WORK, ENERGY & POWER

INTRODUCTION

Figure **(a)** show a block of stone. starting from rest at the top of a uniform slope. What`s the skier`s speed at the bottom ? You can solve this problem by applying Newton`s second law to find the block constant acceleration and then the speed what about the block in figure **(b)** ? Here the slope is continuously changing and so is the acceleration. Constant-acceleration equations are not applicable here, so solving for the details of the block motion is difficult.

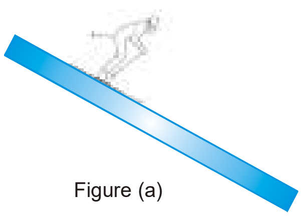


Figure (a)

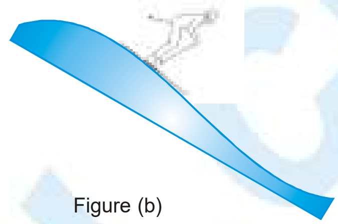


Figure (b)

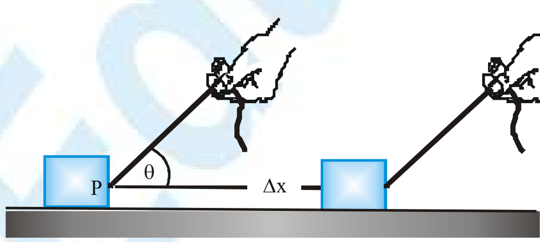
There are many cases where motion involves changing forces and acceleration. In this chapter, we introduce the important physical concepts of work and energy. These powerful concepts enables us to "shortcut" the detailed application of Newton`s law to analyze these more complex situations we begin with the concept of work.

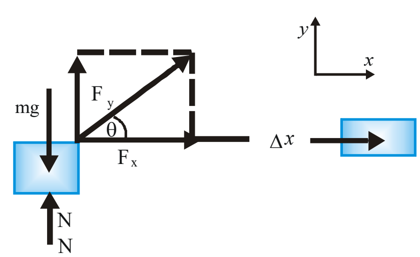
WORK OF A FORCE

In everyday life by the word "work" we refer to a vast category of Jobs. This means is not precise enough to be sued as a physical quantity. It was the practical need of scientists and engineers of the late 18th century at the start of Industrial Revolution that made necessary to define work quantitatively as a physical quantity. Physical concept of work involves a force and displacement produces.

Work done by Constant Force on a Body in Rectilinear Motion

To understand concept of work, consider a block being pulled with the help of a string on frictionless horizontal ground. Let pull $\vec{F}$ of the string on the box is constant in magnitude as well as direction the vertical component F_y of $\vec{F}$, the weight (mg) and the normal reaction N all act on the box in vertical direction but none of them can moves it unless F_y becomes greater than the weight (mg). Consider that is smaller than the weight of the box. Under this condition, the box moves along the plane only due to the horizontal components F_x of the force $\vec{F}$ the weight mg, the normal reaction. N from the ground and vertical component F_y all are perpendicular to the displacement therefore have no contribution in its displacement. Therefore, work is done on the box only by the horizontal component F_x of the force $\vec{F}$.





Here we must take car of one more point that is the box, which is a rigid body and undergoes translation motion therefore, displacement of every particle of the body including that on which the force is applied are equal. The particle of a body on which force acts is known as point of application of the force.

Now we observe that block is displaced & its speed is increase. And work W of the force $\vec{F}$ on the block is proportional to the product of its components in the direction of the displacement and the magnitude of the displacement Δx.

$$W \propto F_x . \Delta x = F \cos\theta . \Delta x$$

If we chose one unit of work as newton-meter, the constant of proportionality becomes unity and we have

$$W = F \cos\theta \ . \Delta x = \vec{F} . \Delta\vec{x}$$

The work W done by the force $\vec{F}$ is defined as scalar product of the force $\vec{F}$ and displacement $\Delta\vec{x}$ of point of application of the force.

ETOOS KEY POINTS

(i) If $\vec{F} = F_x = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

$\vec{S} = \Delta_x\hat{i} + \Delta_y\hat{j} + \Delta_z\hat{k}$

then

$w = \vec{F}.\vec{S} = F_x(\Delta_x) + F_y(\Delta_y) + F_z(\Delta_z)$

(ii) Work done force is frame dependents as displacement is frame dependent.

(iii) Work can be positive or negative or zero. When a force speed up the practice, it does positive work. A force acting at 90° to the motion does no work. And when a force slow down the motion, it does negative work.

Special Cases in work

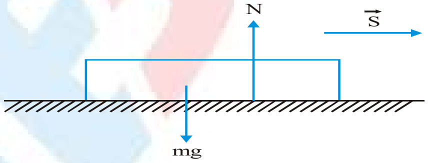
Case - I

When θ = 90° then W = FS cos90° = 0

So , work done by a force is zero if the body is displaced in a direction perpendicular to the direction of the force.

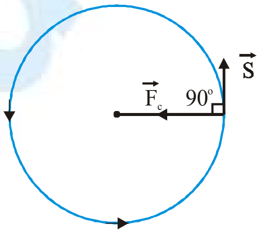
Examples :

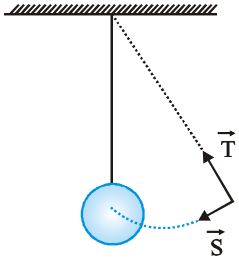
1. Consider a body sliding over a horizontal surface. The work done by the force of gravity and the reaction of the surface will be zero. This is because both the force of gravity and the reaction act normally to the displacement.



The same argument can be applied to a man carrying a load on his head and walking on a railway platform.

2. Consider a body moving in a circle with constant speed. At every point of the circular path, the centripetal force and the displacement are mutually perpendicular (Figure). So, the work done by the centripetal force is zero. The same argument can be applied to a satellite moving in a circular orbit. In this case, the gravitational force is always perpendicular to displacement. So work done by gravitational force is zero.





3. The tension in the string of a simple pendulum is always perpendicular to displacement (Figure). So, work done by the tension is zero.

Case - II

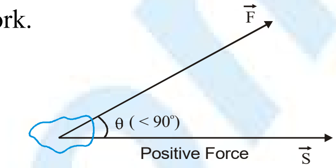
When S = 0, then W = 0.
So, work done by a force is zero if the body suffers no displacement on the application of a force.

Example :

A person carrying a load on his head and standing at a given place does no work.

Case - III

When $0^o \leq \theta < 90^o$ [Figure], then $\cos\theta$ is positive. Therefore.
W (= FS cosθ) is positive.
work done by a force is said to be positive if the applied force has a component in the direction of the displacement.



Examples :

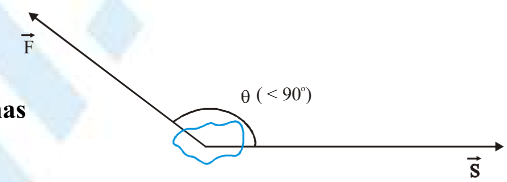
1. When a horse pulls a cart, the applied force and the displacement are in the same direction. So, work done by the horse is positive.

2. When a load is lifted, the lifting force and the displacement act in the same direction. So, work done by the lifting force is positive.

3. When a spring is stretched, both the stretching and the displacement act in the same direction. So, work done by the stretching force is positive.

Case - IV

When $90^o < \theta \leq 180^o$ (Figure), then cosq is negative. Therefore W (= FS cosθ) is negative.
Work done by a force is said to be negative if the applied force has component in a direction opposite to that of the displacement



Examples :

1. When brakes are applied to a moving vehicle, the work done by the braking force is negative. This is because the braking force and the displacement act in opposite directions.

2. When a body is dragged along a rough surface, the work done by the frictional force is negative. This is because the frictional force acts in a direction opposite to that of the displacement.

3. When a body is lifted, the work done by the gravitational force is negative. This is because the gravitational force acts vertically downwards while the displacement is in the vertically upwards direction.

Ex. Figure shows four situations in which a force acts on a box while the box slides rightward a distance d across a frictionless floor. The magnitudes of the forces are identical, their orientations are as shown. Rank the situations according to the work done on the box during the displacement, from most positive to most negative.

Ans. D, C, B, A

Sol. In **(D)** $\theta = 0^o$, $\cos\theta = 1$ (maximum value). So, work done is maximum.

In **(C)** $\theta < 90^o$, $\cos\theta$ is positive. Therefore, W is positive.

In **(B)** $\theta = 90^o$, $\cos\theta$ is zero. W is zero.

In **(A)** θ is obtuse, $\cos\theta$ is negative. W is negative.

WORK DONE BY MULTIPLE FORCES

If several forces act on a particle, then we can replace $\vec{F}$ in equation W = $\vec{F}\cdot\vec{S}$ by the net force $\sum\vec{F}$ where

$$\sum\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots\dots\dots$$

$\therefore$ $$w = \left[\sum\vec{F}\right]\cdot\vec{S} \qquad \text{....(i)}$$

This gives the work done by the net force during a displacement $\vec{S}$ of the particle.

We can rewrite equation **(i)** as :

$$W = \vec{F}_1.\vec{S} + \vec{F}_2.\vec{S} + \vec{F}_3.\vec{S} + \dots\dots$$

or $$W = W_1 + W_2 + W_3 + \dots\dots$$

So, the work done on the particle is the sum of the individual work by all the forces acting on the particle.

Ex. A particle is moving along a straight line from point A to point B. The position vectors for points A and B are $\left(2\hat{i}+7\hat{j}+3\hat{k}\right)$m and $\left(5\hat{i}+3\hat{j}+6\hat{k}\right)$m respectively. One of the force acting on the particle is $\vec{F} = 20\hat{i}-30\hat{j}+15\hat{k}$ N. Find the work done by this force.

Sol. $$\vec{S} = \left(5\hat{i}-3\hat{j}+6\hat{k}\right)-\left(2\hat{i}+7\hat{j}-3\hat{k}\right)$$

$$= 3\hat{i}-10\hat{j}+3\hat{k}\ \text{m}$$

$$\vec{F} = 20\hat{i}-30\hat{j}+15\hat{k}$$

Now $$W = \vec{F}.\vec{S}$$

$$= 60 + 300 - 45 = 315\ \text{J}$$

ETOOS KEY POINTS

(i) Work is defined for an interval or displacement. There is no term like instantaneous work similar to instantaneous velocity.

(ii) For a particular displacement, work done by a force is independent of type of motion i.e. whether it moves with constant velocity, constant acceleration or retardation etc.

(iii) For a particular displacement work is independent of time. Work will be same for displacement whether the time taken is small or large

(iv) When several forces act, work done by a force for a particular displacement is independent of other forces.

(v) A force is independent from reference frame. Its displacement depends on frame so work done by a force is frame dependent therefore work done by a force can be different in different reference frame.

(vi) Effect of work is change in kinetic energy of the particle or system.

(vii) Work is done by the source or agent that applies the force.

Units of work :

I. In cgs system, the unit of work is erg.

One erg of work is said to be done when a force of one dyne displaces a body through one centimeter in its own direction.

$\therefore$ 1 erg = 1 dyne × 1 cm = 1g cm s^{-2} × 1 cm = 1 g cm^2 s^{-2}

Note : Erg is also called dyne centimeter.

II. In SI i.e., International System of units, the unit of work is joule (abbreviated as J). It is named after the famous British physicist James Personal Joule (1818 – 1869).

One joule of work is said to be done when a force of one Newton displaces a body through one metre in its own direction.

1 joule = 1 Newton × 1 metre = 1 kg ×1 m/s^2 × 1 m = 1kg m^2 s^{-2}

Note : Another name for joule is Newton metre.

Relation between joule and erg

1 joule = 1 Newton × 1 metre

1 joule = 10^5 dyne × 10^2 cm = 10^7 dyne cm

1 joule = 10^7 erg

1 erg = 10^{-7} joule

Dimensions of Work

[Work] = [Force] [Distance] = [MLT^{-2}] [L] = [ML^{2}T^{-2}]

Work has one dimension in mass, two dimensions in length and '–2' dimensions in time,

On the basis of dimensional formula, the unit of work is kg m^2 s^{-2}.

Note that 1 kg m^2 s^{-2} = (1 kg m s^{-2}) m

Ex. There is an elastic ball and a rigid wall. Ball is thrown towards the wall. The work done by the normal reaction exerted by the wall on the ball is -

(A) +ve **(B)** – ve (C*) zero **(D)** None of these

Sol. As the point of application of force does not move, the w.d by normal reaction is zero.

Ex. Work done by the normal reaction when a person climbs up the stairs is -

(A) +ve **(B)** – ve (C*) zero **(D)** None of these

Sol. As the point of application of force does not move, the w.d by normal reaction is zero.

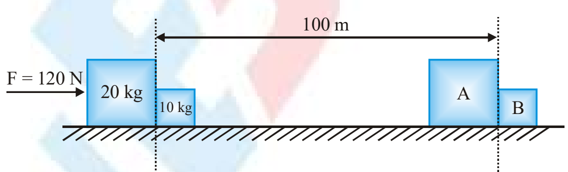
Ex. Work done by static friction force when a person starts running is ________ .

Sol. As the point of application of force does not move, the w.d by static friction is zero.

WORK DONE BY VARIOUS REAL FORCES

1. Work done by normal reaction

Ex.



(i) Find work done by force F on A during 100 m displacement.

(ii) Find work done by force F on B during 100 m displacement.

(iii) Find work done by normal reaction on B and A during the given displacement.

(iv) Find out the kinetic energy of block A & B finally.

Sol. **(i)** $(W_F)_{on\,A}$ = FDS cosθ

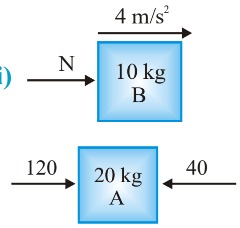
= 120 × 100 × cos0°

= 12000 J

(ii) $(W_F)_{on\,B}$ = 0

✈ F does not act on B

(iii)



$N = 10 \times 4 = 40\ N$

$(W_N)_{on\ B} = 40 \times 100 \times \cos 0^\circ = 4000\ J$

$(W_N)_{on\ A} = 40 \times 100 \times \cos 180^\circ = -4000\ J$

(iv) $v^2 = u^2 + 2as \qquad u = 0$

$\therefore\ v^2 = 2 \times 4 \times 100 \Rightarrow v = 20\sqrt{2}\ m/s$

$\therefore\ KE_A = \frac{1}{2} \times 20 \times 800 = 8000\ J$

$KE_B = \frac{1}{2} \times 10 \times 800 = 4000\ J$

W. D. by normal reaction on system of A & B is zero. i.e. w.d. by internal reaction on a rigid system is zero.

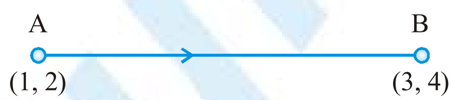
Ex. A particle is displaced from point A(1, 2) to B(3, 4) by applying force $\vec{F}$ to move the particle from point A to B.

Sol. $W = \vec{F}.\Delta\vec{S}$

$\Delta S = (3-1)\hat{i} + (4-2)\hat{j}$

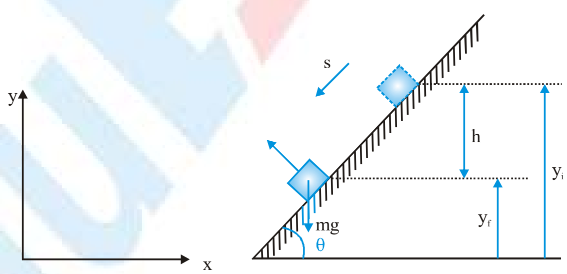
$= (2\hat{i} + 3\hat{j}).(2\hat{i} + 2\hat{j})$

$= 2 \times 2 + 3 \times 2 = 10$ units



2. Work done by Gravity

Consider a block of mass m which slides down a smooth inclined plane of angle q as shown in figure.



Let us assume the coordinate axes as shown in the figure to specify the components of the two vector although the value of work will not depend on the orientation of the axes.

Now, the force of gravity, $\vec{F}_g = -mg\,\hat{j}$

and the displacement is given by

$\vec{s} = \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}$

The work done by gravity is

$$W_g = \vec{F}_s.\vec{s} = -mg\,\hat{j}\left(\Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}\right)$$

or $$W_g = \vec{F}_s.\vec{s} = -mg(\Delta y)$$

Since $\Delta y = y_f - y_i = -h$

$\therefore$ $W_g = +mgh$

If the block moves in the upward direction, then the work done by gravity is negative and is given by

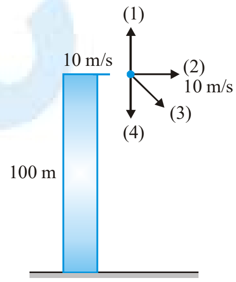
$W_g = -mgh$

ETOOS KEY POINTS

(i) The work done by the force of gravity depends only on the initial and final vertical coordinates, not on the path taken.

(ii) The work done by gravity is zero for path that returns to its initial point.

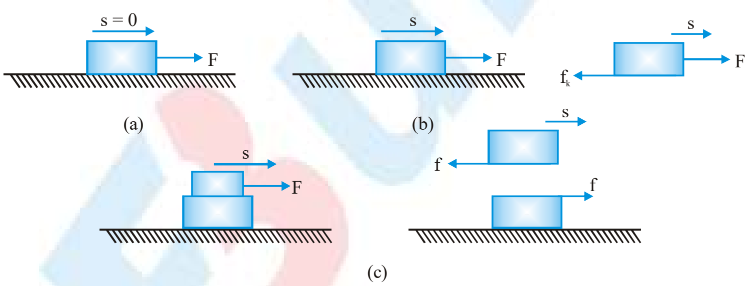
Ex. The mass of the particle is 2 kg. It is projected as shown in four different ways with same speed of 10 m/s. Find out the work done by gravity by the time the stone falls on ground.



Sol. $W = |\vec{F}||\vec{S}| \cos\theta = 2000$ J in each case.

3. Work done by Friction

There is a misconception that the force of friction always does negative work. In reality, the work done by friction may be zero, positive or negative depending upon the situation as shown in the figure.



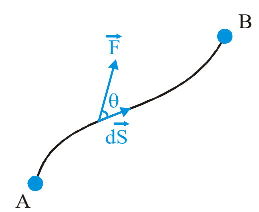
(a) When a block is pulled by a force F and the block does not move, the work done by friction is zero.

(b) When a block is pulled by a force F on a stationary surface, the work done by the kinetic friction is negative.

(c) Block A is placed on the block B. When the block A is pulled with force F, the friction does negative work on block A and positive work on block B, which is being accelerated by a force F, the displacement of A relative to the table is in the forward direction. The work done by kinetic friction on block B is positive.

4. Work done by a Variable Force

Often the force applied to an object varies with position. Important examples include electric and gravitational force, which vary with the distance between interacting objects. The force of a spring that we encountered in previous chapter provides another example; as the spring stretches, the force increases.



In this case we have difficulty to apply $W = \vec{F}.\vec{S}$, since $\vec{F}$ is not same for complete $\vec{S}$.

Thus, we take a very small part $d\vec{S}$ of its path. This displacement $d\vec{S}$ is so small that in variation force may be neglected during it. So we may write, for the work done during this displacement as

$$dW = \vec{F}.d\vec{S}$$
$$= F\,dS\cos\theta$$

The total work done in going from A to B as shown may be calculated by summing up i.e. integrating the work done during its small fractions.

i.e. $W_{A\to B} = \int_A^B \vec{F}.d\vec{S} = \int_A^B (F\cos\theta)\,dS$

In terms of rectangular components,

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

and $\quad d\vec{S} = dx\hat{i} + dy\hat{j} + dz\hat{k}$

therefore, $W_{A\to B} = \int_{x_A}^{x_B} F_x dx + \int_{y_A}^{y_B} F_y dy + \int_{z_A}^{z_B} F_z dz$

Ex. A force $\vec{F} = x\hat{i} + y^2\hat{j}$ N acts on a particle and the particle moves from (1, 2) m to (–3, 4) m. Find work done by the force $\vec{F}$.

Sol. $dW = \vec{F}.d\vec{S}$

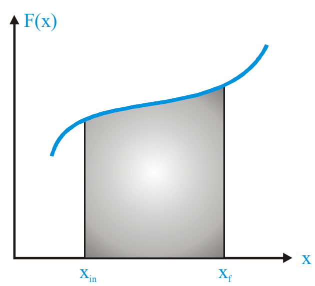
where $\quad d\vec{S} = dx\hat{i} + dy\hat{j}$

$\therefore \quad dW = xdx + y^2 dy$

and $\quad W = \int dW = \int_1^{-3} xdx + \int_2^4 y^2 dy = \left.\frac{x^2}{2}\right|_1^{-3} + \left.\frac{y^3}{3}\right|_2^4 = \frac{68}{3}$ J

5. Work done as Area under the Force Displacement Graph

Suppose of particle moving along a straight line and a force acting on it varies with its displacement x as shown.



$$W = \int_{x_{in}}^{x_f} F.dx$$

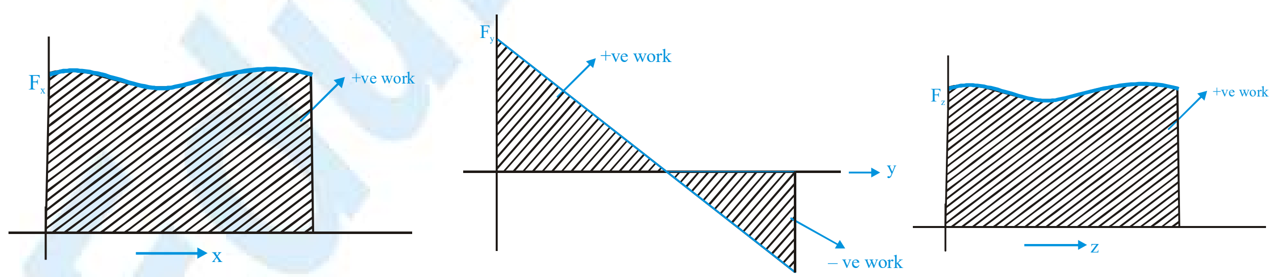
= Area under F vs x graph from y = x_{in} to x_f

In general, the work done by a point x_{in} to final point x_f is given by the area under the force-displacement curve as shown in the figure.

Area (work) above the x-axis is taken as positive, and below x-axis as negative.

AREA UNDER FORCE DISPLACEMENT CURVE

Graphically area under the force-displacement is the work done



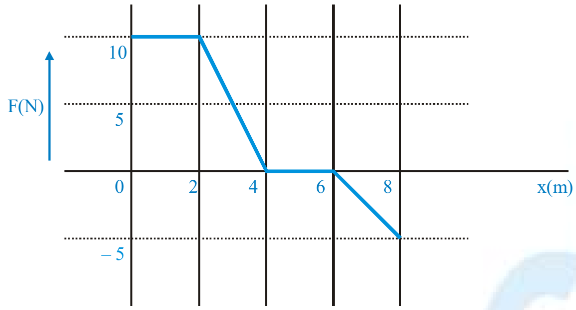
The work done can be positive or negative as per the area above the x-axis or below the x-axis respectively.

Ex. A force which varies with position coordinate x according to equation $F_x = (4x + 2)$ N. Here x is in meters. Calculate work done by this force in carrying a particle from position $x_i = 1$ m to $x_f = 2$m.

Sol. Using the equation $W_{i\to f} = \int_{x_i}^{x_f} F_x.dx$, we have $W_{i\to f} = \int_1^2 (4x-2).dx = 8\,J$

The above problem can also be solved by using graph

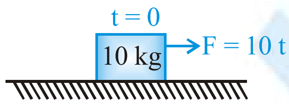
Ex. A 5 kg block moves in a straight line on a horizontal frictionless surface under the influence of a force that varies with position as shown in the figure. Find the work done by this force as the block moves from the origin to x = 8m.



Sol. The work from x = 0 to x = 8 m is the area under the curve.

$$W = 10\times 2 + \frac{1}{2}(10)(4-2) + 0 + \frac{1}{2}(-5)(8-6) = 25\,J$$

Ex. A time dependent force F = 10 t is applied on 10 kg block as shown in figure.



Find out the work done by F in 2 seconds.

Sol. $dW = \vec{F}.d\vec{s}$

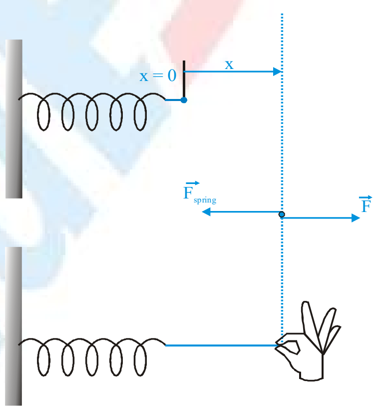
$dW = 10\,t.\,dx$

$dW = 10\,t\,v\,dt$ ....**(1)** ✈ $dx = vdt$

also $10 \times \frac{dv}{dt} = 10\,t$

6. Work done by Spring Force

A spring provides an important example of a force that varies with position. We have seen that an ideal spring exerts a force proportional to its displacement from equilibrium : F = – kx, where k is the spring constant and the minus sign shows that the spring force is opposite the direction of the displacement.



∴ Work done (W_s) by spring force when its deformation changes from x_{in} to x_f is

$$W_s = -k\int_{x_i}^{x_f} x.dx \quad \Rightarrow \quad W_s = -\frac{1}{2}k\left(x_f^2 - x_{in}^2\right)$$

Note: Work done by a spring force to stretch it form its undeformed length to deform it upto x is

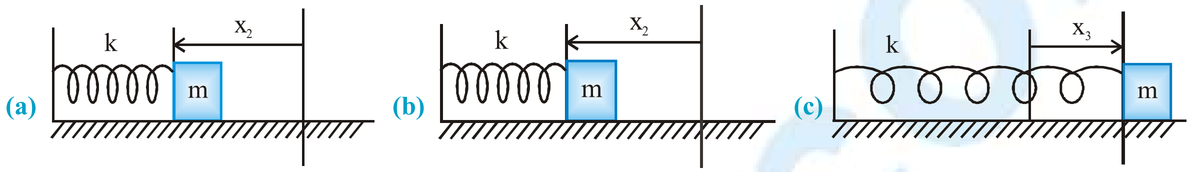
$$W = -\frac{1}{2}k\left(x^2 - 0\right) = -\frac{1}{2}kx^2$$

Work done by a spring force may be negative ($x_f > x_{in}$) or may be positive (if $x_f < x_{in}$) or may be zero

Work done by the spring force depends only on initial and final deformation.

In the equation, $W = -\frac{1}{2}k\left(x_f^2 - x_{in}^2\right)$

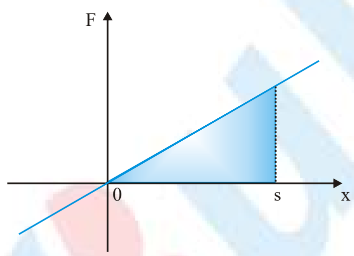
x_{in} & x_f are magnitudes of deformations no matter if these are compressions or extensions.



Work done by spring force from position **(a)** to **(b)** and that from **(a)** to **(c)** are same and equal to $-\frac{1}{2}k\left(x_2^2 - x_1^2\right)$

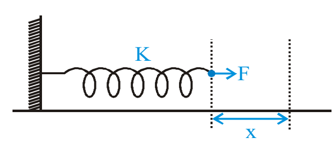
USE OF GRAPH

The variation in F with extension x in the spring is linear therefore area under the force extension graph can easily be calculated. This area equals to the work done by the applied force. The graph showing variation in F with x is shown in the adjoining figure.



$$W_F = \int_0^s \vec{F}\cdot d\vec{x} = \text{Area of the shaded portion} = \frac{1}{2}ks^2$$

Ex. Initially spring is relaxed. A person starts pulling the spring by applying a variable force F. Find out the work done by F to stretch it slowly to a distance by x.



Sol. $\int dW = \int F \cdot ds = \int_0^x Kx\,dx \Rightarrow W = \left(\frac{Kx^2}{2}\right)_2^x = \frac{Kx^2}{2}$

Ex. In the above example

(i) Where has the work gone ?

(ii) Work done by spring on wall is zero. Why ?

(iii) Work done by spring force on man is _______.

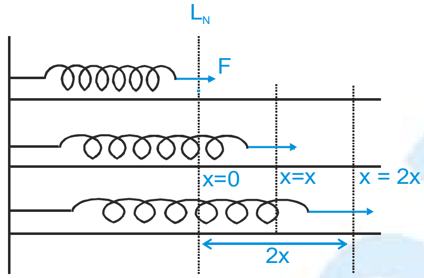
Sol. **(i)** It is stored in the form of potential energy in spring.

(ii) Zero, as displacement is zero.

(iii) $-\frac{1}{2}Kx^2$

Ex. Find out work done by applied force to slowly extend the spring from x to 2x.

Sol. Initially the spring is extended by x



$$W = \vec{F} \cdot \vec{ds}$$

$$W = \int_x^{2x} K\, x . dx$$

$$W = \left[\frac{Kx^2}{2}\right]_x^{2x} = \frac{3}{2} Kx^2$$

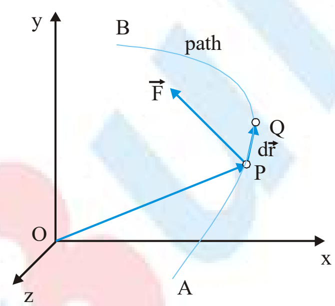
It can also found by difference of PE.

i.e. $U_f = \frac{1}{2} K (2x)^2 = 2kx^2 \Rightarrow U_i = \frac{1}{2} kx^2 \Rightarrow U_f - U_i = \frac{3}{2} kx^2$

WORK OF A VARIABLE FORCE ON A BODY IN CURVILINEAR TRANSLATION MOTION

Till now we have learnt how to calculate work of a force in rectilinear motion. We can extend this idea to calculate work of a variable force on any curvilinear path. To understand this let us consider a particle moving from point A to B. There may be several forces acting on it but here we show only that force whose work we want to calculate. This force is denoted by $\vec{F}$. Consider an infinitely small path length PQ. Over this infinitely small length, the force can be assumed constant. Work of this force $\vec{F}$ over this path length PQ is given by

$$dW = \vec{F} . d\vec{r}$$



The whole path from A to B can be divided in several such infinitely small elements and work done by the force over the whole path from A to B is sum of work done over every such infinitely small element. This we can calculate by integration. Therefore, work done $W_{A \to B}$ by the force $\vec{F}$ is given by the following equation.

$$W_{A \to B} = \int_{\vec{r}_A}^{\vec{r}_B} \vec{F} . d\vec{r}$$

Work of a variable Force

For a generalization, let point A be the initial point and point B be the final point. Now we can express work $W_{i \to f}$ of a force $\vec{F}$ when its point of application moves from position vector $\vec{r}_i$ to $\vec{r}_f$ over a path by the following equation.

$$W_{i \to f} = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} . d\vec{r}$$

The integration involved in the above equation must be carried over the path followed. Such kind of integration is known as path integrals.

Work of a Constant Force

In simple situations where force $\vec{F}$ is constant, the above equation reduces to a simple form.

$$W_{i\to f} = \vec{F}.\left(\int_{\vec{r}_i}^{\vec{r}_f} d\vec{r}\right) = \vec{F}\left(\vec{r}_f - \vec{r}_i\right) = \vec{F}.\Delta\vec{r}$$

Ex. Calculate work done by the force $\vec{F} = \left(3\hat{i} + 2\hat{j} + 4\hat{k}\right)$ N in carrying a particle from point (– 2m, 1m, 3m) to (3m, 6m,–2m).

Sol. The force $\vec{F}$ is a constant force, therefore we can use equation $W_{i\to f} = \vec{F}.\Delta\vec{r}$

$$W = \vec{F}.\Delta\vec{r} = \left(3\hat{i} - 2\hat{j} + 4\hat{k}\right).\left(5\hat{i} + 5\hat{j} - 5\hat{k}\right) = -15\text{ J}$$

Ex. A particle is shifted form point (0, 0, 1m) to (1m, 1m, 2m) under simultaneous action of several forces. Two of the forces are $\vec{F}_1 = \left(2\hat{i} + 3\hat{j} - \hat{k}\right)$N and $\vec{F}_2 = \left(\hat{i} - 2\hat{j} + 2\hat{k}\right)$N . Find work done by these two forces.

Sol. Work doe by a constant force equals to dot products of the force and displacement vectors.

$$W = \vec{F}.\Delta\vec{r} \to W = \left(\vec{F}_1 - \vec{F}_2\right).\Delta\vec{r}$$

Substituting given values, we have

$$W = \left(3\hat{i} + \hat{j} + \hat{k}\right).\left(\hat{i} + \hat{j} + \hat{k}\right) = 3 + 1 + 1 = 5\text{ J}$$

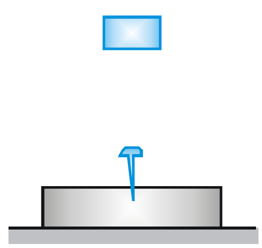
Work of a Force Depends on Frame of Reference

A force does not depends on frame of reference and assumes same value in all frame of references, but displacement depends on frame of reference and may assume different values relative to different reference frames. Therefore, work of a force depends on choice of reference frame. For example, consider a man holding a suitcase stands in a lift that is moving up. In the reference frame attached with the lift, the , man applies a force equal to weight of the bag but the displacement of the bag is zero., therefore work of this force on the bag is zero. However, in a reference frame attached with the ground the bag has displacement equal to that of the lift and the force applied by the man does a nonzero work.

WORK AND ENERGY

Suppose you have to push a heavy box on a rough horizontal floor. You apply a force on the box it moves and you do work. If you continue pushing, after some time you get tired and become unable to maintain your speed and eventually become unable to push the box further. You take rest and next day you can repeat the experiment and same thing happens. Why you get tired and eventually become unable to pull the box further ? The explanation lies in fact that you have a capacity to do work, and when it is used up, you become unable to do work further. Next day you recollect this capacity and repeat the experiment. This capacity of doing work is known as energy. Here it comes from chemical reactions occurring with food in our body and is called chemical energy.

Consider another experiment in which we drop a block on a nail as shown in the figure. When set free, weight of the block accelerates it through the distance it falls and when it strikes the nail, its motion vanishes and what appears are the work that drives the nail, heat that increases temperature of surrounding, and sound that causes air molecules to oscillate. If the block were placed on the nail and pressed hard, it would not have been so effective. Actually, the weight and the distance through which the hammer falls on the nail decide its effectiveness. We can explain these events by assuming that the block possesses energy due to its position at height against gravity.



This energy is known as gravitational potential energy. When the block falls, this potential energy is converted into another form that is energy due to motion. This energy is known as kinetic energy. Moreover, when the block strikes the nail this kinetic energy is converted into work driving the nail, increasing temperature and producing sound.

Potential, Kinetic and Mechanical Energy

If a material-body is moved against a force like gravitational, electrostatic, or spring, a work must be done. In addition, if the force continues to acts even after the displacement, the work done can be recovered in form of energy, if the is set loose. This recoverable stored energy by virtue of position in a force field is defined as potential energy, a name given by William Rankine.

All material bodies have energy due to their motion. This energy is known as kinetic energy, a name given by Lord Kelvin.

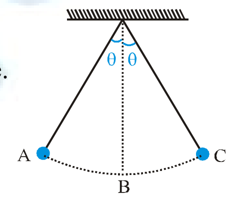
These two forms of energies- the kinetic energy and the potential energy are directly connected with motion of the body and force acting on the body respectively. The are collectively known as mechanical energy.

Other forms of Energy

Thermal energy, sound energy, chemical energy, electrical energy and nuclear energy are examples of some other forms of energy. Actually, in very fundamental way every form of energy is either kinetic or potential in nature. The real energy which is contribution of kinetic energy of chaotic motion of molecules in a body and potential energy due to intermolecular forces within the body. Sound energy is contribution kinetic energy of oscillating molecules and potential energy due intermolecular forces within the medium in which sound oscillating molecules and potential energy due intermolecular forces within the medium in which sound propagates. Chemical energy is contribution of potential energy due to inter-atomic forces. Electric energy is kinetic energy of moving charge carries in conductors. In addition, nuclear energy is contribution of electrostatic potential energy of nucleons.

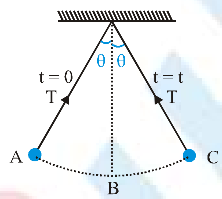
In fact, every physical phenomenon involves in some way conversion of one form of energy into other. Whenever mechanical energy is converted into other forms or vice versa it always occurs through forces and displacements of tier point of applications i.e. work. Therefore, we can say that work is measure of transfer of mechanical energy from one body to other. That is why the unit of energy is usually chosen equal to the unit of work.

WORK DONE BY TENSION



Ex. A bob of pendulum is released at rest from extreme position as shown in figure. Find work done by tension from A to B to C and C to A.

Sol. Zero because $F_T \perp dS$ at all time.



Ex. In the above question find out work done by gravity from A to B and B to C.

Sol. $W_g = \vec{F}.\Delta\vec{S}$

$= mg\ \Delta S\ \cos\theta$

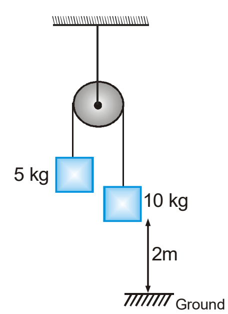
$W_g = mg(1 - 1\cos\theta)$ for A to B

$W_g = mg(1 - 1\cos\theta)$ for B to C

Ex. The system is released from rest. When 10 kg block reaches at ground then find :

(i) Work done by gravity on 10 kg **(ii)** Work done by gravity on 5 kg

(iii) Work done by tension on 10 kg **(iv)** Work done by tension on 5 kg.



Sol. **(i)** $(W_g)_{10\,kg} = 10\,g \times 2$

$= 200$ J

(ii) $(W_g)_{5kg} = 5\,g \times 2 \times \cos 180°$

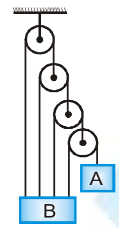
$= -100$ J

(iii) $(W_T)_{10\,kg} = \frac{200}{3} \times 2 \times \cos 180° = \frac{-400}{3}$ J

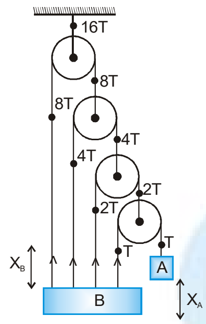
(iv) $(W_T)_{5\,kg} = \frac{200}{3} \times 2 \times \cos 0° = \frac{400}{3}$ J

Net w.d. by tension is zero. Work done by internal tension i.e. (tension acting within system) on the system is always zero if the length remains constant.

Ex. The velocity block A of the system shown in figure is V_A at any instant. Calculate velocity of bock B at that instant.



Sol. Work done by internal tension is zero.

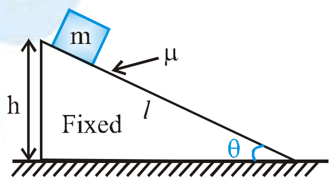


$\therefore 15\,T \times X_B - T \times X_A = 0$

$X_A = 15\,X_B$

$\therefore V_A = 15\,V_B$

Ex. A block of mass m is released from top of an incline plane of inclination θ. The coefficient of friction between the block and incline surface is μ (μ < tanθ). Find work done by normal reaction, gravity & friction, when block moves from top to the bottom.



Sol. $W_N = 0$ $\quad\therefore F_N \perp \Delta S$

$W_g = mg\,1\ \sin\theta$

$W_f = -\mu mg\cos\theta . 1$

Ex. What is kinetic energy of block of mass m at bottom in above problem.

Sol. $V^2 = u^2 + 2as$

$V^2 = 2(g\sin\theta - \mu g\cos\theta)\,(\bullet)$

$\therefore KE = \frac{1}{2} m\, 2\,(g\sin\theta - \mu g\cos\theta)\,\bullet \;= mg\,\bullet\,(\sin\theta - \mu\cos\theta)$

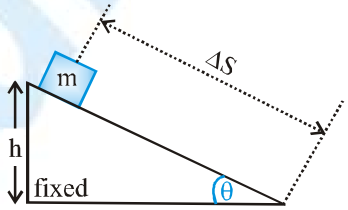
Ex. What is kinetic energy of block of mass m at bottom in above problem.

Sol. $V^2 = u^2 + 2as$

$V^2 = 2(g\sin\theta - \mu g\cos\theta)\,(1)$

WORK DONE BY CONSTANT FORCES

Ex. A block of mass m is released from top of a smooth fixed inclined plane of inclination θ.



Find out work done by normal reaction & gravity during the time block comes to bottom.

Sol. $W_N = 0$ as $F \perp \Delta S$

$Wg = \vec{F} . \vec{\Delta S} = mg.\ \Delta S . \cos(90-\theta) = mg\,\Delta S\ \sin\theta = mgh$

Ex. Find out the speed of the block at the bottom and its kinetic energy.

Sol. $V^2 = u^2 + 2as$

$$V^2 = 0 + 2\,(g \sin\theta)\,\frac{h}{\sin\theta} \Rightarrow V^2 = 2gh \Rightarrow V = \sqrt{2gh}$$

$$KE = \frac{1}{2}mv^2 = mgh$$

Work done by Internal Force

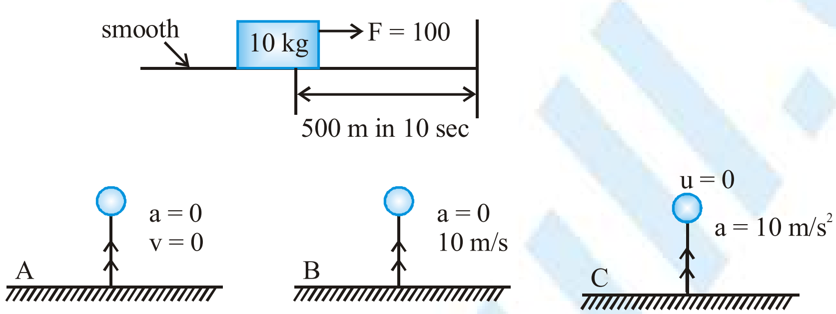
$F_{AB} = -F_{BA}$ i.e. sum of internal forces is zero.

But it is not necessary that work done by internal force is zero. There must be some deformation or reformation between the system to do internal work. In case of a rigid body work done by internal force is zero.

Work done by PSEUDO Force

Kinetic energy of a body frame dependent as velocity is a frame dependent quantity. Therefore pseudo force work has to be considered.

Ex. A block of mass 10 kg is pulled by force F = 100 N. It covers a distance 500 m in 10 sec. From initial point. This motion is observed by three observers A, B and C as shown in figure.



Find out work done by the force F in 10 seconds as observed by A, B & C.

Sol. $(W_F)_{\text{on block w.r.t. A}} = 100 \times 500\text{ J} = 50{,}000\text{ J}$

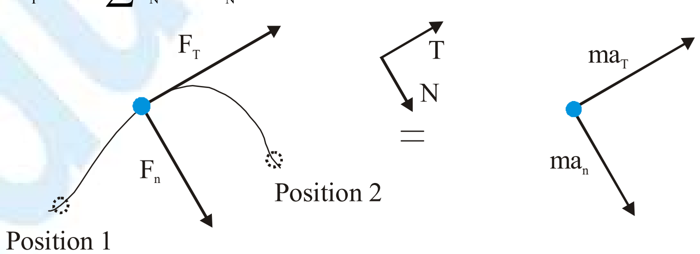
$(W_F)_{\text{on block w.r.t. B}} = 100\ [500 - 10 \times 10] = 40{,}000\text{ J}$

$(W_F)_{\text{on block w.r.t. C}} = 100\ [500 - 500] = 0$

WORK ENERGY THEOREM

Consider the situation described in the figure. The body shown is in translation motion on a curvilinear path with increasing speed. The net force acting on the body must have two components - the tangential component necessary to increase the speed and the normal components necessary to change the direction of motion. Applying Newton`s laws of motion in an inertial frame, we have

$$\sum F_T = ma_T \text{ and } \sum F_N = ma_N$$



Let the body starts at position 1 with speed v1 and reaches position 2 with speed v_2. If an infinitely small path increment is represented by vector $d\vec{s}$, the work done by the net force during the process is

$$W_{1\to2} = \int_1^2 \vec{F}.d\vec{s} = \int_1^2 \left(\vec{F}_T - \vec{F}_N\right).d\vec{s} = \int_1^2 F_T.ds \quad \Rightarrow \quad W_{1\to2} = \int_1^2 ma_T ds = \int_1^2 mvdv = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

The terms $\frac{1}{2}mv_1^2$ and $\frac{1}{2}mv_2^2$ represent the kinetic energies K_1 and K_2 of the particle at position-1 and 2 respectively.

With this information the above equation reduces to $W_{1\to2} = K_2 - K_1$

The above equation expresses that the work done by all external forces acting on a body in carrying it from one position to another equals to the change in the kinetic energy of the body between these positions. This statement is known as the work kinetic energy theorem.

How to apply Work Kinetic Energy Theorem

The work kinetic energy theorem is deduced here for a single body is moving relative to an inertial frame, therefore it is recommended at present to use it for a single body in inertial frame. To use work kinetic energy theorem the following steps should be followed.

(i) Identify the initial and final positions as position 1 and 2 and write expressions for kinetic energies, whether known or unknown.

(ii) Draw the free body diagram of the body at any intermediate stage between positions 1 and 2. The forces shown will help in deciding their work. Calculate work by each force and add them to obtain work $W_{1\to2}$ by all the forces.

(iii) Use the wrok obtained in step 2 kinetic energies in step 1 into $W_{1\to2} = K_2 - K_1$.

Note: $W_{net} = \frac{1}{2}m\left[V_f^{\,2} - V_{in}^{\,2}\right] \quad \Rightarrow \quad W_{net} = \Delta K.E.$

Thus change in an object`s kinetic energy is equal to the net work done on the object, this is called work energy theorem.

ETOOS KEY POINTS

(i) As mass m and V2 $\left(\vec{V}.\vec{V}\right)$ are always positive kinetic energy is always positive, kinetic energy is always positive scalar i.e. kinetic energy can never be negative.

(ii) The kinetic energy of an object is a measure of the amount of work needed to increases it speed from zero to a given value.

(iii) The kinetic energy of a particle is the work it can do on its surroundings in coming to rest.

(iv) If work done by net force is positive, kinetic energy of the system increases. If net work done is negative K.E. decreases and if net work is zero, K.E. remains constant.

(v) Since the velocity and displacement of a particle depends on the frame of reference, the numerical values of the work and the kinetic energy also depends on the frame.

KINETIC ENERGY OF GROUP OF PARTICLES OR BODIES

The speed v may be acquired by the body in any manner. The kinetic energy of a group of particles or bodies is the sum of the kinetic energies of the individual particles. Consider a system consisting of n particles of masses $m_1, m_2, \ldots\ldots, m_n$. Let $\vec{V}_1, \vec{V}_2, \ldots\ldots \vec{V}_n$ be their respectively velocities. Then , total kinetic energy E_k of the system is given by

$$E_k = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \ldots\ldots\ldots + \frac{1}{2}m_nv_n^2$$

If m is measured in gram and v in cm s^{-1}, then the kinetic energy is measured in erg. If m is measured in kilogram and v in m s^{-1}, then the kinetic energy is measured in joule. It may be noted that the units of kinetic energy are the same as those of work. Infect, this is true of all forms of energy since they are inte-convertible.

Typical Kinetic Energies (K)

S. No	Object	Mass (kg)	Speed (m s^{-1})	K (J)
1	Air molecule	$\approx 10^{-26}$	500	$\approx 10^{-21}$
2	Rain drop at terminal speed	3.5×10^{-5}	9	1.4×10^{-3}
3	Stone dropped from 10 m	1	14	10^{2}
4	Bullet	5×10^{-2}	200	10^{3}
5	Running athlete	70	10	3.5×10^{3}
6	Car	2000	25	6.3×10^{5}

Relation between Momentum and Kinetic Energy

Consider a body of mass m moving both velocity v. Linear momentum of the body, p = mv

Kinetic energy of the body, $E_k = \frac{1}{2}mv^2$

$E_k = \frac{1}{2m}(m^2v^2)$ or $E_k = \frac{p^2}{2m}$ or $p = \sqrt{2mE_k}$

Ex. The kinetic energy of a body is increased by 21 %. What is the percentage increase in the magnitude of linear momentum of the body ?

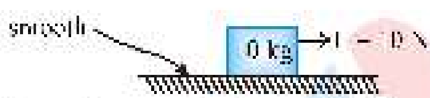
Sol. $E_k = \frac{121}{100}E_k$ or $\frac{1}{2}mv_2^2 = \frac{121}{100}\frac{1}{2}mv_1^2$ or $V_2 = \frac{11}{10}V$

or $mv_2 = \frac{11}{10}mv_1$ or $p_2 = \frac{11}{10}p_1$ or $\frac{p_2}{p_1} - 1 = \frac{11}{10} - 1 = \frac{1}{10}$

or $\frac{p_2 - p_1}{p_1} \times 100 = \frac{1}{10} \times 100 = 10$

So, the percentage increase in the magnitude of linear momentum is 10 %.

Ex.



Force shown acts for 2 seconds. Find out w.d. out by force F on 10 kg in 2 seconds.

Sol. $w = \vec{F}_{ext}.\Delta\vec{S} \Rightarrow w = F_{ext}\Delta S\cos 0° \Rightarrow w = 10\Delta S_{com}$

Now $10 = 10a$ ∴ $a = 1\ m/s^2$ → $S = \frac{1}{2}at^2 = \frac{1}{2} \times 1 \times 2^2 = 2m$

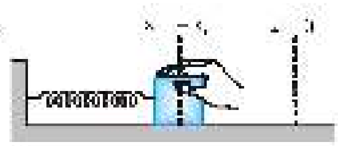
$w = 10 \times 2 = 20\ J$

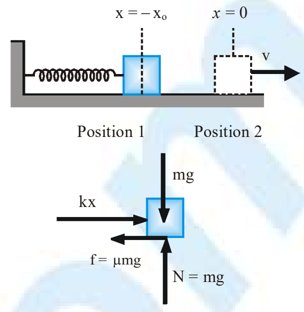
Ex. Find kinetic energy after 2 seconds.

Sol. $V = 0 + at \Rightarrow V = 1 \times 2 = 2\ m/s$

∴ $KE = \frac{1}{2} \times 10 \times 2^2 = 20\ J$

Ex. A box of mass m is attached to one end of a coiled spring of force constant k. The other end of the spring is fixed and the box can slide on a rough horizontal surface, where the coefficient of friction is μ. The box is held against the spring force compressing the spring by a distance x_0. The spring force in this position is more than force of limiting friction. Find the speed of the box when it passes the equilibrium position, when released.





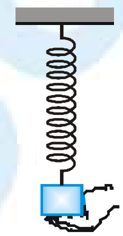
Sol. Before the equilibrium position, when the box passes the position coordinate –x, forces acting on it are its weight mg, normal reaction N from the horizontal surface, the force of kinetic friction f, and spring force F = kx as shown in the free body diagram. Let the box passes the equilibrium position with a speed v_o.

Applying work energy theorem on the box when it moves from position 1 ($-x_o$) to position 2 (x = 0), we have

$$W_{1\to2} = K_2 - K_1 \Rightarrow W_{F,1\to2} + W_{f,1\to2} = K_2 - K_1$$

$$\tfrac{1}{2}kx_o^2 - \mu mgx_o = \tfrac{1}{2}mv_o^2 - 0 \Rightarrow v_o = \sqrt{\left(kx_o^2 - 2\mu mgx_o\right)}$$

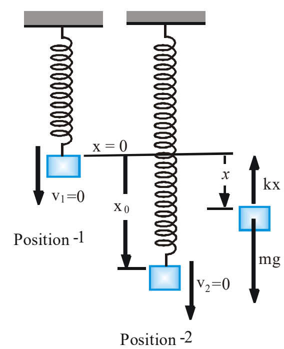
Ex. A block of mass m is suspended from a spring of force constant k. It is held to keep the spring
in its relaxed length as shown in the figure.



(a) The applied force is decreased gradually so that the block moves downward at negligible speed. How far below the initial position will the block stop?

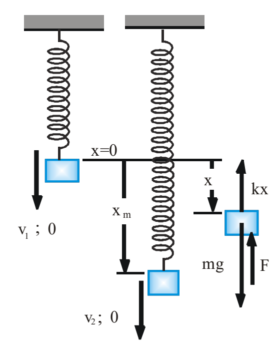
(b) The applied force is removed suddenly. How far below the initial position, will the block come to an instantaneous rest?

Sol.



(a) As the applied force (F) is decreased gradually, everywhere in its downward motion the block remains in the state of translational equilibrium and moves with negligible speed. Its weight (mg) is balanced by the upward spring force (kx) and the applied force. When the applied force becomes zero the spring force becomes equal to the weight and the block stops below a distance x_o from the initial position. The initial and final positions and free body diagram of the block at any intermediate position are shown in the adjoining figure. Applying the conditions of equilibrium, we have

$$x_o = \frac{mg}{k}$$



(b) In the previous situation the applied force was decreased gradually keeping the block everywhere in equilibrium. If the applied force is removed suddenly, the block will accelerate downwards. As the block moves, the increase in spring extension increases the upward force, due to which acceleration decreases until extension becomes x_o. At this extension, the block will acquire its maximum speed and it will move further downward. When extension becomes more than x_o spring force becomes more than the weight (mg) and the block decelerates and ultimately stops at a distance x_m below the initial position. The initial position-1, the final position-2, and the free body diagram of the block at some intermediate position when spring extension is x are shown in the adjoining figure.

Kinetic energy in position-1 is $K_1 = 0$

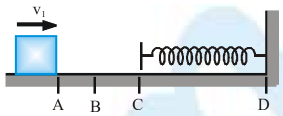
Kinetic energy in position-2 is $K_2 = 0$

Work done $W_{1\to2}$ by gravity and the spring force is $W_{1\to2} = W_{g,1\to2} + W_{spring,1\to2} = mgx_m - \tfrac{1}{2}kx_m^2$

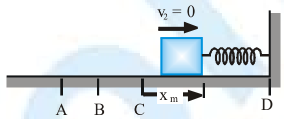
Using above values in the work energy theorem, we have $W_{1\to2} = K_2 - K_1 \to mgx_m - \tfrac{1}{2}kx_m^2 = 0$

$$x_m = 2mg/k$$

Ex. A block of mass m = 0.5 kg slides from the point A on a horizontal track with an initial sped of v_1 = 3 m/s towards a weightless horizontal spring of length 1 m and force constant k = 2 N/m. The part AB of the track is frictionless and the part BD has the coefficients of static and kinetic friction as 0.22 and 0.2 respectively. If the distance AB and BC are 2 m and 2.14 m respectively, find the total distance through which the block moves before it comes to rest completely. g = 10 m/s².



Sol.



Since portion AB of the track is smooth, the block reaches B with velocity v_1. Afterward force of kinetic friction starts opposing its motion. As the block passes the point C the spring force also starts opposing its motion in addition to the force of kinetic friction. The work done by these forces decrease the kinetic energy of the block and stop the block momentarily at a distance x_m after the point C.

Kinetic energy of the block at position-1 is $K_1 = \frac{1}{2}mv_1^2 = 2.25$ J.

Kinetic energy of the block at position-2 is $K_2 = \frac{1}{2}mv_2^2 = 0$ J.

Work $W_{f,1\to2}$ done by the frictional force before the block stops is $W_{f,1\to2} = \mu mg(BC + x_m) = 2.14 + x_m$

Work $W_{s,1\to2}$ done by the spring force before the block stops is $W_{s,1\to2} = \int_{x=0}^{x_m} kx dx = \frac{1}{2}kx_m^2 = x_m^2$

Using above information and the work energy principle, we have

$$W_{1\to2} = K_2 - K_1 \Rightarrow 2.14 + x_m + x_m^2 = 2.25 \Rightarrow x_m = 0.1 \text{ m}$$

The motion of block after it stops momentarily at position-2 depends upon the condition whether the spring force is more than or less than the force of limiting friction. If the spring force in position-2 is more than the force of limiting friction the block will move back and if the spring force in position-2 is less than the force of static friction the block will not move back and stop permanently.

Spring force F_s at position-2 is $F_s = kx_m = 0.2$ N.

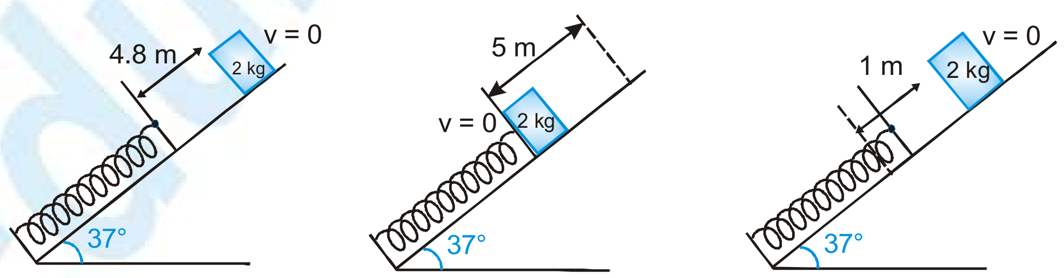
The force of limiting friction f_m is $f_m = \mu_s mg = 1.1$ N.

The force of limiting friction is more than the spring force therefore the block will stop at position-2 permanently.

The total distance traveled by the block = AB + BC + x_m =4.24 m.

Ex. A spring is fixed at the bottom end of an incline of inclination 37o. A small block is released from rest on an incline from a point 4.8 m away from the spring. The block compresses the spring by 20 cm., stops momentarily and then rebounds through a distance of 1 m up the incline. Find **(a)** the friction coefficient between the plane and the block and **(b)** the spring constant of the spring. Take g = 10 m/s².

Sol.



fig(a) just after release fig(b) when stopped for the first time fig(c) when stopped for the second time

Applying work energy theorem for motion from **(a)** to **(b)**

$$W_{gravity} + W_{friction} + W_{spring} = \Delta K.E = \frac{1}{2}m(0,0) = 0$$

$$\therefore \quad 20 \times 5 \sin 37^o - \mu(20 \cos 37^o)5 - \frac{1}{2}k[(0.2)^2 - 0] = 0 \qquad \text{.......(i)}$$

Applying work energy for motion form **(b)** to **(c)**

$$-20 \times 1 \times \sin 37^o - \mu(20 \cos 37^o) \times 1 - \frac{1}{2}k[0 - (0,2)^2] = 0 \qquad \text{.......(i)}$$

Adding equation **(i)** and **(ii)**

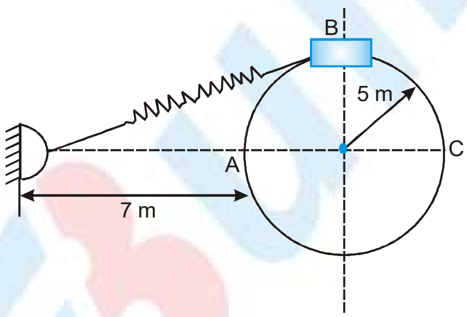
$$\therefore \quad -20(5-1) \times \frac{3}{5} - \mu\left(20 \times \frac{4}{5}\right)(5+1) = 0 \quad \Rightarrow \quad \mu = 0.5$$

Putting this value in equation **(i)**, we get

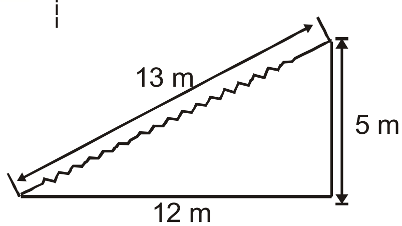
$k = 1000$ N/m

Here velocity is maximum at equilibrium since before this, spring force was less than the weight of the block and the block was accelerating and after this, the spring force is greater than the weight thus retarding the block to zero velocity upto the lowest position.

Ex. A collar B of mass 2 kg is constrained to move along a horizontal smooth and fixed circular track of radius 5 m as shown in figure. The spring lying in the plane of the circular track and having spring constant 200 N/m is undeformed when the collar is at A. If the collar starts from rest from B, then find the normal reaction exerted by the track on the collar when it passes through A.



Sol. Initially,



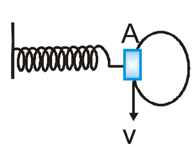
Length of spring = 13 m

undeformed length = 7 m

Initial extension (x_{in}) = 13 – 7 = 6

final extension (x_f) = 0

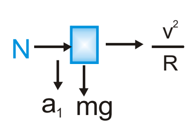
Applying work energy theorem for motion from B to A



$$\frac{1}{2}k\left(x_{in}^2 - x_f^2\right) = \frac{1}{2}mv^2$$

$mv^2 = 200\,(y^2 - 0^2) = 200 \times 49$

$mv^2 = 9800$



At point A, along radial direction, $F_{net} = N = m\dfrac{v^2}{R}$

$$\therefore \quad N = \frac{mv^2}{R} = \frac{9800}{5} \qquad = \frac{200}{5}[0 - (6)^2] \qquad = 1960 \text{ Newton}$$

Note: According to work-energy theorem the work done by all the forces on a particle is equal to the change in its kinetic energy.

$W_C + W_{NC} + W_{Ps} = \Delta K$

Where, W_C is the work done by the all conservative forces.

W_{NC} is the work done by all non-conservative forces.

W_{Ps} is the work done by all pseudo forces.

Modified form of Work-Energy Theorem

We know that conservative forces are associated with the concept of potential energy. That is

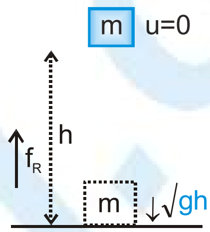
$W_C = -\Delta U$

So, work-energy theorem may be modified as

$W_{NC} + W_{ps} = \Delta k + \Delta U$

$W_{NC} + W_{ps} = \Delta E$

Ex. A body of mass m when released from rest from a height h, hits the ground with speed $\sqrt{gh}$. Find work done by resistive force.



Sol. Identify initial and final state and identify all forces.

$W_g + W_{air\ res.} + W_{int\ force} = \Delta K$

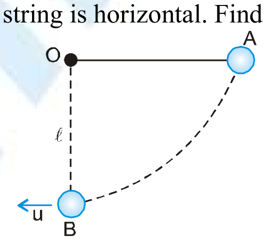
$$mgh + W_{air\ res} + 0 = \frac{1}{2} m \left(\sqrt{gh}\right)^2 - 0$$

$$\Rightarrow \qquad W_{air\ res.} = -\frac{mgh}{2}$$

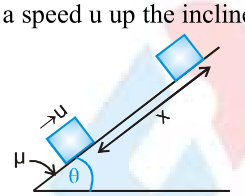
Ex. The bob of a simple pendulum of length l is released when the string is horizontal. Find its speed at the bottom.

Sol. $W_g + W_T = \Delta K$

$$mg\bullet + 0 = \frac{1}{2} mu^2 - 0 \Rightarrow u = \sqrt{2g\lambda}$$



Ex. A block is given a speed u up the inclined plane as shown.

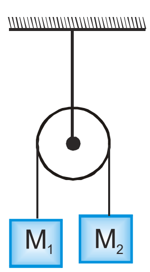


Using work energy theorem find out x when the block stops moving.

Sol. $W_g + W_f + W_N = \Delta K$

$$-mg\,x \sin\theta - \mu\, mgx \cos\theta + 0 = 0 - \frac{1}{2} mu^2 \quad \Rightarrow \quad x = \frac{u^2}{2g(\sin\theta + \mu\cos\theta)}$$

Ex. The masses M_1 and M_2 ($M_2 > M_1$) are released from rest. Using work energy theorem find out velocity of the blocks when they move a distance x.



Sol. $(W_{all\ F})_{system} = (\Delta K)_{system}$

$$(W_g)_{sys} + (W_T)_{sys} = (\Delta K)_{sys} \text{ as } \quad (W_T)_{sys} = 0$$

$$M_2gx - M_1gx = \frac{1}{2}(M_1 + M_2)V^2 - 0 \quad \text{.......... (1)}$$

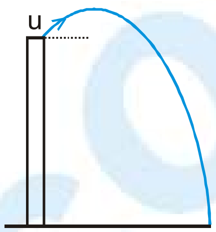
$$V = \sqrt{\frac{2(M_2 - M_1)gx}{M_1 + M_2}}$$

Ex. In the above question find out acceleration of blocks.

Sol. $(M_2g - M_1g) = \frac{1}{2}(M_1 + M_2)\, 2v\frac{dv}{dx}$ [Differentiating equation (1) above]

$$\Rightarrow \left(\frac{M_2 - M_1}{M_1 + M_2}\right)g = v\frac{dv}{dx} = a$$

Ex. A stone is projected with initial velocity u from a building of height h. After some time the stone falls on ground. Find out speed with it strikes the ground.



Sol. $W_{\text{all forces}} = \Delta K$

$W_g = \Delta K$

$mgh = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$v = \sqrt{u^2 + 2gh}$

POWER

Power is defined as the time rate of doing work.

When the time taken to complete a given amount of work is important, we measure the power of the agent of doing work.

The average power ($\overline{P}$ or p_{av}) delivered by an agent is given by

$$\overline{P} \text{ or } p_{av} = \frac{W}{t}$$ where W is the amount of work done in time t.

Power is the ratio of two scalars-work and time. So, power is a scalar quantity. If time taken to complete a given amount of work is more, then power is less. For a short duration dt, if P is the power delivered during this duration,

then $$P = \frac{\vec{F}.d\vec{S}}{dt} = \vec{F}.\frac{d\vec{S}}{dt} = \vec{F}.\vec{v}$$

This is instantaneous power. It may be +ve, –ve or zero.

By definition of dot product,

$$P = Fv\cos\theta$$

where θ is the smaller angle between $\vec{F}$ and $\vec{v}$.

This P is called as instantaneous power if dt is very small.

Ex. A block moves in uniform circular motion because a cord tied to the block is anchored at the centre of a circle. Is the power of the force exerted on the block by the cord is positive, negative or zero?

Sol. $\vec{F}$ and $\vec{v}$ are perpendicular. $\therefore$ Power $= \vec{F}.\vec{v} = Fv\cos 90^\circ = $ zero.

Unit of Power

A unit power is the power of an agent which does unit work in unit time.

The power of an agent is said to be one watt if it does one joule of work in one second.

1 watt = 1 joule/second = 10^7 erg/second

Also, 1 watt $= \frac{1\,\text{newton} \times 1\,\text{meter}}{1\,\text{second}} = 1\ \text{N m s}^{-1}$.

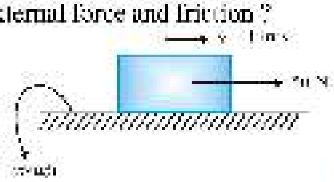
Dimensional formula of power

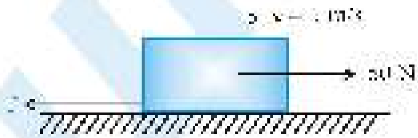
$$[\text{Power}] = \frac{[\text{Work}]}{[\text{Time}]} = \frac{[ML^2T^{-2}]}{[T]} = [ML^2T^{-3}]$$

Power has 1 dimension in mass, 2 dimensions in length and – 3 dimensions in time.

S.No	Human Activity	Power (W)
1	Heart beat	1.2
2	Sleeping	83
3	Sitting	120
4	Riding in a car	140
5	Walking (4.8 km h^{-1})	265
6	Cycling (15 km h^{-1})	410
7	Playing Tennis	440
8	Swimming (breaststroke, 1.6 km h^{-1})	475
9	Skating	535
10	Climbing stairs (116 steps min^{-1})	685
11	Cycling (21.3 km h^{-1})	700
12	Playing Basketball	800
13	Tube light	40
14	Fan	60

Ex. A block moves with constant velocity 1 m/s under the action of horizontal force 50 N on a horizontal surface. What is the power of external force and friction ?





Sol. Since a = 0 i.e. f = 50 N

$P_{ext} = 50 \times 1 = 50$ W

$P_f = -50 \times 1 = -50$ W

Power is also the rate at which energy is supplied.

Net power = $P_1 + P_2 + P_3$ ....

$P_{net} = \frac{dW_1}{dt} + \frac{dW_2}{dt}$ .... $\Rightarrow P_{net} = \frac{d(W_1 + W_2 ....)}{dt}$ $\Rightarrow P_{net} = \frac{dK}{dt}$ $\Rightarrow \Sigma W_{net} = \Delta K$

$\therefore$ Rate of change of kinetic energy is also power.

Ex. A stone is projected with velocity u at an angle θ with horizontal. Find out

(i) Average power of the gravity during time t.

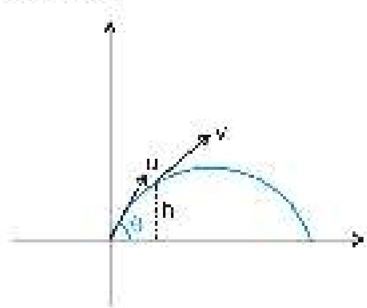
(ii) Instantaneous power due to gravitational force at time t where t is time of flight.

(iii) When is average power zero ?

(iv) When is $P_{ins.}$ zero ?

(v) When is $P_{ins.}$ negative ?

(vi) When is $P_{ins.}$ positive ?



Sol. (i) $\langle P \rangle = \frac{W}{T} = \frac{-mgh}{t} = \frac{-mg\left[u\sin\theta t - \frac{1}{2}gt^2\right]}{t}$

$\langle P \rangle = mg\left[\frac{gt}{2} - u\sin\theta\right]$

(ii) Instantaneous power

$P = \vec{F} \cdot \vec{v}$

$= (-mg\,\hat{j})\,[u\cos\theta\,\hat{i} + (u\sin\theta - gt)\,\hat{j}\,)\,]$

$= -mg(u\sin\theta - gt)$

(iii) $\dfrac{gt}{2} = u\sin\theta \quad \Rightarrow \quad t = \dfrac{2u\sin\theta}{g}$, i.e. time of flight.

(iv) When $\vec{F}$ & $\vec{V}$ are $\perp$ i.e. at $t = \dfrac{u\sin\theta}{g}$ which is at the highest point.

(v) From base to highest point.

(vi) From highest point to base.

CONSERVATION & NON CONSERVATION FORCE

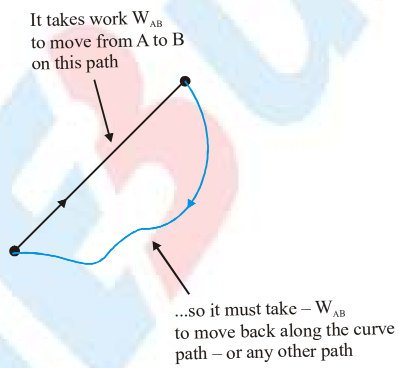
If we throw a body up along smooth incline plane with some speed v_0, then it moves along incline till it becomes stationary for a moment and then moves down the incline. It is observed the, when it reaches the point of projection, it speed is v_0 again, which proves that during the journey the net work done on the block is zero. Two forces act on the block during its motion. One is Normal force (N) which is continuously perpendicular to the block`s motion, so its work for any part of the path is zero. Another one is weight (mg) which does negative work while upward motion and positive work of same magnitude during downward motion, does zero net work when the body reaches the initially position, its speed is lesser than the speed of projection since friction does negative work for motion during up and as well as down the incline.

Thus, here we find two categories of force.

Conservative Force

When the total work done by a force F acting as an moves over any closed path is zero, the force is conservative. Mathematically.

$$\oint \vec{F}.d\vec{r} = 0 \qquad \text{(conservative force)}$$



Suppose we move an object along the straight path between point A and B shown in figure, along which a conservative force acts; let the work done by the conservative force be W_{AB}. Since the work done over any closed path is zero, the work W_{BA} done in moving back from B to A must be $- W_{AB}$, regardless of the path taken.

In other words : The work done by a points is independent of the path taken: mathematically $\int_A^B \vec{F}.d\vec{r}$ depends only on the endpoints A and B, not on the path between them.

These include force due to gravity (mg), spring force, electrostatic force etc.

Examples of Conservative forces

(i) Gravitational force, not only due to the Earth but in its general form as given by the universal law of gravitation, is a conservative force.

(ii) Elastic force in a stretched or compressed spring is a conservative force.

When the total work done by a force F acting as an moves over any closed path is zero, the force is conservative.

(iii) Electrostatic force between two electric charges is a conservative force.

(iv) Magnetic force between two magnetic poles is a conservative forces.

Infact, all, fundamental forces of nature are conservative in nature.

Forces acting along the line joining the centres of two bodies are called central forces. Gravitational force and Electrostatic forces are two important examples of central forces. Central forces are conservative forces.

Properties of conservative forces

(i) Work done by or against a conservative force depends only on the initial and final positions of the body.

(ii) Work done by or against a conservative force does no depends upon the nature of the path between initial and final position of the body.

If the work done by a force in moving a body from an initial location to a final location is independent of the path taken between the two points, then the force is conservative.

(iii) Work done by or against a conservative force in a round trip is zero.

If a body moves under the action of a force that does no total work during any round trip, then the force is conservative ; otherwise it is non-conservative.

The concept of potential energy exists only in the case of conservative forces.

(iv) The work done by a conservative force is completely recoverable.

Complete recoverability is an important aspect of the work of a conservative force.

NON CONSERVATION FORCE

A force is said to be non-conservative if work done by or against the force in moving a body depends upon the path between the initial and final positions.

The frictional forces are non-conservative forces. This is because the work done against friction depends on the length of the path along which a body is moved. It is does not depends only on the initial and final positions. Note that the work done by frictional force in a round trip is not zero.

The velocity-dependent forces such as air resistance, viscous force etc., are non conservative forces.

S.No	Conservative forces	Non-Conservative forces
1	Work done does not depends upon path	Work done depends on path
2	work done in round trip is zero	Work done in a round trip is not zero
3	Central in nature	Forces are velocity-dependent and retarding in nature.
4	When only a conservative force acts within a system, the kinetic energy and potential energy can change. However their sum, the mechanical energy of the system does not change.	Work done against a non-conservative force may be dissipated as heat energy
5	Work done is completely recoverable.	Work done in not completely recoverable

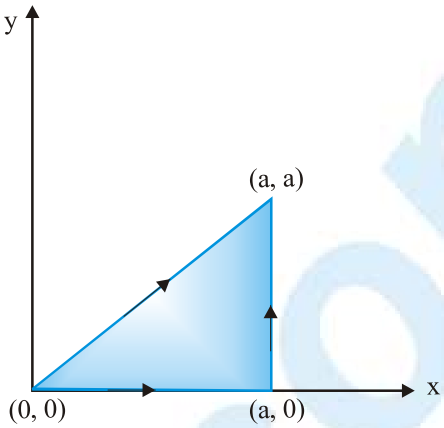
Ex. A particle is moves in x-y plane from (0, 0) to (a, a) and is acted upon by a force $\vec{F} = k(y\hat{i} + x\hat{j})$ N, where k is a constant and x and y are coordinates in meter. Find work done by this force, if the particle moves along

(i) Two straight lines first from (0, 0) to (a, 0) and then from (a, 0) to (a, a)

(ii) A single straight line.

Sol. $dW = \vec{F}.\vec{ds} = k\left(y^2\hat{i} + x^2\hat{j}\right).\left(dx\hat{i} + dy\hat{j}\right)$

$= ky^2\ dx + k\,x^2\,dy$



(i) when it moves from (0, 0) to (a, 0)

$y = \text{constant} = 0$

$\Rightarrow\quad dy = 0$

$\therefore\quad dW = k(0)\,dx + kx^2(0)$

$= 0$

$\Rightarrow\quad W_A = \int dw_A = 0$

When it moves from (a, 0) to (a, a)

$x = \text{constant} = a \Rightarrow dx = 0$

and y changes from 0 to a

$\therefore\quad dW_B = ky^2(0) + ka^2\,dy = ka^2\,dy$

$\therefore\quad W_B = ka^2\int_0^a dy = ka^3$

$\therefore\quad W = W_A + W_B = ka^3$

(ii) When moves from (0, 0) to (a, a) as shown in above figure, along path C which is a straight line for which

$y = x$

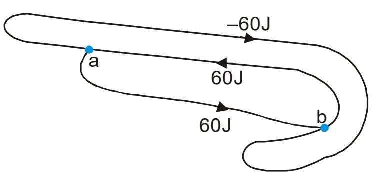
$\Rightarrow\quad dy = dx$

$\therefore\quad dW = kx^2\,dx + k\,x^2\,dx = 2k\,x^2\,dx$

$\therefore\quad w = \int dW = 2k\int_0^a x^2 dx = \frac{2ka^3}{3}$

In above illustration, the work done by the force is different for different paths for different paths taken, so it provides an example of non-conservative force.

Ex. The figure shows three paths connecting points a and b. A single force F does the indicated work on a particle moving along each path in the indicated direction. On the basis of this information, is force F conservative ?



Ans. No

Sol. For a conservative force, the work done in a round trip should be zero.

Ex. Find the work done by a force $\vec{F} = x\hat{i} + y\hat{j}$ acting on a particle to displace it from point A(0, 0) to B(2, 3).

Sol. $dW = \vec{F}.ds = \left(x\hat{i} + y\hat{j}\right).\left(dx\hat{i} + dy\hat{j}\right)$

$$W = \int_0^2 x dx + \int_0^3 y dy = \left[\frac{x^2}{2}\right]_0^2 + \left[\frac{y^2}{2}\right]_0^3 = \frac{13}{2}\ \text{units}$$

Ex. In case of a non conservative force work done along two different paths will always be different .

Ans. False

Ex. In case of non conservative force work done along two different paths may be different.

Ans. True

Ex. In case of non conservative force work done along all possible paths cannot be same.

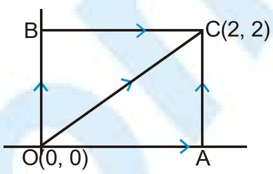
Ans. True

Ex. Find work done by a force $\vec{F} = x\hat{i} + xy\hat{j}$ acting on a particle to displace it from point O (0, 0) to C(2, 2).

Sol. $\int dW = \int_0^2 x\,dx \quad + \quad \int_0^2 xy\,dy$

Can be found — cannot be found until x is known in terms of y i.e. until equation of path is known.

Ex. Find the work done by $\vec{F}$ from O to C for above example if paths are given as below.



Sol. **(i)** OAC $\Rightarrow$ OA + AC

for OA $y = 0$ $\quad \therefore dy = 0$

$\therefore \int dW_{OA} = \int_0^2 x\,dx + 0 \qquad \therefore W_{OA} = 2\text{ J}$

for AC $x = 2$ $\quad dx = 0$

$\int dW_{AC} = 0 + 2\int_0^2 y\,dy \qquad \therefore \quad W_{AC} = 4\text{J}$

$\therefore \quad W_{OAC} = W_{OA} + W_{AC} = 2 + 4 = 6\text{J}$

(ii) OBC $\Rightarrow$ OB + BC

for OB $x = 0$ $\quad dx = 0 \qquad \therefore \quad W_{OB} = 0$

for BC $y = 2$ $\quad dy = 0$

$\therefore \quad \int dW = \int x\,dx \qquad \therefore \quad W = \left[\frac{x^2}{2}\right]_0^2 = 2\text{ J}$

$\therefore \quad W_{OAC} \neq W_{OBC}$

Hence the force is non-conservative.

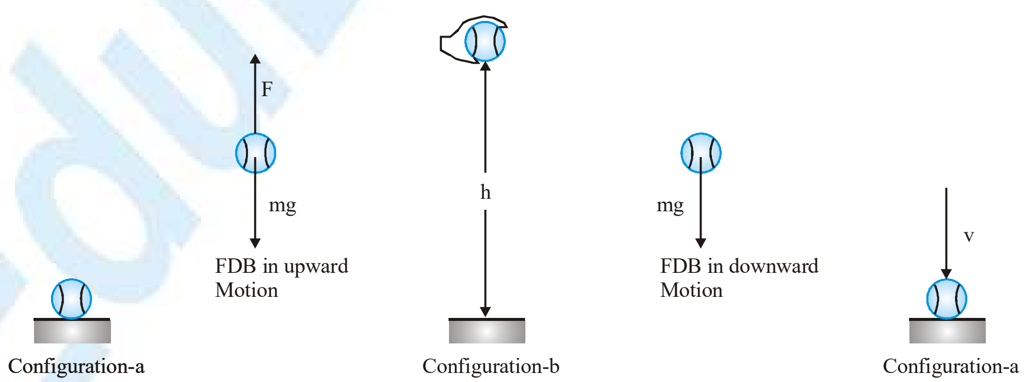
(iii) For W_{OC} $\quad dW = xdy + xydx$

for OC $\quad x = y \quad dx = dy$

$dW = \int_0^2 x\,dx + \int_0^2 y^2\,dy \qquad W = \frac{14}{3}$ unit

POTENTIAL ENERGY

Consider a ball of mass m placed on the ground and someone moves it at negligible speed through a height h above the ground as shown in figure. The all remains in the state of equilibrium therefore the upward force F applied on it everywhere equals to the weight (mg) of the ball. The work $W_{a\to b} = \vec{F}.\vec{h} = mgh$.



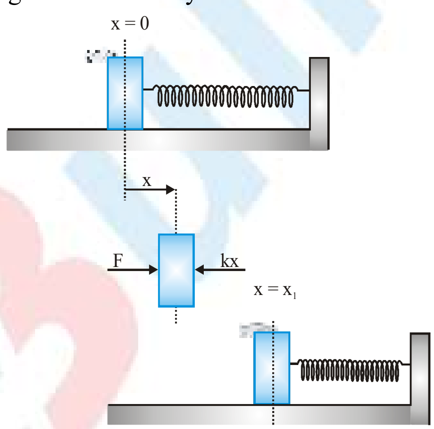
Now if the balls is dropped from the height h it starts moving downwards due to its weight and strikes the ground with speed v. The work $W_{b\to a}$ done by its downward motion imparts it a kinetic energy K_c which is obtained by using work energy principle and the above equation as

$$W_{b\to a} = K_a - K_b \Rightarrow mgh = \frac{1}{2}mv^2 = mgh$$

Instead of raising the ball to height, if it were thrown upwards with a speed v it would have reached the height h and returned to the ground with the same speed. Now if we assume a new form of energy that depends on the separation between the ball and the ground, the above phenomena can be explained. This new form of energy is known as potential energy of the earth-ball system. When ball moves up, irrespective of the path or method how the ball has been moved, potential energy of the earth-ball system increases. This increase equals to work done by applied force F in moving the ball to height h or negative of work done by gravity. When the ball descends, potential energy of the earth ball system decreases; and is recovered as the kinetic energy of the ball when separation vanishes. During descend of the ball gravity does positive work, which equals to decrease in potential energy.

Potential energy of the ball system is due to gravitation force and therefore is call gravitation potential energy. Change in gravitational potential energy equals to negative of wrok done by gravitational force. It is denoted by ΔU .

Infact, when the ball is released both the ball and the earth move towards each other and acquire momenta of equal magnitude but the mass of the earth is infinitely large as compared to that of the ball, the earth acquires negligible kinetic energy. It is the ball, that acquires almost all the kinetic energy and therefore sometimes the potential energy is erroneously assigned with the ball and called the potential energy of the ball. Nevertheless, It must be kept in mind that the potential energy belongs to the entire system.

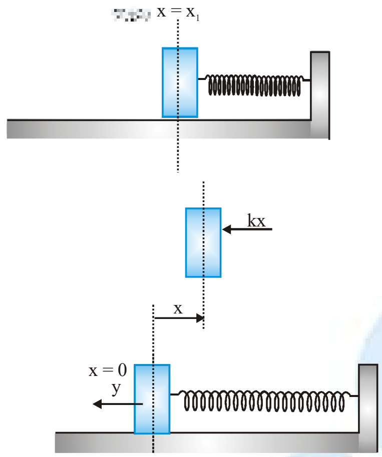


As another instance, consider a block of mass m placed on a smooth horizontal plane and connected to one end of a spring of force constant k, whose other end is connected to a fixed support. Initially, when the spring is relaxed, no net force acts on the block and it is in equilibrium at position x = 0. If the block is pushed gradually against the spring force and moves at negligible speed without acceleration, at every position x, the applied force F balances the spring force kx. The work done $W_{0\to 1}$ by this force in moving the block from position x = 0 to x = x_1 is

$$W_{0\to 1} = \int_{x=0}^{x=x_1} \vec{F}.d\vec{x} = \tfrac{1}{2}kx_1^2$$

If the applied force is removed, the block moves back and reaches its initial position with a kinetic energy K_0 which is obtained by applying work energy theorem together with the above equation.

$$W_{1\to 0} = K_0 - K_1 \Rightarrow K_0 = \tfrac{1}{2}mv^2 = \tfrac{1}{2}kx_1^2$$



The above equation shows that the work done on the block by the applied force in moving it from x = 0 to $x = x_1$ is stored in the spring block system as increase in potential energy and when the block returns to its initial position x = 0 this stored potential energy decreases and is recovered as the kinetic energy of the block. The same result would have been obtained if the block were pulled elongating the spring and then released. The equals to negative of work done by the spring force.

In both the above cases force involved were conservative. In fact, work done against all conservative forces is recoverable. With every conservative force, we can associate a potential energy, whose change equals to negative of work done by the conservative force. For an infinitely small change in configuration, change in potential energy dU equals to the negative of work done dW_C by conservative forces.

$$dW = dU = -dW_C$$

Since a force is the interaction between two bodies, on very fundamental level potential energy is defined for every pair of bodies interaction with conservative forces. The potential energy is defined for every pair of bodies interacting with conservative force. The potential energy of a system consisting of a large number of bodies thus will be sum of potential energies of all possible pairs of bodies constituting the system.

Because only change in potential energy has significance, we can chose potential energy of any configuration as reference value.

1. GRAVITATIONAL POTENTIAL ENERGY

(a) Gravitational potential energy for uniform gravitational force (Near the Earth`s Surface)

The work done by gravity on a particle of mass m whose vertical coordinate changes from y_A to y_B is

$W_g = -mg(y_B - y_A)$

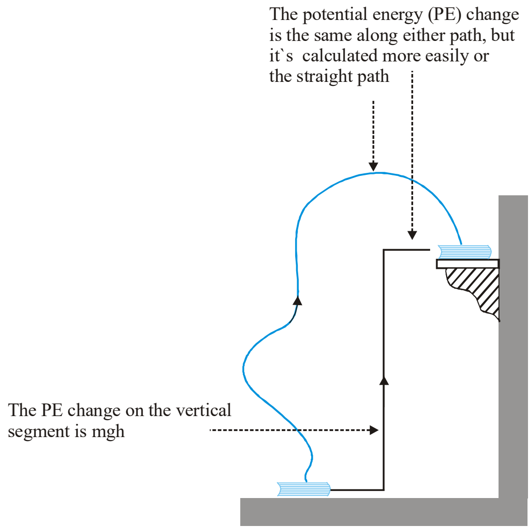
From equation, we have $Wg = -\Delta U_g = -(U_B - U_A)$

Thus gravitational potential energy at the point B near the surface of the Earth is given by

$$U_B = U_A + mgh$$

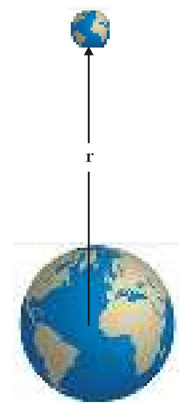
If we assume potential energy at the point A to be zero, then potential energy at the point B is given by

$$U_B = 0 + mgh = mgh$$



(b) Gravitational potential energy for non-uniform gravitational force

When motion of a body of mass m involves distances from the earth surface large enough, the variation in the gravitational force between the body and the earth cannot be neglected. For such physical situations the configuration is arbitrarily assumed zero ($U_\infty = 0$).



The ball at a distance r from the centre of the earth

If the body is brought at negligible speed to a distance r from infinitely large distance from the earth center, the work done W_g by the gravitational force is given by the following equation.

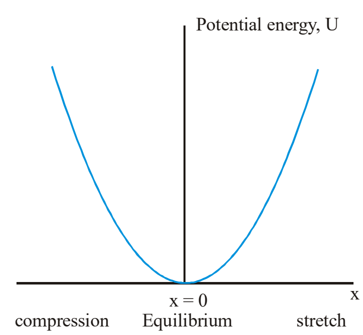
$$W_g = \int_\infty^r \vec{F}_g \cdot d\vec{r} = \left[\frac{GMm}{r}\right]_\infty^r$$

Negative of this work equals to change in potential energy of the system. Denoting potential energies configuration of separation r and ∞ by U_r and U_∞, we have

$$U_r - U_\infty = -W_g \rightarrow U_r = -\frac{GMm}{r}$$

2. Elastic Potential energy

When you stretch or compress a spring, you do work against the spring force, and that work gets stored as elastic potential energy. For an ideal spring, the force is $F = -kx$, where x is the distance the spring is stretched from equilibrium, and the minus sign shows that the force opposes the stretching or compression.

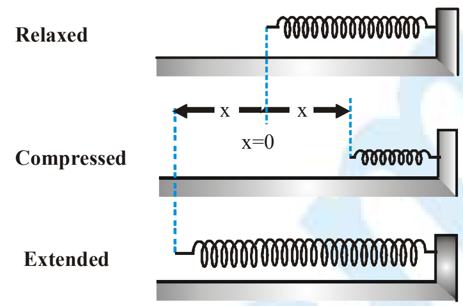


$$\Delta U = -\int_{x_1}^{x_2} F(x)\,dx = -\int_{x_1}^{x_2} (-kx)\,dx = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$$

where x_1 and x_2 are the initial and final values of the stretch. If we take U = 0 when x = 0 that is ,when the spring is neither stretched nor compressed then we can use this result to write the potential energy

$U_2 = U$ at an arbitrary stretch (or compression) $x_2 = x$, $U = \frac{1}{2}kx^2$

Potential energy associated with spring force

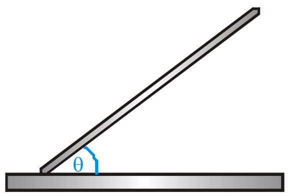


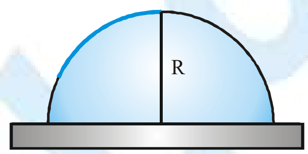
The potential energy associated with a spring force of an ideal spring when compressed or elongated by a distance x from its natural length is defined by the following equation

$$U = \tfrac{1}{2}kx^2$$

Ex. Find the gravitational potential energies in the following physical situations. Assume the ground as the reference potential energy level.

(a) A thin rod of mass m and length L kept at angle θ with one of its end touching the ground.

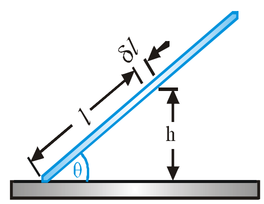




(b) A flexible rope of mass m and length L placed on a smooth hemisphere of radius R and one of the ends of the rope is fixed at the top of the hemisphere.

Sol. In both the above situations, mass is distributed over a range of position coordinates. In such situations calculate potential energy of an infinitely small portion of the body and integrate the expression obtained over the entire range of position coordinates covered by the body.

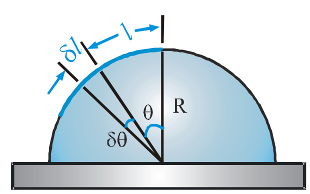
(a) Assume a small portion of length δ● of the rod at distance ● from the bottom end and height of the midpoint of this portion from the ground is h. Mass of this portion is δm. When δ● approaches to zero, the gravitational potential energy dU of the assumed portion becomes



$$dU = \frac{m}{L}ghdl = \frac{m}{L}gl\sin\theta dl$$

The gravitational potential energy U of the rod is obtained by carrying integration of the above equation over the entire length of the rod.

$$U = \int_{l=0}^{L}\frac{m}{L}gl\sin\theta dl = \tfrac{1}{2}mgL\sin\theta$$



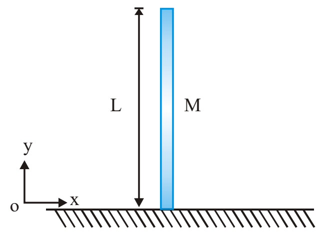
(b) The gravitational potential energy dU of a small portion of length δ● shown in the adjoining figure, when δ● approaches to zero is

$$dU = \frac{m}{L}gR^2\sin\theta d\theta$$

The gravitational potential energy U of the rope is obtained by carrying integration of the above equation over the entire length of the rope.

$$U = \int_{\theta=0}^{\theta=\frac{L}{R}}\frac{m}{L}gR^2\sin\theta d\theta = \frac{m}{L}gR^2\left\{1-\cos\left(\frac{L}{R}\right)\right\}$$

Ex. A uniform rod of mass M and length L is held vertically upright on a horizontal surface as shown in the figure. Find the potential energy of the rod if the zero potential energy level is assumed at the horizontal energy level is assumed at the horizontal surface.

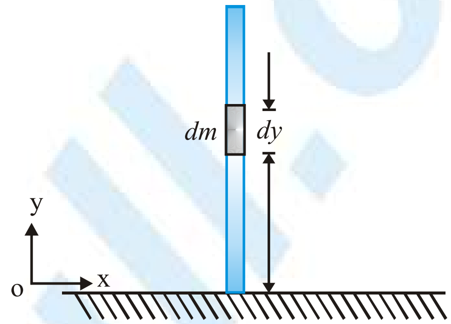


Sol. Since the parts of the rod are at different level with respect to the horizontal surface, therefore, we have to use the integration to find its potential energy. Consider a small element of length dy at a height of the element is

$$dm = \frac{M}{L}dy$$

Its potential energy is given by

$$dU = (dm)gy$$



or $$dU = \frac{M}{L}gydy$$

On integration, we get

$$U = \frac{Mg}{L}\int_0^L ydy$$

or $$U = \frac{Mg}{L}\left|\frac{y^2}{2}\right|_0^L$$

or $$U = \frac{1}{2}MgL$$

Note that the potential energy of the rod is equal to the product of Mg and height of the center of mass $\left(\frac{L}{2}\right)$ from the surface.

Conservation of Mechanical Energy

The work energy theorem, shows that the change ΔKE in a body`s kinetic energy is equal to the net work done on it:

$$\Delta KE = W_{net}$$

Consider separately the work W_c done by conservative force and the work W_{nc} done by nonconservative forces. So we can write

$$\Delta KE = W_c + W_{nc}$$

We `ve defined the change in potential energy ΔU as the mechanical energy. Then equation shown that the change in mechanical energy is equal to the work done by non-conservative force.

i.e. $\Delta E = Wnc$

∴ $\Delta E = 0$ if $W_{nc} = 0$

Thus if wrok done by non conservative forces is zero the mechanical energy. It may also be written as

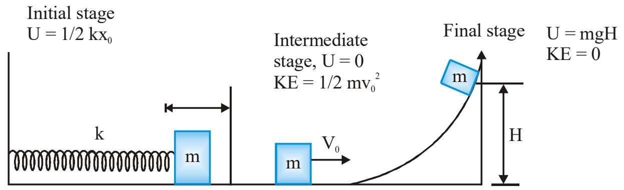
$$\Delta U + \Delta KE = 0$$

or $\Delta U = -\Delta KE$

or $U + KE = \text{constant}$

or $U_{in} + KE_{in} = U_r + KE_f$

The surfaces shown in the figure are frictionless and horizontal surface is taken as reference level.



By conservation of mechanical energy

$$\frac{1}{2}kx_0^2 + 0 = 0 + \frac{1}{2}mv_0^2 = mgH + 0$$

ETOOS KEY POINTS

(i) It is a scalar quantity having dimensions $[ML^2T^{-2}]$ and SI units joule.

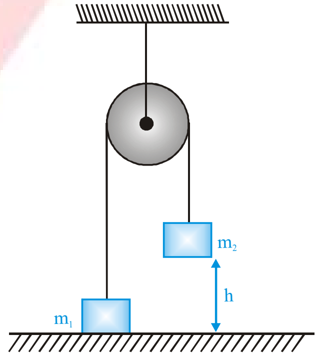
(ii) It depends on frame of reference.

(iii) A body can have mechanical energy without having either kinetic energy or potential energy. However, if both kinetic and potential energies are zero, mechanical energy will be zero. The converse may or may not be true, i.e., if E = 0 either both PE and KE are zero or PE may be negative and KE may be positive such that KE + PE = 0.

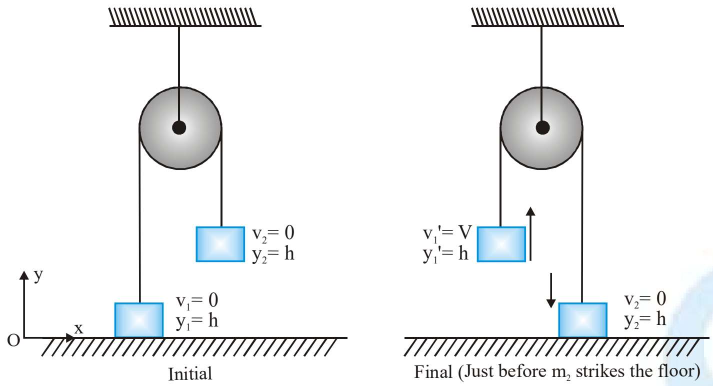
(iv) As mechanical energy E = K + U, i.e., E - U = K. Now as K is always positive, E - U $\geq$ 0,i.e., for existence of a particle in the field, E $\geq$ U.

(v) As mechanical energy E = K + U and K is always positive, so, if 'U' is positive 'E' will be positive. However, if potential energy U is negative, 'E' will be positive if K > |U| and E will be negative if K < |U| i.e., mechanical energy of a body of system can be negative, and negative mechanical energy means that potential energy is negative and in magnitude it is more than kinetic energy. Such a state is called bound state, e.g., electron in an atom or a satellite moving around a planet are in bound state.

Ex. Two block with masses m_1 = 3 kg and m_2 = 5kg are connected by a light string that slides over a frictionless pulley as shown in figure. Initially, m_2 is held 5 m off the floor while m_1 is on the floor. The system is then released. At what speed does m_2 hit the floor ?



Sol. The initial and final configurations are shown in the figure. It is convenient to set U_g = 0 at the floor. Initially only m_2 has potential energy . As it falls, it loses potential energy and gains kinetic energy. At the same time, m_1 gains potential energy and kinetic energy. Just before m_2 lands, it has only kinetic energy. Let v the final speed of each mass. Then, using the law of conservation of mechanical energy.



$$K_f + U_f = K_i + U_i$$

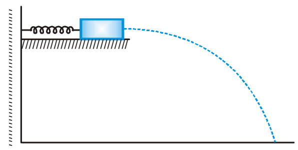
$$\frac{1}{2}(m_1 + m_2)v^2 + m_1gh = 0 + m_2gh$$

$$v^2 = \frac{2(m_2 - m_1)gh}{m_1 + m_2}$$

Putting $m_1 = 3$ kg; $m_2 = 5$ kg; $h = 5$ m and $g = 10$ m/s^2

we get $v^2 = \frac{2(5-3)(10)(5)}{m_1 + m_2}$ or $v = 5$ m/s

Ex. As shown in figure there is a spring block system. Block of mass 500 g is pressed against a horizontal spring fixed at one end to compress the spring through 5.0 cm. The spring constant is 500 N/m. When released, the block moves horizontally till it leaves the spring. Calculate the distance where it will hit the ground 4 m below the spring?



Sol. When block released, the block moves horizontally with speed V till it leaves the spring.

By energy conservation $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$

$$V^2 = \frac{kx^2}{m} \Rightarrow V = \sqrt{\frac{kx^2}{m}}$$

Time of flight $t = \sqrt{\frac{2H}{g}}$

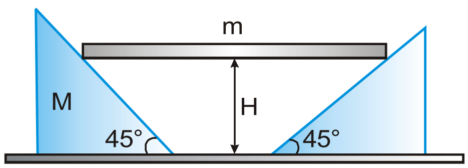
So. horizontal distance travelled from the free end of the spring is $V \times t$

$$= \sqrt{\frac{kx^2}{m}} \times \sqrt{\frac{2H}{g}}$$

$$= \sqrt{\frac{500 \times (0.05)^2}{0.5}} \times \sqrt{\frac{2 \times 4}{10}} = 2 \text{ m}$$

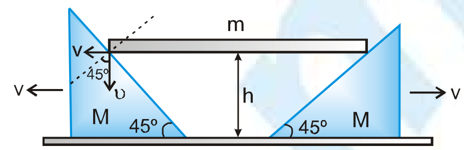
So, At a horizontal distance of 2 m from the free end of the spring.

Ex. A rigid body of mass m is held at a height H on two smooth wedges of mass M each of which are themselves at rest on a horizontal frictionless floor. On releasing the body it moves down pushing aside the wedges. The velocity of recede of the wedges from each other when rigid body is at a height h from the ground is



(A) $\sqrt{\frac{2mg(H-h)}{m+2M}}$ (B) $\sqrt{\frac{2mg(H-h)}{2m+M}}$ (C) $\sqrt{\frac{8mg(H-h)}{m+2M}}$ (D) $\sqrt{\frac{8mg(H-h)}{2m+M}}$

Sol. Let speed of the wedge and the rigid body be V and υ respectively.



Then applying wedge constraint we get

$$V\cos 45° = \upsilon \cos 45°$$

$\therefore$ $V = \upsilon$ ...**(i)**

Using energy conservation,

$$mg(H-h) = 2\left(\frac{1}{2}MV^2\right) + \frac{1}{2}m\upsilon^2 \quad \text{...(ii)}$$

From equation **(i)** and **(ii)**

$$V = \sqrt{\frac{2mg(H-h)}{m+2M}}$$

$\therefore$ The velocity of recede of wedges from each other $= 2 \times V = \sqrt{\frac{8mg(H-h)}{m+2M}}$

So, answer is **(C)**

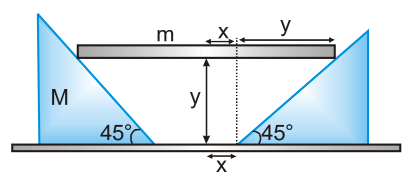
Alter :

Length of rod = ●

$$x + y = \frac{\lambda}{2}$$

$$\frac{dx}{dt} + \frac{dy}{dt} = 0$$

velocity of block = velocity of rod

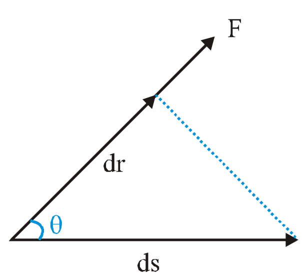


decrease in potential energy = increase in kinetic energy

$$mg(H-h) = \frac{1}{2}mV^2 + \frac{1}{2}MV^2 + \frac{1}{2}MV^2$$

$\therefore$ $V = \sqrt{\frac{2mg(H-h)}{2M+m}}$ $\therefore$ $2V = \sqrt{\frac{8mg(H-h)}{2M+m}}$

Potential energy and the associated conservative force



We know how to find potential energy associated with a conservative force. Now we learn how to obtain the conservative force if potential energy function is known. Consider work done dW by a conservative force in moving a particle through an infinitely small path length $d\vec{s}$ as shown in the figure.

From the above equation, the magnitude F of the conservative force can be expressed.

$$F = \frac{dU}{ds\cos\theta} = -\frac{dU}{dr}$$

If we assume an infinitely small displacement $d\vec{r}$ in the direction of the force, magnitude of the force is given by the following equation.

$$\vec{F} = -\frac{dU}{dr}\hat{e}_r$$

Ex. Force between the atoms of a diatomic molecule has its origin in the interactions between the electrons and the nuclei present in each atom. This force is conservative and associated potential energy U(r) is, to a good approximation, represented by the Lennard - Jones potential.

$$U(r) = U_o\left\{\left(\frac{a}{r}\right)^{12} - \left(\frac{a}{r}\right)^{6}\right\}$$

Here r is the distance between the two atoms and U_o and a are positive constants. Develop expression for the associated force and find the equilibrium separation between the atoms.

Sol. Using equation $F = -\frac{dU}{dr}$, we obtain the expression for the force

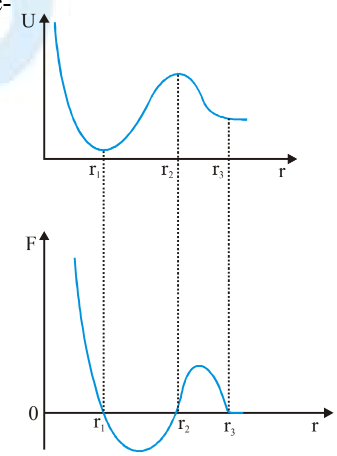
$$F = -\frac{6U_o}{a}\left\{2\left(\frac{a}{r}\right)^{13} - \left(\frac{a}{r}\right)^{7}\right\}$$

At equilibrium the force must be zero. Therefore the equilibrium separation r_o is

$$r_o = 2^{\frac{1}{6}}a$$

Potential Energy and Nature of Equilibrium

The above equation suggest that on every location where the potential energy function assumes a minimum or a maximum value or in every region where the potential energy function assumes a constant value, the associated conservative force becomes zero and a body under the action of only this conservative force becomes zero and a body under the action of only this conservative force must be in the state of equilibrium. Different status of potential energy function in the state of equilibrium suggests us to define three different types of equilibriums - the stable, unstable and neutral equilibrium.



Force is negative of the slope of the potential energy function.

The state of stable an unstable equilibrium is associated with a point location, where the potential energy function assume a minimum and maximum value respectively, and the neutral equilibrium is associated with region of space, where the potential energy function assumes a constant value.

For the sake of simplicity, consider one dimensional potential energy function U of a central force F. Here r is the radial coordinate of a particle. The central force F experienced by the particle equals to the negative of the slope of the potential energy function. Variation in the force with r is also shown in the figure.

At locations $r = r_1$, $r = r_2$, and in the region $r \geq r_3$, where potential energy function assumes a minimum a maximum, and a constant value respectively, the force becomes zero and the particle is in the state of equilibrium.

Stable Equilibrium

At $r = r_1$ the potential energy function is a minima and the force on either side acts towards the point $r = r_1$. If the particle is displaced on either side and released, the force tries to restore it at $r = r_1$. At this location the particle in the state of stable equilibrium. The dip in the potential energy curve at the location of stable equilibrium is known as potential well. A particle when distributed from the state of stable within the potential well starts oscillations about the location of stable equilibrium. At the locations of stable equilibrium we have

$$F(r) = -\frac{\partial U}{\partial r} = 0 \text{ ; and } \frac{\partial F}{\partial r} < 0 \text{ ; and } \frac{\partial^2 U}{\partial r^2} > 0$$

Unstable Equilibrium

At $r = r_2$ the potential energy function is a maxima, the force acts away from the point $r = r_2$. If the particle is displaces slightly on either side, it will not return to the location $r = r_2$. At this location, the particle is in the state of unstable equilibrium we have

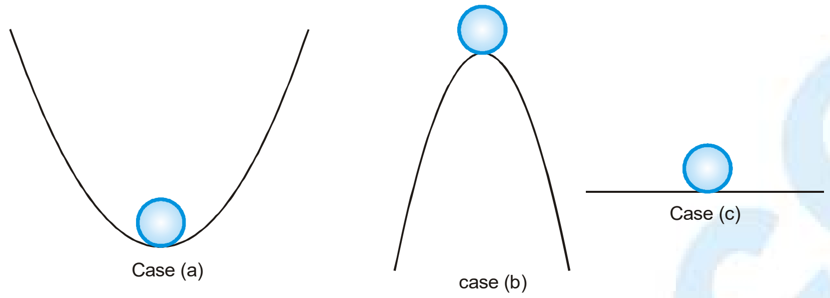
$$F(r) = -\frac{\partial U}{\partial r} = 0 \text{ therefore } \frac{\partial F}{\partial r} > 0 \text{ ; and } \frac{\partial^2 U}{\partial r^2} < 0$$

Neutral Equilibrium

In the region r $\geq$ r3 , the potential energy function is constant and the force is zero everywhere. In this region, the particle is in the state of neutral equilibrium. At the locations of neutral equilibrium we have

$$F(r) = -\frac{\partial U}{\partial r} = 0 \text{ therefore } \frac{\partial F}{\partial r} = 0 \text{ and } \frac{\partial^2 U}{\partial r^2} = 0$$

Ex. Suppose a small ball is placed on a smooth track under three different situations as shown in figure **(a)**, **(b)** and **(c)**. In all the three situation, the ball , is in equilibrium.



Case (a)

When the ball is slightly displaced from its equilibrium position, it tries to attain the shown position again, such type of equilibrium is called stable equilibrium.
Here potential energy in equilibrium position is minimum as compares to its neighbouring, point i.e. under stable equilibrium potential energy is minimum.

$$\text{i.e., } \frac{dU}{dr} = 0 \text{ and } \frac{d^2U}{dr^2} > 0$$

Case (b)

When the ball is slightly displaced from its equilibrium position it tends to move father form the shown equilibrium position. Such type of equilibrium is called unstable equilibrium.
Here potential energy in equilibrium is maximum as compared to its near by points.

$$\text{i.e., } \frac{dU}{dr} = 0 \text{ and } \frac{d^2U}{dr^2} < 0$$

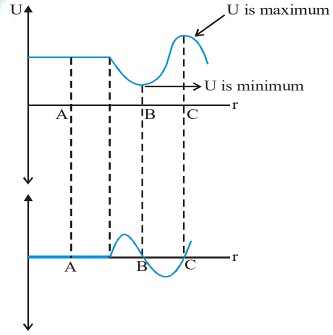
Case (c)

When the ball is displaced, it accepts the new position as equilibrium position, such type of equilibrium. is called neutral equilibrium.
Here potential energy remains uniform for the equilibrium position.

$$\text{i.e., } \frac{dU}{dr} = 0 \text{ and } \frac{d^2U}{dr^2} = 0$$

Although, the above discussion is under the effect of gravity but the result observed is applicable in other situations also where a particle can move under the effect of the conservative force only.

Ex. The above result can be studies with the help of the following graphs.
For a particle whose position (r) varies along a straight line, the graph below show variation of U vs r and F vs r.



At Point A $F = 0$; $\frac{dU}{dr} = 0$, but F = 0 at its nearly points also. So when slightly displaced from A, the new position is also equilibrium. Thus point A shown is the position of neutral equilibrium.

At Point B $F = 0$; $\frac{dU}{dr} = 0$, Now when it is slightly displaced towards left of B, force is positive i.e. towards right and when it is slightly displaced towards right of B, force is negative i.e. towards right. Thus force tries to bring the particle towards B again. This type of force is called restoring force and the point B is the position of stable equilibrium.

At Point C $F = 0$; $\frac{dU}{dr} = 0$, but when particle is displaced slightly form it towards any direction, force acts in that direction only i.e. to move the particle any from C. Thus point C. is the position of unstable equilibrium.

Ex. The potential energy of a conservative system is given by

$$U = ax^2 - bx$$

Where a and b are positive constants. Find the equilibrium position and discuss whether the equilibrium is stable unstable or neutral.

Sol. In a conservative field

$$F = \frac{dU}{dx} = -\frac{d}{dx}\left(ax^2 - bx\right) = -\left(2ax - b\right)$$

$\therefore$ $F = b - 2ax$

For equilibrium $F = 0$

or $b - 2ax = 0$ $\therefore$ $x = \frac{b}{2a}$

From the given equation we can see that $\frac{d^2U}{dx^2} = 2a$ (positive), i.e. U minimum.

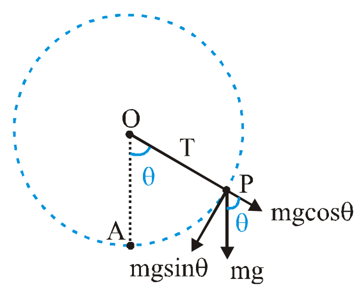
Therefore, $x = \frac{b}{2a}$ is the stable equilibrium position.

CIRCULAR MOTION IN VERTICAL PLANE

Suppose a particle of mass m is attached to an inextensible light string of length R. The particle is moving in a vertical circle of radius R about a fixed point O. It is imparted a velocity u in horizontal direction at lowest point A. Let v be its velocity at point P of the circle as shown in figure. Here,

$h = R(1 - \cos\theta)$ ....**(i)**

From conservation of mechanical energy



$\frac{1}{2}m\left(u^2 - v^2\right) = mgh \Rightarrow v^2 = u^2 - 2gh$ ....**(ii)**

The necessary centripetal force is provided by the resultant of tension T and mg cosq

$T - mg\cos\theta = \frac{mv^2}{R}$ ....**(iii)**

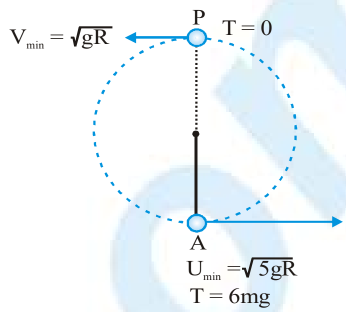
Since speed of the particle decreases with height, hence tension is maximum at the bottom, where $\cos\theta = 1$ (as $\theta = 0^o$)

$\Rightarrow T_{max} = \frac{mv^2}{R} + mg$; $T_{min} = \frac{mv'^2}{R} - mg$ at the top. Here v` = speed of the particle at the top.

1. CONDITION OF LOOPING THE LOOP $\left(u \geq \sqrt{5gR}\right)$

The particle will complete the circle if the string does not slack even at the highest point ($\theta = \pi$). Thus tension in the string should be greater than or equal to zero ($T \geq 0$) at $\theta = \pi$. In critical case substituting $T = 0$ and $\theta = \pi$

in Eq. **(iii)**, we get $mg = \frac{mv^2_{min}}{R} \Rightarrow v_{min} = \sqrt{gR}$ (at highest point)



Substituting $\theta = \pi$ in Eq. **(i)** , Therefore from Eq. **(ii)**

$$u^2_{min} = v^2_{min} + 2gh = gR + 2g(2R) = 5gR \Rightarrow u_{min} = \sqrt{5gR}$$

Thus, if $u \geq \sqrt{5gR}$, the particle will complete the circle. At $u = \sqrt{5gR}$,

velocity at highest point is $v = \sqrt{gR}$ and tension in the string is zero.

Substituting as $\theta = 0^o$ and $v = \sqrt{5gR}$ in Eq. **(iii)** we get T = 6 mg or in the critical condition tension in the string at lowest position is 6 mg. This is shown in figure. If $u < \sqrt{5gR}$, following two cases are possible.

2. CONDITION OF LEAVING THE CIRCLE $\left(\sqrt{2gR} < U < \sqrt{5gR}\right)$

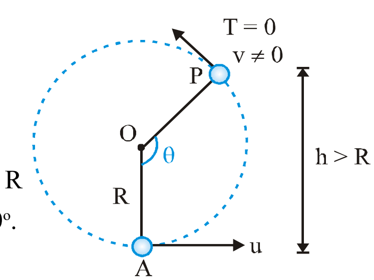
If $u < \sqrt{5gR}$, tension in the string will become zero before reaching the highest point. From Eq. **(iii)** tension in the string becomes zero (T = 0) where, $\cos\theta = \frac{-v^2}{Rg} \Rightarrow \cos\theta = \frac{2gh - u^2}{Rg}$

Substituting, this value of cos θ in Eq. **(i)** we get $\frac{2gh - u^2}{Rg} = 1 - \frac{h}{R} \Rightarrow h = \frac{u^2 + Rg}{3g} = h_1$ (say) ......**(iv)**

or we can say that at height h1 tension in the string becomes zero. Further, if $u < \sqrt{5gR}$, velocity of the particle

becomes zero when $0 = u^2 - 2gh \Rightarrow h = \frac{u^2}{2g} = h_2$ (say) .....**(v)** i.e., at height h2 velocity is not zero. or T = 0 but $v \neq 0$. This is possible only when h1 < h2

$$\frac{u^2 + Rg}{3g} < \frac{u^2}{2g} \Rightarrow 2u^2 + 2Rg < 3u^2 \Rightarrow u^2 > 2Rg \Rightarrow u > \sqrt{2Rg}$$



Therefore, if $\sqrt{2gR} < u < \sqrt{5gR}$, the particle, will leave the circle when h > R if $u^2 > 2gR$. Thus, the particle, will leave the circle when h > R or $90^o < \theta < 180^o$.
This situation is shown in the figure

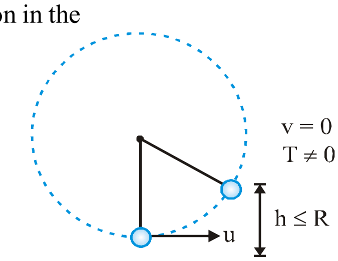
$$\sqrt{2gR} < u < \sqrt{5gR} \quad \text{or} \quad 90^o < \theta < 180^o.$$

Note: After leaving the circle, the particle will follow a parabolic path.

3. CONDITION OF OSCILLATION $\left(0 < U < \sqrt{2gR}\right)$

The particle will oscillate if velocity of the particle becomes zero but tension in the string is not zero or v = 0, but
$T \neq 0$. This is possible when $h_2 < h_1$

$$\Rightarrow \frac{u^2}{2g} < u < \frac{u^2 + Rg}{3g} \Rightarrow 3u^2 < 2u^2 + 2Rg \Rightarrow u^2 < 2Rg \Rightarrow u < \sqrt{2Rg}$$



Moreover, if $h_1 = h_2$, $u = \sqrt{2gR}$ and tension and velocity both becomes zero simultaneously. Further, from Eq. **(iv)**, we can see that $h \le R$ if

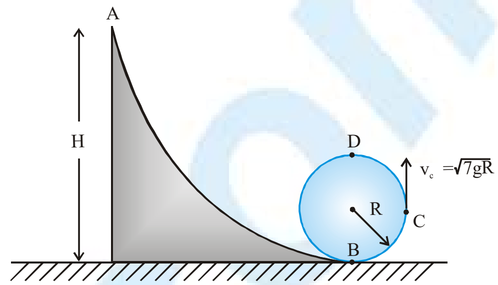
$u \le \sqrt{2gR}$.

Thus, for $0 < u \le \sqrt{2gR}$, particle oscillates in lower half of the circle ($0^o < \theta < 90^o$)

This situation is shown in the figure. $0 < u \le \sqrt{2gR}$ or $0^o < \theta < 90^o$

Ex. Calculate following for shown situation

(a) Speed at D **(b)** Normal reaction at D **(c)** Height H



Sol.

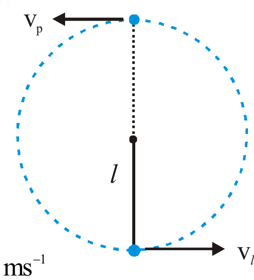
(a) $v_D{}^2 = v_C{}^2 - 2gR = 5gR \Rightarrow v_D = \sqrt{5gR}$

(b) $mg + N_D = \frac{mv_D^2}{R} \Rightarrow N_D = \frac{m(5gR)}{R} - mg = 4mg$

(c) by energy conservation between point A & C

$$mgH = \frac{1}{2}mv_D^2 + mgR = \frac{1}{2}m(5gR) + mg2R = \frac{9}{2}mgR \Rightarrow H = \frac{9}{2}R$$

Ex. A stone of mass 1 kg tied to a light string of length $l = \frac{10}{3}$ m is whirling in a circular path in vertical plane. If the ratio of the maximum to minimum tension in the string is 4, find the speed of the stone at the lowest and highest point.



Sol. $\therefore \frac{T_{max}}{T_{min}} = 4$ $\therefore \frac{\frac{mv_l^2}{l} + mg}{\frac{mv_p^2}{l} - mg} = 4 \Rightarrow \frac{v_l^2 + gl}{v_p^2 - gl} = 4$

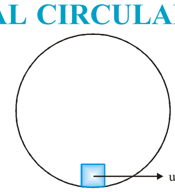
we know $v_l^2 = v_p^2 + 4gl \Rightarrow \frac{v_l^2 + 5gl}{v_p^2 - gl} = 4 \Rightarrow 3v_p{}^2 = 9g$●

$\Rightarrow v_p = \sqrt{3gl} = \sqrt{3 \times 10 \times \frac{10}{3}} = 10ms^{-1} \Rightarrow v_l = \sqrt{7gl} = \sqrt{7 \times 10 \times \frac{10}{3}} = 15.2 ms^{-1}$

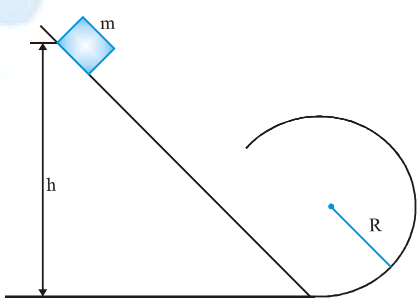
$\Rightarrow T = (mg + ma) + 2m(g + a)(1 - \cos\theta) = m(g + a)(3 - 2\cos\theta)$

MOTION OF A PARTICLE AT THE INNER SURFACE OF A VERTICAL CIRCULAR TRACK

Same results as above are obtained when a body is given some speed u at the bottom most point inside a fixed circular loop as shown. Here instead of tension; normal force due to inner surface of the loop comes into action.



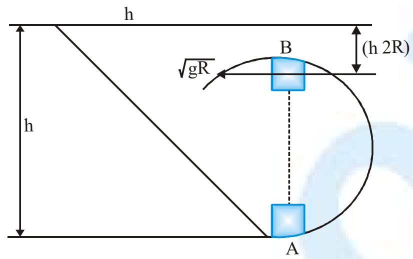
Ex. Figure shows a an incline which ends into a circular track of radius R. What should be the minimum value of height (h). so that the small object shown after release, is able to complete the loop. Neglect friction.



Sol. It completes the loop, if its speed is atleast $\sqrt{gR}$ at B and $\sqrt{5gR}$ at A. Applying work energy theorem, for motion starting point to the point B.

$$mg\,(h-2R)=\frac{1}{2}m\ [\left(\sqrt{gR}\right)^2-0]$$

$\Rightarrow$ $h=\frac{5R}{2}$

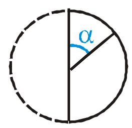


Alternatively : Applying work - energy theorem for motion from starting point to A.

$$mgh=\frac{1}{2}m\left[\left(\sqrt{5gR}\right)^2-0\right]$$

$\Rightarrow$ $h=\frac{5R}{2}$

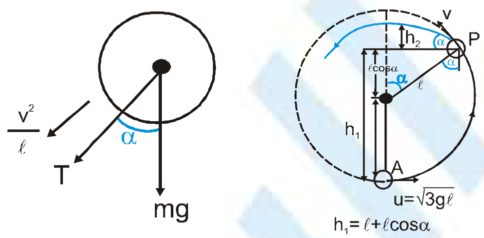
Ex. The BOB of a simple pendulum of length ● is given a sharp hit to impart it a horizontal speed of $\sqrt{3gl}$. When it was at its lowermost position. Find **(i)** angle α shown of the string from upside of vertical and speed of the particle when the string becomes slack. **(ii)** maximum height (from the bottom).



Sol. **(i)** Since $\sqrt{2gl}<u<\sqrt{5gl}$, the string slacks somewhere between horizontal point and the topmost point.

Let string slack at P, where speed is say v.

At point P,



$\Rightarrow$ $$T+mg\cos\alpha=\frac{mv^2}{1}$$

As the string slacks, T = 0

$\Rightarrow$ $$mg\cos\alpha=\frac{mv^2}{1}$$

$\Rightarrow$ $v=\sqrt{g1\cos\alpha}$ .... **(i)**

Applying work energy theorem for motion from A to P

$$-mg\ h_1=\frac{1}{2}m(v^2-u^2)$$ $\therefore$ from equation **(i)**

$$-mg\ (1+\cos\alpha)=\frac{1}{2}m[g(1\cos\alpha)-3g1]$$

$\Rightarrow$ $$\cos\alpha=\frac{1}{3}$$

$\Rightarrow$ $$\alpha=\cos^{-1}\left(\frac{1}{3}\right)$$

$\therefore$ equation **(i)**, $u=\sqrt{\frac{g1}{3}}$

(ii) Now, after slackening of the string, the motion of the bob is under gravity only, for which the maximum height from P is given by

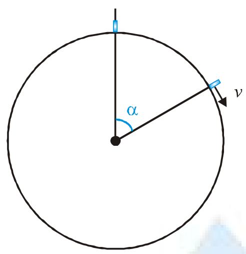
$$h_2=\frac{v^2\sin^2\alpha}{2g}$$

where $v_2 = \frac{gl}{3}$ and $\sin^2\alpha = 1-\cos^2\alpha = 1-\left(\frac{1}{3}\right)^2 = \frac{8}{9}$

$\therefore \qquad h_2 = \frac{\left(\frac{gl}{3}\right)\left(\frac{8}{9}\right)}{2g} = \frac{41}{27}$

$\therefore \qquad$ maximum height from A is $= h_1 + h_2 = l\left(1+\frac{1}{3}\right)+\frac{41}{27} = \frac{401}{27}$

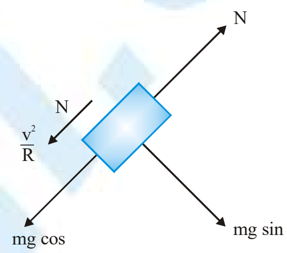
Ex. A particle slides on the surface of a fixed smooth sphere starting from the topmost point. Find the angle rotated by radius through the particle, where it leaves contact with the sphere. Also find speed at that instant.



Sol. Let it rotates by angle α

$$mg\cos\alpha - N = \frac{mv^2}{R}$$

It looses contact i.e. N = 0

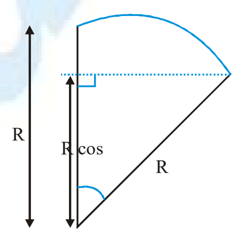


$\Rightarrow \qquad mg\cos\alpha = \frac{mv^2}{R}$

$\Rightarrow \qquad v^2 = gR\cos\alpha \qquad$ .....**(i)**

Also applying work energy theorem

$$mgh = \frac{1}{2}m\left(v^2-0\right)$$



$\Rightarrow \qquad mgR = (1-\alpha) = \frac{1}{2}m\left(gR\cos\alpha - 0\right)$

$\Rightarrow \qquad \cos\alpha = \frac{1}{3}$

$\Rightarrow \qquad \alpha = \cos^{-1}\left(\frac{1}{3}\right)$

Also from equation **(i)**, $\qquad v = \sqrt{\frac{gR}{3}}$

• Etoos Tips & Formulas •

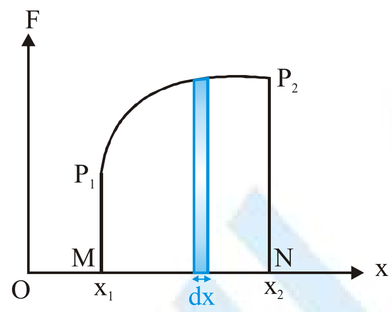
1. **Work done** $W = \int dW = \int \vec{F}.d\vec{r} = \int Fdr\cos\theta$ [where θ is the angle between $\vec{F}$ & $d\vec{r}$]

(a) For constant force $W = \vec{F}.\vec{d} = Fd\cos\theta$

(b) For Unidirectional force

$$W = \int dW = \int Fdx = \text{Area between F -x curve and x- axis.}$$

2. **Calculation of work done from force-displacement graph**



Total work done, $W = \sum_{x_1}^{x_2} dW = \sum_{r_1}^{r_2} Fdx = \text{Area of } P_1P_2NM = \int_{x_1}^{x_2} Fdx$

3. **Nature of work done**

Although work done is a scalar quantity, yet its value may be positive, negative or even zero

(a) If $\vec{F}$ is a conservative force then $\vec{\nabla} \times \vec{F} = \vec{0}$ (i.e. curl of $\vec{F}$ is zero)

4. **Conservative Forces**

(a) Work done does not depend upon path.
(b) Work done in a round trip is zero.
(c) Central forces, spring forces etc, are conservative forces
(d) When only a conservative forces acts within a system, the kinetic energy and potential energy can change into each other. However, their sum the mechanical energy of the system, doesn`t change.
(e) Work done is completely recoverable.

5. **Non-conservative Forces**

(a) Work done depends upon path.
(b) Work done in a round trip is not zero.
(c) Force are velocity-dependent & retarding in nature e.g. friction, viscous force etc.
(d) Work done against a non-conservative force may be dissipated as heat energy
(e) Work done is not recoverable.

6. **Kinetic energy**

(a) The energy possessed by a body by virtue of its motion is called kinetic energy.

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m(\vec{v}.\vec{v})$$

(b) Kinetic energy is a frame dependent quantity because velocity is a frame depends.

7. Potential energy

(a) The energy which a body has by virtue of its position or configuration in a conservative force field

(b) Potential energy is a relative quantity.

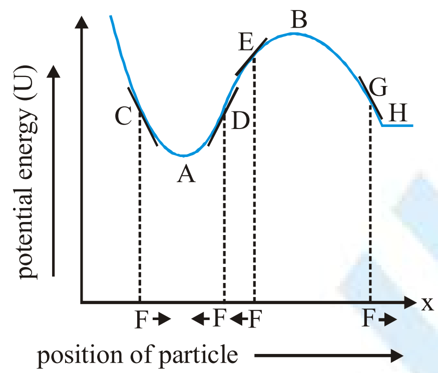
(c) Potential energy is defined only for conservative force field and potential energy :

$$\vec{F} = -\nabla U = -\text{grad}(U) = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} - \frac{\partial U}{\partial z}\hat{k}$$

(d) If force vaires only one dimension (along x-axis) then

$$F = -\frac{dU}{dx} \Rightarrow U = -\int_{x_1}^{x_2} F dx$$

8. Potential energy curve and equilibrium



It is a curve which shows change in potential energy with position of a particle.

9. Stable Equilibrium

When a particle is slightly displaced from equilibrium position and it tend to come back towards equilibrium then it is said to be in stable equilibrium

At point **C** : slope $\frac{dU}{dx}$ is negative so F is positive

At point **D** : slope $\frac{dU}{dx}$ is positive so F is negative

At point **A** : It is the point of stable equilibrium.

$$U = U_{min}, \ \frac{dU}{dx} = 0 \text{ and } \frac{d^2U}{dx^2} = \text{positive}$$

10. Unstable equilibrium

When a particle is slightly displaced from equilibrium and it tends to move away from equilibrium position then it is said to be in unstable equilibrium

At point **E** : slope $\frac{dU}{dx}$ is positive so F is negative

At point **G** : slope $\frac{dU}{dx}$ is negative so F is positive

At point **B** : slope It is the point of unstable equilibrium.

$$U = U_{max}, \ \frac{dU}{dx} = 0 \text{ and } \frac{d^2U}{dx^2} = \text{negative}$$

11. Neutral equilibrium

When a particle is slightly displaced from equilibrium position and no force acts on it then equilibrium is said to be neutral equilibrium. Point H is at neutral equilibrium $\Rightarrow$ U = constant ; $\frac{dU}{dx} = 0, \frac{d^2U}{dx^2} = 0$

12. Work energy theorem

$W = \Delta KE$

Change in kinetic energy = work done by all force

13. For conservative force

$F(x) = -\frac{dU}{dx}$

Change in potential energy $DU = -\int F(x)dx$

14. Law of conservation of Mechanical energy

Total mechanical (kinetic + potential) energy of a system remains constant if only conservative forces are acting on the system of particles or the work done by all other forces is zero. From work energy theorem $W = \Delta KE$

Proof : For internal conservative forces $W_{int} = -\Delta U$

So $W = W_{ext} + W_{int} = 0 + W_{int} = -\Delta U \Rightarrow -\Delta U = -\Delta KE$

$\Rightarrow$ (KE + U) = 0 $\Rightarrow$ KE + U = (constant)

Spring force $F = -kx$, Elastic potential energy astored in spring $U(x) = \frac{1}{2}kx^2$

Mass and energy are equivalent and are related by $E = mc^2$

15. Power

Power is a scalar quantity with dimension $M^1 L^2 T^{-3}$

SI unit of power is J/s or watt

1 horsepower = 746 watt = 550 ft-lb/sec.

Average power $P_{av} = W/t$

Instantaneous power $P = \frac{dW}{dt} = \frac{\vec F . d\vec r}{dt} = \vec F . \vec v$

For a system of varying mass $\vec F = \frac{d}{dt}(m\vec v) = m\frac{d\vec v}{dt} + \vec v = \frac{dm}{dt}$

If $\vec v$ = constant then $\vec F = \vec v \frac{dm}{dt}$ then $P = \vec F . \vec v = v^2 \frac{dm}{dt}$

In rotatory motion : $P = \tau \frac{d\theta}{dt} = \tau\omega$

16. A body may gain kinetic energy and potential energy simultaneously because principle of conservation of mechanical energy may not be valid every time.

17. Comets moves around the sun in elliptical orbits. The gravitational force on the comet due to sun is not normal to the comet`s velocity but the work done by the gravitational force is zero in complete round trip beacuse gravitational force is a conservative force.

18. Work done by static friction may be positive beacuse static friction may acts along the direction of motion of an object.