ELECTROMAGNETIC WAVES

INTRODUCTION

We have seen that in certain situations light may be described as a wave. The wave equation for light propagating in x-direction in vacuum may be written ass

$$E = E_0 \sin \omega(t - x/c)$$

where E is the sinusoidally varying electric field at the position x at time t. The constant c is the speed of light in vacuum. The electric field E is in the Y-Z plane, that is perpendicular to the direction of propagation.

There is also a sinusoidally varying magnetic field associated with the electric field when light propagates. This magnetic is perpendicular to the direction of propagation as well as to the electric field E. It is given by

$$B = B_0 \sin \omega(t - x/c)$$

Such a combination of mutually perpendicular electric and magnetic fields is referred to as an electromagnetic wave in vacuum. The theory of electromagnetic wave was mainly developed by Maxwell around 1864.

DISPLACEMENT CURRENT

We have seen that an electrical current produces a magnetic field around it. Maxwell showed that for logical consistency, a changing electric field must also produce a magnetic field. This effect is of great importance because it explains the existence of radio waves, gamma rays and visible light, as well as all other forms of electromagnetic waves.

To see how a changing electric field gives rise to a magnetic field, let us consider the process of charging of a capacitor and apply Ampere's circuital law given by

$$\oint B.\,dl = \mu_0 (t) \qquad \text{........ (1)}$$

to find magnetic field at a point outside the capacitor. Figure 1 (a) shows a parallel plate capacitor C which is a part of circuit through which a time-dependent current i (t) flows. Let us find the magnetic field at a point such as P, in a region outside the parallel plate capacitor. For this, we consider a plane circular loop of radius r whose plane is perpendicular to the direction of the current - Carrying wire, and which is centered symmetrically with respect to the wire. From symmetry, the magnetic field is directed along the circumference of the circular loop and is the same in magnitude at all points on the loop so that if B is the magnitude of the field, the left side of equation. (8.1) is B ($2\pi r$). So we have

$$B\,(2\pi r) = \mu_0\, i\,(t) \qquad \text{......... (2)}$$

Now, consider a different surface, which has the same boundary. This is a pot like surface (Fig.1 (b)] which nowhere touches the current, but has its bottom between the capacitor plates; its mouth is the circular loop mentioned above. Another such surface is shaped like a tiffin box (without the lid) [Fig. 1 (c)]. On applying Ampere's circuital law to such surfaces with the same perimeter, we find that the left hand side of Eq. (1) has not changed but the right hand side is zero and not μ_0 i, since no current passes through the surface of Fig 1 (b) and (c). So we have a contradiction; calculated one way, there is a magnetic field at a point P; calculated another way, the magnetic field at P is zero. Since the contradiction arises from our use of Ampere's circuital law, this law must be missing something. The missing term must be such that one gets the same magnetic field at point P, no matter what surface is used.

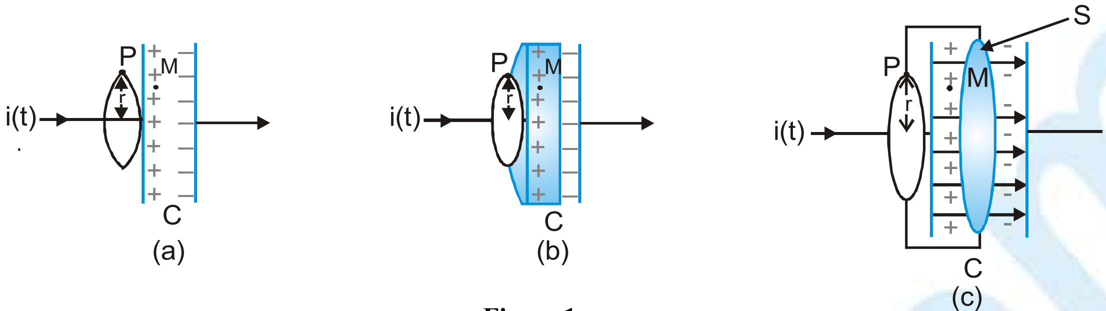


Figure 1

We can actually guess the missing term by looking carefully at Fig. 1 (c). Is there anything passing through the surface S between the plates of the capacitor ? Yes, of course, the electric field . If the plates of the capacitor have an area A, and a total charge Q, the magnitude of the electric field **E** between the plates is (Q/A)/ ε_0. The field is perpendicular to the surface S of Fig.1 (c). It has the same magnitude over the area A of the capacitor plates, and vanishes outside it. So what is the electric flux Φ_E through the surface S ? Using Gauss's law, it is

$$\Phi_E = |E|\, A = \frac{1}{\varepsilon_0}\frac{Q}{A} A = \frac{Q}{\varepsilon_0} \qquad .... (3)$$

Now if the charge Q on the capacitor plates changes with time, there is a current i = (dQ / dt), so that using Eq. (3), we have

$$\frac{d\Phi_E}{dt} = \frac{d}{dt}\left(\frac{Q}{\varepsilon_0}\right) = \frac{1}{\varepsilon_0}\frac{dQ}{dt}$$

This implies that for consistency,

$$\varepsilon_0\left(\frac{d\Phi_E}{dt}\right) = i \qquad .... (4)$$

This is the missing term in Ampere's circuital law. If we generalis this law by adding to the total current carried by conductors through the surface, another term which is ε_0 times the rate of change of electric flux through the same surface, the total has the same value of current i for all surfaces. If this is done, there is no contradiction in the value of B obtained anywhere using the generalized Ampere's law. B at the point P is non-zero no matter which surface is used for calculating it. B at a point P outside the plates [Fig.1 (a)] is the same as at a point M just inside, as it should be. The current carried by conductors due to flow of charges is called conduction current. The current, given by Eq. (4), is a new term, and is due to changing electric field (or electric displacement). It is therefore called displacement current or Maxwell's displacement current. Figure 2 shows the electric and magnetic fields inside the parallel plates capacitor discussed above. The generalisation made by Maxwell then is the following. The source of a magnetic field is not just the conduction electric current due to flowing charges, but also the time rate of change of electric field. More precisely, the total current i is the sum of the conduction current denoted by i_C, and the displacement current denoted by i_d (= ε_0 ($d\Phi_E$)/ dt). So we have

$$i = i_e + i_d = i_c + \varepsilon_0 \frac{d\Phi_E}{dt} \qquad .... (5)$$

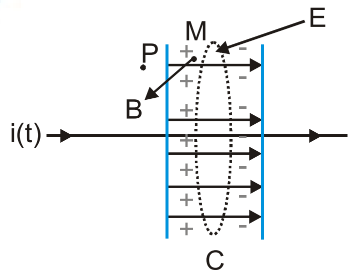


Figure 2 (a)

In explicit terms, this means that outside the capacitor plates, we have only conduction current $i_c = i$, and no displacement current, i.e., $i_d = 0$. On the other hand, inside the capacitor, there is no conduction current, i.e., $i_c = 0$, and there is only displacement current, so that $i_d = i$.

The generalised (and correct) Ampere's circuital law has the same form as Eq, **(1)**, with one difference: " the total current passing through any surface of which the closed loop is the perimeter" is the sum of the conduction current and the displacement current The generalised law is

$$\oint B \bullet d\lambda = \mu_0 i_c + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \qquad \text{..........} \textbf{(6)}$$

and is known as Ampere-Maxwell law.

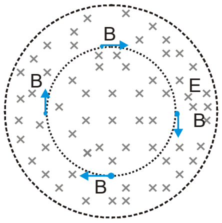


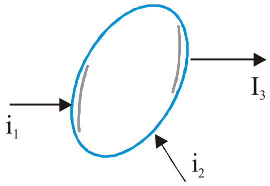
Figure 2(b)

In all respects, the displacement current has the same physical effects as the conduction current. In some cases, for example, steady electric fields in a conducting wire, the displacement current may be zero since the electric field E does not change with time. In other cases, for example, the charging capacitor above, both conduction and displacement currents may be present in different regions of space. In most of the cases, they both may be present in the same region of space, as there exist no perfectly conducting or perfectly insulating medium. Most interestingly, there may be large regions of space where there is no conduction current, but there is only a displacement current due to time-varying electric fields. In such a region, we expect a magnetic field, though there is no (conduction) current source nearby. The prediction of such a displacement current can be verified experimentally. For example, a magnetic field (say at point M) between the plates of the capacitor in Fig. 2 (a) can be measured and is seen to be the same as that just outside (at P).

The displacement current has (literally) far reaching consequences. One thing we immediately notice is that the laws of electricity and magnetism are now more symmetrical Faraday's law of induction states that there is an induced emf equal to the rate of change of magnetic flux. Now, since the emf between two points 1 and 2 is the work done per unit charge in taking it from 1 to 2, the existence of an emf implies the existence of an electric field. So,we can rephrase Faraday's law of electromagnetic induction by saying that a magnetic field, changing with time, gives rise to an electric field. Then, the fact that an electric field changing with time gives rise to a magnetic field, is the symmetrical counterpart, and is a consequence of the displacement current being a source of a magnetic field. Thus, time-dependent electric and magnetic field give rise to each other. Faraday's law of electromagnetic induction and Ampere-Maxwell law give a quantitative expression of this statement, with the current being the total current as in Eq. **(5)**. One very important consequence of this symmetry is the existence of electromagnetic waves, which we discuss qualitatively in the next section.

CONTINUITY OF ELECTRIC CURRENT

Consider a closed surface enclosing a volume (in figure). Suppose charges are entering into the volume and are also leaving it. If no charge is accumulated inside the volume, the total charge going into the volume in any time is equal to the total charge leaving it, during the same time. The conduction current is then continuous.



If charge is accumulated inside the volume, this continuity breaks. However, if we consider the conduction current plus the displacement current, the total current is still still continuous. Any loss of conduction current i_c appears as displacement current i_d. This can be shown as follows.

Suppose a total conduction current i_1 goes into the volume and a total conduction current i_2 goes out of it. The charge goiing into the volume in a time dt is i_1 dt and that coming out is i_2 dt. The charge accumulated inside the volume is

$$d(q_{inside}) = i_1 dt - i_2 dt$$

or, $$\frac{d}{dt}(q_{inside}) = i_1 - i_2 \qquad \text{......... (i)}$$

From Gayss's law,

$$\Phi_1 = \oint \vec{E}.\vec{dS} = \frac{q_{inside}}{\varepsilon_0}$$

or, $$\varepsilon_0 \frac{d\Phi_E}{dt} = \frac{d}{dt}(q_{inside})$$

Comparing with **(i)**,

$$i_i - i_2 = i_d$$

$$i_1 = i_2 + i_d$$

Thus, the total current (conduction + displacement) going into the volume is equal to the total current counting out of it.

MAXWELL'S EQUATIONS AND PLANE ELECTROMAGNETIC WAVES

The whole subject of electricity and magnetism may be described mathematically with the help of four fundamental equations :

Gauss's law for electricity $$\Phi_1 = \oint \vec{E}.\vec{dS} = \frac{q}{\varepsilon_0}$$

Gauss's law for magnetism $$\Phi_1 = \oint \vec{B}.\vec{dS} = 0$$

Faraday's law $$\Phi_1 = \oint \vec{E}.\vec{dl} = -\frac{d\Phi_E}{dt}$$

Ampere's law $$\Phi_1 = \oint \vec{B}.\vec{dl} = \mu_0 i + \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt}$$

These equations are correctively known as Maxwell's equations.

In vacuum there are no charges and hence no conduction currents. Faraday's law and Ampere's law take the form

$$\Phi_1 = \oint \vec{B}.\vec{dl} = -\frac{d\Phi_B}{dt} \qquad \text{........ (i)}$$

and $$\Phi_1 = \oint \vec{B}.\vec{dl} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \qquad \text{........ (ii)}$$

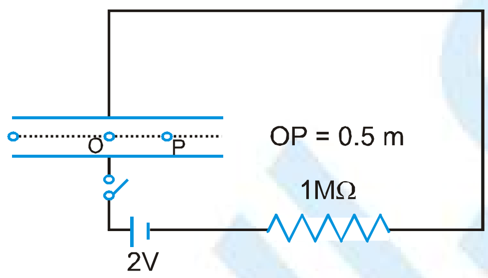
respectively.

Let us check if these equations are satisfied by a plane electromagnetic wave given by

$$\left.\begin{array}{l} E = E_y = E_0 \sin \omega(t - x/c) \\ B = B_z = B_0 \sin \omega(t - x/c) \end{array}\right] \quad \text{and} \qquad \text{..........(7)}$$

The wave described above propagates along the positive x-direction, the electric field remains along the y-direction and the magnetic field along the z-direction. The magnitudes of the fields oscillate between $\pm E_0$ and $\pm B_0$ respectively. It is a linearly polarized light, polarized along the y-axis.

Ex. A parallel plate capacitor with circular plates of radius 1 m has a capacitance of 1 nF. At t = 0, it is connected for charging in series with a resistor R = 1 M Ω across a 2 V battery (as shown). Calculate the magnetic field at a point P. halfway between the centre and the periphery of the plates, after $t = 10^{-3}$ s. (The charge on the capacitor at time t is q (t) = CV [1 – exp (–t / τ)], where the time constant τ is equal to CR)



Sol. The time constant of the CR circuit is $\tau = CR = 10^{-3}$ s. Then we have

$$q(t) = CV\,[1 - \exp(-t/\tau)]$$
$$= 2 \times 10^{-9}\,[1 - \exp(-t/10^{-3})]$$

The electric field in between the plates at time t is

$$E = \frac{q(t)}{\varepsilon_0 A} = \frac{q}{\pi\varepsilon_0}\ ;\ A = \pi\,(1)^2\ \text{m}^2 = \text{area of the plates.}$$

Consider now a circular loop of radius (1 / 2) m parallel to the plates passing through P. The magnetic field **B** at all points on the loop is along the loop and of the same value. The flux Φ_E through this loop is

$\Phi_E = E \times$ area of the loop

$$= E \times \pi \times \left(\frac{1}{2}\right)^2 = \frac{\pi E}{4} = \frac{q}{4\varepsilon_0}$$

The displacement current

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \frac{1}{4}\frac{dq}{dt} = 0.5 \times 10^{-6} \exp(-1)$$

at $t = 10^{-3}$ s. Now, applying Ampere-Maxwell law to the loop, we get

$$B \times 2\pi \times \left(\frac{1}{2}\right) = \mu_0\,(i_c + i_d) = \mu_0\,(0 + i_d) = 0.5 \times 10^{-6}\,\mu_0 \exp(-1)$$

or, $\qquad B = 0.74 \times 10^{-3}$ T

Sources of Electromagnetic Waves

How are electromagnetic waves produced ? Neither stationary charges Nor charges in uniform motion (steady currents) can be sources of electromagnetic waves. The former produces only electrostatic field, while the latter produces magnetic fields that, however, do not vary with time. It is an important result of Maxwell's theory that accelerated charges radiate electromagnetic waves. The proof of this basic result is beyond the scope of this book, but we can accept it on the basis of rough qualitative reasoning. Consider a charge oscillating with some frequency. (An oscillating charge is an example of accelerating charge.) This produces an oscillating electric field in space, which produces an oscillating magnetic field, which in turn, is a source of oscillating electric field, and so on. The oscillating electric and magnetic fields thus regenerate each other as the waves propagates through the space. The frequency of electromagnetic wave naturally equals the frequency of oscillation of the charge. The energy associated with the propagating wave comes at the expense of the energy of the source-the accelerated charge.

Nature of Electromagnetic Wave

It can be shown from Maxwell's equations that electric and magnetic field in an electromagnetic wave are perpendicular to each other and to the direction of propagation. It appears reasonable, say from our discussion of the displacement current. Consider Fig 2. The electric field inside the plates of the capacitor is directed perpendicular to the plates. The magnetic field this gives rise to via the displacement current is along the perimeter of a circle parallel to the capacitor plates. So B and E are perpendicular in this case. This is a general feature.

In Fig 4, we show a typical example of a plane electromagnetic wave propagating along the z direction (the fields are shown as a function of the z coordinate, at a given time t). The electric field E_x is along the x-axis, and varies sinusoidally with z, at a given time. The magnetic field B_y is along the y-axis and again varies sinusoidally with z. The electric and magnetic fields E_x and B_y are perpendicular to each other, and to the direction z of propagation. We can write E_x and B_y as follows :

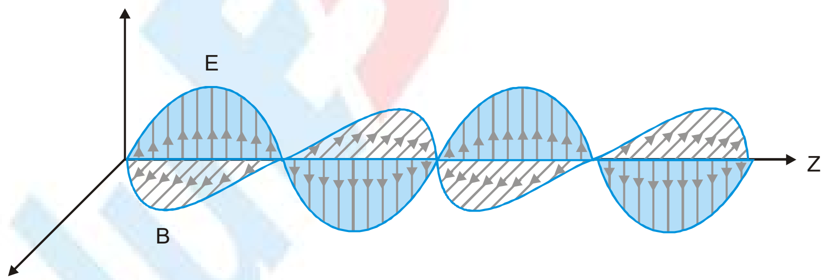
$$E_x = E_0 \sin(kz - \omega t) \qquad \text{............ Eq. 7(a)}$$

$$B_y = B_0 \sin(kz - \omega t) \qquad \text{............ Eq. 7 (b)}$$

Here k is related to the wave length λ of the wave by the usual equation

$$k = \frac{2\pi}{\lambda} \qquad \text{............ Eq. 8}$$

and ω is the angular frequency. k is the magnitude of the wave vector (or propagation vector) k and its irection describes the direction of propagation of the wave. The speed of propagation of the wave is (ω/k). Using Eqs. [7 (a) and (b)] for E_x and B_y and Maxwells equation we finds that



$$\omega = cK, \text{ where, } c = 1/\sqrt{\mu_0 \varepsilon_0} \qquad \text{...........Eq.9 (a)}$$

The relation ω = cK is the standard one for waves. This relation is often written in terms of frequency.

ν (=ω/ 2π) and wavelength. λ (= 2π / k) as

$$2\pi\nu = c\left(\frac{2\pi}{\lambda}\right) \text{ or}$$

$$\nu\lambda = c \qquad \text{...........Eq. 9(b)}$$

It is also seen from Maxwell's equations that the magnitude of the electric and the magnetic fields in an electromagnetic waves are related as $B_0 = E_0 / c$. Electromagnetic waves are self-sustaining oscillations of electric and magnetic fields in free space, or vacuum. They differ from all the other wave we have studied so far, in respect that no material medium is involved in the vibrations of the electric and magnetic fields. Sound waves in air are longitudinal waves of compression and rarefaction. Transverse wave on water surface spreads horizontally and radially out wards. Transverse elastic (sound) waves can also propagate in a solid, which is rigid and that resists shear. Scientists in the nineteenth century were so much used to this mechanical picture that they thought that there must be some medium pervading all space and all matter, which responds to electric and magnetic fields just as any elastic medium does. They called this medium ether. We now accept that no such physical medium is needed. The famous experiment of Michelson and Morley in 1887 demolished conclusively the hypothesis of ether. Electric and magnetic fields oscillating in space and time, can sustain each other in vacuum.

But what if a material medium is actually there ? We known that light, and electromagnetic wave, does propagate through glass, for example. We have seen earlier that the total electric and magnetic fields inside a medium are described in terms of a permittivity ε and a magnetic permeability μ (These describe the factors by which the total fields differ from the external fields). These replace ε_0 and μ_0 in the description to electric and magnetic fields in maxwell's equation with the result that in a material medium of permittivity ε and magnetic permeability μ. The velocity of light becomes,

$$\upsilon = \frac{1}{\sqrt{\mu\varepsilon}} \qquad \text{.............}(10)$$

Thus, the velocity of light depends on electric and magnetic properties of the medium. The velocity of electromagnetic waves in free space or vacuum is an important fundamental constant. It has been shown by experiments on electromagnetic waves of different wavelength that this velocity is the same (independent of wavelength) to within a few metres per second, out of a value of 3×10^8 m/s. The constancy of the velocity of em waves in vacuum is so strongly supported by experiments and the actual value is so well known now that this is used to define a standard of length Hertz not only showed the existence of electromagnetic waves, but also that the light waves, could be diffracted, refracted and polarised.Thus, he conclusively established the wave nature of the radiation. Further, he produced stationary electromagnetic waves and determined their wavelength by measuring the distance between two successive nodes. Since the frequency of the wave was known (being equal to the frequency of the oscillator), he obtained the speed of the wave using the formula

$v = \nu\lambda$ and found that the waves travelled with the same speed as the speed of light.

Do electromagnetic waves carry energy and momentum like other waves ? Yes, they do. In a region of free space with electric field E, there is an energy density ($\varepsilon_0 E^2/ 2$). Similarly, as seen associated with a magnetic field B is a magnetic energy density ($B^2 / 2\mu_0$). As electromagnetic wave contains both electric and magnetic fields, there is a non-zero energy density associated with it. Now consider a plane perpendicular to the direction of propagation of the electromagnetic wave (Fig. 4). If there are, on this plane, electric charges the will be set and sustained in motion by electric and magnetic fields of the electromagnetic wave. The charges thus acquire energy and momentum from the waves. This just illustrates the fact that an electromagnetic wave (like other waves.) carries energy and momentum. Since it carries momentum, an electromagnetic wave also exerts pressure called radiation pressure. If the total energy transferred to a surface in time t is U. It can be shown that the magnitude of the total momentum delivered to this surface (for complete absorption) is,

$$p = \frac{U}{c} \qquad \text{.......}(11)$$

Light carries energy from the sun to the earth, thus making life possible on the earth.

Ex. A plane electromagnetic wave of frequency 25 MHz travels in free space along the x-direction. At a particular point in space and time, $\mathbf{E} = 6.3\ \hat{j}$ V/m. What is B at this point ?

Sol. Using Eq. (8.10), the magnitude of B is

$$B = \frac{E}{c}$$

$$= \frac{6.3 V/m}{3 \times 10^8 m/s} = 2.1 \times 10^{-8} T$$

To find the direction, we note that E is along y-direction and the wave propagate along x-axis. Therefore, B should be in a direction perpendicular to both x- and y-axes. Using vector algebra, E × B should be along x-direction. Since, $(+\hat{j}) \times (+\hat{K}) = \hat{i}$, B is along the z-direction

Thus, $B = 2.1 \times 10^{-8}$ kT

Ex. The magnetic field in a plane electromagnetic wave is given by $B_y = 2 \times 10^{-7} \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)$ T.

(a) What is the wavelength and frequency of the wave ?

(b) Write an expression for the electric field.

Sol. **(a)** Comparing the given equation with

$$B_y = B_0 \sin\left[2\pi\left(\frac{x}{y} + \frac{t}{T}\right)\right]$$

we get, $\lambda = \frac{2\pi}{0.5 \times 10^3}$ m = 1.26 cm,

and $\frac{1}{T} = \nu = (1.5 \times 10^{11}) / 2\pi = 23.9$ GHz

(b) $E_0 = B_0 c = 2 \times 10^{-7}$ T × 3×10^8 m/s = 60 V/m

The electric field component is perpendicular to the direction of propagation and the direction of magnetic field. Therefore, the electric field component along the z-axis is obtained as

$$E_z = 60 \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t) \text{ V/m}$$

Ex. Light with an energy flux of 18 W/cm² falls on a nonreflecting surface at normal incidence. If the surface has an area of 20cm², find the average force exerted on the surface during a 30 minute time span.

Sol. The total energy falling on the surface is

$$U = (18 \text{ W/cm}^2) \times (20\text{cm}^2) \times (30 \times 60)$$
$$= 6.48 \times 10^5 \text{ J}$$

Therefore, the total momentum delivered (for complete absorption) is

$$p = \frac{U}{c} = \frac{6.48 \times 10^5 J}{3 \times 10^8 m/s} = 2.16 \times 10^{-3} \text{ kg m/s}$$

The average force exerted on the surface is

$$F = \frac{p}{t} = \frac{2.16 \times 10^{-3}}{0.18 \times 14^4} = 1.2 \times 10^{-6} N$$

Ex. Calculate the electric and magnetic fields produced by the radiation coming from a 100 W bulb at a distance of 3 m. Assume that the efficiency of the bulb is 2.5% and it is a point source.

Sol. The bulb, as a point source, radiates light in all directions uniformly. At a distance of 3 m, the surface area of the surrouding sphere is

$$A = 4\pi r^2 = 4\pi(3)^2 = 113 \text{ m}^2$$

The intensity at this distance is

$$I = \frac{\text{Power}}{\text{Area}} = \frac{100\,\text{W} \times 2.5\%}{113\,\text{m}^2} = 0.022 \text{ W/m}^2$$

Half of this intensity is provided by the electric field and half by the magnetic field.

$$\frac{1}{2}I = \frac{1}{2}\left(\varepsilon_0 E_{rms}^2 c\right) = \frac{1}{2}(0.022 \text{ W/m}^2)$$

$$E_{rms} = \sqrt{\frac{0.022}{(8.85\times10^{-12})(3\times10^{8})}}\,\text{V/m} = 2.9 \text{ V/m}$$

The value of E found above is the root mean square value of the electric field. Since the electric field in a light beam is sinusoidal, the peak electric field, E_0 is

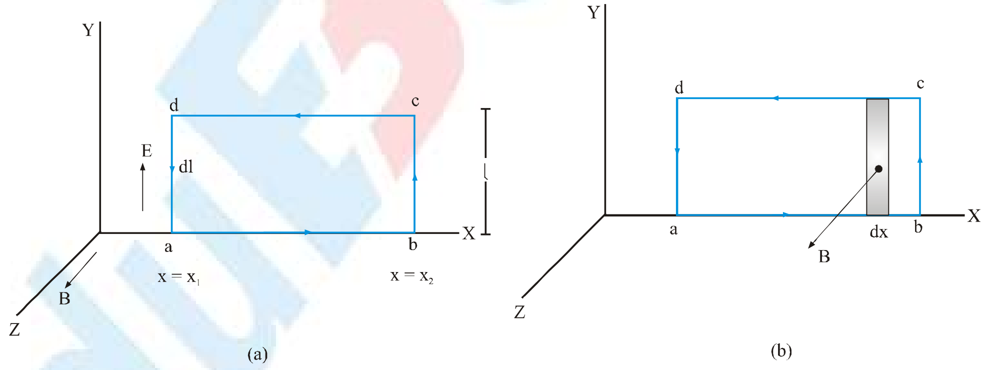
$$E_0 = \sqrt{2}\,E_{rms} = \sqrt{2} \times 2.9 \text{ V/m} = 4.07 \text{ V/m}$$

Electric field strength of light is fairly large

$$B_{rms} = \frac{E_{rms}}{c} = \frac{2.9\,\text{Vm}^{-1}}{3\times10^{8}\,\text{ms}^{-1}} = 9.6 \times 10^{-9}\,\text{T}$$

Agains, since the field in the light beam is sinusoidal, the peak magnetic field is $B_0 = \sqrt{2}\,B_{rms} = 1.4 \times 10^{-8}$ T. Note that although the energy in the magnetic field is equal to the energy in the electric field, the magnetic field strength is evidently very weak.

Faraday's Law



Let us consider the rectangular path abcd in the x–y plane as shown in figure. Let us evaluate the terms in the Faraday's law on this path. The electric field is parallel to the y-axis. The circulation of E is

$$\oint \vec{E}.\vec{dl} = \int_a^b \vec{E}.\vec{dl} + \int_b^c \vec{E}.\vec{dl} + \int_c^d \vec{E}.\vec{dl} + \int_d^a \vec{E}.\vec{dl}$$

$$= 0 + E(x_2)l + 0 + E(x_1)(-l)$$

$$= E_0 l[\sin\omega(t - x_2/c) - \sin\omega(t - x_1/c)] \quad \text{.......... (i)}$$

Next, let us calculate the flux of the magnetic field Φ_B, through the same rectangle abcd (in figure (b)). The flux through a strip of width dx at x is

$$B(x)l\ dx = B_0[\sin\omega(t - x/c)]l\ dx$$

The flux through the rectangle abcd is

$$\Phi_B = \int_{x_1}^{x_2} B_0 l \sin(t - x/c)dx$$

$$= \frac{c}{\omega}B_0 l[-\cos\omega(t - x_2/c) + \cos\omega(t - x_1/c)]$$

Thus,

$$\frac{d\Phi_B}{dt} = cB_0\ l[\sin\omega(t - x_2/c) - \sin\omega(t - x_1/c)] \qquad \text{...... (ii)}$$

The Faraday's law for vacuum is

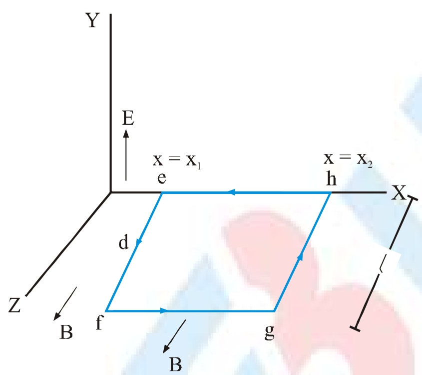
$$\oint \vec{E}.\vec{dl} = \frac{d\Phi_B}{dt}$$

Putting from (i) and (ii) in this equation, we see that Faraday's law is satisfied by the wave given by equation if

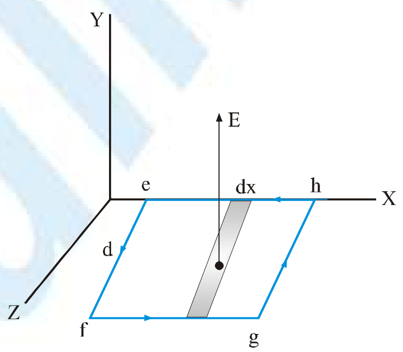
$$E_0 = cB_0 \qquad \text{......... (12)}$$

Ampere's Law

Let us consider the rectangular path efgh in the x-z plane as shown in figure (a).



(a)



(b)

The circulation of $\vec{B}$ is

$$\oint \vec{B}.\vec{dl} = \int_e^f \vec{B}.\vec{dl} + \int_f^g \vec{B}.\vec{dl} + \int_g^h \vec{B}.\vec{dl} + \int_h^e \vec{B}.\vec{dl}$$

$$= B(x_1)l + 0 - B(x_2)l + 0$$

$$= B_0 l[\sin\omega(t - x_1/c) - \sin\omega(t - x_2/c)] \qquad \text{............. (i)}$$

The flux of the electric field through theb same rectangle efgh (figure (b)) is

$$\Phi_E = \int \vec{E}.\vec{dS}$$

$$= \int_{x_1}^{x_2} E(x)l\ dx$$

$$= E_0\, l \int_{x_1}^{x_2} \sin\omega(t - x/c)\, dx$$

$$= -\frac{c}{\omega} E_0 l[-\cos\omega(t - x_2/c) + \cos\omega(t - x_1/c)]$$

or, $$\frac{d\Phi_E}{dt} = -cE_0 l[\sin\omega(t - x_2/c) - \sin\omega(t - x_1/c)] \qquad \text{...... (ii)}$$

The Ampere's law for vacuum is

$$\oint \vec{B}.d\vec{l} = \mu_0\varepsilon_0 \frac{d\Phi_E}{dt}$$

Putting from **(i)** and **(ii)** in this equation, we see that Ampere's law is satisfied if

$$B_0 = \mu_0\varepsilon_0\; c\, E_0$$

or, $$\mu_0\varepsilon_0 = \frac{B_0}{E_0 c}$$

Using equation **(iii)**,

$$\mu_0\varepsilon_0 = \frac{1}{c^2}$$

or, $$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \qquad \text{...... (13)}$$

Thus, Maxwell's equations have a solution giving a plane electromagnetic wave of the form (7) with $E_0 = cB_0$ and the speed of this wave is $\frac{1}{\sqrt{\mu_0\varepsilon_0}}$.

In older days, μ_0 and ε_0 were defined in terms of electric and magnetic measurements. Putting these values of μ_0 and ε_0, the speed of electromagnetic waves came out to be $c = 2.99793 \times 10^8$ m/s which was the same as the measured speed of light in vacuum. This provided a confirmatory proof that light is an electromagnetic wave.

it may be revalled that the speed of electromagnetic waves, which is the same as the speed of light, is now an exactly defined constant. Similarly, the constant $\mu_0 = 4\pi \times 10^{-7}$ T – m/A is an exactly defined constant. The quantity ε_0 is defined by the equation **(13)**.

Ex. The maximum electric field in a plane electromagnetic wave is 600 N C^{-1}. The wave is going in the x-direction and the electric field is in the y-direction. Find the maximum magnetic field in the wave and its direction.

Sol. We have $B_0 = \frac{E_0}{c} = \frac{600\ NC^{-1}}{3 \times 10^8\ ms^{-1}} = 2 \times 10^{-6}\ T$

As $\vec{E}, \vec{B}$ and the direction of propagation are mutually perpendicular, $\vec{B}$ should be along the z-direction.

ENERGY DENSITY AND INTENSITY

The electric and magnetic field in a plane electromagnetic wave are given by

$$E = E_0 \sin\omega(t - x/c)$$

and $$B = B_0 \sin\omega(t - x/c)$$

In any small volume dV, the energy of the electric field is

$$U_E = \frac{1}{2}\varepsilon_0 E^2 dV$$

and the energy of the magnetic field is

$$U_B = \frac{1}{2\mu_0} B^2 dV$$

Thus, the total energy is

$$U = \frac{1}{2}\varepsilon_0 E^2 dV + \frac{1}{2\mu_0} B^2 dV$$

The energy density is $u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0} B^2$

$$= \frac{1}{2}\varepsilon_0 E_0^2 \sin^2 \omega(t - x/c) + \frac{1}{2\mu_0} B^2 \sin^2 \omega(t - x/c)$$

If we take the average over a long time, the $\sin^2$ terms have an average value of 1/2.

Thus, $$u_{au} = \frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4\mu_0} B_0^2$$

From equations **(12)** and **(13)**,

$$\frac{1}{4\mu_0} B_0^2 = \frac{\varepsilon_0 c^2}{4}\left(\frac{E_0}{c}\right)^2 = \frac{1}{4}\varepsilon_0 E_0^2$$

Thus, the electric energy density is equal to the magnetic energy density in average.

or, $$u_{au} = \frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4}\varepsilon_0 E_0^2 = \frac{1}{2}\varepsilon_0 E_0^2 \qquad \text{....... (14)}$$

Also, $$\frac{1}{4\mu_0} B_0^2 + \frac{1}{4\mu_0} B_0^2 = \frac{1}{2\mu_0} B_0^2 \qquad \text{....... (15)}$$

Ex. The electric field in an electromagnetic wave is given by

$$E = (50\,N\,C^{-1}) \sin \omega(t - x/c)$$

Find the energy contained in a cylinder of cross-section 10 cm^2 and length 50 cm along the x-axis.

Sol. The energy density is

$$u_{au} = \frac{1}{2}\varepsilon_0 E_0^2 \times (8.85 \times 10^{-12} C^2 N^{-1} m^{-2}) \times (50 NC^{-1})^2$$

$$= 1.1 \times 10^{-8}\ J\ m^{-3}$$

The volume of the cylinder is

$$V = 10\ cm^2 \times 50\ cm = 5 \times 10^{-4}\ m^3$$

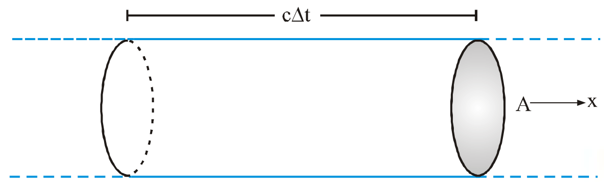
The energy contained in this volume is

$$U = (1.1 \times 10^{-8}\ J\ m^{-3}) \times (5 \times 10^{-4} m^3)$$

$$= 5.5 \times 10^{-12}\ J$$

Intensity

The energy crossing per unit area per unit time perpendicular to the direction of propagation is called the intensity of a wave.



Consider a cylindrical volume with area of cross section A and length c Δt along the X-axis (in figure). The energy contained in this cylinder crosses the area A in time Δt as the wave propagates at speed c. The energy contained is

$$U = u_{av}(c\,\Delta t)A$$

The intensity is $I = \frac{U}{A\Delta t} = u_{av}c$

In terms of maximum electric field,

$$I = \frac{1}{2}\varepsilon_0 E_0^2 c \qquad \text{...... (14)}$$

Ex. Find the intensity of the wave discussed in above example.

Sol. The intensity is

$$I = \frac{1}{2}\varepsilon_0 E_0^2 c = (1.1\times10^{-8}\,Jm^{-3})\times(3\times10^{8}\,ms^{-1})$$

$= 3.3\ W\ m^{-2}$

Electromagnetic spectrum

At the time Maxwell predicted the existence of electromagnetic waves, the only familiar electromagnetic waves were the visible light waves. The existence of ultraviolet and infrared waves was barely established. By the end of the nineteenth century, X -rays and gamma rays had also been discovered. We now know that, electromagnetic waves include visible light waves, X-rays gamma rays, radio waves, microwaves, ultraviolet and infrared waves. The classification of electromagnetic waves according to frequency is the electromagnetic spectrum (Fig. 5). There is no sharp division between one kind of wave and the next. The classification is based roughly on how the waves are produced and / or detected.

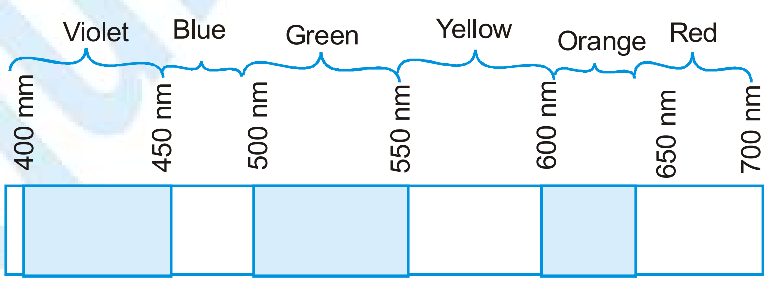


Figure - spectrum of visible light

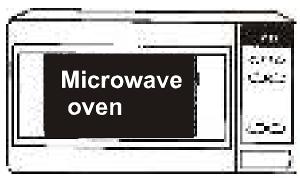
We briefly describe these different types of electromagnetic waves, in order of decreasing wavelengths.

Radio waves

Radio waves are produced by the accelerated motion of charges in conducting wires. They are used in radio and television communication systems. They are generally in the frequency range from 500 kHz to about 1000 MHz. The AM (amplitude modulated) band is from 530 kHz to 1710 kHz. Higher frequencies upto 54 MHz are used for short wave bands. TV waves range from 54 MHz to 890 MHz. The FM (frequency modulated) radio band extends from 88 MHz to 108 MHz. Cellular phones use radio waves to transmit voice communication in the ultrahigh frequency (UHF) band.

Microwaves

Microwaves (short-wavelength radio waves), with frequencies in the gigahertz (GHz) range, are produced by special vacuum tubes (called klystrons, magnetrons and Gunn diodes). Due to their short wavelengths, they are suitable for the radar systems used in aircraft navigation. Microwave ovens are an interesting domestic application of these waves. In such ovens, the frequency of the microwaves is selected to match the resonant frequency of water molecules so that energy from the waves is transferred efficiently to the kinetic energy of the molecules. This raises the temperature of any food containing water.



Infrared waves

Infrared waves are produced by hot bodies and molecules. This band lies adjacent to the low-frequency or long-wave length end of the visible spectrum. Infrared waves are sometimes referred to as heat waves. This is because water molecules present in most materials readily absorb infrared waves). After absorption, their thermal motion increases, that is they heat up and heat their surroundings. Infrared lamps are used in physical therapy. I R radiation maintain the earth's warmth or average temperature through the greenhouse effect. Incoming visible light (which passes relatively easily through the atmosphere) is absorbed by the earth's surface and reradiated as infrared (longer wavelength) radiations. This radiation is trapped by green house gases such as carbon dioxide and water vapour. Infrared detectors are used in earth satellites, both for military purpose and to observe growth of crops. Electronic devices (for example semiconductor light emitting diodes) also emit infrared and are widely used in the remote switches of household electronic systems such as TV sets, video recorders and hi-fi systems.

Visible rays

It is the most familiar form of electromagnetic waves. It is the part of the spectrum that is detected by the human eye. It runs from about 4×10^{14} Hz to about 7×10^{14} Hz or a wavelength range of about 700- 400 nm. Visible light emitted or reflected from objects around us provides us information about the world. Our eyes are sensitive to this range of wavelengths. Different animals are sensitive to different range of wavelengths. For example, snakes can detect infrared waves, and the 'visible' range of many insects extends well into the ultraviolet.

Ultraviolet rays

It covers wavelengths ranging from about 4×10^{-7} m (400 nm) down to 6×10^{-10} m (0.6 nm). Ultraviolet (UV) radiation is produced by special lamps and very hot bodies. The sun is an important source of ultraviolet light. But fortunately, most of it is absorbed in the ozone layer in the atmosphere at an altitude of about 40-50 km. UV light in large quantities has harmful effects on humans. Exposure to UV radiation induces the production of more melanin, causing tanning of the skin. UV radiation is absorbed by ordinary glass. Hence, one cannot get tanned or sunburn through glass windows.

Welders wear special glass goggles or face masks with glass windows to protect their eyes from large amount of UV produced by welding arcs. Due to its shorter wavelength, UV radiations can be focussed into very narrow beams for high precision application such as LASIK (Laser assisted in situ keratomileusis) eye surgery. UV lamps are used to kill germs in water purifiers.

Ozone layer in the atmosphere plays a protective role, and hence its depletion by chlorofluorocarbons (CFCs) gas (such as freon) is a matter of international concern.

X-rays

Beyond the UV region of the electromagnetic spectrum lies the X-ray region. We are familiar with X-rays because of its medical applications. It covers wavelengths from about 10^{-8} m (10 nm) down to 10^{-13} m (10^{-4} nm). One common way to generate X-rays is to bombard a metal target by high energy electrons. X-rays are used as a diagnostic tool in medicine and as a treatment for certain forms of cancer. Because X-rays damage or destroy living tissues and organisms, care must be taken to avoid unnecessary or over exposure.

Gamma rays

They lie in the upper frequency range of the electromagnetic spectrum and have wavelength of from about 10^{-10} m to less than 10^{-14} m. This high frequency radiation is produced in nuclear reactions and also emitted by radioactive nuclei. They are used in medicine to destroy cancer cells.

Table 8.1 summarises different types of electromagnetic waves, their production and detections. As mentioned earlier, the demarcation between different region is not sharp and there are over laps.

Different Type of Electromagnetic waves

Type	Wavelength range	Production	Detection
Radio	> 0.1 m	Rapid acceleration and decelerations of electrons in aerials	Receiver's aerials
Microwave	0.1 m to 1 mm	Klystron valve or magnetron valve	Point contact diodes
Infra-red	1 mm to 700 nm	Vibration of atoms and molecules	Thermopiles Bolometer, Infrared photographic film
Light	700 nm to 400 nm	Electrons in atoms emit light when they move from one energy level to a lower energy	The eye, photocells, Photographic film
Ultraviolet	400 nm to 1 nm	Inner shell electrons in atoms moving from one energy level to a lower level	Photocells Photographic film
X-rays	1 nm to 10^{-3} nm	X-ray tubes or inner shell electrons	Photographic film, Geiger tubes, Ionisation chamber
Gamma rays	< 10^{-3} nm	Radioactive decay of the nucleus	-do-

• Etoos Tips & Formulas •

1. Cathode rays :

(a) Generated in a discharge tube in which a high vaccum is maintained.

(b) They are electrons accelerated by high potential difference (10 to 15 kV)

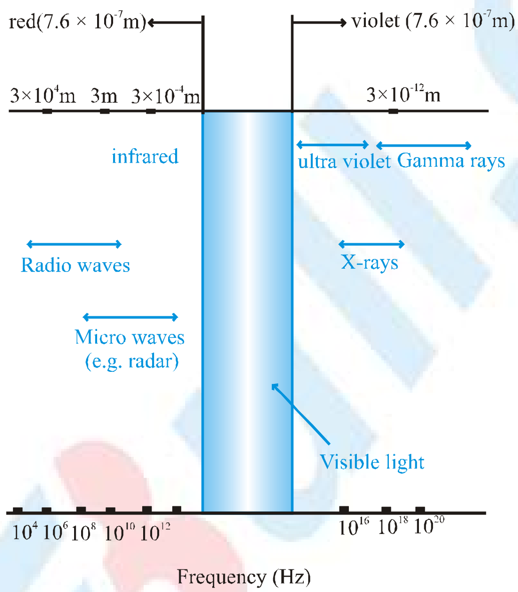
(c) K.E. of C.R. particle accelerated by a p.d V is $eV = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

(d) Can be deflected by Electric & magnetic fields.

2. Electromagnetic Spectrum

Ordered arrangement of the big family of electro magneti waves (EMW) either in ascending order of frequencies or decending order of wave lengths.

Speed of E.M.W. in vaccum : $c = 3 \times 10^8$ m/s $= v\lambda$



3. Plancks Quantam Theory

A beam of EMW is a stream of discrete packets of energy called photons; each photon having a frequancy v and energy $= E = hv$

where h = planck's constant $= 6.63 \times 10^{-34}$ J-s.

(a) According to Planck the energy of a photon is directly proportional to the frequency of the radiation.

$$E = \frac{hc}{\lambda} = \frac{12400}{\lambda} eV - \overset{o}{A} \left[\because \frac{hc}{e} = 12400 (\overset{o}{A} - eV)\right]$$

(b) Effective mass of photon $m = \frac{E}{c^2} = \frac{hc}{c^2\lambda} = \frac{h}{c\lambda}$ i.e. $m \propto \frac{1}{\lambda}$

So mass of violet light photon is greater than the mass of red light photon.

$(\because \lambda_R > \lambda_V)$

(c) Linear momentum of photon $p = \frac{E}{c} = \frac{hv}{c} = \frac{h}{\lambda}$

4. **Intensity of light :** $I = \frac{E}{At} = \frac{P}{A}$ ....(i)

Here P = power of source, A = Area, t = time taken

E = energy incident in t time = Nhν N = no. of photon incident in t time

Intensity $I = \frac{N(h\nu)}{At} = \frac{n(h\nu)}{A}$ ....(ii) $\left[\because n = \frac{N}{t} = \text{no. of photon per sec.}\right]$

From equation (i) and (ii), $\frac{P}{A} = \frac{n(h\nu)}{A} \Rightarrow n = \frac{P}{h\nu} = \frac{P\lambda}{hc} = 5\times10^{24}\,J^{-1}m^{-1}\times P\times\lambda$

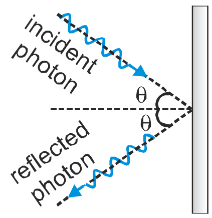
5. **Force exerted on perfectly reflecting surface**

$\therefore F = n\left(\frac{2h}{\lambda}\right) = \frac{2P}{c}$ and Pressure $= \frac{F}{A} = \frac{2P}{cA} = \frac{2I}{c}$ $\left[\because I = \frac{P}{A}\right]$

Force exerted on perfectly absorbing surface

$F = \frac{P}{c}\left(\because n = \frac{P\lambda}{hc}\right)$ and Pressure $= \frac{F}{A} = \frac{P}{Ac} = \frac{I}{c}$

When a beam of light is incident at angle θ on perfectly reflector surface



$$F = \frac{2IA\cos^2\theta}{c}$$

When a beam of light is incident at angle θ on perfectly absorbing surface $F = \frac{IA\cos\theta}{c}$