{e} = \frac{8 \times 60}{1.6 \times 10^{-19}} = 3 \times 10^{21}$ electrons

Ex. An electron moves in a circle of radius 10 cm with a constant speed of 4×10^6 m/s. Find the electric current at a point on the circle.

Sol. Consider a point A on the circle. The electron crosses this point once in every revolution. The number of revolutions

made by electron in one second is $n = \frac{v}{2\pi r} = \frac{4 \times 10^6}{2\pi \times 10 \times 10^{-2}} = \frac{2}{\pi} \times 10^7$ rot/s.

\therefore Current $I = \frac{ne}{t} = \frac{2}{\pi} \times 10^7 \times 1.6 \times 10^{-19}$ ($\rightarrow t = 1$ s.) $= \frac{3.2}{\pi} \times 10^{-12} \cong 1 \times 10^{-12}$ A



Ex. A current of 1.34 A exists in a copper wire of cross-section 1.0 mm^2 . Assuming each copper atom contributes one free electron. Calculate the drift speed of the free electrons in the wire. The density of copper is 8990 kg/m^3 and atomic mass $= 63.50$.

Sol. Mass of 1 m^3 volume of the copper is $= 8990 \text{ kg} = 8990 \times 10^3 \text{ g}$

Number of moles in $1 \text{ m}^3 = \frac{8990 \times 10^3}{63.5} = 1.4 \times 10^5$

Since each mole contains 6×10^{23} atoms therefore number of atoms in 1 m^3

$n = (1.4 \times 10^5) \times (6 \times 10^{23}) = 8.4 \times 10^{28}$

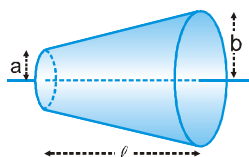
$\rightarrow I = neAv_d \quad \therefore v_d = \frac{I}{neA} = \frac{1.34}{8.4 \times 10^{28} \times 1.6 \times 10^{-19} \times 10^{-6}} = 10^{-4} \text{ m/s} = 0.1 \text{ mm/s}$ ($\rightarrow 1 \text{ mm}^2 = 10^{-6} \text{ m}^2$)

Ex. The current through a wire depends on time as $i = (2 + 3t) \text{ A}$. Calculate the charge crossed through a cross section of the wire in 10 s.

Sol. $I = \frac{dq}{dt} \Rightarrow dq = (2 + 3t)dt \Rightarrow \int_0^{10} dq = \int_0^{10} (2 + 3t) dt \Rightarrow q = \left(2t + \frac{3t^2}{2} \right)_0^{10}$

$q = 2t + \frac{3}{2} \times 100 = 20 + 150 = 170 \text{ C}$

Ex. Figure shows a conductor of length ℓ carrying current I and having a circular cross-section. The radius of cross section varies linearly from a to b . Assuming that $(b - a) \ll \ell$. Calculate current density at distance x from left end.



Sol. Since radius at left end is a and that of right end is b , therefore increase in radius over length ℓ is $(b - a)$.

Hence rate of increase of radius per unit length $= \left(\frac{b - a}{\ell} \right)$ Increase in radius over length $x = \left(\frac{b - a}{\ell} \right) x$

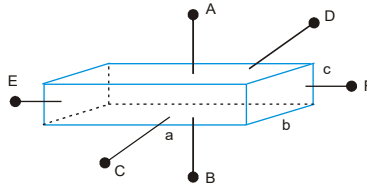


Since radius at left end is a so radius at distance x , $r = a + \left(\frac{b-a}{l}\right)x$

$$\text{Area at this particular section } A = \pi r^2 = \pi \left[a + \left(\frac{b-a}{l}\right)x \right]^2$$

$$\text{Hence current density } J = \frac{I}{A} = \frac{I}{\pi r^2} = \frac{I}{\pi \left[a + \frac{x(b-a)}{l} \right]^2}$$

Ex. The dimensions of a conductor of specific resistance ρ are shown below. Find the resistance of the conductor across AB, CD and EF.



Sol. For a condition

$$R = \frac{\rho \lambda}{A} = \frac{\text{Resistivity} \times \text{length}}{\text{Area of cross section}}$$

$$R_{AB} = \frac{\rho c}{ab}, R_{CD} = \frac{\rho b}{ac}, R_{EF} = \frac{\rho a}{bc}$$

Ex. If a wire is stretched to double its length, find the new resistance if original resistance of the wire was R .

Sol. As we know that $R = \frac{\rho \lambda}{A}$

$$\text{in case } R' = \frac{\rho \lambda'}{A'} \Rightarrow \lambda' = 2\lambda$$

$$A' \lambda' = A \lambda \quad (\text{volume of the wire remains constant})$$

$$A' = \frac{A}{2} \Rightarrow R' = \frac{\rho \times 2\lambda}{A/2} = 4 \frac{\rho \lambda}{A} = 4R$$

Ex. The wire is stretched to increase the length by 1% find the percentage change in the Resistance.

Sol. As we known that

$$\therefore R = \frac{\rho \lambda}{A} \Rightarrow \frac{\Delta R}{R} = \frac{\Delta \rho}{\rho} + \frac{\Delta \lambda}{\lambda} - \frac{\Delta A}{A} \quad \text{and} \quad \frac{\Delta \lambda}{\lambda} = \frac{\Delta A}{A}$$

$$\frac{\Delta R}{R} = 0 + 1 + 1 = 2$$

Hence percentage increase in the Resistance = 2%

Ex. The resistance of a thin silver wire is 1.0Ω at 20°C . The wire is placed in liquid bath and its resistance rises to 1.2Ω . What is the temperature of the bath? (Here $\alpha = 10^{-2} / ^\circ\text{C}$)

Sol. Here change in resistance is small so we can apply

$$R = R_0(1 + \alpha \Delta\theta)$$

$$\Rightarrow 1.2 = 1 \times (1 + 10^{-2} \Delta\theta) \Rightarrow \Delta\theta = 20^\circ\text{C}$$

$$\Rightarrow \theta - 20 = 20 \Rightarrow \theta = 40^\circ\text{C} \quad \text{Ans.}$$

Ex. A conductive wire has resistance of 10 ohm at 0°C, and α is $\frac{1}{273} / ^\circ\text{C}$, then determine its resistance at 273°C.

Sol. In such a problem, term $\alpha \Delta T$ will have a larger value so could not be used directly in $R = R_0 (1 + \alpha \Delta T)$. We need to go for basics as

As we know that $\alpha = \frac{dR}{RdT}$

$$\Rightarrow \int \frac{dR}{R} = \int \alpha dT \quad \Rightarrow \quad \ln \frac{R_2}{R_1} = \alpha(T_2 - T_1)$$

$$\Rightarrow R_2 = R_1 e^{\alpha(T_2 - T_1)} \quad \Rightarrow \quad R_2 = 10e^1$$

$$\Rightarrow R_2 = 10 e \Omega \quad \text{Ans.}$$

ELECTRIC CURRENT IN RESISTANCE

In a resistor current flows from high potential to low potential

High potential is represented by positive (+) sign and low potential is represented by negative (-) sign.

$$V_A - V_B = iR$$

If $V_1 > V_2$

then current will flow from A to B

and $i = \frac{V_1 - V_2}{R}$

If $V_1 < V_2$

then current will go from B to A and $i = \frac{V_2 - V_1}{R}$

Ex. Calculate current (i) flowing in part of the circuit shown in figure?



Sol. $V_A - V_B = i \times R \quad \Rightarrow \quad i = \frac{6}{2} = 3\text{A}$

Specific use of conducting materials

- (i) The **heating element** of devices like heater, geyser, press etc are made of **microhm** because it has high resistivity and high melting point. It does not react with air and acquires steady state when red hot at 800°C.
- (ii) **Fuse wire** is made of **tin lead alloy** because it has low melting point and low resistivity. The fuse is used in series, and melts to produce open circuit when current exceeds the safety limit.
- (iii) **Resistances** of resistance box are made of **manganin** or **constantan** because they have moderate resistivity and very small temperature coefficient of resistance. The resistivity is nearly independent of temperature.
- (iv) The **filament of bulb** is made up of **tungsten** because it has low resistivity, high melting point of 3300 K and gives light at 2400 K. The bulb is filled with inert gas because at high temperature it reacts with air forming oxide.
- (v) The **connection wires** are made of **copper** because it has low resistance and resistivity.

Colour Coding of Resistance

To know the value of resistance colour code is used. These code are printed in form of set of rings or strips. By reading the values of colour bands, we can estimate the value of resistance.

The carbon resistance has normally four coloured rings or bands say A, B, C and D as shown in following figure.

Colour band A and B : Indicate the first two significant figures of resistance in ohm.

Band C : Indicates the decimal multiplier i.e, the number of zeros that follows the two significant figures A and B.

Band D : Indicates the tolerance in percent about the indicated value or in other words it represents the percentage accuracy of the indicated value.

The tolerance in the case of gold is $\pm 5\%$ and silver is $\pm 10\%$. If only three bands are marked on carbon resistance, then it indicate a tolerance of 20%.

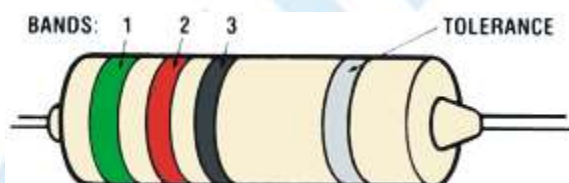


TABLE : COLOUR CODING FOR CARBON RESISTORS

Colour	Strip A	Strip B	Strip C	Strip D (Tolerance)
Black	0	0	10^0	
Brown	1	1	10^1	
Red	2	2	10^2	
Orange	3	3	10^3	
Yellow	4	4	10^4	
Green	5	5	10^5	
Blue	6	6	10^6	
Violet	7	7	10^7	
Grey	8	8	10^8	
White	9	9	10^9	
Gold	-	-	10^{-1}	$\pm 5 \%$
Silver	-	-	10^{-2}	$\pm 10 \%$
No colour	-	-	-	$\pm 20 \%$

May be remembered as
BBROY
Great Britain
Very Good Wife.

PHYSICS FOR JEE MAIN & ADVANCED

Ex. Draw a colour code for $42 \text{ k } \Omega \pm 10\%$ carbon resistance.

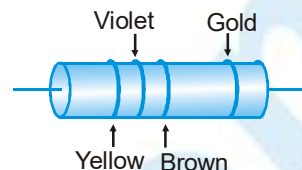
Sol. According to colour code colour for digit 4 is yellow, for digit 2 it is red, for 3 colour is orange and 10% tolerance is represented by silver colour. So colour code should be yellow, red, orange and silver.

Ex. What is resistance of following resistor.

Sol. Number for yellow is 4, Number of violet is 7

Brown colour gives multiplier 10^1 , Gold gives a tolerance of $\pm 5\%$

So resistance of resistor is $47 \times 10^1 \Omega \pm 5\% = 470 \pm 5\% \Omega$.



ELECTRICAL POWER :

Energy liberated per second in a device is called its power. The electrical power P delivered or consumed by an electrical device is given by $P = VI$, where V = Potential difference across the device and

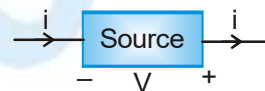
I = Current.

If the current enters the higher potential point of the device then electric power is consumed by it (i.e. acts as load). If the current enters the lower potential point then the device supplies power (i.e. acts as source).

$$\text{Power} = \frac{V \cdot dq}{dt}$$

$$= VI$$

$$P = VI$$



If power is constant then energy = $P t$

If power is variable then

$$\text{Energy} = \int p dt$$

Power consumed by a resistor

$$P = I^2 R = VI = \frac{V^2}{R}$$

When a current is passed through a resistor energy is wasted in overcoming the resistance of the wire. This energy is converted into heat.

$$W = VIt = I^2 Rt = \frac{V^2}{R} t$$

The heat generated (in joules) when a current of I ampere flows through a resistance of R ohm for t second is given by:

$$H = I^2 Rt \text{ Joule} = \frac{I^2 Rt}{4.2} \text{ Calorie}$$

1 unit of electrical energy = 1 Kilowatt hour = $1 \text{ KWh} = 3.6 \times 10^6 \text{ Joule}$.

Ex. If bulb rating is 100 watt and 220 V then determine

(a) Resistance of filament

(b) Current through filament

(c) If bulb operate at 110 volt power supply then find power consumed by bulb.



Sol. Bulb rating is 100 W and 220 V bulb means when 220 V potential difference is applied between the two ends then the power consumed is 100 W

Here $V = 220$ Volt

$P = 100$ W

$$\frac{V^2}{R} = 100 \quad \text{So} \quad R = 484 \, \Omega$$

Since Resistance depends only on material hence it is constant for bulb

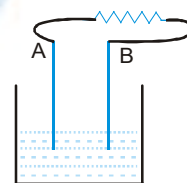
$$I = \frac{V}{R} = \frac{220}{22 \times 22} = \frac{5}{11} \text{ Amp.}$$

power consumed at 110 V

$$\therefore \text{power consumed} = \frac{110 \times 110}{484} = 25 \text{ W}$$

Battery (Cell)

A battery is a device which maintains a potential difference across its two terminals A and B. Dry cells, secondary cells, generator and thermocouple are the devices used for producing potential difference in an electric circuit. Arrangement of cell or battery is shown in figure. Electrolyte provides continuity for current.



It is often prepared by putting two rods or plates of different metals in a chemical solution. Some internal mechanism exerts force (F_n) on the ions (positive and negative) of the solution. This force drives positive ions towards positive terminal and negative ions towards negative terminal. As positive charge accumulates on anode and negative charge on cathode a potential difference and hence an electric field E is developed from anode to cathode. This electric field exerts an electrostatic force $F = qE$ on the ions. This force is opposite to that of F_n . In equilibrium (steady state)

$F_n = F_e$ and no further accumulation of charge takes place.

When the terminals of the battery are connected by a conducting wire, an electric field is developed in the wire. The free electrons in the wire move in the opposite direction and enter the battery at positive terminal. Some electrons are withdrawn from the negative terminal. Thus, potential difference and hence, F_e decreases in magnitude while F_n remains the same. Thus, there is a net force on the positive charge towards the positive terminal. With this the positive charge rush towards positive terminal and negative charge rush towards negative terminal. Thus, the potential difference between positive and negative terminal is maintained.

Internal Resistance (R)

The potential difference across a real source in a circuit is not equal to the emf of the cell. The reason is that charge moving through the electrolyte of the cell encounters resistance. We call this the internal resistance of the source.

* The internal resistance of a cell depends on the distance between electrodes ($r \propto d$), area of electrodes

($r \propto \frac{1}{s}$) and nature, concentration ($r \propto c$) and temperature of electrolyte ($r \propto \frac{1}{\text{Temp.}}$).

Ex. What is the meaning of 10 Amp. hr ?

Sol. It means if the 10 A current is withdrawn then the battery will work for 1 hour.

$$10 \text{ Amp} \longrightarrow 1 \text{ hr}$$

$$1 \text{ Amp} \longrightarrow 10 \text{ hr}$$

$$\frac{1}{2} \text{ Amp} \longrightarrow 20 \text{ hr}$$

Electromotive Force : (E.M.F.)

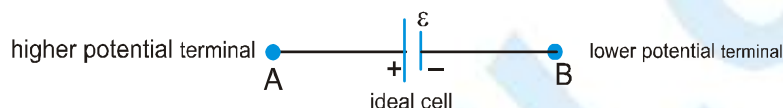
Definition I : Electromotive force is the capability of the system to make the charge flow.

Definition II : It is the work done by the battery for the flow of 1 coulomb charge from lower potential terminal to higher potential terminal inside the battery.

Representation for Battery :

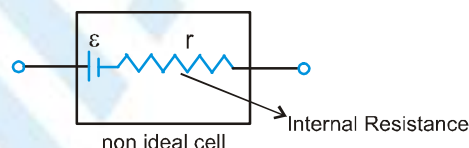
(a) Ideal cell :

Cell in which there is no heating effect.



(b) Non Ideal Cell :

Cell in which there is heating effect inside due to opposition to the current flow internally



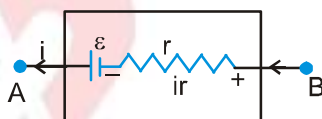
Case I : Battery Acting as a Source (or Battery is Discharging)

$$V_A - V_B = \varepsilon - ir$$

$$V_A - V_B$$

⇒ it is also called terminal voltage.

The rate at which the chemical energy of the cell is consumed = εi



The rate at which heat is generated inside the battery or cell = $i^2 r$

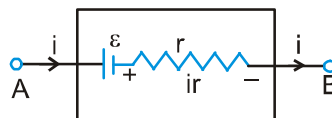
electric power output = $\varepsilon i - i^2 r$

$$= (\varepsilon - ir) i$$

Case II : Battery Acting as a Load (or Battery Charging)

$$V_A - V_B = \varepsilon + ir$$

the rate at which chemical energy stored in the cell = εi



thermal power inside the cell = $i^2 r$

electric power input = $\varepsilon i + i^2 r = (\varepsilon + ir) i = (V_A - V_B) i$

Definition III :

Electromotive force of a cell is equal to potential difference between its terminals when no current is passing through the circuit.

Case III :

When cell is in open circuit

$i = 0$ as resistance of open circuit is infinite (∞).

So $V = \mathcal{E}$, so open circuit terminal voltage difference is equal to emf of the cell.

Case IV :

Short circuiting : Two points in an electric circuit directly connected by a conducting wire are called short circuited, under such condition both points are at same potential.

When cell is short circuited

$$i = \frac{\mathcal{E}}{r} \text{ and } V = 0, \text{ short circuit current of a cell is maximum.}$$

- The potential at all points of a wire of zero resistance will be same.

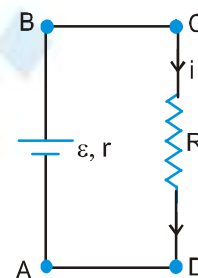
Earthing : If some point of circuit is earthed then its potential is assumed to be zero.

Relative Potential

While solving an electric circuit it is convenient to choose a reference point and assigning its voltage as zero, then all other potentials are measured with respect to this point. This point is also called the common point.

Ex. In the given electric circuit find

- current
- power output
- relation between r and R so that the electric power output (that means power given to R) is maximum.
- value of maximum power output.
- plot graph between power and resistance of load
- From graph we see that for a given power output there exists two values of external resistance, prove that the product of these resistances equals r^2 .
- what is the efficiency of the cell when it is used to supply maximum power.



Sol. (a) In the circuit shown if we assume that potential at A is zero then potential at B is

$$\mathcal{E} - ir. \text{ Now since the connecting wires are of zero resistance}$$

$$\therefore V_D = V_A = 0 \Rightarrow V_C = V_B = \mathcal{E} - ir$$

Now current through CD is also i

(\rightarrow it's in series with the cell).

$$\therefore i = \frac{V_C - V_D}{R} = \frac{(\mathcal{E} - ir) - 0}{R} \quad \text{Current } i = \frac{\mathcal{E}}{r + R}$$

- After learning the concept of series combination we will be able to calculate the current directly

$$(b) \quad \text{Power output } P = i^2 R = \frac{\mathcal{E}^2}{(r + R)^2} \cdot R$$

$$(c) \quad \frac{dP}{dR} = \frac{\mathcal{E}^2}{(r + R)^2} - \frac{2\mathcal{E}^2 R}{(r + R)^3} = \frac{\mathcal{E}^2}{(R + r)^3} [R + r - 2R]$$

for maximum power supply

$$\frac{dP}{dR} = 0 \Rightarrow r + R - 2R = 0 \Rightarrow r = R$$

Here for maximum power output outer resistance should be equal to internal resistance

(d) $P_{\max} = \frac{\varepsilon^2}{4r}$

(e) Graph between 'P' and R
maximum power output at $R = r$

$$P_{\max} = \frac{\varepsilon^2}{4r} \Rightarrow i = \frac{\varepsilon}{r + R}$$

(f) Power output

$$P = \frac{\varepsilon^2 R}{(r + R)^2}$$

$$P(r^2 + 2rR + R^2) = \varepsilon^2 R$$

$$R^2 + (2r - \frac{\varepsilon^2}{P})R + r^2 = 0$$

above quadratic equation in R has two roots R_1 and R_2 for given values of ε , P and r such that

$$\therefore R_1 R_2 = r^2 \quad (\text{product of roots}) \Rightarrow r^2 = R_1 R_2$$

(g) Power of battery spent

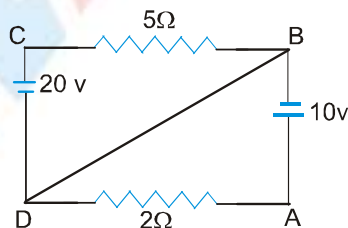
$$= \frac{\varepsilon^2}{(r + r)^2} \cdot 2r = \frac{\varepsilon^2}{2r}$$

power (output)

$$= \left(\frac{\varepsilon}{r + r} \right)^2 \times r = \frac{\varepsilon^2}{4r}$$

$$\text{Efficiency} = \frac{\text{power output}}{\text{total power spent by cell}} = \frac{\frac{\varepsilon^2}{4r} \times 100}{\frac{\varepsilon^2}{2r}} = \frac{1}{2} \times 100 = 50\%$$

Ex. In the figure given beside find out the current in the wire BD

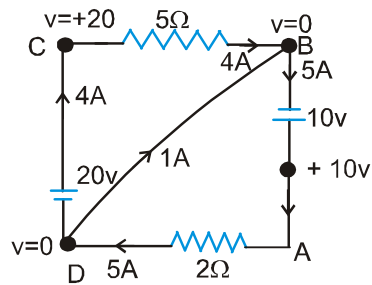


Sol. Let at point D potential = 0 and write the potential of other points then

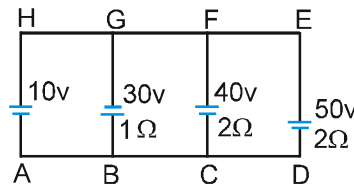
$$\text{current in wire AD} = \frac{10}{2} = 5 \text{ A from A to D} \quad \text{current in wire CB} = \frac{20}{5} = 4 \text{ A}$$

from C to B

\therefore current in wire BD = 1 A from D to B



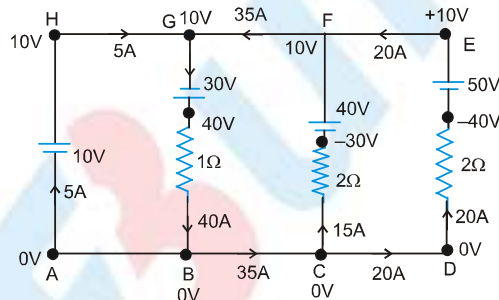
Ex. Find the current in each wire



Sol. Let potential at point A is 0 volt then potential of other points is shown in figure.

$$\text{current in BG} = \frac{40 - 0}{1} = 40 \text{ A from G to B}$$

$$\text{current in FC} = \frac{0 - (-30)}{2} = 15 \text{ A from C to F}$$



$$\text{current in DE} = \frac{0 - (-40)}{2} = 20 \text{ A from D to E}$$

$$\text{current in wire AH} = 40 - 35 = 5 \text{ A from A to H}$$

KIRCHHOFF'S LAWS

1. Kirchhoff's Current Law (Junction law)

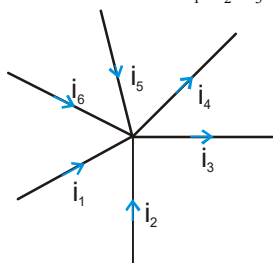
This law is based on law of conservation of charge. It states that "The algebraic sum of the currents meeting at a point of the circuit is zero" or total currents entering a junction equals total current leaving the junction.

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

It is also known as KCL (Kirchhoff's current law).

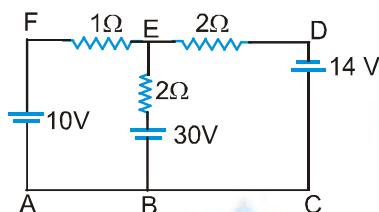
PHYSICS FOR JEE MAIN & ADVANCED

Ex. Find relation in between current i_1, i_2, i_3, i_4, i_5 and i_6 .



Sol. $i_1 + i_2 - i_3 - i_4 + i_5 + i_6 = 0$

Ex. Find the current in each wire



Sol. Let potential at point B = 0. Then potential at other points are mentioned.

∴ Potential at E is not known numerically.

Let potential at E = x

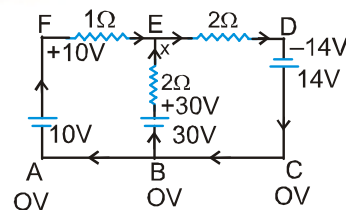
Now applying kirchhoff's current law at junction E. (This can be applied at any other junction also).

$$\frac{x-10}{1} + \frac{x-30}{2} + \frac{x+14}{2} = 0 \Rightarrow 4x = 36 \Rightarrow x = 9$$

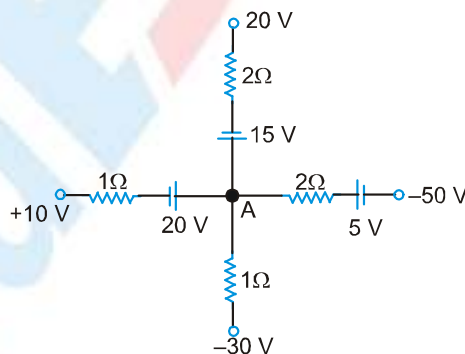
Current in EF = $\frac{10-9}{1} = 1$ A from F to E

Current in BE = $\frac{30-9}{2} = 10.5$ A from B to E

Current in DE = $\frac{9-(-14)}{2} = 11.5$ A from E to D



Ex. Find the potential at point A



Sol. Let potential at A = x, applying kirchhoff current law at junction A

$$\frac{x-20-10}{1} + \frac{x-15-20}{2} + \frac{x+45}{2} + \frac{x+30}{1} = 0 \Rightarrow \frac{2x-60+x-35+x+45+2x+60}{2} = 0$$

$$\Rightarrow 6x + 10 = 0 \Rightarrow x = -5/3 \Rightarrow \text{Potential at A} = -\frac{5}{3} \text{ V}$$



2. Kirchhoff's Voltage Law (LOOP LAW)

"The algebraic sum of all the potential differences along a closed loop is zero.

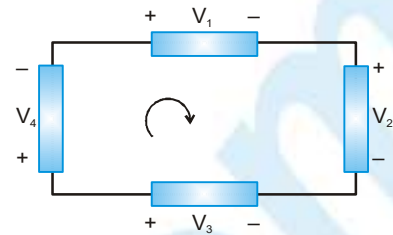
So $IR + \Sigma \text{EMF} = 0$ ".

The closed loop can be traversed in any direction. While traversing a loop if potential increases, put a positive sign in expression and if potential decreases put a negative sign. (Assume sign convention)

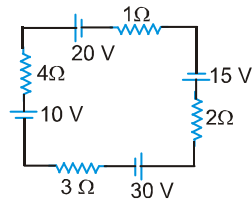
$$-V_1 - V_2 + V_3 - V_4 = 0.$$

Boxes may contain resistor or battery or any other element (linear or nonlinear).

It is also known as **KVL**



Ex. Find current in the circuit



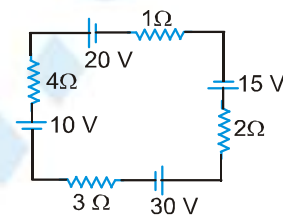
Sol. all the elements are connected in series
current is all of them will be same

let current = i

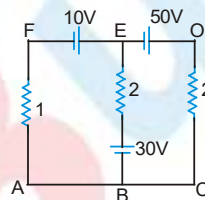
Applying kirchhoff voltage law in ABCDA loop

$$10 + 4i - 20 + i + 15 + 2i - 30 + 3i = 0$$

$$10i = 25 \Rightarrow i = 2.5 \text{ A}$$



Ex. Find the current in each wire applying only kirchhoff voltage law



Sol. Applying kirchhoff voltage law in loop ABEFA

$$i_1 + 30 + 2(i_1 + i_2) - 10 = 0$$

$$3i_1 + 2i_2 + 20 = 0 \quad \text{..... (i)}$$

Applying kirchhoff voltage law in BEDCB

$$+30 + 2(i_1 + i_2) + 50 + 2i_2 = 0$$

$$4i_2 + 2i_1 + 80 = 0$$

$$2i_2 + i_1 + 40 = 0 \quad \text{..... (ii)}$$

Solving (i) and (ii)

$$3[-40 - 2i_2] + 2i_2 + 20 = 0$$

$$-120 - 4i_2 + 20 = 0$$

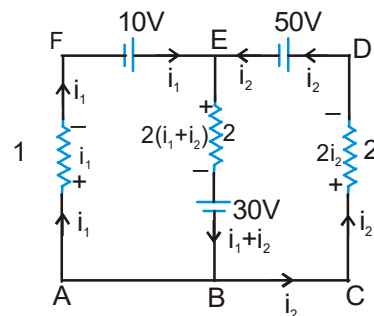
$$i_2 = -25 \text{ A}$$

and $i_1 = 10 \text{ A}$

$$\therefore i_1 + i_2 = -15 \text{ A}$$

current in wire AF = 10 A from A to F

current in wire DE = 25 A from E to D.



current in wire EB = 15 A from B to E

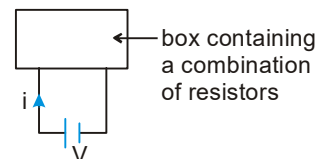
ETOOS KEY POINTS

- (i) If a wire is stretched to n times of its original length, its new resistance will be n^2 times.
- (ii) If a wire is stretched such that its radius is reduced to $\frac{1}{n}$ th of its original values, then resistance will increase n^4 times similarly resistance will decrease n^4 times if radius is increased n times by contraction.
- (iii) To get maximum resistance, resistance must be connected in series and in series the resultant is greater than largest individual.
- (iv) To get minimum resistance, resistance must be connected in parallel and the equivalent resistance of parallel combination is lower than the value of lowest resistance in the combination.
- (v) Ohm's law is not a fundamental law of nature. As it is possible that for an element :—
 - (i) V depends on I non linearly (e.g. vacuum tubes)
 - (ii) Relation between V and I depends on the sign of V for the same value [Forward and reverse Bias in diode]
 - (iii) The relation between V and I is non unique. That is for the same I there is more than one value of V .
- (vi) In general
 - (i) Resistivity of alloys is greater than their metals.
 - (ii) Temperature coefficient of alloys is lower than pure metals.
 - (iii) Resistance of most of non metals decreases with increase in temperature. (e.g. carbon)
 - (iv) The resistivity of an insulator (e.g. amber) is greater than the metal by a factor of 10^{22}
- (vii) Temperature coefficient (α) of semi conductor including carbon (graphite), insulator and electrolytes is negative.

Combination of Resistances

A number of resistances can be connected and all the complicated combinations can be reduced to two different types, namely series and parallel.

The equivalent resistance of a combination is defined as $R_{eq} = \frac{V}{i}$



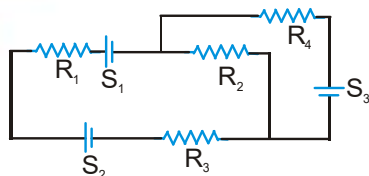
Resistances in Series :

When the resistances (or any type of elements) are connected end to end then they are said to be in series. The current through each element is same.



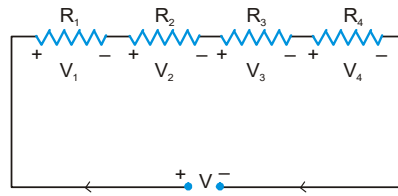
Resistances in series carry equal current but reverse may not be true.

Ex. Which electrical elements are connected in series.



Sol. Here S_1, S_2, R_1, R_3 connected in one series and R_4, S_3 connected in different series

Equivalent of Resistors



The effective resistance appearing across the battery (or between the terminals A and B) is

$$R = R_1 + R_2 + R_3 + \dots + R_n \text{ (this means } R_{eq} \text{ is greater than any resistor)}$$

and $V = V_1 + V_2 + V_3 + \dots + V_n$

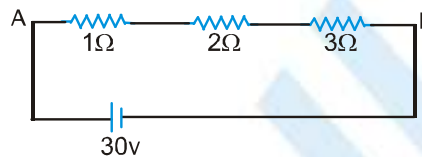
The potential difference across a resistor is proportional to the resistance. Power in each resistor is also proportional to the resistance

$$\rightarrow V = IR \text{ and } P = I^2 R$$

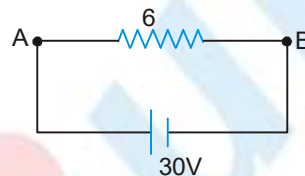
where I is same through any of the resistor.

$$V_1 = \frac{R_1}{R_1 + R_2 + \dots + R_n} V; V_2 = \frac{R_2}{R_1 + R_2 + \dots + R_n} V; \text{ etc}$$

Ex. Find the current in the circuit

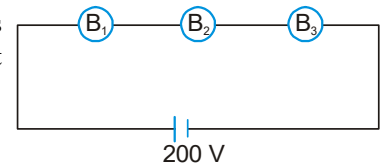


Sol. $R_{eq} = 1 + 2 + 3 = 6 \Omega$ the given circuit is equivalent to



$$\text{current } i = \frac{v}{R_{eq}} = \frac{30}{6} = 5 \text{ A} \quad \text{Ans.}$$

Ex. In the figure shown B_1 , B_2 and B_3 are three bulbs rated as (200V, 50 W), (200V, 100W) and (200 V, 25W) respectively. Find the current through each bulb and which bulb will give more light?



Sol. $R_1 = \frac{(200)^2}{50};$

$$R_2 = \frac{(200)^2}{100}; \quad R_3 = \frac{(200)^2}{25}$$

the current following through each bulb is

$$= \frac{200}{R_1 + R_2 + R_3} = \frac{200}{(200)^2 \left[\frac{2+1+4}{100} \right]} = \frac{100}{200 \times 7} = \frac{1}{14} \text{ A}$$

Since $R_3 > R_1 > R_2$

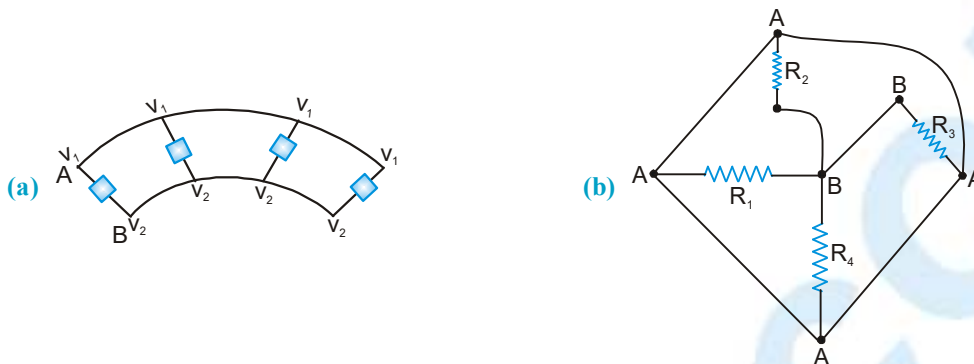
\therefore Power consumed by bulb = $i^2 R$

\therefore if the resistance is of higher value then it will give more light.

\therefore Here Bulb B_3 will give more light.

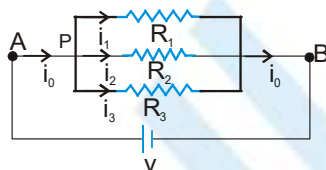
Resistances in Parallel

A parallel circuit of resistors is one in which the same voltage is applied across all the components in a parallel grouping of resistors $R_1, R_2, R_3, \dots, R_n$.



In the figure (a) and (b) all the resistors are connected between points A and B so they are in parallel.

Equivalent resistance



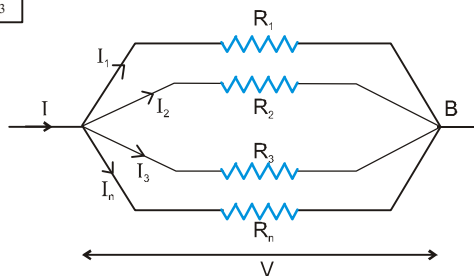
Applying kirchhoff's junction law at point P

$$i_0 = i_1 + i_2 + i_3$$

Therefore,
$$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \quad \boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

in general,

$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_n}}$$



Conclusions: (about parallel combination)

(a) Potential difference across each resistor is same.

(b) $I = I_1 + I_2 + I_3 + \dots + I_n$.

(c) Effective resistance (R) then $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_n}$.

(R is less than each resistor).

(d) Current in different resistors is inversely proportional to the resistance.

$$I_1 : I_2 : \dots : I_n = \frac{1}{R_1} : \frac{1}{R_2} : \frac{1}{R_3} : \dots : \frac{1}{R_n}$$

$$I_1 = \frac{G_1}{G_1 + G_2 + \dots + G_n} I, I_2 = \frac{G_2}{G_1 + G_2 + \dots + G_n} I, \text{ etc.}$$

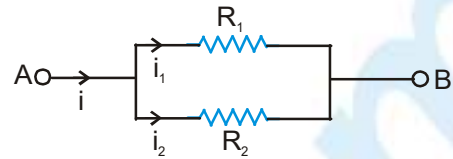
where $G = \frac{1}{R}$ = Conductance of a resistor. [Its unit is Ω^{-1} or \mathcal{U} (mho)]

Ex. When two resistors are in parallel combination then determine i_1 and i_2 , if the combination carries a current i ?

Sol. $\therefore i_1 R_1 = i_2 R_2$

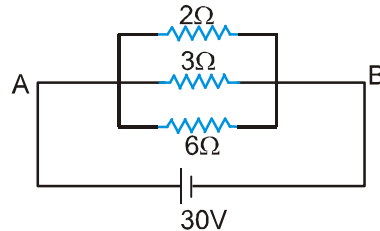
or $\frac{i_1}{i_2} = \frac{R_2}{R_1}$

$$i_1 = \frac{R_2 i}{R_1 + R_2} \Rightarrow i_2 = \frac{R_1 i}{R_1 + R_2}$$



- Remember this law of $i \propto \frac{1}{R}$ in the resistors connected in parallel. It can be used in problems.

Ex. Find current passing through the battery and each resistor.



Sol. Method (I)

It is easy to see that potential difference across each resistor is 30 V.

\therefore current in each resistor are $\frac{30}{2} = 15 \text{ A}$, $\frac{30}{3} = 10 \text{ A}$ and $\frac{30}{6} = 5 \text{ A}$

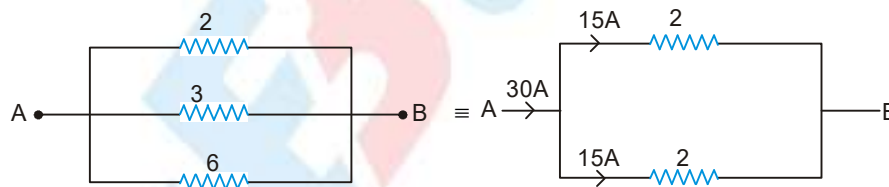
\therefore Current through battery is $= 15 + 10 + 5 = 30 \text{ A}$.

Method (II)

By ohm's law $i = \frac{V}{R_{eq}} \Rightarrow \frac{I}{R_{eq}} = \frac{I}{2} + \frac{I}{3} + \frac{I}{6} = 1\Omega$

$R_{eq} = 1 \Omega \Rightarrow i = \frac{30}{1} = 30 \text{ A}$

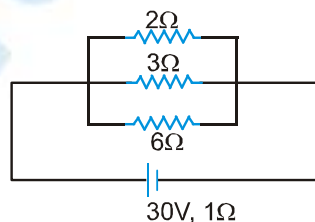
Now distribute this current in the resistors in their inverse ratio.



Current total in 3 Ω and 6 Ω is 15 A it will be divided as 10 A and 5 A.

- The method (I) is better. But you will not find such an easy case every where.

Ex. Find current which is passing through battery.



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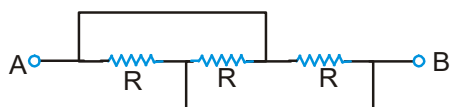
Sol. Here potential difference across each resistor is not 30 V

→ battery has internal resistance. Here the concept of combination of resistors is useful.

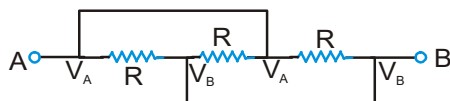
$$R_{eq} = 1 + 1 = 2 \Omega$$

$$i = \frac{30}{2} = 15 \text{ A.}$$

Ex. Find equivalent Resistance

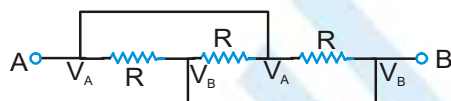


Sol.

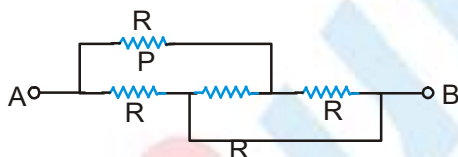


Here all the Resistance are connected between the terminals A and B
Modified circuit is

So $R_{eq} = \frac{R}{3}$

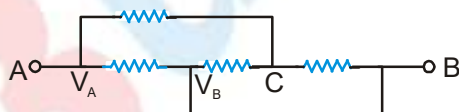


Ex. Find the current in Resistance P if voltage supply between A and B is V volts



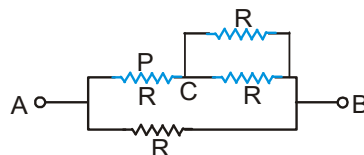
Sol

$$R_{eq} = \frac{3R}{5}$$



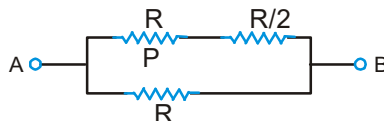
$$I = \frac{5V}{3R}$$

Modified circuit

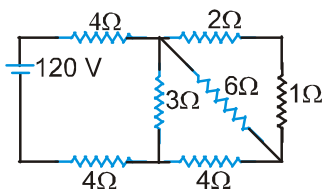


$$\text{Current in P} = \frac{R \times \frac{5V}{3R}}{1.5R + R}$$

$$= \frac{2V}{3R}$$



Ex. Find the current in 2Ω resistance



Sol. $2\Omega, 1\Omega$ in series $= 3\Omega$

$$3\Omega, 6\Omega \text{ in parallel} = \frac{18}{9} = 2\Omega$$

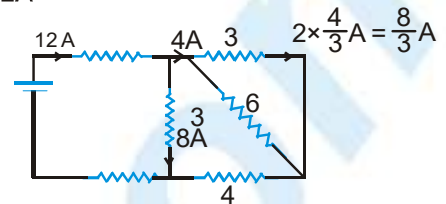
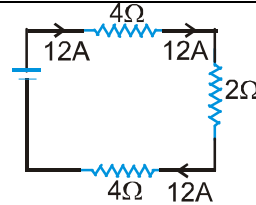
$$2\Omega, 4\Omega \text{ in series} = 6\Omega$$

$$6\Omega, 3\Omega \text{ in parallel} = 2\Omega$$

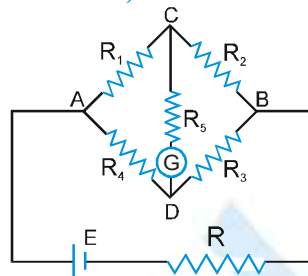
$$R_{eq} = 4 + 4 + 2 = 10\Omega$$

$$i = \frac{120}{10} = 12A$$

$$\text{So current in } 2\Omega \text{ Resistance} = \frac{8}{3} A$$



Wheatstone Network : (4 terminal network)



The arrangement as shown in figure, is known as Wheat stone bridge

Here there are four terminals in which except two all are connected to each other through resistive elements.

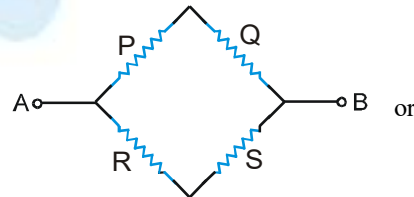
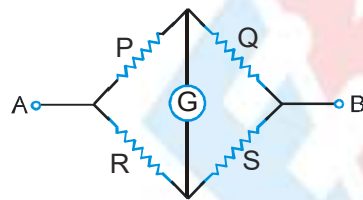
In this circuit if $R_1 R_3 = R_2 R_4$ then $V_C = V_D$ and current in $R_5 = 0$ this is called balance point or null point

When current through the galvanometer is zero (null point or balance point) $\frac{P}{Q} = \frac{R}{S}$, then $PS = QR \Rightarrow$ Here

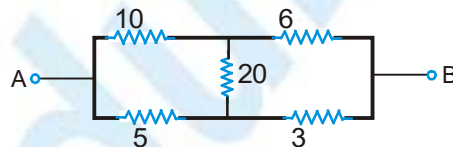
in this case products of opposite arms are equal. Potential difference between C and D at null point is zero.

The null point is not affected by resistance R_5 , E and R. It is not affected even if the positions of Galvanometer and battery (E) are interchanged.

hence, here the circuit can be assumed to be following,



Ex. Find equivalent resistance of the circuit between the terminals A and B.

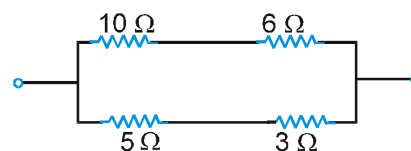


Sol. Since the given circuit is wheat stone bridge and it is in balance condition.

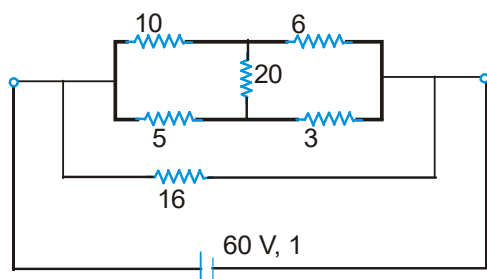
$$10 \times 3 = 30 = 6 \times 5$$

hence this is equivalent to

$$R_{eq} = \frac{16 \times 8}{16 + 8} = \frac{16}{3} \Omega$$



Ex.



Find (a) Equivalent resistance (b) and current in each resistance

Sol. (a) $R_{eq} = \left(\frac{1}{16} + \frac{1}{8} + \frac{1}{16} \right)^{-1} = 5 \Omega$

(b) $i = \frac{60}{4+1} = 12 \text{ A}$

Hence 12 A will flow through the cell.

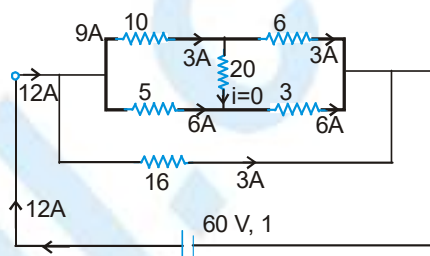
By using current distribution law.

Current in resistance 10Ω and $6\Omega = 3 \text{ A}$

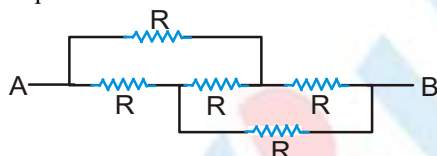
Current in resistance 5Ω and $3\Omega = 6 \text{ A}$

Current in resistance $20\Omega = 0$

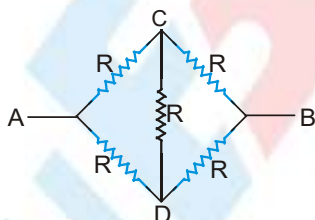
Current in resistance $16\Omega = 3 \text{ A}$



Ex. Find the equivalent resistance between A and B

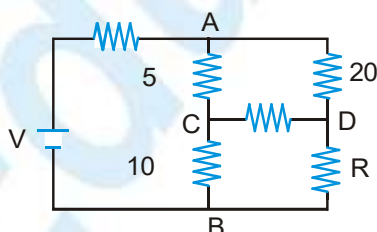


Sol This arrangement can be modified as shown in figure since it is balanced wheat stone bridge



$$R_{eq} = \frac{2R \times 2R}{2R + 2R} = R$$

Ex. Determine the value of R in the circuit shown in figure, when the current is zero in the branch CD.



Sol The current in the branch CD is zero, if the potential difference across CD is zero. That means, voltage at point C = voltage at point D.

Since no current is flowing, the branch CD is open circuited. So the same voltage is applied across ACB and ADB

$$V_{10} = V \times \frac{10}{15} \Rightarrow V_R = V \times \frac{R}{20+R}$$

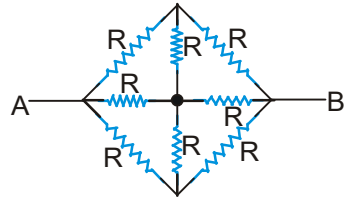
$$\therefore V_{10} = V_R \text{ and } V \times \frac{10}{15} = V \times \frac{R}{20+R}$$

$$\therefore R = 40 \, \Omega$$

Symmetrical Circuits

Some circuits can be modified to have simpler solution by using symmetry if they are solved by traditional method of KVL and KCL then it would take much time.

Ex. Find the equivalent Resistance between A and B



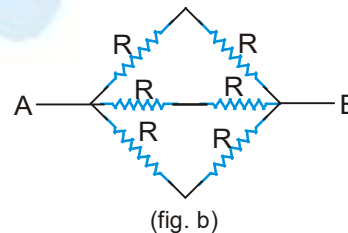
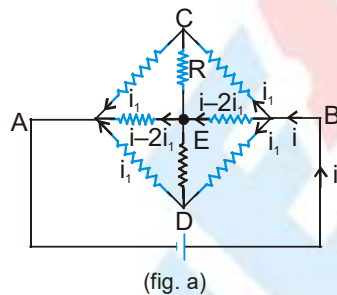
Sol I Method :

Here no two resistors appear to be in series or parallel no Wheatstone bridge here. This circuit will be

solved by using $R_{eq} = \frac{V}{I}$. The branches AC and AD are symmetrical

\therefore current through them will be same.

The circuit is also similar from left side and right side current distribution while entering through B and an exiting from A will be same. Using all these facts the currents are as shown in the figure. It is clear that current in resistor between C and E is 0 and also in ED is 0. It's equivalent as shown in figure (b)



$$R_{eq} = \frac{2R}{3}$$

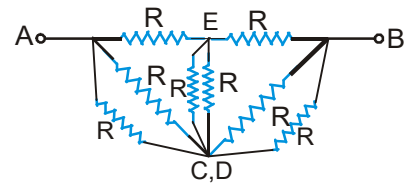
II Method

→ The potential difference in R between (B, C) and between (B.D.) is same $V_C = V_D$

Hence the point C and D are same hence circuit can be simplified as this called folding.

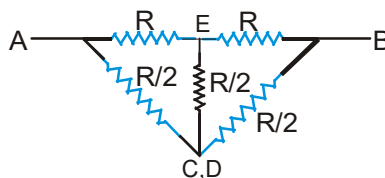
Now, it is Balanced Wheatstone bridge

$$R_{eq} = \frac{2R \times R}{2R + R} = \frac{2R}{3}$$

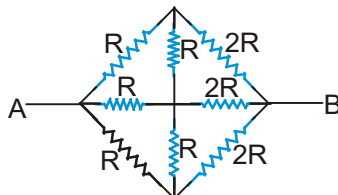


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- In II Method it is not necessary to know the currents in CA and DA.



Ex. Find the equivalent Resistance between A and B



Sol. In this case the circuit has symmetry in the two branches AC and AD at the input

\therefore current in them are same but from input and from exit the circuit is not similar

(\rightarrow on left R and on right 2R)

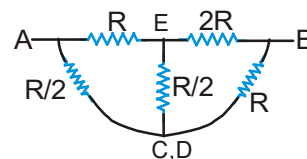
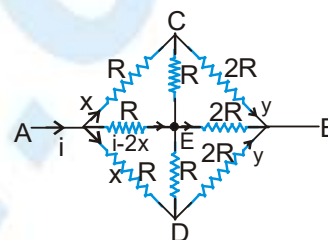
\therefore on both sides the distribution of current will not be similar.

Here $V_c = V_d$

hence C and D are same point

the circuit can be simplified that as shown

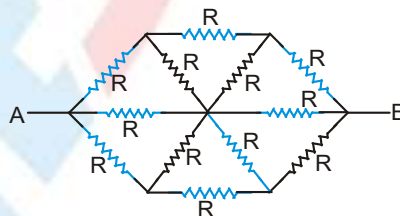
Now it is balanced wheat stone bridge



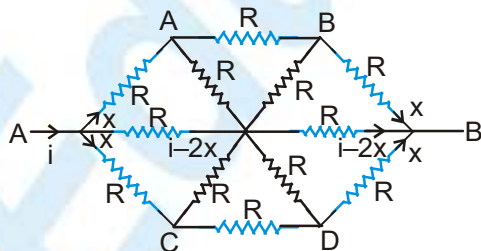
$$R_{eq} = \frac{3R \times \frac{3R}{2}}{3R + \frac{3R}{2}}$$

$$= \frac{\frac{9}{2}R}{\frac{9}{2}} = R.$$

Ex. Find the equivalent Resistance between A and B

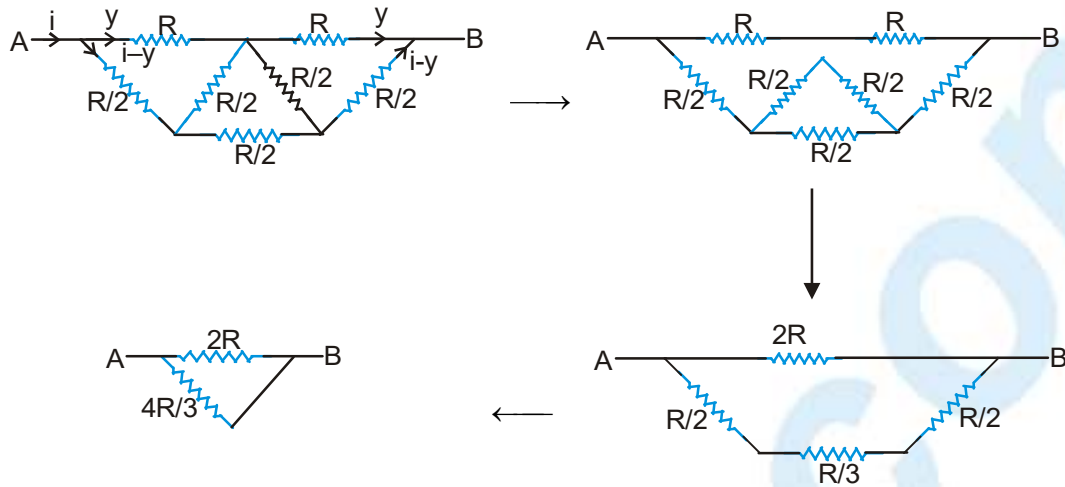


Sol.



Here $V_A = V_C$ and $V_B = V_D$

Here the circuit can be simplified as
this circuit can be simplified as

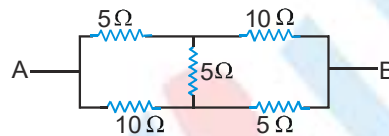


$$R_{eq} = \frac{2R \times \frac{4R}{3}}{\frac{10R}{3}}$$

$$= \frac{4R}{5}$$

Ans.

Ex. Find the equivalent Resistance between A and B



Sol. It is wheat stone bridge but not balanced. No series parallel connections. But similar values on input side and output. Here we see that even after using symmetry the circuit does not reduce to series parallel combination as in previous examples.

∴ applying kirchoff voltage law

$$v - 10(i - x) - 5x = 0$$

$$v - 10i + 5x = 0 \quad \text{.....(1)}$$

$$10(i - x) - 5(2x - i) - 5x = 0$$

$$10i - 10x - 10x + 5i - 5x = 0$$

$$15i - 25x = 0$$

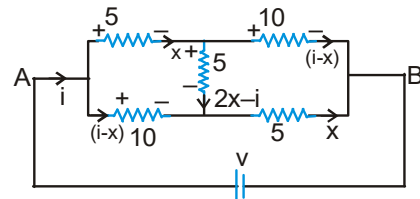
$$x = \frac{15}{25} i \quad 5x = 3i \quad \text{.....(2)}$$

Using (2) and (1)

$$\therefore v - 10i + 3i = 0$$

$$\frac{v}{i} = 7\Omega$$

$$R_{eq} = 7\Omega$$



Combination of Cells

1. Cells in Series



Equivalent EMF

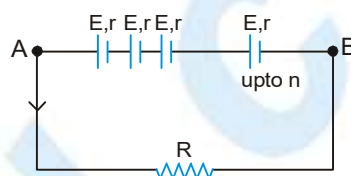
$$E_{eq} = E_1 + E_2 + \dots + E_n \quad [\text{write EMF's with polarity}]$$

Equivalent internal resistance

$$r_{eq} = r_1 + r_2 + r_3 + r_4 + \dots + r_n$$

If n cells each of emf E , arranged in series and if r is internal resistance of each cell, then total emf $= nE$
so current in the circuit

$$I = \frac{nE}{R + nr}$$



If $nr \ll R$ then $I = \frac{nE}{R} \rightarrow$ Series combination is advantageous.

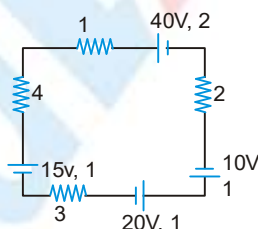
If $nr \gg R$ then $I = \frac{E}{r} \rightarrow$ Series combination is not advantageous.

ETOOS KEY POINTS

- (i) If polarity of m cells is reversed, then equivalent emf $= (n-2m)E$ while the equivalent resistance is still $nr+R$, so current in R will be

$$i = \frac{(n-2m)E}{nr + R}.$$

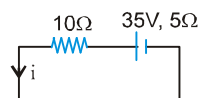
Ex. Find the current in the loop.



Sol. The given circuit can be simplified as

$$i = \frac{35}{10+5} = \frac{35}{15}$$

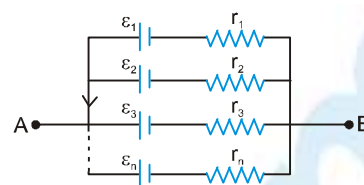
$$= \frac{7}{3} \text{ A} \quad \Rightarrow \quad I = \frac{7}{3} \text{ A}$$



2. Cells in Parallel

$$E_{eq} = \frac{\frac{\epsilon_1}{r_1} + \frac{\epsilon_2}{r_2} + \dots + \frac{\epsilon_n}{r_n}}{\frac{1}{r_1} + \frac{1}{r_2} + \dots + \frac{1}{r_n}} \quad [\text{Use emf's with polarity}]$$

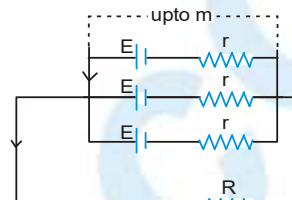
$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \dots + \frac{1}{r_n}$$



If m cells each of emf E and internal resistance r be connected in parallel and if this combination is connected to an external resistance then equivalent emf of the circuit = E .

Internal resistance of the circuit = $\frac{r}{m}$.

and
$$I = \frac{E}{R + \frac{r}{m}} = \frac{mE}{mR + r}$$



If $mR \ll r$; $I = \frac{mE}{r} \rightarrow$ Parallel combination is advantageous.

If $mR \gg r$; $I = \frac{E}{R} \rightarrow$ Parallel combination is not advantageous.

3. Cells in Multiple Arc

mn = number of identical cells.

n = number of rows

m = number of cells in each row.

The combination of cells is equivalent to single cell of
emf = mE

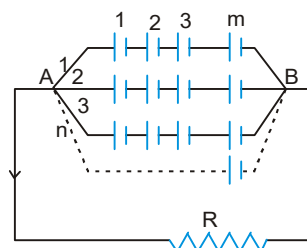
and internal resistance = $\frac{mr}{n}$

Current
$$I = \frac{mE}{R + \frac{mr}{n}}$$

For maximum current $nR = mr$

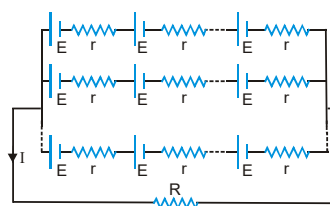
or $R = \frac{mr}{n}$ = internal resistance of the equivalent battery.

$$I_{\max} = \frac{nE}{2r} = \frac{mE}{2R}$$



4. Mixed Combination

If n cells connected in series and there are m such branches in the circuit then total number of identical cell in this circuit is nm . The internal resistance of the cells connected in a row = nr . Since there are such m rows,



Total internal resistance of the circuit $r_{\text{net}} = \frac{nr}{m}$

Total e.m.f. of the circuit = total e.m.f. of the cells connected in a row $E_{\text{net}} = nE$

Current in the circuit $I = \frac{E_{\text{net}}}{R + r_{\text{net}}} = \frac{nE}{R + \frac{nr}{m}}$

Current in the circuit is maximum when external resistance in the circuit is equal to the total internal resistance of the

cells $R = \frac{nr}{m}$

ETOOS KEY POINTS

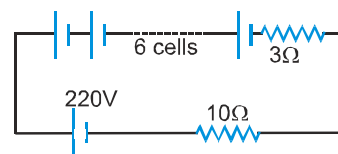
- (i) At the time of charging a cell when current is supplied to the cell, the terminal voltage is greater than the e.m.f. E , $V = E + Ir$
- (ii) Series combination is useful when internal resistance is less than external resistance of the cell.
- (iii) Parallel combination is useful when internal resistance is greater than external resistance of the cell.
- (iv) Power in R (given resistance) is maximum, if its value is equal to net resistance of remaining circuit.
- (v) Internal resistance of ideal cell = 0
- (vi) If external resistance is zero then current given by circuit is maximum.

Ex. A battery of six cells each of e.m.f. 2 V and internal resistance 0.5Ω is being charged by D. C. mains of e.m.f. 220 V by using an external resistance of 10Ω . What will be the charging current.

Sol. Net e.m.f. of the battery = 12V and total internal resistance = 3Ω

Total resistance of the circuit = $3 + 10 = 13 \Omega$

Charging current $I = \frac{\text{Net e.m.f.}}{\text{total resistance}} = \frac{220 - 12}{13} = 16 \text{ A}$



Ex. A battery of six cells each of e.m.f. 2 V and internal resistance 0.5Ω is being charged by D. C. mains of e.m.f. 220 V by using an external resistance of 10Ω . What is the potential difference across the battery?

Sol. In case of charging of battery, terminal potential $V = E + Ir = 12 + 16 \times 3 = 60 \text{ volt}$.

Ex. Four identical cells each of e.m.f. 2V are joined in parallel providing supply of current to external circuit consisting of two 15Ω resistors joined in parallel. The terminal voltage of the equivalent cell as read by an ideal voltmeter is 1.6V calculate the internal resistance of each cell.

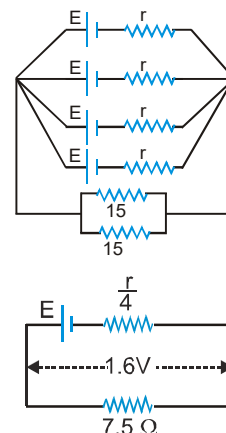
Sol. Total internal resistance of the combination $r_{\text{eq}} = \frac{r}{4}$

Total e.m.f. $E_{\text{eq}} = 2\text{V}$

Total external resistance $R = \frac{15 \times 15}{15 + 15} = \frac{15}{2} = 7.5 \Omega$

Current drawn from equivalent cell $I = \frac{\text{terminal potential}}{\text{external resistance}} = \frac{1.6}{7.5} \text{ A}$

$$\rightarrow E - I\left(\frac{r}{4}\right) = 1.6 \quad \therefore 2 - \frac{1.6}{7.5}\left(\frac{r}{4}\right) = 1.6 \Rightarrow r = 7.5 \Omega$$



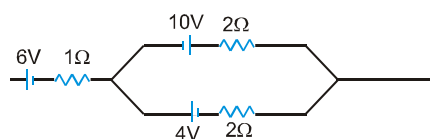
Ex. The e.m.f. of a primary cell is 2 V, when it is shorted then it gives a current of 4 A. Calculate internal resistance of primary cell.

Sol. $I = \frac{E}{r + R}$, If cell is shorted then $R = 0$, $I = \frac{E}{r} \quad \therefore r = \frac{E}{I} = \frac{2}{4} = 0.5 \Omega$

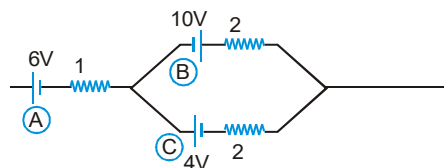
Ex. n rows each containing m cells in series, are joined in parallel. Maximum current is taken from this combination in a 3Ω resistance. If the total number of cells used is 24 and internal resistance of each cell is 0.5Ω , find the value of m and n.

Sol. Total number of cell $mn = 24$, For maximum current $\frac{mr}{n} = R \Rightarrow 0.5 m = 3 n$, $m = \frac{3n}{0.5} = 6n$
 $\therefore 6n \times n = 24 \Rightarrow n = 2$ and $m \times 2 = 24 \Rightarrow m = 12$

Ex. Find the emf and internal resistance of a single battery which is equivalent to a combination of three batteries as shown in figure.



Sol.



Battery (B) and (C) are in parallel combination with opposite polarity. So, their equivalent

$$\varepsilon_{BC} = \frac{\frac{10}{2} + \frac{-4}{2}}{\frac{1}{2} + \frac{1}{2}} = \frac{5 - 2}{1} = 3V \quad \Rightarrow \quad r_{BC} = 1\Omega.$$

Now, $\varepsilon_{ABC} = 6 - 3 = 3V \quad \Rightarrow \quad r_{ABC} = 2\Omega.$ **Ans.**

1. Galvanometer

Galvanometer is represented as follow :



It consists of a pivoted coil placed in the magnetic field of a permanent magnet. Attached to the coil is a spring. In the equilibrium position, with no current in the coil, the pointer is at zero and spring is relaxed. When there is a current in the coil, the magnetic field exerts a torque on the coil that is proportional to current. As the coil turns, the spring exerts a restoring torque that is proportional to the angular displacement. Thus, the angular deflection of the coil and pointer is directly proportional to the coil current and the device can be calibrated to measure current.

When coil rotates the spring is twisted and it exerts an opposing torque on the coil.

There is a resistive torque also against motion to damp the motion. Finally in equilibrium

$$\tau_{\text{magnetic}} = \tau_{\text{spring}} \Rightarrow BINA \sin \theta = C\phi$$

But by making the magnetic field radial $\theta = 90^\circ$.

$$\therefore BINA = C\phi$$

$$I \propto \phi$$

here $B =$ magnetic field

$I =$ Current

$N =$ Number of turns

$A =$ Area of the coil

$C =$ torsional constant

$\phi =$ angle rotate by coil.

• **Current Sensitivity**

The ratio of deflection to the current i.e. deflection per unit current is called current sensitivity (C.S.) of the

galvanometer $CS = \frac{\phi}{I} = \frac{BNA}{C}$

Shunt

The small resistance connected in parallel to galvanometer coil, in order to control current flowing through the galvanometer, is known as shunt.

• **Merits of shunt**

- (i) To protect the galvanometer coil from burning.
- (ii) Any galvanometer can be converted into ammeter of desired range with the help of shunt.
- (iii) The range an ammeter can be changed by using shunt resistance of different values.

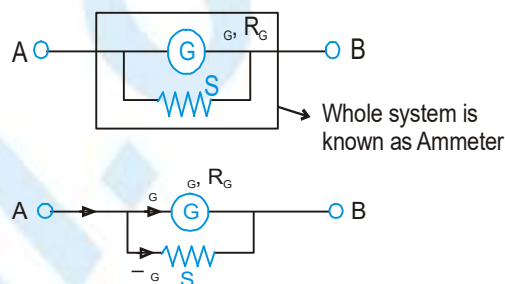
• **Demerits of shunt**

Shunt resistance decreases the sensitivity of galvanometer.

2. **Ammeter**

A shunt (small resistance) is connected in parallel with galvanometer to convert it into ammeter; An ideal ammeter has zero resistance

Ammeter is represented as follow -



If maximum value of current to be measured by ammeter is I then

$$I_G \cdot R_G = (I - I_G)S$$

$$S = \frac{I_G \cdot R_G}{I - I_G}$$

$$S = \frac{I_G \times R_G}{I} \quad \text{when } I \gg I_G$$

where I = Maximum current that can be measured using the given ammeter.

For measuring the current the ammeter is connected in series.

In calculation it is simply a resistance



Resistance of ammeter

$$R_A = \frac{R_G \cdot S}{R_G + S}$$

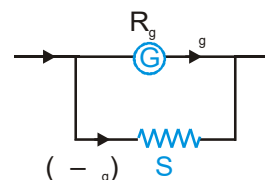
for $S \ll R_G \Rightarrow R_A = S$

Ex. What is the value of shunt which passes 10% of the main current through a galvanometer of 99 ohm ?

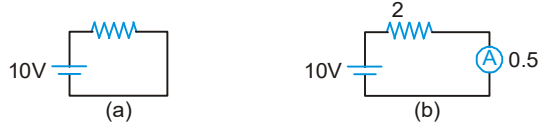
Sol. As in figure $R_G I_g = (I - I_g)S$

$$\Rightarrow 99 \times \frac{I}{10} = \left(I - \frac{I}{10}\right) \times S$$

$$\Rightarrow S = 11 \Omega$$



Ex. Find the current in the circuit (a) & (b) and also determine percentage error in measuring the current through an ammeter.



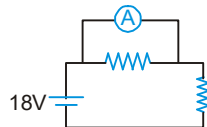
Sol. In A $I = \frac{10}{2} = 5\text{A}$

In B $I = \frac{10}{2.5} = 4\text{A}$

Percentage error is $= \frac{i - i'}{i} \times 100 = 20\%$ **Ans.**

Here we see that due to ammeter the current has reduced. A good ammeter has very low resistance as compared with other resistors, so that due to its presence in the circuit the current is not affected.

Ex. Find the reading of ammeter ? Is this the current through $6\ \Omega$?



Sol. $R_{eq} = \frac{3 \times 6}{3 + 6} + 1 = 3\ \Omega$

Current through battery

$$I = \frac{18}{3} = 6\text{ A}$$

So, current through ammeter

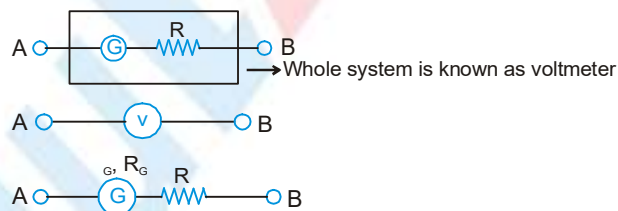
$$= 6 \times \frac{6}{9} = 4\text{ A}$$

No, it is not the current through the $6\ \Omega$ resistor.

- Ideal ammeter is equivalent to zero resistance wire for calculation potential difference across it is zero.

3. Voltmeter

A high resistance is put in series with galvanometer. It is used to measure potential difference across a resistor in a circuit.



For maximum potential difference

$$V = I_G \cdot R + I_G R_G \quad R = \frac{V}{I_G} - R_G$$

$$\text{If } R_G \ll R \Rightarrow R_s \approx \frac{V}{I_G}$$

For measuring the potential difference a voltmeter is connected across that element. (parallel to that element it measures the potential difference that appears between terminals 'A' and 'B'.)

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For calculation it is simply a resistance



Resistance of voltmeter $R_v = R_G + R \approx R$

$$I_g = \frac{V_o}{R_g + R}. \quad R \rightarrow \infty \Rightarrow \text{Ideal voltmeter.}$$

A good voltmeter has high value of resistance.

Ideal voltmeter \rightarrow which has high value of resistance.

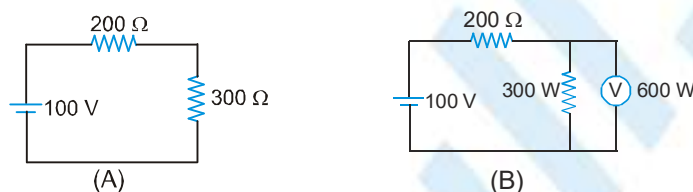
- For calculation purposes the current through the ideal voltmeter is zero.
- Percentage error in measuring the potential difference by a voltmeter is $= \frac{V - V'}{V} \times 100$

Ex. A galvanometer has a resistance of G ohm and range of V volt. Calculate the resistance to be used in series with it to extend its range to nV volt.

Sol. Full scale current $i_g = \frac{V}{G}$
to change its range

$$V_1 = (G + R_s)i_g \Rightarrow nV = (G + R_s) \frac{V}{G} \Rightarrow R_s = G(n - 1) \quad \text{Ans.}$$

Ex. Find potential difference across the resistance 300Ω in A and B.



Sol. In (A) : Potential difference $= \frac{100}{200 + 300} \times 300 = 60$ volt

$$\text{In (B) : Potential difference} = \frac{100}{200 + \frac{300 \times 600}{300 + 600}} \times \frac{300 \times 600}{300 + 600} = 50 \text{ volt}$$

We see that by connecting voltmeter the voltage which was to be measured has changed. Such voltmeters are not good. If its resistance had been very large than 300Ω then it would not have affected the voltage by much amount.

Current sensitivity

The ratio of deflection to the current i.e. deflection per unit current is called current sensitivity (C.S.) of the

$$\text{galvanometer CS} = \frac{\theta}{I}$$

- Shunting a galvanometer decreases its current sensitivity.

Ex. A galvanometer with a scale divided into 100 equal divisions, has a current sensitivity of 10 division per mA and voltage sensitivity of 2 division per mV. What adaptations are required to use it (a) to read 5A full scale and (b) 1 division per volt ?

Sol. Full scale deflection current $i_g = \frac{\theta}{cs} = \frac{100}{10} \text{ mA} = 10 \text{ mA}$

$$\text{Full scale deflection voltage } V_g = \frac{\theta}{vs} = \frac{100}{2} \text{ mv} = 50 \text{ mv}$$

$$\text{So galvanometer resistance } G = \frac{V_g}{i_g} = \frac{50\text{mV}}{10\text{mA}} = 5 \Omega$$



- (a) to convert the galvanometer into an ammeter of range 5A, a resistance of value $S\Omega$ is connected in parallel with it such that

$$(I - i_g) S = i_g G$$

$$(5 - 0.01) S = 0.01 \times 5$$

$$S = \frac{5}{499} \cong 0.01 \Omega \quad \text{Ans.}$$

- (b) To convert the galvanometer into a voltmeter which reads 1 division per volt, i.e. of range 100 V,

$$V = i_g (R + G)$$

$$100 = 10 \times 10^{-3} (R + 5)$$

$$R = 10000 - 5$$

$$R = 9995 \Omega \cong 9.995 \text{ k}\Omega \quad \text{Ans.}$$

4. POTENTIOMETER

(i) Necessity of potentiometer

Practically voltmeter has a finite resistance. (ideally it should be ∞) in other words it draws some current from the circuit. To overcome this problem potentiometer is used because at the instant of measurement, it draws no current from the circuit. It means its effective resistance is infinite.

(ii) Working principle of potentiometer

Any unknown potential difference is balanced on a known potential difference which is uniformly distributed over entire length of potentiometer wire. This process is named as zero deflection or null deflection method.

(iii) Potentiometer wire

Made up of alloys of manganin, constantan, Eureka. Specific properties of these alloys are high specific resistance, negligible temperature co-efficient of resistance (α). Invariability of resistance of potentiometer wire over a long period.

Circuits of Potentiometer

- (a) **Primary circuit** contains constant source of voltage rheostat or Resistance Box

- (b) **Secondary, Unknown or galvanometer circuit**

Let ρ = Resistance per unit length of potentiometer wire

- (c) **Potential gradient (x) (V/m)**

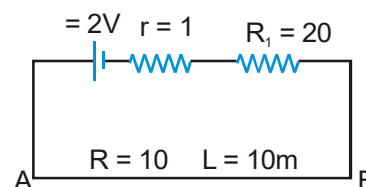
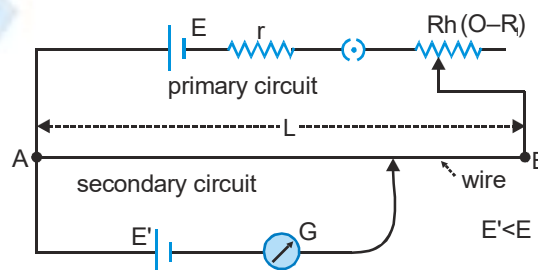
The fall of potential per unit length of potentiometer wire is called potential gradient.

$$x = \frac{V}{L} = \frac{\text{current} \times \text{resistance of potentiometer wire}}{\text{length of potentiometer wire}} = I \left(\frac{R}{L} \right)$$

The potential gradient depends only on primary circuit and is independent of secondary circuit.

Ex. Primary circuit of potentiometer is shown in figure determine :

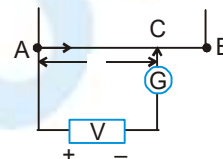
- (A) current in primary circuit
 (B) potential drop across potentiometer wire AB
 (C) potential gradient (means potential drop per unit length of potentiometer wire)
 (D) maximum potential which we can measure above potentiometer



- Sol.** (a) $i = \frac{\varepsilon}{r + R_1 + R} = \frac{2}{1 + 20 + 10} \Rightarrow i = \frac{2}{31} \text{ A}$
- (b) $V_{AB} = iR = \frac{2}{31} \times 10 \Rightarrow V_{AB} = \frac{20}{31} \text{ volt}$
- (c) $x = \frac{V_{AB}}{L} = \frac{2}{31} \text{ volt/m}$
- (d) Maximum potential which we can measure by it = potential drop across wire AB = $\frac{20}{31} \text{ volt}$

Ex. How to measure an unknown voltage using potentiometer.

Sol. The unknown voltage V is connected across the potentiometer wire as shown in figure. The positive terminal of the unknown voltage is kept on the same side as of the source of the top most battery. When reading of galvanometer is zero then we say that the meter is balanced. In that condition $V = x \bullet$.



Application of Potentiometer

- (a) To find emf of unknown cell and compare emf of two cells.

In Case I,

In figure, (2) is joint to (1) then balance length = \bullet_1

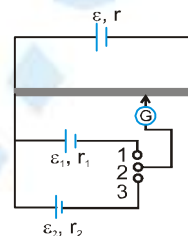
$$\varepsilon_1 = x \bullet_1 \quad \dots(1)$$

In Case II,

In figure, (3) is joint to (2) then balance length = \bullet_2

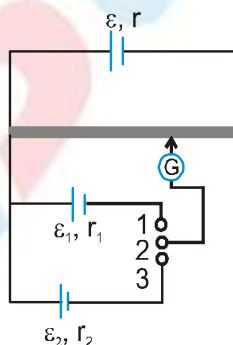
$$\varepsilon_2 = x \bullet_2 \quad \dots(2)$$

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{\lambda_1}{\lambda_2}$$



If any one of ε_1 or ε_2 is known the other can be found. If x is known then both ε_1 and ε_2 can be found

Ex. In an experiment to determine the emf of an unknown cell, its emf is compared with a standard cell of known emf $\varepsilon_1 = 1.12 \text{ V}$. The balance point is obtained at 56cm with standard cell and 80 cm with the unknown cell. Determine the emf of the unknown cell.



Sol. Here, $\varepsilon_1 = 1.12 \text{ V}$; $\bullet_1 = 56 \text{ cm}$; $\bullet_2 = 80 \text{ cm}$

Using equation

$$\varepsilon_1 = x \bullet_1 \quad \dots(1)$$

$$\varepsilon_2 = x \bullet_2 \quad \dots(2)$$

we get $\frac{\varepsilon_1}{\varepsilon_2} = \frac{\lambda_1}{\lambda_2} \Rightarrow \varepsilon_2 = \varepsilon_1 \left(\frac{\lambda_2}{\lambda_1} \right)$

or $\varepsilon_2 = 1.12 \left(\frac{80}{56} \right) = 1.6 \text{ V Ans}$

(b) To find current if resistance is known

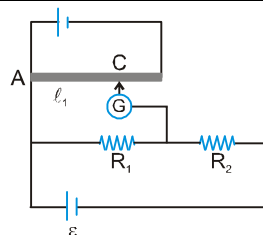
$$V_A - V_C = x \bullet_1$$

$$IR_1 = x \bullet_1$$

$$I = \frac{x \lambda_1}{R_1}$$

Similarly, we can find the value of R_2 also.

Potentiometer is ideal voltmeter because it does not draw any current from circuit, at the balance point.



Ex. A standard cell of emf $\epsilon_0 = 1.11$ V is balanced against 72 cm length of a potentiometer. The same potentiometer is used to measure the potential difference across the standard resistance $R = 120 \Omega$. When the ammeter shows a current of 7.8 mA, a balanced length of 60 cm is obtained on the potentiometer.

(i) Determine the current flowing through the resistor.

(ii) Estimate the error in measurement of the ammeter.

Sol. Here, $\bullet_0 = 72$ cm; $\bullet = 60$ cm; $R = 120 \Omega$ and $\epsilon_0 = 1.11$ V

(i) By using equation $\epsilon_0 = x \bullet_0$ (i)

$V = IR = x \bullet$ (ii)

From equation (i) and (ii)

$$I = \frac{\epsilon_0}{R} \left(\frac{\lambda}{\lambda_0} \right) \quad \therefore \quad I = \frac{1.11}{120} \left(\frac{60}{72} \right) = 7.7 \text{ mA}$$

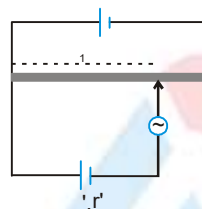
(ii) Since the measured reading 7.8 mA (> 7.7 mA) therefore, the instrument has a positive error.

$$\Delta I = 7.8 - 7.7 = 0.1 \text{ mA},$$

$$\frac{\Delta I}{I} = \frac{0.1}{7.7} \times 100 = 1.3 \%$$

(c) To find the internal resistance of cell.

1st arrangement



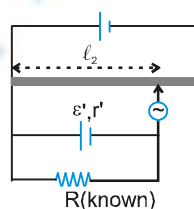
by first arrangement $\epsilon' = x \bullet_1$ (1)

by second arrangement $IR = x \bullet_2$

$$I = \frac{x \lambda_2}{R},$$

$$\text{Also } I = \frac{\epsilon'}{r' + R} \Rightarrow \frac{\epsilon'}{r' + R} = \frac{x \lambda_2}{R} \Rightarrow \frac{x \lambda_1}{r' + R} = \frac{x \lambda_2}{R} \Rightarrow r' = \left[\frac{\lambda_1 - \lambda_2}{\lambda_2} \right] R$$

2nd arrangement

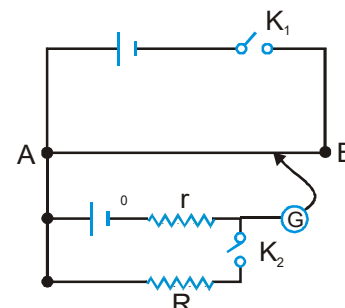


Ex. The internal resistance of a cell is determined by using a potentiometer. In an experiment, an external resistance of 60Ω is used across the given cell. When the key is closed, the balance length on the potentiometer decreases from 72 cm to 60 cm. calculate the internal resistance of the cell.

Sol. According to equation $\epsilon_0 = x \bullet_0$ (i)

$V = IR = x \bullet$ (ii)

$I = \frac{\epsilon_0}{R + r}$ (iii)



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From equation (i), (ii) and (iii) we get

$$r = R \left(\frac{\lambda_0}{\lambda} - 1 \right)$$

Here $\bullet_0 = 72 \text{ cm}$; $\bullet = 60 \text{ cm}$; $R = 60 \Omega$ $\therefore r = (60) \left(\frac{72}{60} - 1 \right)$ or $r = 12 \Omega$.

Applications of Potentiometer

- (i) To measure potential difference across a resistance.
- (ii) To find out emf of a cell.
- (iii) Comparison of two emfs E_1/E_2
- (iv) To find out internal resistance of a primary cell.
- (v) Comparison of two resistance.
- (vi) To find out an unknown resistance which is connected series with the given resistance.
- (vii) To find out current in a given circuit.
- (viii) Calibration of an ammeter or to have a check on reading of (A).
- (ix) Calibration of a voltmeter or to have a check on reading of (V).
- (x) To find out thermocouple emf (e) (mV or mV)

Difference between potentiometer and voltmeter	
Potentiometer	Voltmeter
(a) It measures the unknown emf very accurately	It measures the unknown emf approximately.
(b) While measuring emf it does not draw any current from the driving source of known emf.	While measuring emf it draws some current from the source of emf.
(c) While measuring unknown potential difference the resistance of potentiometer becomes infinite.	While measuring unknown potential difference the resistance of voltmeter is high but finite.
(d) It is based on zero deflection method.	It is based on deflection method.
(e) It has a high sensitivity.	Its sensitivity is low.
(f) it is used for various applications like measurement of internal resistance of cell, calibration of ammeter and voltmeter, measurement of thermo emf, comparison of emf's etc.	It is only used to measure emf or unknown potential difference.

5. Metre Bridge (Use to measure unknown resistance)

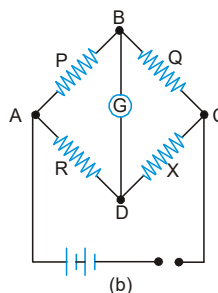
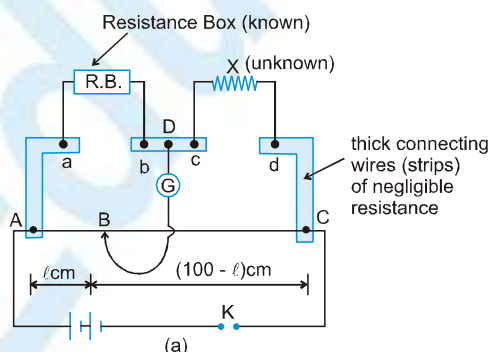
If $AB = \bullet \text{ cm}$, then $BC = (100 - \bullet) \text{ cm}$.

Resistance of the wire between A and B $R \propto \bullet$

[\rightarrow Specific resistance ρ and cross-sectional area A are same for whole of the wire]

or $R = \sigma \bullet$... (1)

where σ is resistance per cm of wire.



Similarly, if Q is resistance of the wire between B and C, then

$$Q \propto 100 - \lambda$$

$$\therefore Q = \sigma(100 - \lambda) \quad \dots(2)$$

Dividing (1) by (2),
$$\frac{P}{Q} = \frac{\lambda}{100 - \lambda}$$

Applying the condition for balanced Wheatstone bridge, we get

$$R Q = P X$$

$$\therefore X = R \frac{Q}{P} \quad \text{or} \quad X = \frac{100 - \lambda}{\lambda} R$$

Since R and λ are known, therefore, the value of X can be calculated.

- For better accuracy, R is so adjusted that λ lies between 40 cm and 60 cm.

Ex. In a meter bridge experiment, the value of unknown resistance is 2Ω . To get the balancing point at 40cm distance from the same end, the resistance in the resistance box will be :

- (A) 0.5Ω (B) 3Ω (C) 20Ω (D) 80Ω

Sol. Apply condition for balance wheat stone bridge,

$$\frac{P}{Q} = \frac{\lambda}{100 - \lambda} = \frac{P}{2} = \frac{100 - 40}{40}$$

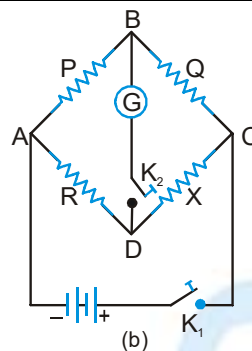
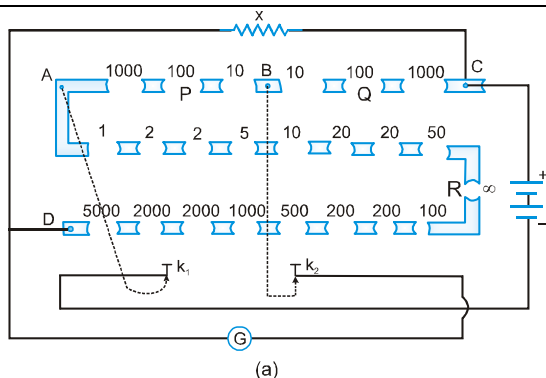
$$P = 3\Omega.$$

6. Post-office Box

Introduction. It is so named because its shape is like a box and it was originally designed to determine the resistances of electric cables and telegraph wires. It was used in post offices to determine the resistance of transmission lines.

Construction. A post office box is a compact form of Wheatstone bridge with the help of which we can measure the value of the unknown resistance correctly up to 2nd decimal place, i.e., up to 1/100th of an ohm correctly. Two types of post office box are available - plug type and dial type. In the plug-type instrument shown in figure (a), each of the arms AB and BC contains three resistances of 10, 100 and 1000 ohm. These arms are called the ratio arms. While the resistance P can be introduced in the arm AB, the resistance Q can be introduced in the arm BC. The third arm AD, called the resistance arm, is a complete resistance box containing resistances from 1Ω to $5,000\Omega$. In this arm, the resistance R is introduced by taking out plugs of suitable values. The unknown resistance X constitutes the fourth arm CD. Thus, the four arms AB, BC, CD and AD are in fact the four arms of the Wheatstone bridge (figure (b)). Two tap keys K_1 and K_2 are also provided. While K_1 is connected internally to the terminal A, K_2 is connected internally to B. These internal connections are shown by dotted lines in figure (a).

A battery is connected between C and key K_1 (battery key). A galvanometer is connected between D and key K_2 (galvanometer key). Thus, the circuit is exactly the same as that shown in figure (b). It is always the battery key which is pressed first and then the galvanometer key. This is because a self-induced current is always set up in the circuit whenever the battery key is pressed or released. If we first press the galvanometer key, the balance point will be disturbed on account of induced current. If the battery key is pressed first, then the induced current becomes zero by the time the galvanometer key is pressed. So, the balance point is not affected.



Working : The working of the post office box involves broadly the following four steps :

I. Keeping R zero, each of the resistances P and Q are made equal to $10\ \Omega$ by taking out suitable plugs from the arms AB and BC respectively. After pressing the battery key first and then the galvanometer key, the direction of deflection of the galvanometer coil is noted. Now, making R infinity, the direction of deflection is again noted. If the direction is opposite to that in the first case, then the connections are correct.

II. Keeping both P and Q equal to $10\ \Omega$, the value of R is adjusted, beginning from $1\ \Omega$, till $1\ \Omega$ increase reverses the direction of deflection. The 'unknown' resistance clearly lies somewhere between the two final values of R .

$$\left[X = R \frac{Q}{P} = R \frac{10}{10} = R \right]$$

As an illustration, suppose with $3\ \Omega$ resistance in the arm AD , the deflection is towards left and with $4\ \Omega$, it is towards right. The unknown resistance lies between $3\ \Omega$ and $4\ \Omega$.

III. Making P $100\ \Omega$ and keeping Q $10\ \Omega$, we again find those values of R between which direction of deflection is reversed. Clearly, the resistance in the arm AD will be 10 times the resistance X of the wire.

$$\left[X = R \frac{Q}{P} = R \frac{10}{100} = \frac{R}{10} \right]$$

In the illustration considered in step II, the resistance in the arm AD will now lie between $30\ \Omega$, and $40\ \Omega$. So, in this step, we have to start adjusting R from $30\ \Omega$ onwards. If $32\ \Omega$ and $33\ \Omega$ are the two values of R which give opposite deflections, then the unknown resistance lies between $3.2\ \Omega$ and $3.3\ \Omega$.

IV. Now, P is made $1000\ \Omega$ and Q is kept at $10\ \Omega$. The resistance in the arm AD will now be 100 times the 'unknown' resistance.

$$\left[X = R \frac{10}{1000} = \frac{R}{100} \right]$$

In the illustration under consideration, the resistance in the arm AD will lie between $320\ \Omega$ and $330\ \Omega$. Suppose the deflection is to the right for $326\ \Omega$, towards left for $324\ \Omega$ and zero deflection for $325\ \Omega$. Then, the unknown resistance is $3.25\ \Omega$.

The post office box method is a less accurate method for the determination of unknown resistance as compared to a metre bridge. This is due to the fact that it is not always possible to arrange resistance in the four arms to be of the same order. When the arms ratio is large, large resistance are required to be introduced in the arm R .

ETOOS KEY POINTS

- (i) To increase the range of an ammeter a shunt is connected in parallel with the galvanometer.
- (ii) To convert an ammeter of range I ampere and resistance $R_g \Omega$ into an ammeter of range nI ampere, the value of resistance to be connected in parallel will be $R_g(n-1)$
- (iii) To increase the range of a voltmeter a high resistance is connected in series with it.
- (iv) To convert a voltmeter of resistance $R_g \Omega$ and range V volt into a voltmeter of range nV volt, the value of resistance to be connected in series will be $(n-1)R_g$.
- (v) Resistance of ideal ammeter is zero & resistance of ideal voltmeter is infinite.
- (vi) The bridge is most sensitive when the resistance in all the four branches of the bridge is of same order.

Ex. The post office box works on the principle of :

- (A) Potentiometer
- (B) Wheatstone bridge
- (C) Matter waves
- (D) Ampere's law

Ans. : (B)

Ex. While using a post office box the keys should be switched on in the following order :

- (A) first cell key the and then galvanometer key.
- (B) first the galvanometer key and then cell key.
- (C) both the keys simultaneously.
- (D) any key first and then the other key.

Ans. : (A)

Ex. In a post office box if the position of the cell and the galvanometer are interchanged, then the :

- (A) null point will not change
- (B) null point will change
- (C) post office box will not work
- (D) Nothing can be said.

Ans. : (A)

Heating effect of current

Cause of heating

The potential difference applied across the two ends of conductor sets up electric field. Under the effect of electric field, electrons accelerate and as they move, they collide against the ions and atoms in the conductor, the energy of electrons transferred to the atoms and ions appears as heat.

Joules's Law of Heating

When a current I is made to flow through a passive or ohmic resistance R for time t , heat Q is produced such that

$$Q = I^2 R t = P \times t = V I t = \frac{V^2}{R} t$$

Heat produced in conductor does not depend upon the direction of current.

• **SI unit** : joule ;

$$1 \text{ kWh} = 3.6 \times 10^6 \text{ joule} = 1 \text{ unit}$$

$$\bullet \text{Power} : P = V I = \frac{V^2}{R} = I^2 R$$

Practical Units : 1 kilowatt hour (kWh)

$$1 \text{ BTU (British Thermal Unit)} = 1055 \text{ J}$$

• **SI unit** : Watt

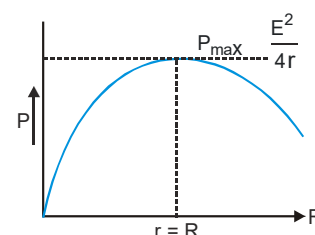
The watt-hour meter placed on the premises of every consumer records the electrical energy consumed.

Power transferred to load by cell :

$$P = I^2 R = \frac{E^2 R}{(r + R)^2} \Rightarrow P = P_{\max} \quad \text{if} \quad \frac{dP}{dR} = 0 \Rightarrow r = R$$

Power transferred by cell to load is maximum when

$$r = R \text{ and } P_{\max} = \frac{E^2}{4r} = \frac{E^2}{4R}$$



PHYSICS FOR JEE MAIN & ADVANCED

• Series combination of resistors (bulbs)

$$\text{Total power consumed } P_{\text{total}} = \frac{P_1 P_2}{P_1 + P_2} \quad \text{If } n \text{ bulbs are identical } P_{\text{total}} = \frac{P}{n}$$

In series combination of bulbs Brightness \propto Power consumed by bulb $\propto V \propto R \propto \frac{1}{P_{\text{rated}}}$

Bulb of lesser wattage will shine more. For same current $P = I^2 R$ $P \propto R$ $R \uparrow \Rightarrow P \uparrow$

• Parallel combination of resistors (bulbs)

$$\text{Total power consumed } P_{\text{total}} = P_1 + P_2 \quad \text{If } n \text{ bulbs are identical } P_{\text{total}} = nP$$

In parallel combination of bulbs

Brightness \propto Power consumed by bulb $\propto I \propto \frac{1}{R}$ Bulb of greater wattage will shine more.

For same V more power will be consumed in smaller resistance $P \propto \frac{1}{R}$

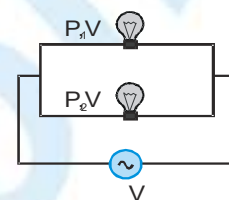
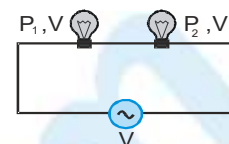
- Two identical heater coils gives total heat H_s when connected in series and H_p when connected in parallel then

$$\frac{H_p}{H_s} = 4 \quad [\text{In this, it is assumed that supply voltage is same}]$$

- If a heater boils m kg water in time T_1 and another heater boils the same water in T_2 , then both connected in series

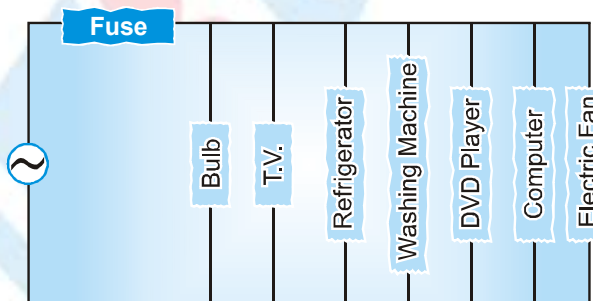
$$\text{will boil the same water in time } T_s = T_1 + T_2 \text{ and in parallel } T_p = \frac{T_1 T_2}{T_1 + T_2} \quad [\text{Use time taken } \propto \text{Resistance}]$$

- Instruments based on heating effect of current, works on both A.C. and D.C. Equal value of A.C. (RMS) and D.C. produces, equal heating effect. That why brightness of bulb is same whether it is operated by A.C. or same value D.C.



Fuse wire

The fuse wire for an electric circuit is chosen keeping in view the value of safe current through the circuit.



- The fuse wire should have high resistance per unit length and low melting point.
- However the melting point of the material of fuse wire should be above the temperature that will be reached on the passage of the current through the circuit
- A fuse wire is made of alloys of lead (Pb) and tin (Sn).
- Length of fuse wire is immaterial.
- The material of the filament of a heater should have high resistivity and high melting point.
- The temperature of the wire increases to such a value at which, the heat produced per second equals heat lost per

$$\text{second due to radiation from the surface of wire } I^2 \left(\frac{\rho l}{\pi r^2} \right) = H \times 2 \pi r l \quad I^2 \propto r^3$$

H = heat lost per second per unit area due to radiation.



Ex. An electric heater and an electric bulb are rated 500 W, 220 V and 100 W, 220 V respectively. Both are connected in series to a 220 V a.c. mains. Calculate power consumed by (i) heater (ii) bulb.

Sol. $P = \frac{V^2}{R}$ or $R = \frac{V^2}{P}$, For heater. Resistance $R_h = \frac{(220)^2}{500} = 96.8 \Omega$, For bulb resistance $R_L = \frac{(220)^2}{100} = 484 \Omega$

Current in the circuit when both are connected in series $I = \frac{V}{R_L + R_h} = \frac{220}{484 + 96.8} = 0.38 \text{ A}$

(i) Power consumed by heater $= I^2 R_h = (0.38)^2 \times 96.8 = 13.98 \text{ W}$

(ii) Power consumed by bulb $= I^2 R_L = (0.38)^2 \times 484 = 69.89 \text{ W}$

Ex. A heater coil is rated 100 W, 200 V. It is cut into two identical parts. Both parts are connected together in parallel, to the same source of 200 V. Calculate the energy liberated per second in the new combination.

Sol. $\rightarrow P = \frac{V^2}{R}$ $\therefore R = \frac{V^2}{P} = \frac{(200)^2}{100} = 400 \Omega$

Resistance of half piece $= \frac{400}{2} = 200 \Omega$

Resistance of pieces connected in parallel $= \frac{200}{2} = 100 \Omega$

Energy liberated/second $P = \frac{V^2}{R} = \frac{200 \times 200}{100} = 400 \text{ W}$

Ex. The power of a heater is 500W at 800°C. What will be its power at 200°C. If $\alpha = 4 \times 10^{-4}$ per °C?

Sol. $P = \frac{V^2}{R}$ $\therefore \frac{P_{200}}{P_{800}} = \frac{R_{800}}{R_{200}} = \frac{R_0(1 + 4 \times 10^{-4} \times 800)}{R_0(1 + 4 \times 10^{-4} \times 200)} \Rightarrow P_{200} = \frac{500 \times 1.32}{1.08} = 611 \text{ W}$

Ex. When a battery sends current through a resistance R_1 for time t , the heat produced in the resistor is Q . When the same battery sends current through another resistance R_2 for time t , the heat produced in R_2 is again Q . Determine the internal resistance of battery.

Sol. $\left[\frac{E}{R_1 + r} \right]^2 R_1 = \left[\frac{E}{R_2 + r} \right]^2 R_2 \Rightarrow r = \sqrt{R_1 R_2}$

Ex. A fuse with a circular cross-sectional radius of 0.15 mm blows at 15A. What is the radius of a fuse, made of the same material which will blow at 120 A?

Sol. For fuse wire $I \propto r^{3/2}$

So $\frac{r_2}{r_1} = \left(\frac{I_2}{I_1} \right)^{2/3} = \left(\frac{120}{15} \right)^{2/3} = (8)^{2/3} = 4 \Rightarrow r_2 = 4r_1 = 0.60 \text{ mm}$

• Etoos Tips & Formulas •

1. **Electric Current :**

Electric charges in motion constitute an electric current. Any medium having practically free electric charges, free to migrate is a conductor of electricity. The electric charge flows from higher potential energy state to lower potential energy state. Positive charge flows from higher to lower potential and negative charge flows from lower to higher. Metals such as gold, copper, aluminium etc. are good conductors.

2. **Electric Current in a Conductor :**

In absence of potential difference across a conductor, no net current flows through a cross section. When a potential difference is applied across a conductor the charge carriers (electrons in case of metallic conductors) flow in a definite direction which constitutes a net current in it. These electrons are accelerated by electric field in the conductor produced by potential difference across the conductor. They move with a constant drift velocity. The direction of current is along the flow of positive charge (or opposite to flow of negative charge). $i = n v_d e A$, where v_d = drift velocity.

3. **Charge Current and Current Density**

The strength of the current i is the rate at which the electric charges are flowing. If a charge Q coulomb passes through a given cross section of the conductor in t second the current I through the conductor is given by

$$I = \frac{Q \text{ coulomb}}{t \text{ second}} = \text{ampere}$$

Ampere is the unit of current. If i is not constant then $i = \frac{dq}{dt}$, where dq is net charge transported at a section in time

dt . In a current carrying conductor we can define a vector which gives the direction as current per unit normal, cross sectional area & is known as current density.

$$\text{Thus } \vec{J} = \frac{I}{S} \hat{n} \text{ or } I = \vec{J} \cdot \vec{S}$$

Where \hat{n} is the unit vector in the direction of the flow of current.

For random J or S , we use $I = \int \vec{J} \cdot d\vec{s}$

4. **Relation in J , E and V_d :**

In conductors drift velocity of electrons is proportional to the electric field inside the conductor as; $V_d = \mu E$ where μ is the mobility of electrons

$$\text{current density is given as } J = \frac{I}{A} = ne V_d = ne(\mu E) = \sigma E$$

where $\sigma = ne\mu$ is called conductivity of material and we can also write

$$\rho = \frac{1}{\sigma} \rightarrow \text{resistivity of material.}$$

Thus $E = \rho \cdot J$. It is called as differential form of Ohm's Law.

5. **Sources of Potential Difference & Electromotive Force :**

Dry cells, secondary cells, generator and thermo couple are the devices used for producing potential difference in an electric circuit. The potential difference between the "Electromotive force" or "EMF" of the source. The unit of potential difference is volt.

$$1 \text{ volt} = 1 \text{ Ampere} \times 1 \text{ Ohm.}$$



6. **Electrical Resistance :**

The property of a substance which opposes the flow of electric current through it, is termed as electrical resistance. Electrical resistance depends on the size, geometry, temperature and internal structure of the conductor.

7. **Law of Resistance**

The resistance R offered by a conductor depends on the following factors :

$$R \propto l \text{ (length of the conductor)} ; R \propto \frac{1}{A} \text{ (cross section area of the conductor) at a given temperature } R = \rho \frac{l}{A}.$$

where ρ is the resistivity of the material of the conductor at the given temperature. It is also known as **specific resistance** of the material & it depends upon nature of conductor.

8. **Dependence of Resistance on Temperature :**

The resistance of most conductors and all pure metals increases with temperature, but there are a few in which resistance decreases with temperature.

If R_0 & R be there are few in which resistance of a conductor at 0°C and $\theta^\circ\text{C}$, then it is found that $R = R_0 (1 + \alpha\theta)$. Here we assume that the dimensions of resistance do not change with temperature if expansion coefficient of material is considerable. Then instead of resistance we use same property for resistivity as $\rho = \rho_0 (1 + \alpha\theta)$. The material for which resistance decreases with temperature, the temperature coefficient of resistance is negative.

Where α is called the temperature co-efficient of resistance. The unit of α is K^{-1} or $^\circ\text{C}^{-1}$. Reciprocal of resistivity is called conductivity and reciprocal of resistance is called conductance (G). S.I. unit of G is mho.

9. **Ohm's Law :** Ohm's law is the most fundamental law of all the laws in electricity. It says that the current through the cross section of the conductor is proportional to the applied potential difference under the given physical condition. $V = RI$. Ohm's law is applicable to only metallic conductors.

10. **KRICHHOFF'S LAW'S :**

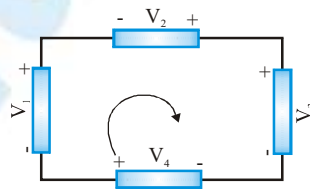
(a) **I - Law (Junction law or Nodal Analysis) :** This law is based on law of conservation of charge. It states that "The algebraic sum of the currents meeting at a point is zero" or total currents entering a junction equal s total current leaving the junction.

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

It is also known as KCL (Kirchoff's current law).

(b) **II - Law (Loop analysis) :** The algebraic sum of all the voltages in closed circuit is zero.

$\sum IR = \sum \text{EMF} = 0$ in a closed loop. The closed loop can be traversed in any direction. While traversing a loop if higher potential point is entered, put a +ve sign in expression or if lower potential point is entered put a negative sign.



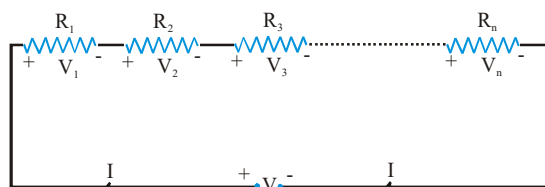
$$-V_1 - V_2 + V_3 - V_4 = 0. \text{ Boxes may contain resistor or battery or any other element (linear or non-linear).}$$

It is also known as KVL (Kirchoff's voltage law).

PHYSICS FOR JEE MAIN & ADVANCED

11. COMBINATION OF RESISTANCES:

A number of resistance can be connected and all the complicated combinations can be reduced to two different types, namely series and parallel.



(a) Resistance in Series :

When the resistance are connected end to end then they are said to be in series. The current through each resistor is same the effective resistance appearing across the battery.

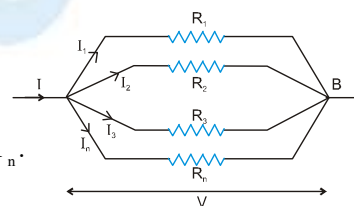
$$R = R_1 + R_2 + R_3 + \dots + R_n \text{ and } V = V_1 + V_2 + V_3 + \dots + V_n$$

The voltage across a resistor is proportion to the resistance

$$V_1 = \frac{R_1}{R_1 + R_2 + \dots + R_n} V; V_2 = \frac{R_2}{R_1 + R_2 + \dots + R_n} V; \text{ etc}$$

(b) Resistance in Parallel :

A parallel circuit of resistors is one, in which the same voltage is applied across all the components in a parallel grouping of resistors $R_1, R_2, R_3, \dots, R_n$.



12. Conclusion :

(a) Potential difference across each resistor is same.

(b) $I = I_1 + I_2 + I_3 + \dots + I_n$

(c) Effective resistance (R) then $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_n}$

(d) Current in different resistors is inversally proportional to the resistance.

$$I_1 : I_2 : \dots : I_n = \frac{1}{R_1} : \frac{1}{R_2} : \frac{1}{R_3} : \dots : \frac{1}{R_n}$$

$$I_1 = \frac{G_1}{G_1 + G_2 + \dots + G_n} I, I_2 = \frac{G_2}{G_1 + G_2 + \dots + G_n} I, \text{ etc.}$$

where $G = \frac{1}{R}$ = Conductance of a resistor.

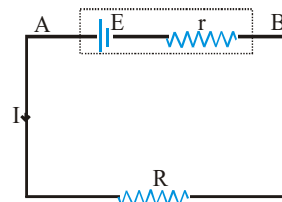
13. EMF of a Cell & its Internal Resistance :

If a cell emf E and internal resistance r be connected with a resistance R the total resistance of the circuit is $(R + r)$.

$$I = \frac{E}{R + r}; V_{AB} = \frac{E}{R + r} R$$

where V_{AB} = Terminal voltage of the battery.

If $r \rightarrow 0$, cell is Ideal & $V \rightarrow E$ & $r = R \left(\frac{E}{V} - 1 \right)$



14. **Grouping of Cells :**

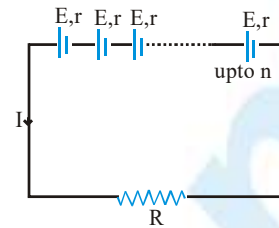
(a) **Cells in Series :** Let there be n cells each of emf E , arranged in series.

Let r be the internal resistance of each cell. The total emf = nE . Current

in the circuit $I = \frac{nE}{R + nr}$.

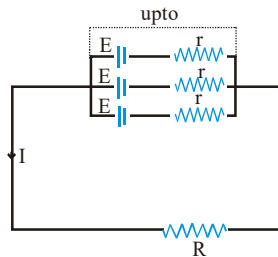
If $nr \ll R$ then $I = \frac{nE}{R} \rightarrow$ Series combination should be used

If $nr \gg R$ then $I = \frac{E}{R} \rightarrow$ Series combination should not be used.



(b) **Cells in Parallel :** If m cells each of emf E & internal resistance r be connected in parallel and if this combination be connected to an external resistance then the emf of the circuit = E .

Internal resistance of the circuit = $\frac{r}{m}$.



$$I = \frac{E}{R + \frac{r}{m}} = \frac{mE}{mR + r}$$

If $mR \ll r$ then $I = \frac{mE}{r} \rightarrow$ Parallel combination should be used.

If $mR \gg r$ then $I = \frac{E}{R} \rightarrow$ Parallel combination should be used.

(c) **Cells in Multiple Arc :**

mn = number of identical cells.

n = number of rows

m = number of cells in each rows.

The combination of cells in each equivalent to single cell of :

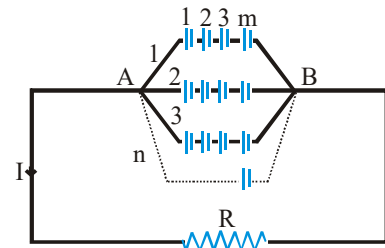
(a) emf = mE & (b) internal resistance = $\frac{mr}{n}$

Current $I = \frac{mE}{R + \frac{mr}{n}}$

For maximum current

$nR = mr$ or $R = \frac{mr}{n}$ so $I_{\max} = \frac{nE}{2r} = \frac{mE}{2R}$

For a cell to deliver maximum power across the load internal resistance = load resistance



15. **Wheat Stone Network :**

When current through the galvanometer is zero

$$\text{(null point or balance point)} \quad \frac{P}{Q} = \frac{R}{S}.$$

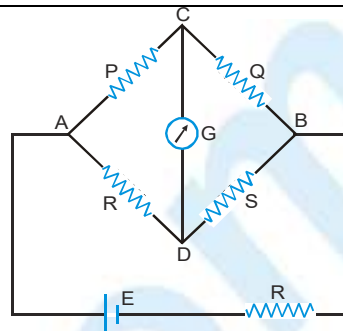
When

$$PS > QR, V_c < V_d \text{ \& } PS < QR, V_c > V_d$$

or $PS = QR \Rightarrow$ product of opposite arms are equal. Potential difference between

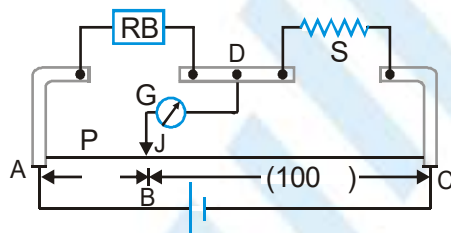
C & D at null point is zero. The null point is not affected by resistance of G & E. It is not affected even if the position of G & E are interchanged.

$$I_{CD} \propto (QR - PS)$$



16. **Metre Bridge**

$$\text{At balance condition : } \frac{P}{Q} = \frac{R}{S} \Rightarrow \frac{rl}{r(100-l)} = \frac{R}{S} \Rightarrow S = \frac{(100-l)}{l} R$$

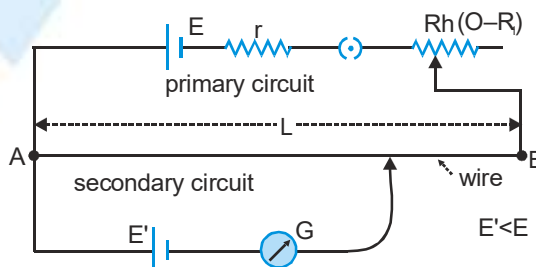


17. **Potentiometer :**

A potentiometer is a linear conductor of uniform cross-section with a steady current set up in it. This maintains a uniform potential gradient along the length of the wire. Any potential difference which is less than the potential difference maintained across the potentiometer wire can be measured using this. The potentiometer equation is

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}.$$

Circuits of potentiometer :



$$x = \frac{V}{L} = \frac{\text{current} \times \text{resistance of potentiometer wire}}{\text{length of potentiometer wire}} = I \left(\frac{R}{L} \right)$$

18. Ammeter :

It is a modified form of suspended coil galvanometer, it is used to measure current. A shunt (small resistance) is connected in parallel with galvanometer to convert into ammeter.

$$S = \frac{I_g R_g}{I - I_g}$$

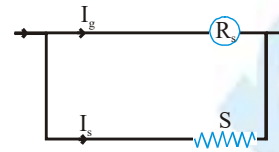
where

R_g = galvanometer resistance

I_g = Maximum current that can flow through the galvanometer.

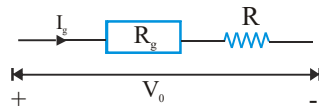
I = Maximum current that can be measured using the given ammeter.

An ideal ammeter has zero resistance



19. Voltmeter :

A high resistance is put in series with galvanometer. It is used to measure potential difference.



$$I_g = \frac{V_0}{R_g + R}; R \rightarrow \infty, \text{ Ideal voltmeter}$$

20. Electrical Power :

The energy liberated per second in a device is given by $P = VI$, where V = potential difference across device & I = current enters the higher potential point difference across device then power is consumed by it (i.e. acts as load). If the current enters the lower potential point then the device supplies power (i.e., acts as source)

$$\text{Power consumed by a resistor } P = I^2 R = VI = \frac{V^2}{R}$$

21. Heating Effect of Electric Current :

When a current is passed through a resistor energy is wasted in over coming the resistance of the wire. The energy is converted into heat

$$W = VIt = I^2 R t = \frac{V^2}{R} t$$

22. Joules Law of Electric Heating :

The heat generated (in joules) when a current of I ampere flows through a resistance of R ohm for T second is given by :

$$H = I^2 R T \text{ joule} = \frac{I^2 R t}{4.2} \text{ calories.}$$

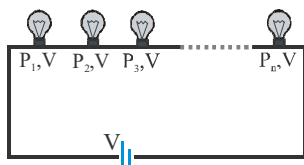
If current is variable passing through the conductor then we use for heat produced in resistance in time 0 to T is :

$$H = \int_0^T I^2 R dt$$

23. Unit of Electrical Energy Consumption :

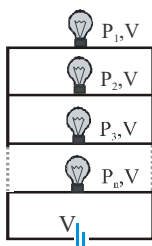
1 unit of electrical energy = kilowatt hour = 1 kWh = 3.6×10^6 joules

(a) **Series combination of Bulbs**



$$\frac{1}{P_{\text{total}}} = \frac{1}{P_1} + \frac{1}{P_2} + \frac{1}{P_3} + \dots$$

(b) **Parallel combination of Bulbs**



$$P_{\text{total}} = P_1 + P_2 + P_3 + \dots$$

24. A current flows through a conductor only when there is an electric field within the conductor because the drift velocity of electrons is directly proportional to the applied electric field.
25. Electric field outside the conducting wire which carries a constant current is zero because net charge on a current carrying conductor is zero.
26. A metal has a resistance and gets often heated by flow of current because when free electrons drift through a metal, they make occasional collisions with the lattice. These collisions are inelastic and transfer energy to the lattice as internal energy.
27. Ohm's law holds only for small current in metallic wire, not for high currents because resistance increased with increase in temperature.
28. Potentiometer is an ideal instrument to measure the potential difference because potential gradient along the potentiometer wire can be made very small.
29. An ammeter is always connected in series whereas a voltmeter is connected in parallel because an ammeter is a low-resistance galvanometer while a voltmeter is a high-resistance galvanometer.
30. Current is passed through a metallic wire, heating it red, when cold water is poured over half of the portion, rest of the portion becomes more hot because resistance decreases due to decrease in temperature so current through wire increases.