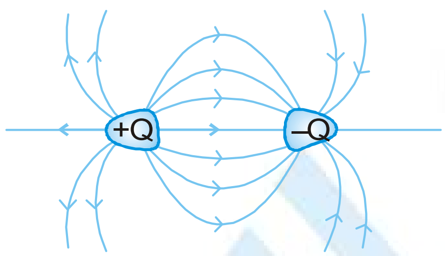
CAPACIATNCE

INTRODUCTION

Capacitor is an arrangement of two conductors generally carrying charges of equal magnitudes and opposite sign and separated by an insulating medium. A capacitor is a device which stores electric charge. Capacitors vary in shape and size, but the basic configuration is two conductors carrying equal but opposite chagres (Figure). Capacitors have many important applications in electronics. Some examples include storing electric potential energy, delaying voltage changes when coupled with resistors, filtering out unwanted frequency signals, forming resonant circuits and making frequency- dependent and independent voltage divides when combined with resistors, Some of these applications will be discussed in latter chapters.



ELOF due to positive and negative charges

When charges are pulled apart, energy is associated with the pulling apart of charges, just like energy is involved in stretching a spring. Thus, some energy is stored in capacitors.

In the uncharged staste, the charge on eitherone of the conductors in the capacitor is zero. During the charging process, a charge Q is moved from one conductor to the other one, giving one conductor a charge +Q, and the other one a charge _Q. A potential difference ΔV is created, with the positively charged conductor at a higher potential than the negatively charged conductor. Note that whether charged or uncharged, the net charge on the capacitor as a whole is zero.

Note :

1. The net charge on the capacitor as a whole is zero. When we say that a capacitor has a charge Q, we mean that the positively charged conductor has charge +Q and negatively charged conductor has a charge –Q.
2. In a circuit, a capacitor is represented by the symobol : 

Limitations on Charging a Conductor

How much electric charge can be placed on a conductor ?

As more air is pumped into the tank, the pressure opposing the flowadditional air becomes greater so it becomes further diffifult to pump more air. Similarly, as more charge Q is transferred to the conductor, the potential V of the conductor becomes higher, making it increasingly difficult to transfer more charge. Suppose we try to place an indefinite quantify of charge Q on a spherical conductor of radius r. the air surrounding the conductor is an insulator, sometimes called a dielectric, which contains few charges free to move. The electric field intensity E and the potential V at the surface of the sphere are given by

$$E = \frac{kQ}{r^2} \quad \text{and} \quad V = \frac{kQ}{r}$$

Since the radius r is constant, bothe the field intensity and the potential at the surface of the sphere increase in direct proportion to the charge Q. There is a limit, however, to the field intensity that can exist on a conductor without ionizing the surrounding air. When this occurs, the air essentially becomes a conductor, and any additional charge placed onthe sphere will “leak off” to the air. This limiting value of electric field intensity for which a material loses its insulation properties is called the dielectric strength of that material.

The dielectric strength for a given material is that electric field intensity for which the material ceases to bhe an insulator and becomes a conductor.

The dielectric strength for dry air at 1 atm pressure is around 3MC/C. Since the dielectric strength of a material varies considerably with environmental conditions, such as pressure and humidity, it is diffifult to compute accurate values.

Note that the amount of charge that can be placed on a spherical conductor decreases with the radius of the sphere. thus, smaller conductors can usually hold less charge. But the shape of a conductor also influences its ability to retain charge. Consider the charged conductros. If these conductor are tested with an electroscope, it will be discovered that the charge on the surface of conductor is concentrated at points of greatest curvature. Because of the greater charge density in these regions, the electric field intensity is also greater in regions of higher curvature. If the surface is reshaped to a sharp point, the field intensity may become great enough to ionize the surrounding air. A show leakage of charge sometimes occurs at these locations, producing a corona discharge, which is often observed as a faint violet glow in the vicinity of the sharply pointed conductor. It is important to remove all sharp edges from electrical equipment to minimize this leakage of charge.

Ex. What is the maximum charge that may be placed on a spherical conductor 1m in diameter? Assume it is surrounded by air. Assume the dielectric strength for dry air at 1 atm pressure is around 3MN/C.

Sol. $= \frac{1}{12} \times 10^{-3} C$

Concept of Capacitance

Capacitance of a conductor is a measure of ability of the conductor to store charge on it. When a conductor is charged then its potential rises. The increase in potential is directly proportional to the charge given to the conductor. $Q \propto V \Rightarrow Q = CV$

The constant C is known as the capacity of the conductor.

Capacitance is a scalar quantity with dimension $C = \frac{Q}{V} = \frac{Q^2}{W} = \frac{A^2T^2}{M^1L^2T^{-2}} = M^{-1}L^{-2}T^4A^2$

Unit :- farad, coulomb/volt

The capacity of a conductor is independent of the charge given or its potential raised. It is also independent of nature of material and thickness of the conductor. Theoretically infinite amount of charge can be given to a conductor. But practically the electric field becomes so large that it causes ionisation of medium surrounding it. The charge on conductor leaks reducing its potential.

Capacitance of an Isolated Conductor

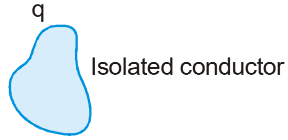
When a conductor is charged its potential increases. It is found that for an isolated conductor (conductor should be of finite dimension, so that potential of infinity can be assumed to be zero) potential of the conductor is proportional to charge given to it.

q = charge on conductor

V = potential of conductor

$q \propto V$

$\Rightarrow q = CV$



Where C is proportionality constant called capacitance of the conductor.

(1) Definition of Capacitance

Capacitance of conductor is defined as charge required to increase the potential of conductor by one unit.

(2) Important points about the capacitance of an isolated conductor

(i) It is a scalar quantity.

(ii) Unit of capacitance is farad in SI units and its dimensional formula is $M^{-1} L^{-2} I^2 T^4$

(iii) **1 Farad :** 1 Farad is the capacitance of a conductor for which 1 coulomb charge increases potential by 1 volt.

$$1 \text{ Farad} = \frac{1 \text{ Coulomb}}{1 \text{ Volt}}$$

$1\ \mu F = 10^{-6}$ F, 1nF = 10^{-9} F or 1 pF = 10^{-12} F

(iv) **Capacitance of an isolated conductor depends on following factors :**

(a) Shape and size of the conductor :

On increasing the size, capacitance increases.

(b) On surrounding medium :

With increase in dielectric constant K, capacitance increases.

(c) Presence of other conductors :

When a neutral conductor is placed near a charged conductor, capacitance of conductors increases.

(v) Capacitance of a conductor do not depend on

(a) Charge on the conductor

(b) Potential of the conductor

(c) Potential energy of the conductor.

Potential Energy or Self Energy of an isolated Conductor

Work done in charging the conductor to the charge on it against its own electric field or total energy stored in electric field of conductor is called self energy or self potential energy of conductor.

Electric Potential Energy (Self Energy)

Work done in charging the conductor

$$W = \int_0^q \frac{q}{C} dq = \frac{q^2}{2C}$$

$$W = U = \frac{q^2}{2C} = \frac{1}{2} CV^2 = \frac{qV}{2}.$$

q = Charge on the conductor

V = Potential of the conductor

C = Capacitance of the conductor.

Self energy is stored in the electric field of the conductor with energy density (Energy per unit volume)

$$\frac{dU}{dV} = \frac{1}{2} \in_0 E^2 \text{ [The energy density in a medium is } \frac{1}{2} \in_0 \in_r E^2 \text{]}$$

where E is the electric field at that point.

In case of charged conductor energy stored is only out side the conductor but in case of charged insulating material it is outside as well as inside the insulator.

Capacitance of an Isolated Spherical Conductor

The capacitance of an isolated spherical conductor of radius R.
Let there is charge Q on sphere.

⇒ Potential $V = \dfrac{KQ}{R}$

Hence by formula : $Q = CV$

$$Q = \frac{CKQ}{R}$$

$$C = 4\pi\epsilon_0 R$$

Capacitance of an isolated spherical conductor

$$C = 4\pi\epsilon_0 R$$

(i) If the medium around the conductor is vacuum or air.

$C_{Vacuum} = 4\pi\epsilon_0 R$

R = Radius of spherical conductor. (may be solid or hollow.)

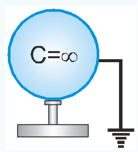
(ii) If the medium around the conductor is a dielectric of constant K from surface of sphere to infinity.

$C_{medium} = 4\pi\epsilon_0 KR$

(iii) $\dfrac{C_{medium}}{C_{air/vaccum}} = K$ = dielectric constant.

ETOOS KEY POINTS

(i) As the potential of the Earth is assumed to be zero, capacity of earth or a conductor connected to earth will be infinite $C = \dfrac{q}{V} = \dfrac{q}{0} = \infty$



(ii) Actual capacity of the Earth $C = 4\pi\varepsilon_0 R = \dfrac{1}{9\times10^9} \times 64 \times 10^5 = 711\ \mu F$

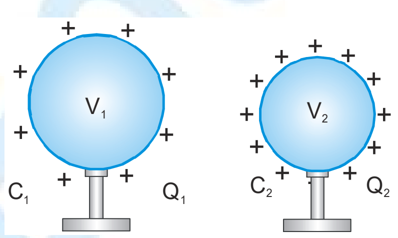
(iii) Work done by battery W_b = (charge given by battery) × (emf) = QV but Energy stored in conductor $= \dfrac{1}{2}QV$ so 50% energy supplied by the battery is lost in form of heat.

Ex. Find out the capacitance of the earth ? (Radius of the earth = 6400 km)

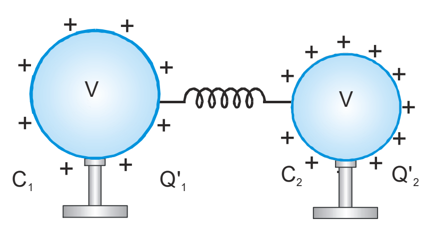
Sol. $C = 4\pi\epsilon_0 R = \dfrac{6400\times10^3}{9\times10^9} = 711\ \mu F$

Redistribution of Charges and Loss of Energy

When two charged conductors are connected by a conducting wire then charge flows from a conductor at higher potential to that at lower potential. This flow of charge stops when the potential of two conductors became equal. Let the amounts of charges after the conductors are connected are Q_1' and Q_2' respectively and potential is V then



(Before connection)



(After connection)

Common Potential

According to law of Conservation of charge $Q_{\text{before connection}} = Q_{\text{after connection}} \Rightarrow C_1V_1 + C_2V_2 = C_1V + C_2V$

Common potential after connection $V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$

Charges After Connection

$$Q_1' = C_1V = C_1\left(\frac{Q_1 + Q_2}{C_1 + C_2}\right) = \left(\frac{C_1}{C_1 + C_2}\right)Q \quad \text{(Q : Total charge on system)}$$

$$Q_2' = C_2V = C_2\left(\frac{Q_1 + Q_2}{C_1 + C_2}\right) = \left(\frac{C_2}{C_1 + C_2}\right)Q$$

Ratio of the charges after redistribution $\frac{Q_1'}{Q_2'} = \frac{C_1V}{C_2V} = \frac{R_1}{R_2}$ (in case of spherical conductors)

Loss of Energy in Redistribution

When charge flows through the conducting wire then **energy is lost mainly on account of Joule effect**, electrical energy is converted into heat energy, so change in energy of this system,

$$\Delta U = U_f - U_i \Rightarrow \left(\frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2\right) - \left(\frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2\right) \Rightarrow \Delta U = -\frac{1}{2}\left(\frac{C_1C_2}{C_1 + C_2}\right)(V_1 - V_2)^2$$

Here negative sign indicates that energy of the system decreases in the process.

Ex. A conductor gets a charge of 50 μC when it is connected to a battery of e.m.f. 5 V. Calculate capacity of the conductor.

Sol. Capacity of the conductor $C = \frac{Q}{V} = \frac{50 \times 10^{-6}}{5} = 10\,\mu F$

Ex. The capacity of a spherical capacitor in air is 50 μF and on immersing it into oil it becomes 110 μF. Calculate the dielectric constant of oil.

Sol. Dielectric constant of oil $\varepsilon_r = \frac{C_{medium}}{C_{air}} = \frac{110}{50} = 2.2$

Ex. A radio active source in the form of a metal sphere of diameter 10^{-3}m emits β particles at a constant rate of 6.25×10^{10} particles per second. If the source is electrically insulated, how long will it take for its potential to rise by 1.0 volt, assuming that 80% of emitted β particles escape from the surface.

Sol. Capacitance of sphere $C = 4\pi\varepsilon_0 R = \frac{0.5 \times 10^{-3}}{9 \times 10^9} = \frac{1}{18} \times 10^{-12}\,F$

Rate to escape of charge from surface $= \frac{80}{100} \times 6.25 \times 10^{10} \times 1.6 \times 10^{-19} = 8 \times 10^{-9}$ C/s

therefore $q = (8 \times 10^{-9})\,t$ and $q = CV \Rightarrow 8 \times 10^{-9} \times t = \frac{1}{18} \times 10^{-12} \times 1 \Rightarrow t = \frac{10^{-12}}{8 \times 10^{-9} \times 18} = \frac{10^{-3}}{144} = 6.95\,\mu s$

Ex. The plates of a capacitor are charged to a potential difference of 100 V and then connected across a resister. The potential difference across the capacitor decays exponentially with respect to time. After one second the potential difference between the plates of the capacitor is 80 V. What is the fraction of the stored energy which has been dissipated ?

Sol. Energy losses $\Delta U = \frac{1}{2}CV_0^2 - \frac{1}{2}CV^2$

Fractional energy loss $\frac{\Delta U}{U_0} = \frac{\frac{1}{2}CV_0^2 - \frac{1}{2}CV^2}{\frac{1}{2}CV_0^2} = \frac{V_0^2 - V^2}{V_0^2} = \frac{(100)^2 - (80)^2}{(100)^2} = \frac{20 \times 180}{(100)^2} = \frac{9}{25}$

Ex. Two uniformly charged spherical drops at potential V coalesce to form a larger drop. If capacity of each smaller drop is C then find capacity and potential of larger drop.

Sol. When drops coalesce to form a larger drop then total charge and volume remains conserved. If r is radius and q is charge on smaller drop then $C = 4\pi\varepsilon_0 r$ and $q = CV$

Equating volume we get $\frac{4}{3}\pi R^3 = 2 \times \frac{4}{3}\pi r^3 \Rightarrow R = 2^{1/3} r$

Capacitance of larger drop $C' = 4\pi\varepsilon_0 R = 2^{1/3} C$

Charge on larger drop $Q = 2q = 2CV$

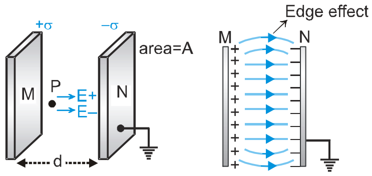
Potential of larger drop $V' = \frac{Q}{C'} = \frac{2CV}{2^{1/3}C} = 2^{2/3} V$

Parallel Plate Capacitor

(i) Capacitance

It consists of two metallic plates M and N each of area A at separation d.

Plate M is positively charged and plate N is earthed. If ε_r is the dielectric constant of the material medium and E is the field at a point P that exists between the two plates, then



I Step : Finding electric field $E = E_+ + E_- = \frac{\sigma}{2\varepsilon} + \frac{\sigma}{2\varepsilon} = \frac{\sigma}{\varepsilon} = \frac{\sigma}{\varepsilon_0\varepsilon_r}$ $[\varepsilon = \varepsilon_0\varepsilon_r]$

II Step : Finding potential difference $V = Ed = \frac{\sigma}{\varepsilon_0\varepsilon_r} d = \frac{qd}{A\varepsilon_0\varepsilon_r}$ $(\because E = \frac{V}{d}$ and $\sigma = \frac{q}{A})$

III Step : Finding capacitance $C = \frac{q}{V} = \frac{\varepsilon_r\varepsilon_0 A}{d}$

If medium between the plates is air or vacuum, then $\varepsilon_r = 1 \Rightarrow C_0 = \frac{\varepsilon_0 A}{d}$

So $C = \varepsilon_r C_0 = KC_0$ (where $\varepsilon_r = K$ = dielectric constant)

(ii) Force between the plates

The two plates of capacitor attract each other because they are oppositely charged.

Electric field due to positive plate $E = \dfrac{\sigma}{2\varepsilon_0} = \dfrac{Q}{2\varepsilon_0 A}$

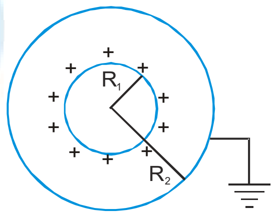
Force on negative charge –Q is $F = -QE = -\dfrac{Q^2}{2\varepsilon_0 A}$

Magnitude of force $F = \dfrac{Q^2}{2\varepsilon_0 A} = \dfrac{1}{2}\varepsilon_0 A E^2$

Force per unit area or energy density or electrostatic pressure $= \dfrac{F}{A} = u = p = \dfrac{1}{2}\epsilon_0 E^2$

Spherical Capacitor

(i) Outer sphere is earthed



When a charge Q is given to inner sphere it is uniformly distributed on its surface A charge –Q is induced on inner surface of outer sphere. The charge +Q induced on outer surface of outer sphere flows to earth as it is grounded.

$E = 0$ for $r < R_1$ and $E = 0$ for $r > R_2$

Potential of inner sphere $V_1 = \dfrac{Q}{4\pi\varepsilon_0 R_1} + \dfrac{-Q}{4\pi\varepsilon_0 R_2} \Rightarrow \dfrac{Q}{4\pi\varepsilon_0}\left(\dfrac{R_2 - R_1}{R_1 R_2}\right)$

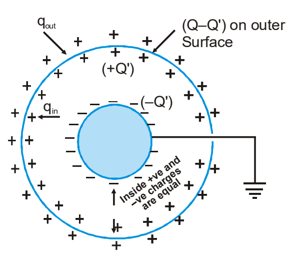
As outer surface is earthed so potential $V_2 = 0$

Potential difference between plates $V = V_1 - V_2 = \dfrac{Q}{4\pi\varepsilon_0}\dfrac{(R_2 - R_1)}{R_1 R_2}$

So $C = \dfrac{Q}{V} = 4\pi\varepsilon_0 \dfrac{R_1 R_2}{R_2 - R_1}$ (in air or vacuum)

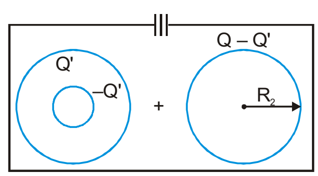
In presence of medium between plate $C = 4\pi\varepsilon_r\varepsilon_0 \dfrac{R_1 R_2}{R_2 - R_1}$

(ii) Inner sphere is earthed



Here the system is equivalent to a spherical capacitor of inner and outer radii R_1 and R_2 respectively and a spherical conductor of radius R_2 in parallel. This is because charge Q given to outer sphere distributes in such a way that for the outer sphere.

Charge on the inner side is $Q' = \dfrac{R_1}{R_2}Q$ and



Charge on the outer side is $Q - \dfrac{R_1}{R_2}Q = \dfrac{(R_2 - R_1)}{R_2}Q$

So total capacity of the system. $C = 4\pi\varepsilon_0 \dfrac{R_1 R_2}{R_2 - R_1} + 4\pi\varepsilon_0 R_2 = \dfrac{4\pi\varepsilon_0 R_2^2}{R_2 - R_1}$

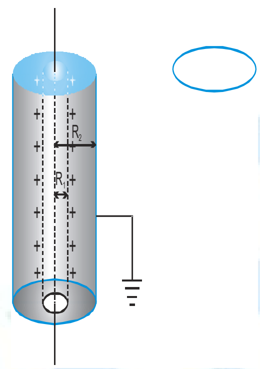
CYLINDRICAL CAPACITOR

When a charge Q is given to inner cylinder it is uniformly distributed on its surface.

A charge –Q is induced on inner surface of outer cylinder. The charge +Q induced on outer surface of outer cylinder flows to earth as it is grounded

Electrical field between cylinders

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{Q/1}{2\pi\varepsilon_0 r}$$



Potential difference between plates

$$V = \int_{R_1}^{R_2} \frac{Q}{2\pi\varepsilon_0 rl} dr = \frac{Q}{2\pi\varepsilon_0 l} \ln\left(\frac{R_2}{R_1}\right)$$

$$\text{Capacitance } C = \frac{Q}{V} = \frac{2\pi\varepsilon_0 l}{\log_e (R_2 / R_1)}$$

$$\text{In presence of medium } C = \frac{2\pi\varepsilon_0 \varepsilon_r l}{\log_e (R_2 / R_1)}$$

Ex. The stratosphere acts as a conducting layer for the earth. If the stratosphere extends beyond 50 km from the surface of earth, then calculate the capacitance of the spherical capacitor formed between stratosphere and earth's surface. Take radius of earth as 6400 km.

Sol. The capacitance of a spherical capacitor is $C = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)$

b = radius of the top of stratosphere layer $= 6400 \text{ km} + 50 \text{ km} = 6450 \text{ km} = 6.45 \times 10^6$ m

a = radius of earth $= 6400 \text{ km} = 6.4 \times 10^6$ m

$$\therefore\ C = \frac{1}{9 \times 10^9} \times \frac{6.45 \times 10^6 \times 6.4 \times 10^6}{6.45 \times 10^6 - 6.4 \times 10^6} = 0.092 \text{ F}$$

Ex. A cylindrical capacitor has two co-axial cylinders of length 15 cm and radii 1.5 cm and 1.4 cm. The outer cylinder is earthed and the inner cylinder is given a charge of 3.5 μC. Determine the capacitance of the system and the potential of the inner cylinder.

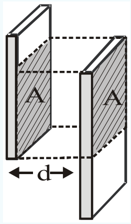
Sol. ● $= 15 \text{ cm} = 15 \times 10^{-2}$ m; $a = 1.4\text{cm} = 1.4 \times 10^{-2}$ m ; $b = 1.5 \text{ cm} = 1.5 \times 10^{-2}$m; $q = 3.5\ \mu C = 3.5 \times 10^{-6}$C

$$\text{Capacitance } C = \frac{2\pi\varepsilon_0 l}{2.303 \log_{10}\left(\frac{b}{a}\right)} = \frac{2\pi \times 8.854 \times 10^{-12} \times 15 \times 10^{-2}}{2.303 \log_{10} \frac{1.5 \times 10^{-2}}{1.4 \times 10^{-2}}} = 1.21 \times 10^{-8} \text{ F}$$

Since the outer cylinder is earthed, the potential of the inner cylinder will be equal to the potential difference between them. Potential of inner cylinder, is $V = \frac{q}{C} = \frac{3.5 \times 10^{-6}}{1.2 \times 10^{-10}} = 2.89 \times 10^4$ V

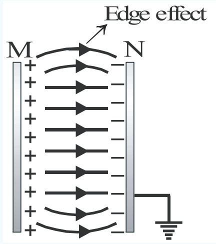
ETOOS KEY POINTS

(i) If one of the plates of parallel plate capacitor slides relatively than C decrease (As overlapping area decreases).



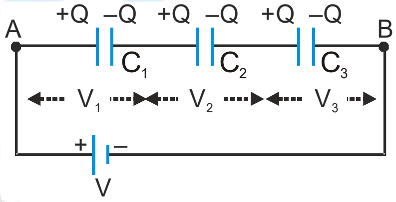
(ii) If both the plates of parallel plate capacitor are touched each other resultant charge and potential became zero.

(iii) Electric field between the plates of a capacitor is shown in figure. Non-uniformity of electric field at the boundaries of the plates is negligible if the distance between the plates is very small as compared to the length of the plates.



COMBINATION OF CAPACITOR

Capacitor in series



In this arrangement of capacitors the charge has no alternative path(s) to flow.

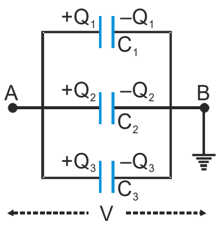
(i) The charges on each capacitor are equal i.e. $Q = C_1V_1 = C_2V_2 = C_3V_3$

(ii) The total potential difference across AB is shared by the capacitors in the inverse ratio of the capacitances $V = V_1 + V_2 + V_3$

If C_S is the net capacitance of the series combination, then

$$\frac{Q}{C_S} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \Rightarrow \frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Capacitors in parallel



In such in arrangement of capacitors the charge has an alternative path(s) to flow.

(i) The potential difference across each capacitor is same and equal the total potential applied. i.e. $V = V_1 = V_2 = V_3 \Rightarrow V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2} = \frac{Q_3}{C_3}$

(ii) The total charge Q is shared by each capacitor in the direct ratio of the capacitances. $Q = Q_1 + Q_2 + Q_3$

If C_p is the net capacitance for the parallel combination of capacitors :

$$C_pV = C_1V + C_2V + C_3V \Rightarrow C_p = C_1 + C_2 + C_3$$

ETOOS KEY POINTS

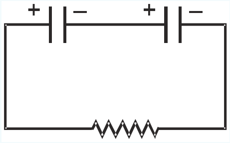
(i) For a given voltage to store maximum energy capacitors should be connected in parallel.

(ii) If N identical capacitors each having breakdown voltage V are joined in

(a) series then the break down voltage of the combination is equal to NV

(b) parallel then the breakdown voltage of the combination is equal to V.

(iii) Two capacitors are connected in series with a battery. Now battery is removed and loose wires connected together then final charge on each capacitor is zero.



(iv) If N identical capacitors are connected then $C_{series} = \frac{C}{N}$, $C_{parallel} = NC$

(v) In DC capacitor's offers infinite resistance in steady state, so there will be no current flows through capacitor branch.

Ex. Capacitor C, 2C, 4C, ... ∞ are connected in parallel, then what will be their effective capacitance ?

Sol. Let the resultant capacitance be $C_{resultant} = C + 2C + 4C + ... \infty = C[1 + 2 + 4 + ... \infty] = C \times \infty = \infty$

Ex. An infinite number of capacitors of capacitance C, 4C, 16C ... ∞ are connected in series then what will be their resultant capacitance ?

Sol. Let the equivalent capacitance of the combination = C_{eq}

$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{4C} + \frac{1}{16C} + \ldots \infty = \left[1 + \frac{1}{4} + \frac{1}{16} + \ldots \infty\right]\frac{1}{C} \text{ (this is G. P.series)}$$

$$\Rightarrow S_\infty = \frac{a}{1-r} \quad \text{first term } a = 1\text{, common ratio } r = \frac{1}{4} \Rightarrow \frac{1}{C_{eq}} = \frac{1}{1-\frac{1}{4}} \times \frac{1}{C} \Rightarrow C_{eq} = \frac{3}{4}C$$

Effect of Dielectric

(i) The insulators in which microscopic local displacement of charges takes place in presence of electric field are known as **dielectrics**.

(ii) Dielectrics are non conductors upto certain value of field depending on its nature. If the field exceeds this limiting value called **dielectric strength** they lose their insulating property and begin to conduct.

(iii) **Dielectric strength** is defined as the maximum value of electric field that a dielectric can tolerate without breakdown. Unit is volt/metre. Dimensions $M^1L^1T^{-3}A^{-1}$

Polar Dielectrics

(i) In absence of external field the centres of positive and negative charge do not coincide-due to asymmetric shape of molecules.

(ii) Each molecule has permanent dipole moment.

(iii) The dipole are randomly oriented so average dipole moment per unit volume of polar dielectric in absence of external field is nearly zero.

(iv) In presence of external field dipoles tends to align in direction of field.

Ex. Water, Alcohol, CO_2, HC●, NH_3

Non Polar Dielectrics

(i) In absence of external field the centre of positive and negative charge coincides in these atoms or molecules because they are symmetric.

(ii) The dipole moment is zero in normal state.

(iii) In presence of external field they acquire induced dipole moment.

Ex. Nitrogen, Oxygen, Benzene, Methane

Polarisation

The alignment of dipole moments of permanent or induced dipoles in the direction applied electric field is called polarisation.

Polarisation Vector $\vec{P}$

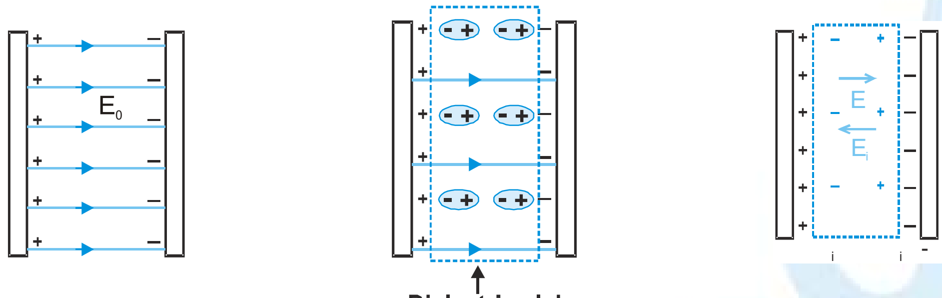
This is a vector quantity which describes the extent to which molecules of dielectric become polarized by an electric field or oriented in direction of field.

$\vec{P}$ = the dipole moment per unit volume of dielectric = $n\vec{p}$

where n is number of atoms per unit volume of dielectric and $\vec{p}$ is dipole moment of an atom or molecule.

$\vec{P} = n\vec{p} = \frac{q_i d}{Ad} = \left(\frac{q_i}{A}\right) = \sigma_i$ = induced surface charge density.

Unit of $\vec{P}$ is C/m^2 Dimension is $L^{-2}T^{1}A^{1}$



Dielectric slab

Let E_0, V_0, C_0 be electric field, potential difference and capacitance in absence of dielectric. Let E, V, C are electric field, potential difference and capacitance in presence of dielectric respectively.

Electric field in absence of dielectric $E_0 = \frac{V_0}{d} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$

Electric field in presence of dielectric $E = E_0 - E_i = \frac{\sigma - \sigma_i}{\varepsilon_0} = \frac{Q - Q_i}{\varepsilon_0} = \frac{V}{d}$

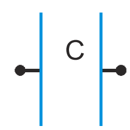
Capacitance in absence of dielectric $C_0 = \frac{Q}{V_0}$

Capacitance in presence of dielectric $C = \frac{Q - Q_i}{V}$

The dielectric constant or relative permittivity K or $\varepsilon_r = \frac{E_0}{E} = \frac{V_0}{V} = \frac{C}{C_0} = \frac{Q}{Q - Q_i} = \frac{\sigma}{\sigma - \sigma_i} = \frac{\varepsilon}{\varepsilon_0}$

From $K = \frac{Q}{Q - Q_i} \Rightarrow Q_i = Q\left(1 - \frac{1}{K}\right)$ and $K = \frac{\sigma}{\sigma - \sigma_i} \Rightarrow \sigma_i = \sigma\left(1 - \frac{1}{K}\right)$

Capacity of Different Configuration



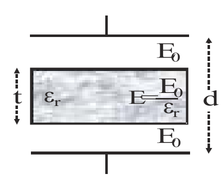
In case of parallel plate capacitor $C = \frac{\varepsilon_0 A}{d}$

If capacitor is partially filled with dielectric

When the dielectric is filed partially between plates, the thickness of dielatric slab is $t (t < d)$.

If no slab is introduced between the plates of the capacitor, then a field E_0 given by $E_0 = \frac{\sigma}{\varepsilon_0}$, exists in a space d.

On inserting the slab of thickness t, a field $E = \frac{E_0}{\varepsilon_r}$ exists inside the slab of thickness t and a field E_0 exists in remaining space $(d - t)$. If V is total potential then $V = E_0(d - t) + E\,t$



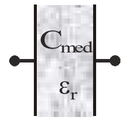
$$\Rightarrow V = E_0\left[d - t + \left(\frac{E}{E_0}\right)t\right] \quad \frac{E_0}{E} = \varepsilon_r = \text{Dielectric constant}$$

$$V = \frac{\sigma}{\varepsilon_0}\left[d - t + \frac{t}{\varepsilon_r}\right] = \frac{q}{A\varepsilon_0}\left[d - t + \frac{t}{\varepsilon_r}\right] \Rightarrow C = \frac{q}{V} = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{\varepsilon_r}\right)} = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{\varepsilon_r}\right)} \quad \text{...(i)}$$

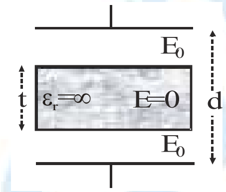
If medium is fully present between the space.

✈ $t = d$

Now from equation **(i)** $\quad C_{medium} = \dfrac{\varepsilon_0 \varepsilon_r A}{d}$



If capacitor is partialy filled by a conducting slab of thickness (t < d).



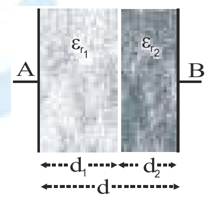
$$\because \varepsilon_r = \infty \text{ for conductor } \quad C = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{\infty}\right)} = \frac{\varepsilon_0 A}{(d-t)}$$

DISTANCE AND AREA DIVISION BY DIELECTRIC

Distance Division

(i) Distance is divided and area remains same.

(ii) Capacitors are in series.



(iii) Individual capacitances are $C_1 = \dfrac{\varepsilon_0 \varepsilon_{r_1} A}{d_1}$, $C_2 = \dfrac{\varepsilon_0 \varepsilon_{r_2} A}{d_2}$

These two in series $\dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2} \Rightarrow \dfrac{1}{C} = \dfrac{d_1}{\varepsilon_0 \varepsilon_{r_1} A} + \dfrac{d_2}{\varepsilon_0 \varepsilon_{r_2} A} \Rightarrow \dfrac{1}{C} = \dfrac{1}{\varepsilon_0 A}\left[\dfrac{d_1 \varepsilon_{r_2} + d_2 \varepsilon_{r_1}}{\varepsilon_{r_1} \varepsilon_{r_2}}\right] \Rightarrow C = \varepsilon_0 A\left[\dfrac{\varepsilon_{r_1} \varepsilon_{r_2}}{d_1 \varepsilon_{r_2} + d_2 \varepsilon_{r_1}}\right]$

Special case : $\quad$ If $d_1 = d_2 = \dfrac{d}{2} \Rightarrow C = \dfrac{\varepsilon_0 A}{d}\left[\dfrac{2\varepsilon_{r_1} \varepsilon_{r_2}}{\varepsilon_{r_1} + \varepsilon_{r_2}}\right]$

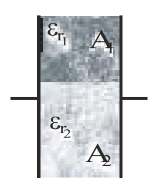
Area Division

(i) Area is divided and distance remains same.

(ii) Capacitors are in parallel.

(iii) Individual capacitances are $C_1 = \dfrac{\varepsilon_0 \varepsilon_{r_1} A_1}{d}$ $C_2 = \dfrac{\varepsilon_0 \varepsilon_{r_2} A_2}{d}$

These two in parallel



So $\quad C = C_1 + C_2 = \dfrac{\varepsilon_0 \varepsilon_{r_1} A_1}{d} + \dfrac{\varepsilon_0 \varepsilon_{r_2} A_2}{d} = \dfrac{\varepsilon_0}{d}(\varepsilon_{r_1} A_1 + \varepsilon_{r2} A_2)$

Variable Dielectric Constant

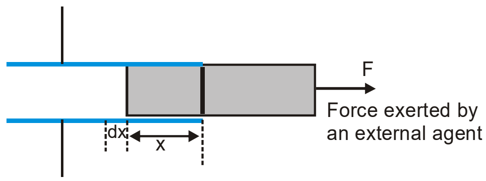
If the dielectric constant is variable, then equivalent capacitance can be obtained by selecting an element as per the given condition and then integrating.

(i) If different elements are in parallel, then $C = \int dC$, where dC = capacitance of selected differential element.

(ii) If different element are in series, then $\dfrac{1}{C} = \int d\left(\dfrac{1}{C}\right)$ is solved to get equivalent capacitance C.

Force on a Dielectric in a Capacitor

Consider a differential displacement dx of the dielectric as shown in figure always keeping the net force on it zero so that the dielectric moves slowly without acceleration. Then, $W_{Electrostatic} + W_F = 0$, where W_F denotes the work done by external agent in displacement dx

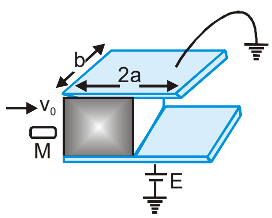


$$W_F = -W_{Electrostatic} \quad W_F = \Delta U \Rightarrow -F.dx = \frac{Q^2}{2} d\left[\frac{1}{C}\right] \quad \left[W = \frac{Q^2}{2C}\right] \Rightarrow -F.dx = \frac{-Q^2}{2C^2} dC \Rightarrow F = \frac{Q^2}{2C^2}\left(\frac{dC}{dx}\right)$$

This is also true for the force between the plates of the capacitor. If the capacitor has battery connected to it, then

as the p.d. across the plates is maintained constant. $V = \frac{Q}{C} \Rightarrow F = \frac{1}{2}V^2\frac{dC}{dx}$.

Ex. A parallel plate capacitor is half filled with a dielectric (K) of mass M. Capacitor is attached with a cell of emf E. Plates are held fixed on smooth insulating horizontal surface. A bullet of mass M hits the dielectric elastically and its found that dielectric just leaves out the capacitor. Find speed of bullet.



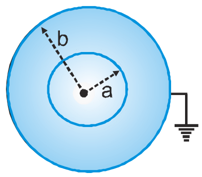
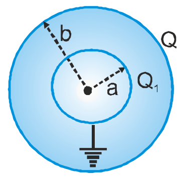
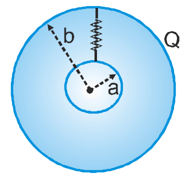
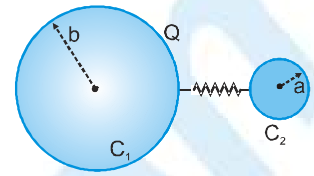
Sol. Since collision is elastic ∴ Velocity of dielectric after collision is v_0.

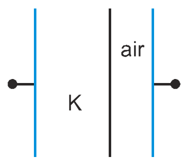
Dielectric will move and when it is coming out of capacitor a force is applied on

it by the capacitor $F = \frac{-dU}{dx} = \frac{-E^2\varepsilon_0 b(K-1)}{2d}$

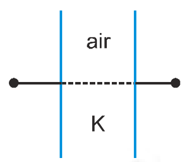
Which decreases its speed to zero, till it comes out it travels a distance a.

$$\frac{1}{2}Mv_0^2 = \frac{E^2\varepsilon_0 b(K-1)a}{2d} \Rightarrow v_0 = E\left[\frac{\varepsilon_0 ab(K-1)}{Md}\right]^{1/2}$$

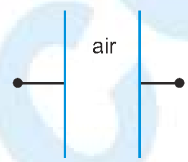
Spherical capacitor outer is earthed	Inner is earthed and outer is given a charge	Connected and outer is given a charge	Connected spheres
			
$C = \frac{4\pi\varepsilon_0 ab}{b-a}$ (b > a)	$C = \frac{4\pi\varepsilon_0 b^2}{b-a}$ (b > a)	$C = 4\pi\varepsilon_0 b$	$C = C_1 + C_2$ $C = 4\pi\varepsilon_0(a+b)$



$C_1 = \left[\frac{2K}{K+1}\right]C$



$C_2 = \left[\frac{K+1}{2}\right]C$



$C_3 = C$
when no dielectric is used

$C_2 > C_1 > C_3$

Ex. A capacitor has two circular plates whose radius are 8cm and distance between them is 1mm. When mica (dielectric constant = 6) is placed between the plates, calculate the capacitance of this capacitor and the energy stored when it is given potential of 150 volt.

Sol. Area of plate $\pi r^2 = \pi \times (8 \times 10^{-2})^2 = 0.0201\ m^2$ and $d = 1mm = 1 \times 10^{-3}\ m$

Capacity of capacitor $C = \frac{\varepsilon_0 \varepsilon_r A}{d} = \frac{8.85 \times 10^{-12} \times 6 \times 0.0201}{1 \times 10^{-3}} = 1.068 \times 10^{-9}\ F$

Potential difference V = 150 volt

Energy stored $U = \frac{1}{2}CV^2 = \frac{1}{2} \times (1.068 \times 10^{-9}) \times (150)^2 = 1.2 \times 10^{-5}\ J$

Ex. A parallel-plate capacitor is formed by two plates, each of area 100 cm^2, separated by a distance of 1mm. A dielectric of dielectric constant 5.0 and dielectric strength 1.9×10^7 V/m is filled between the plates. Find the maximum charge that can be stored on the capacitor without causing any dielectric breakdown.

Sol. If the charge on the capacitor = Q

the surface charge density $\sigma = \frac{Q}{A}$ and the electric field = $\frac{Q}{KA\varepsilon_0}$.

This electric field should not exceed the dielectric strength 1.9×10^7 V/m.

$\therefore$ if the maximum charge which can be given is Q then $\frac{Q}{KA\varepsilon_0} = 1.9 \times 10^7$ V/m

$\because A = 100\ cm^2 = 10^{-2}\ m^2 \Rightarrow Q = (5.0) \times (10^{-2}) \times (8.85 \times 10^{-12}) \times (1.9 \times 10^7) = 8.4 \times 10^{-6}\ C.$

Ex. The distance between the plates of a parallel-plate capacitor is 0.05 m. A field of 3×10^4 V/m is established between the plates. It is disconnected from the battery and an uncharged metal plate of thickness 0.01 m is inserted into the **(i)** before the introduction of the metal plate and **(ii)** after its introduction. What would be the potential difference if a plate of dielectric constant K = 2 is introduced in place of metal plate ?

Sol. **(i)** In case of a capacitor as E = (V/d), the potential difference between the plates before the introduction of metal plate

$$V = E \times d = 3 \times 10^4 \times 0.05 = 1.5 \text{ kV}$$

(ii) Now as after charging battery is removed , capacitor is isolated so q = constant. If C' and V' are the capacity and potential after the introduction of plate q = CV = C'V' i.e., $V' = \frac{C}{C'}V$

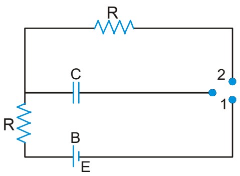
And as $C = \frac{\varepsilon_0 A}{d}$ and $C' = \frac{\varepsilon_0 A}{(d-t)+(t/K)}$, $V' = \frac{(d-t)+(t/K)}{d} \times V$

So in case of metal plate as K = ∞, $V_M = \left[\frac{d-t}{d}\right] \times V = \left[\frac{0.05-0.01}{0.05}\right] \times 1.5 = 1.2 \text{ kV}$

And if instead of metal plate, dielectric with K = 2 is introduced $V_D = \left[\frac{(0.05-0.01)+(0001/2)}{0.05}\right] \times 1.5 = 1.35 \text{ kV}$

CHARGING AND DISCHARGING OF A CAPACITOR

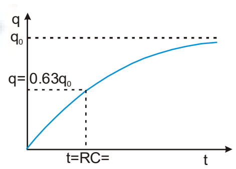
Charging of a Condenser



(i) In the following circuit. If key 1 is closed then the condenser gets charged. Finite time is taken in the charging process. The quantity of charge at any instant of time t is given by $q = q_0[1 - e^{-(t/RC)}]$

Where q_0 = maximum final value of charge at t = ∞.

According to this equations the quantity of charge on the condenser increases exponentially with increase of time.



(ii) If t = RC = τ then

$$q = q_0\,[1 - e^{-(RC/RC)}] = q_0\left[1 - \frac{1}{e}\right]$$

or $\quad q = q_0\,(1 - 0.37) = 0.63\, q_0 = 63\%$ of q_0

(iii) Time t = RC is known as time constant.

i.e. the time constant is that time during which the charge rises on the condenser plates to 63% of its maximum value.

(iv) The potential difference across the condenser plates at any instant of time is given by

$V = V_0[1 - e^{-(t/RC)}]$ volt

(v) The potential curve is also similar to that of charge. During charging process an electric current flows in the circuit for a small interval of time which is known as the transient current. The value of this current at any instant of time is given by

$I = I_0[e^{-(t/RC)}]$ ampere

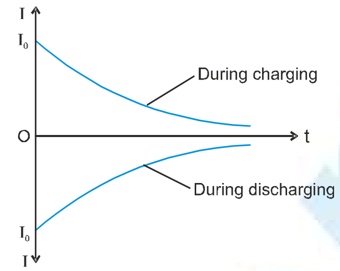
According to this equation the current falls in the circuit exponentially (Fig.).

(vi) If $t = RC = \tau$ = Time constant

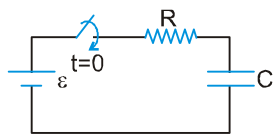
$$I = I_0 e^{(-RC/RC)} = \frac{I_0}{e} = 0.37\, I_0$$

$= 37\%$ of I_0

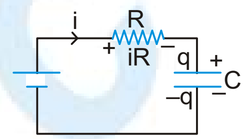
i.e. time constant is that time during which current in the circuit falls to 37% of its maximum value.



Derivation of Formulae for Charging of Capacitor



it is given that initially capacitor is uncharged.
let at any time charge on capacitor is q
Applying kirchoff voltage law



$$\varepsilon - iR - \frac{q}{C} = 0 \quad \Rightarrow \quad iR = \frac{\varepsilon C - q}{C}$$

$$i = \frac{\varepsilon C - q}{CR} \quad \Rightarrow \quad \frac{dq}{dt} = \frac{\varepsilon C - q}{CR}$$

$$\frac{dq}{dt} = \frac{\varepsilon C - q}{CR} \quad \Rightarrow \quad \frac{CR}{\varepsilon C - q}\,.\, dq = dt.$$

$$\int_0^q \frac{dq}{\varepsilon C - q} = \int_0^t \frac{dt}{RC} \quad \Rightarrow \quad -\ln(\varepsilon C - q) + \ln \varepsilon C = \frac{t}{RC}$$

$$\ln \frac{\varepsilon C}{\varepsilon C - q} = \frac{t}{RC}$$

$$\varepsilon C - q = \varepsilon C\,.\, e^{-t/RC}$$

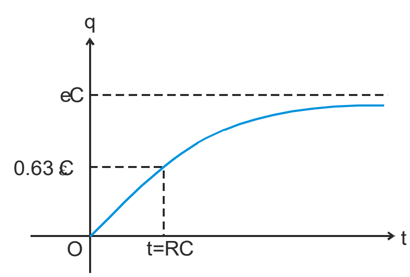
$$q = \varepsilon C(1 - e^{-t/RC})$$

RC = time constant of the RC series circuit.

After one time constant

$$q = \varepsilon C\left(1 - \frac{1}{e}\right)$$

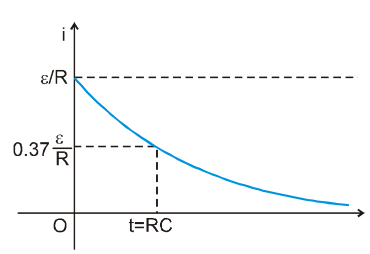
$$= \varepsilon C\,(1 - 0.37) = 0.63\, \varepsilon C.$$



Current at any time t

$$i = \frac{dq}{dt} = \varepsilon C\left(-e^{-t/RC}\left(-\frac{1}{RC}\right)\right)$$

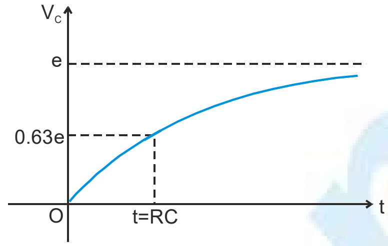
$$= \frac{\varepsilon}{R} e^{-t/RC}$$



Voltage across capacitor after one time constant $V = 0.63\ \varepsilon$

$$Q = CV$$

$$V_C = \varepsilon(1 - e^{-t/RC})$$



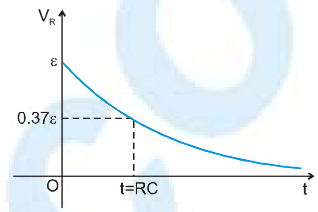
Voltage across the resistor

$$V_R = iR = \varepsilon e^{-t/RC}$$

By energy conservation,

Heat dissipated = work done by battery – ΔUcapacitor

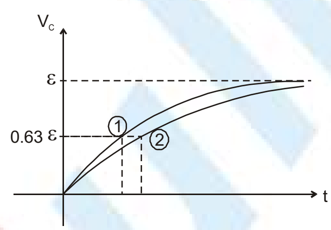
$$= C\varepsilon(\varepsilon) - \left(\frac{1}{2}C\varepsilon^2 - 0\right) = \frac{1}{2}C\varepsilon^2$$



Alternatively :

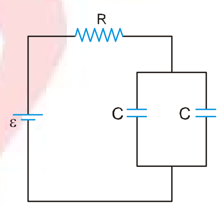
$$\text{Heat} = H = \int_0^\infty i^2 R\,dt = \int_0^\infty \frac{\varepsilon^2}{R^2} e^{-\frac{2t}{RC}} R\,dt \quad = \frac{\varepsilon^2}{R}\int_0^\infty e^{-2t/RC}\,dt$$

$$= \frac{\varepsilon^2}{R}\left[\frac{e^{-\frac{2t}{RC}}}{-2/RC}\right]_0^\infty = -\frac{\varepsilon^2 RC}{2R}\left[e^{-\frac{2t}{RC}}\right]_0^\infty = \frac{\varepsilon^2 C}{2}$$



In the figure time constant of (2) is more than (1)

Ex. Without using the formula of equivalent. Find out charge on capacitor and current in all the branches as a function of time.

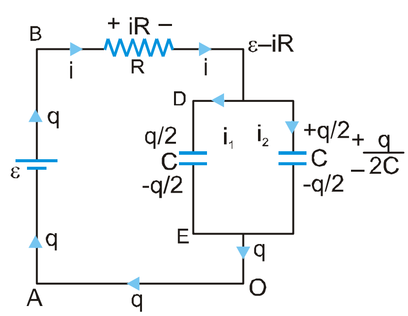


Sol. Applying KVL in ABDEA

$$\varepsilon - iR = \frac{q}{2C}$$

$$i = \frac{\varepsilon}{R} - \frac{q}{2CR}$$

$$= \frac{2C\varepsilon - q}{2CR}$$



$$\frac{dq}{2\varepsilon C - q} = \frac{dt}{2CR} \quad \Rightarrow \quad \frac{dq}{2\varepsilon C - q} = \frac{dt}{2CR}$$

$$\int_0^q \frac{dq}{(2\varepsilon C - q)} = \frac{t}{2CR} \quad \Rightarrow \quad \frac{2\varepsilon C - q}{2\varepsilon C} = e^{-t/2RC}$$

$$q = 2\varepsilon C\,(1 - e^{-t/2RC})$$

$$q_1 = \frac{q}{2} = \varepsilon C\,(1 - e^{-t/2RC}) \quad \Rightarrow \quad i_1 = \frac{\varepsilon}{2R}\,e^{-t/2RC}$$

$$q_2 = \frac{q}{2} = \varepsilon C\,(1 - e^{-t/2RC}) \quad \Rightarrow \quad i_2 = \frac{\varepsilon}{2R}\,e^{-t/2RC}$$

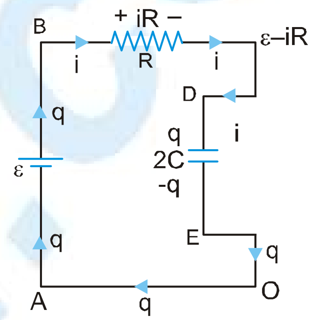
Alternate solution

by equivalent

Time constant of circuit = 2C × R = 2RC

maximum charge on capacitor = 2C × ε = 2Cε

Hence equations of charge and current are as given below



$$q = 2\varepsilon C\,(1 - e^{-t/2RC})$$

$$q_1 = \frac{q}{2} = \varepsilon C\,(1 - e^{-t/2RC}) \quad \Rightarrow \quad i_1 = \frac{\varepsilon}{2R}\,e^{-t/2RC}$$

$$q_2 = \frac{q}{2} = \varepsilon C\,(1 - e^{-t/2RC}) \quad \Rightarrow \quad i_2 = \frac{\varepsilon}{2R}\,e^{-t/2RC}$$

Ex. A capacitor is connected to a 36 V battery through a resistance of 20Ω. It is found that the potential difference across the capacitor rises to 12.0 V in 2μs. Find the capacitance of the capacitor.

Sol. The charge on the capacitor during charging is given by $Q = Q_0(1 - e^{-t/RC})$.

Hence, the potential difference across the capacitor is $V = Q/C = Q_0/C\,(1 - e^{-t/RC})$.

Here, at t = 2 μs, the potential difference is 12V whereas the steady potential difference is

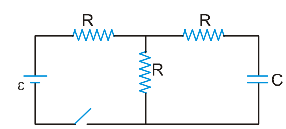
$Q_0/C = 36V$. So, $\Rightarrow$ $12V = 36V(1 - e^{-t/RC})$

or, $1 - e^{-t/RC} = \frac{1}{3}$ or, $e^{-t/RC} = \frac{2}{3}$

or, $\frac{t}{RC} = \ln\left(\frac{3}{2}\right) = 0.405$ or, $RC = \frac{t}{0.405} = \frac{2\ \mu s}{0.45} = 4.936\ \mu s$

or, $C = \frac{4.936\mu s}{20\Omega} = 0.25\ \mu F.$

Ex. Initially the capacitor is uncharged find the charge on capacitor as a function of time, if switch is closed at t = 0.

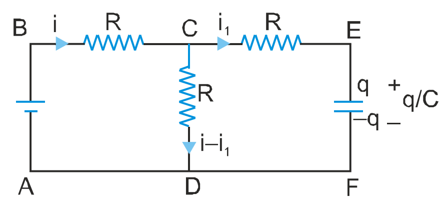


Sol. Applying KVL in loop ABCDA

$$\varepsilon - iR - (i - i_1)\,R = 0$$

$$\varepsilon - 2iR + i_1R = 0 \qquad \text{...(i)}$$

Applying KVL in loop ABCEFDA



$$\varepsilon - iR - i_1R - \frac{q}{C} = 0$$

by eq **(i)** $\frac{2\varepsilon - \varepsilon - i_1R - 2i_1R}{2} = \frac{q}{C} \Rightarrow \varepsilon C - 3i_1RC = 2q$

$\varepsilon C - 2q = 3\frac{dq}{dt}.RC \Rightarrow \int_0^q \frac{dq}{\varepsilon C - 2q} = \int_0^t \frac{dt}{3RC}$

$-\frac{1}{2}\ln\frac{\varepsilon C - 2q}{\varepsilon C} = \frac{t}{3RC} \Rightarrow q = \frac{\varepsilon C}{2}\left(1 - e^{-2t/3RC}\right)$

Method for Objective

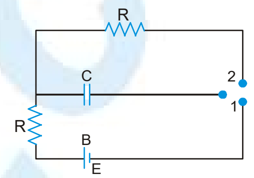
In any circuit when there is only one capacitor then

$q = Q_{st}\left(1 - e^{-t/\tau}\right)$; Q_{st} = steady state charge on capacitor (has been found in article 6 in this sheet)

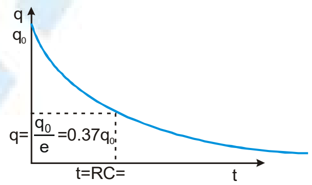
$\tau = R_{eff}.C$

$R_{effective}$ is the resistance between the capacitor when battery is replaced by its internal resistance.

Discharging of a Condenser



(i) In the above circuit (in article 10.1) if key 1 is opened and key 2 is closed then the condenser gets discharged.



(ii) The quantity of charge on the condenser at any instant of time t is given by $q = q_0\, e^{-(t/RC)}$
i.e. the charge falls exponentially.
here q_0 = initial charge of capacitor

(iii) If $t = RC = \tau$ = time constant, then

$q = \frac{q_0}{e} = 0.37q_0 = 37\%$ of q_0

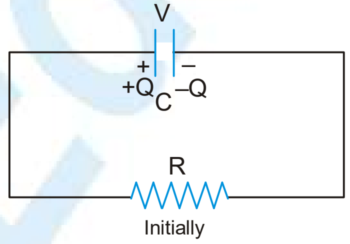
i.e. the time constant is that time during which the charge on condenser plates in discharge process, falls to 37%

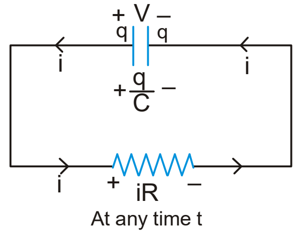
(iv) The dimensions of RC are those of time i.e. $M^oL^oT^1$ and the dimensions of $\frac{1}{RC}$ are those of frequency i.e. $M^0L^0T^{-1}$.

(v) The potential difference across the condenser plates at any instant of time t is given by $V = V_0e^{-(t/RC)}$ Volt.

(vi) The transient current at any instant of time is given by $I = -I_0e^{-(t/RC)}$ ampere.
i.e. the current in the circuit decreases exponentially but its direction is opposite to that of charging current. (– ive only means that direction of current is opposite to that at charging current)

Derivation of Equation of Discharging Circuit





Applying K.V.L.

$$+\frac{q}{C} - iR = 0$$

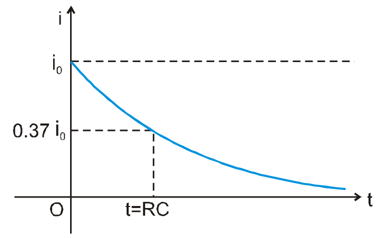
$$i = \frac{q}{CR}$$

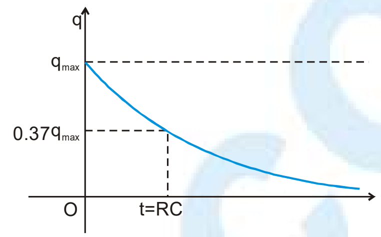
$$\int_Q^q \frac{-dq}{q} = \int_0^t \frac{dt}{CR}$$

$$-\ln\frac{q}{Q} = +\frac{t}{RC}$$

$$q = Q\,.\,e^{-t/RC}$$

$$i = -\frac{dq}{dt} = \frac{Q}{RC}e^{-t/RC} = i_0 e^{-t/RC}$$





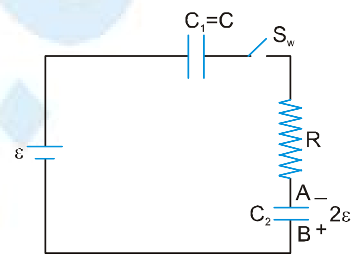
Ex. At t = 0, switch is closed, if initially C_1 is uncharged and C_2 is charged to a potential difference 2ε then find out following

(Given $C_1 = C_2 = C$)

(a) Charge on C_1 and C_2 as a function of time.

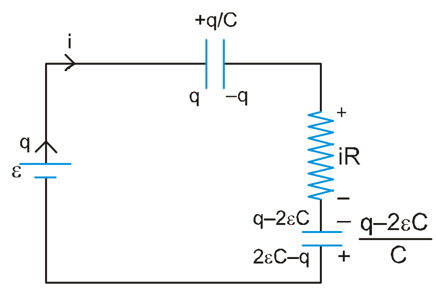
(b) Find out current in the circuit as a function of time.

(c) Also plot the graphs for the relations derived in part (a).



Sol. Let q charge flow in time 't' from the battery as shown.

The charge on various plates of the capacitor is as shown in the figure.
Now applying KVL



$$\varepsilon - \frac{q}{C} - iR - \frac{q-2\varepsilon C}{C} = 0 \Rightarrow \varepsilon - \frac{q}{C} - \frac{q}{C} + 2\varepsilon - iR = 0$$

$$3\varepsilon = \frac{2q}{C} + iR \Rightarrow 3\varepsilon - iR = \frac{2q}{C}$$

$$3\varepsilon C - iRC = 2q \Rightarrow \frac{dq}{dt}RC = 3\varepsilon C - 2q$$

$$\int_0^q \frac{dq}{3\varepsilon C - 2q} = \int_0^t \frac{dt}{RC} \Rightarrow -\frac{1}{2}\ln\left(\frac{3C\varepsilon - 2q}{3C\varepsilon}\right) = \frac{t}{RC}$$

$$\ln\left(\frac{3\varepsilon C - 2q}{3\varepsilon C}\right) = -\frac{2t}{RC} \Rightarrow 3\varepsilon C - 2q = 3\varepsilon C$$

$$e^{-2t/RC}$$

$$3\varepsilon C\,(1 - e^{-2t/RC}) = 2q \Rightarrow q = \frac{3}{2}\varepsilon C\;(1 - e^{-2t/RC})$$

(charge on C, as function of time) **Ans.**

$$i = \frac{dq}{dt} = \frac{3\varepsilon}{R} e^{-2t/RC}$$

Charge on C_2 as function of time :

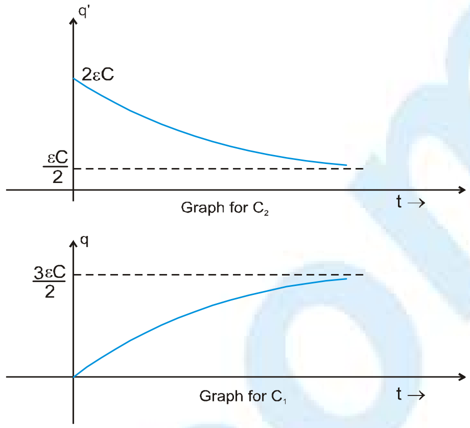
$q' = 2\varepsilon C - q$

$$= 2\varepsilon C - \frac{3}{2}\varepsilon C + \frac{3}{2}\varepsilon C\, e^{-2t/RC}$$

$$= \frac{\varepsilon C}{2} + \frac{3}{2}\varepsilon C e^{-2t/RC}$$

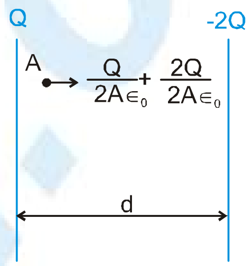
$$= \frac{\varepsilon C}{2}\left[1 + 3e^{-2t/RC}\right]$$

Ans.



Ex. Two parallel conducting plates of a capacitor of capacitance C containing charges Q and –2Q at a distance d apart. Find out potential difference between the plates of capacitors.

Sol. Capacitance = C



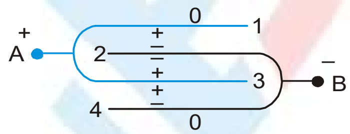
$$\text{Electric field} = \frac{3Q}{2A\in_0}$$

$$V = \frac{3Qd}{2A\in_0} \Rightarrow V = \frac{3Q}{2C}$$

Combination of Parallel Plates

Ex. Find out equivalent capacitance between A and B. (take each plate Area = A and distance between two conjugative plates is d)

Sol. Let numbers on the plates The charges will be as shown in the figure.

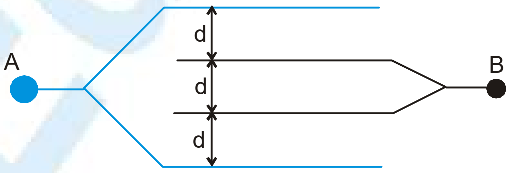


$$V_{12} = V_{32} = V_{34}$$

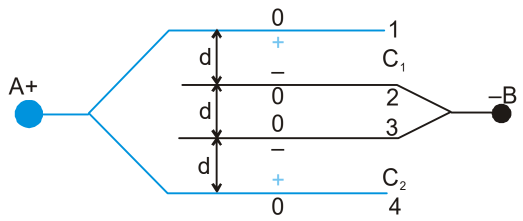
so all the capacitors are in parallel combination.

$$C_{eq} = C_1 + C_2 + C_3 = \frac{3A\in_0}{d}$$

Ex. Find out equivalent capacitance between A and B. (take each plate Area = A)



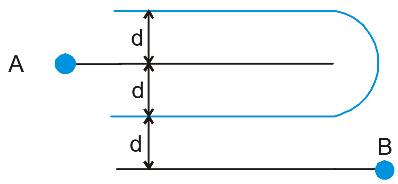
Sol.



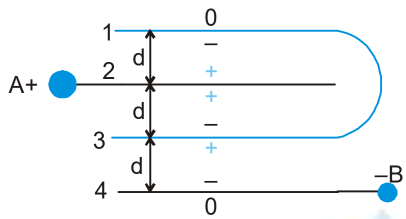
These are only two capacitors.

$$C_{eq} = C_1 + C_2 = \frac{2A\in_0}{d}$$

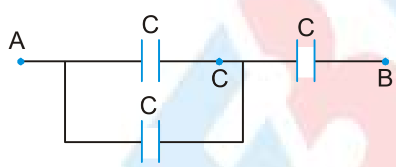
Ex. Find out equivalent capacitance between A and B. (take each plate Area = A)



Sol.

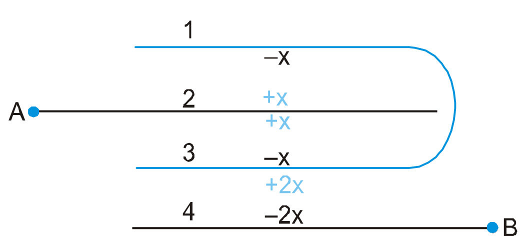


The modified circuit is



$$C_{eq} = \frac{2C}{3} = \frac{2A\in_0}{3d}$$

Other method Let charge density as shown



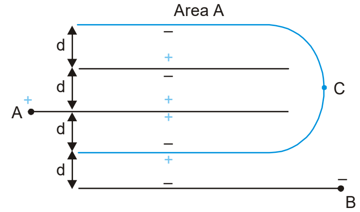
$$C_{eq} = \frac{Q}{V} = \frac{2xA}{V}$$

$$V = V_2 - V_4 = (V_2 - V_3) + (V_3 - V_4)$$

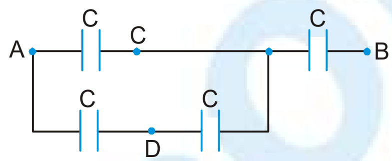
$$= \frac{xd}{\in_0} + \frac{2xd}{\in_0} = \frac{3xd}{\in_0}$$

$$\therefore \quad C_{eq} = \frac{2Ax\in_0}{3xd} = \frac{2A\in_0}{3d} = \frac{2C}{3}.$$

Ex. Find out equivalent capacitance between A and B.

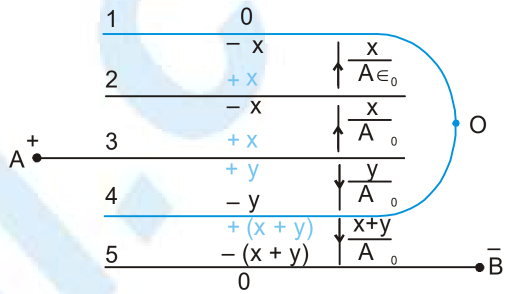


Sol. Let $C = \frac{A \in_0}{d}$ Equivalent circuit : -



$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{2}{3C} = \frac{5}{3C} \Rightarrow C_{eq} = \frac{3C}{5} = \frac{3A \in_0}{5d}$$

Alternative Method : Let charge distribution on plates as shown :



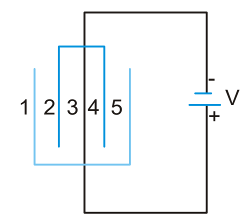
$$C = \frac{Q}{V} = \frac{x+y}{V_{AB}}$$

Potential of 1 and 4 is same

$$\frac{y}{A \in_0} = \frac{2x}{A \in_0} \qquad y = 2x$$

$$V = \left(\frac{2y + x}{A\varepsilon_0}\right) d \Rightarrow C = \frac{(x+2x)A \in_0}{(5x)d} = \frac{3A \in_0}{5d}.$$

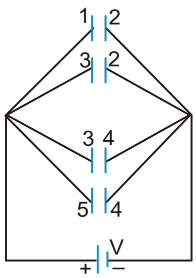
Ex. Five similar condenser plates, each of area A, are placed at equal distance d apart and are connected to a source of e.m.f. V as shown in the following diagram. The charge on the plates 1 and 4 will be-



(A) $\frac{\in_0 A}{d}, \frac{-2 \in_0 A}{d}$ **(B)** $\frac{\in_0 AV}{d}, \frac{-2 \in_0 AV}{d}$

(C) $\frac{- \in_0 AV}{d}, \frac{-3 \in_0 AV}{d}$ **(D)** $\frac{\in_0 AV}{d}, \frac{-4 \in_0 AV}{d}$

Sol. by equivalent circuit diagram Charge on first plate



$Q = CV$

$$Q = \frac{\in_0 AV}{d}$$

Charge on fourth plate

$$Q' = C(-V) \qquad Q' = \frac{- \in_0 AV}{d}$$

As plate 4 is repeated twice, hence charge on 4 will be $Q'' = 2Q'$

$$Q'' = -\frac{2 \in_0 AV}{d}$$

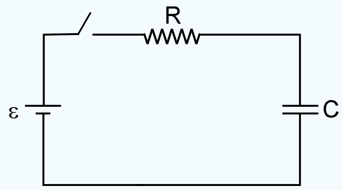
Hence the correct answer will be **(B)**.

Circuit solution for R–C circuit at t = 0 (initial state) and at t = ∞ (final state)

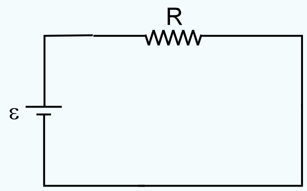
ETOOS KEY POINTS

(i) Charge on the capacitor does not change instantaneously or suddenly if there is a resistance in the path (series) of the capacitor.

(ii) When an uncharged capacitor is connected with battery then its charge is zero initially hence potential difference across it is zero initially. At this time the capacitor can be treated as a conducting wire

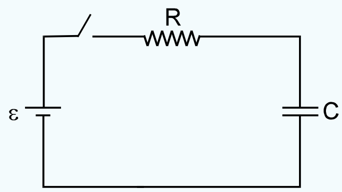


Before connection

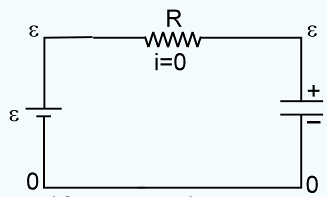


Just after connection at t = 0

(iii) The current will become zero finally (that means in steady state) in the branch which contains capacitor.



Before connection

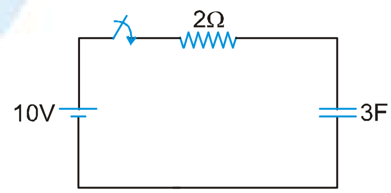


After connection at t = ∞

Ex. Find out current in the circuit and charge on capacitor which is initially uncharged in the following situations.

(a) Just after the switch is closed.

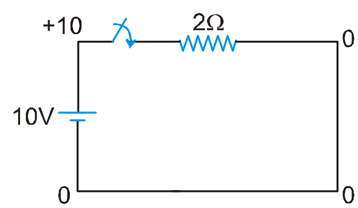
(b) After a long time when switch was closed.



Sol. **(a) For just after closing the switch:**

potential difference across capacitor = 0

$\therefore Q_C = 0 \qquad \therefore i = \frac{10}{2} = 5A$

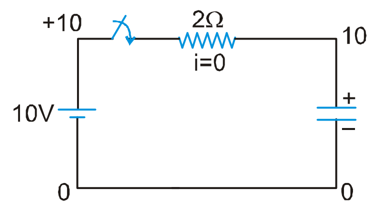


(b) After a long time

at steady state current i = 0

and potential difference across capacitor = 10 V

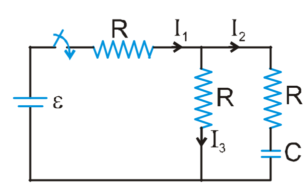
$\therefore Q_C = 3 \times 10 = 30\,C$



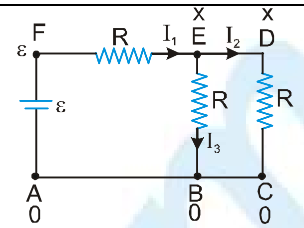
Ex. Find out current I_1, I_2, I_3, charge on capacitor and $\frac{dQ}{dt}$ of capacitor in the circuit which is initially uncharged in the following situations.

(a) Just after the switch is closed

(b) After a long time when switch is closed.



Sol. **(a)** Initially the capacitor is uncharged so its behaviour is like a conductor Let potential at A is zero so at B and C also zero and at F it is ε. Let potential at E is x so at D also x. Apply Kirchhoff's I[st] law at point E :



$$\frac{x-\varepsilon}{R}+\frac{x-0}{R}+\frac{x-0}{R}=0 \Rightarrow \frac{3x}{R}=\frac{\varepsilon}{R}$$

$x=\frac{\varepsilon}{3}$; $Q_c=0$

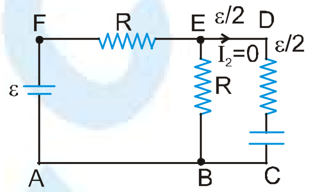
$$\therefore \quad I_1=\frac{-\varepsilon/3+\varepsilon}{R}=\frac{2\varepsilon}{3R} \Rightarrow I_2=\frac{dQ}{dt}=\frac{\varepsilon}{3R} \text{ and } I_3=\frac{\varepsilon}{3R}$$

Alternatively

$$i_1=\frac{\varepsilon}{R_{eq}}=\frac{\varepsilon}{R+\frac{R}{2}}=\frac{2\varepsilon}{3R} \Rightarrow i_2=i_3=\frac{i_1}{2}=\frac{\varepsilon}{3R} \text{ and } \frac{dQ}{dt}=i_2=\frac{\varepsilon}{3R}$$

(b) at $t=\infty$ (finally)

capacitor completely charged so their will be no current through it.



$I_2=0,\quad I_1=I_3=\frac{\varepsilon}{2R}$

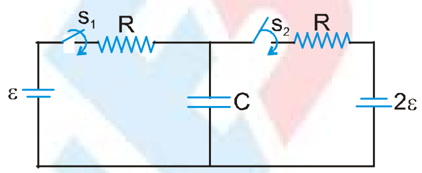
$V_E-V_B=V_D-V_C=(\varepsilon/2R)R=\varepsilon/2$

$\Rightarrow Q_C=\frac{\varepsilon C}{2}$, $\frac{dQ}{dt}=I_2=0$

	I_1	I_2	I_3	Q	dQ/dt
Time t = 0	$\frac{2\varepsilon}{3R}$	$\frac{\varepsilon}{3R}$	$\frac{\varepsilon}{3R}$	0	$\frac{\varepsilon}{3R}$
Finally t = ∞	$\frac{\varepsilon}{2R}$	0	$\frac{\varepsilon}{2R}$	$\frac{\varepsilon C}{2}$	0

Ex. At t = 0 switch S_1 is closed and remains closed for a long time and S_2 remains open. Now S_1 is opened and S_2 is closed.

Find out

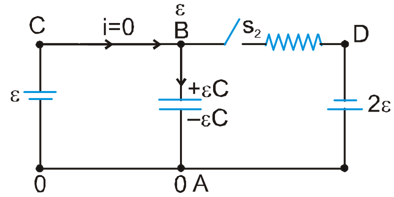


(i) The current through the capacitor immediately after that moment

(ii) Charge on the capacitor long after that moment.

(iii) Total charge flown through the cell of emf 2ε after S_2 is closed.

Sol. **(i)** Let Potential at point A is zero. Then at point B and C it will be ε (because current through the circuit is zero).

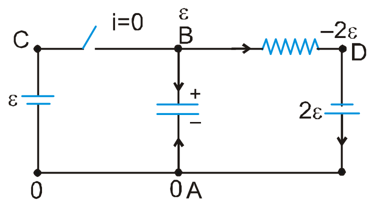


$V_B-V_A=(\varepsilon-0)$

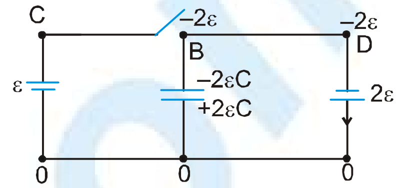
$\therefore$ Charge on capacitor $=C(\varepsilon-0)=C\varepsilon$

Now S_2 is closed and S_1 is open. (p.d. across capacitor and charge on it will not change suddenly)

Potential at A is zero so at D it is – 2ε.



$\therefore$ current through the capacitor $= \dfrac{\varepsilon-(-2\varepsilon)}{R} = \dfrac{3\varepsilon}{R}$ (B to D)



(ii) after a long time, i = 0

$V_B - V_A = V_D - V_A = -2\varepsilon$

$\therefore$ $Q = C(-2\varepsilon - 0) = -2\varepsilon C$

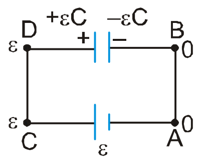
(iii) The charge on the lower plate (which is connected to the battery) changes from $-\varepsilon C$ to $2\varepsilon C$.

$\therefore$ this charge will come form the battery,

$\therefore$ charge flown from that cell is $3\varepsilon C$ downward.

Ex. A capacitor of capacitance C which is initially uncharged is connected with a battery. Find out heat dissipated in the circuit during the process of charging.

Sol. Final status



Let potential at point A is 0, so at B also 0 and at C and D it is ε. finally, charge on the capacitor

$Q_C = \varepsilon C$

$U_i = 0$

$U_f = \frac{1}{2}CV^2 = \frac{1}{2}C\varepsilon^2$

work done by battery $= \int P dt$

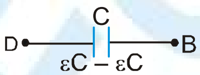
$W = \int \varepsilon i dt = \varepsilon \int i dt = \varepsilon . Q = \varepsilon . \varepsilon C = \varepsilon^2 C$

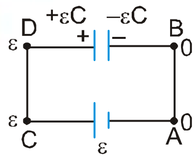
(Now onwards remember that w.d. by battery = εQ if Q has flown out of the cell from high potential and w.d. on battery
is εQ if Q has flown into the cell through high potential)

Heat produced $= W - (U_f - U_i) = \varepsilon^2 C - \frac{1}{2}\varepsilon^2 C = \frac{C\varepsilon^2}{2}$.

Ex. A capacitor of capacitance C which is initially charged upto a potential difference ε is connected with a battery of emf ε such that the positive terminal of battery is connected with positive plate of capacitor. Find out heat loss in the circuit during the process of charging.

Sol.





Since the initial and final charge on the capacitor is same before and after connection.

Here no charge will flow in the circuit so heat loss = 0

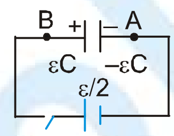
Ex. A capacitor of capacitance C which is initially charged upto a potential difference ε is connected with a battery of emf ε/2 such that the positive terminal of battery is connected with positive plate of capacitor. After a long time

(i) Find out total charge flow through the battery

(ii) Find out total work done by battery

(iii) Find out heat dissipated in the circuit during the process of charging.

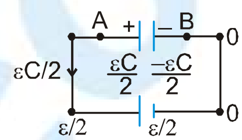
Sol. **(i)** Let potential of A is 0 so at B it is $\frac{\varepsilon}{2}$. So final charge on capacitor = Cε/2



Charge flow through the capacitor = (Cε/2 – Cε) = –Cε/2

So charge is entering into battery.

(ii) finally,



Change in energy of capacitor = $U_{final} - U_{initial}$

$$= \frac{1}{2}C\left(\frac{\varepsilon}{2}\right)^2 - \frac{\varepsilon^2 C}{2}$$

$$= \frac{1}{8}\varepsilon^2 C - \frac{1}{2}\varepsilon^2 C = -\frac{3\varepsilon^2 C}{8}$$

Work done by battery = $\frac{\varepsilon}{2} \times \left(-\frac{\varepsilon C}{2}\right) = -\frac{\varepsilon^2 C}{4}$

(iii) Work done by battery = Change in energy of capacitor + Heat produced

Heat produced = $\frac{3\varepsilon^2 C}{8} - \frac{\varepsilon^2 C}{4} = \frac{\varepsilon^2 C}{8}$

• Etoos Tips & Formulas •

1. CAPACITOR & CAPACITANCE

A capacitor consist of two conductors carrying charges of equal magnitude and opposite sign. The capacitance C of any capacitor is the ratio of the charge Q on either conductor to the potential difference V between them

$C = \frac{Q}{V}$ The capacitance depends only on the geometry of the conductors and not on an external source of charge or potential difference.

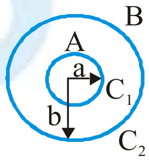
2. Capacitance of an Isolated Spherical Conductor

$C = 4\pi \in_0 \in_r R$ in a medium $C = 4\pi \in_0 R$ in air

This sphere is at infinite distance from all the conductors.

3. Spherical Capacitor :

It consists of two concentric spherical shells. Here capacitance on region between the two shells is C_1 and that outside the shell is C_2. We have



$C_1 = \frac{4\pi \in_0 ab}{b-a}$ and $C_2 = 4\pi \in_0 b$

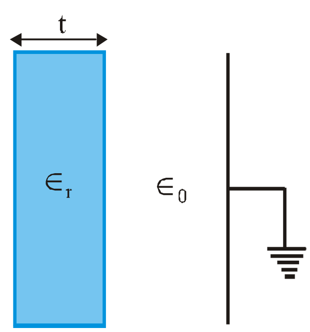
4. Parallel Plate Capacitor :

(a) Uniform Di-Electric Medium : If two parallel plates each of area A & separated by a distance d are charged with equal & opposite charge Q, then the system is called a parallel plate capacitor & capacitance is given by,

$C = \frac{4\pi \in_0 \in_r A}{d}$ in a medium ; $C = \frac{\in_0 A}{d}$ with air as medium

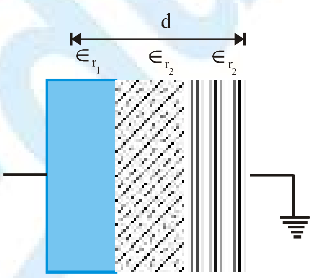
This result is only valid when the electric field between plates of capacitor is constant.

(b) Medium Partly Air : $C = \frac{\in_0 A}{d - \left(t - \frac{t}{\in_r}\right)}$



When a di-electric slab of thickness t & relative permittivity $\in_r$ is introduced between the plates of an air capacitor, then the distance between the plates is effectively reduced by $\left(t - \frac{t}{\in_r}\right)$ irrespective of the position of the di-electric slab.

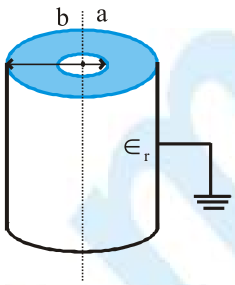
(c) Composite Medium :



$$C = \frac{\in_0 A}{\frac{t_1}{\in_{r_1}} + \frac{t_2}{\in_{r_2}} + \frac{t_3}{\in_{r_3}}}$$

5. Cylindrical Capacitor :

If consists of two co-axial cylinders of radii a & b the outer conductor is earthed. The di-electric constant of the medium filled in the space between the cylinders is $\in_r$.

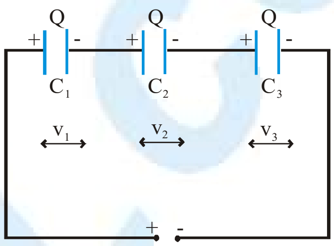


The capacitance per unit length is $C = \dfrac{2\pi \in_0 \in_r}{\ln\left(\dfrac{b}{a}\right)}$

6. Combination of Capacitors :

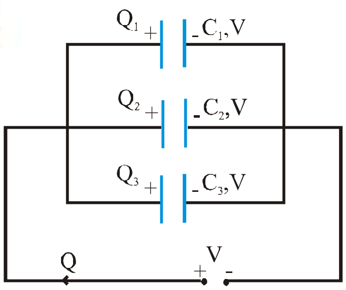
(a) Capacitors in Series : In this arrangement all the capacitors when uncharged get the same charge Q but the potential difference across each will differ (if the capacitance are unequal).

$$\frac{1}{C_{eq.}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ... + \frac{1}{C_n}$$



(b) Capacitors in Parallel : When one plate of each capacitor is connected to the positive terminal of the battery & the other plate of each capacitor is connected to the negative terminals of the battery, then the capacitors are said to be in parallel connection. The capacitors have the same potential difference, V but the charge on each one is different (if the capacitors are unequal).

$$C_{eq.} = C_1 + C_2 + C_3 + ... + C_n$$



7. Energy Stored in a Charged Capacitors :

Capacitors C, charge Q & potential difference V; then energy stored is

$$U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$$

This energy is stored in the electrostatic field set up in the di-electric medium between the conducting plates of the capacitor.

8. Heat Produced in Switching in Capacitive Circuit :

Due to charge flow always some amount of heat is produced when a switch is closed in a circuit which can be obtained by energy conservation as -

Heat = Work done by battery - Energy absorbed by capacitor.

Work done by battery to charge a capacitor $W = CV^2 = QV = \dfrac{Q^2}{C}$

9. Sharing of Charge :

When two charged conductors of capacitance C_1 & C_2 at potential V_1 & V_2 respectively are connected by a conducting wire, the charge flows from higher potential conductor to lower potential conductor, until the potential of the two condensers becomes equal. The common potential (V) after sharing of charges;

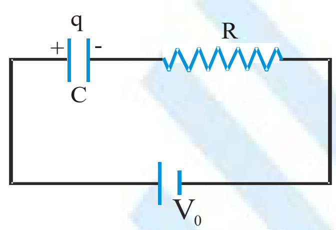
$$V = \frac{\text{net charge}}{\text{net capacitance}} = \frac{q_1 + q_2}{C_1 + C_2} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

charges after sharing $q_1 = C_1V$ & $q_2 = C_2V$. In this process energy is lost in the connecting wire as heat.

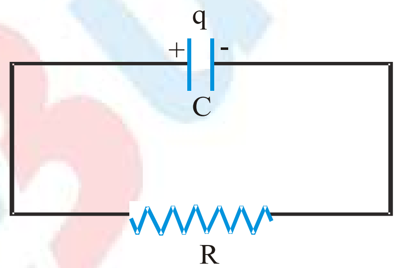
This loss of energy is $U_{\text{initial}} - U_{\text{final}} = \frac{C_1C_2}{2(C_1 + C_2)}(V_1 - V_2)^2$

10. Attractive force between capacitor plate $F = \left(\frac{\sigma}{2\epsilon_0}\right)(\sigma A) = \frac{Q^2}{2\epsilon_0 A}$

11. Charging of a capacitor : $q = q_0(1 - e^{-t/RC})$ where $q_0 = CV_0$



12. Discharging of a capacitor : $q = q_0e^{-t/RC}$



13. The energy of a charged conductor resides outside the conductor in its electric field, where as in a condenser it is stored within the condenser in its electric field.

14. The energy of an uncharged condenser = 0.

15. The capacitance of a capacitor depends only on its size & geometry & the di-electric between the conducting surface. (i.e. independent of the conductor, like whether it is copper, silver, gold etc.)

16. The two adjacent conductors carrying same charge can be at different potential because the conductors may have different sizes and hence difference capacitance.

17. When a capacitor is charged by a battery, both the plates received charge equal in magnitude, no matter sizes of plates are identical or not because the charge distribution on the plates of a capacitor is in accordance with charge conservation principle.

18. On filling the space between the plates of a parallel plates are identical or not because the charge distribution on the plates of a capacitor is increased because the same amount of charge can be stored at a reduced potential.

19. The potential of a grounded object is taken to be zero because capacitance of the earth is very large.