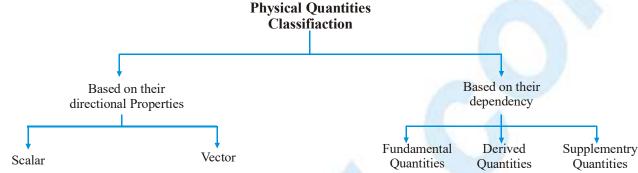
• Unit & dimensions •

INTRODUCTION

The quantities which can be measured by an instrument and by means of which we can describe the law of physics are called physical quantities

CLASSIFICATION

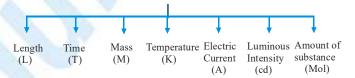
Physical quantities can be classified on the following bases.



- 1. Based on their directional properties
 - (i) Scalars Quantities: Which have only magnitude but not direction.
 - e.g. Density, time, Electric current, mass, volume etc.
 - (ii) Vector Quantities: Which have both magnitude and direction and follow the law of vector algebra
 - e.g. Force, velocity, Displacement etc.
- 2. Based on their dependency:
- (1) Fundamental quantities:

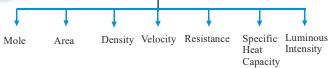
The quantities which do not depends upon other quantities for their complete definition are known as fundamental or base quantities.

- (i) These are the elementary quantities which covers the entire space of physics.
- (ii) Any other quantities can be derived from these.
- (iii) All the basic quantities are chosen such that they should be different, that means independent of each other i.e., distance (d), time (t), and velocity (v) cannot be chosen basic quantities (because they are related as V = d/t). An international organisation named CGMP (General conference on weight and Measure chosen seven physical quantities as basic or fundamental



These are the elementary quantities (in our planet) that's why chosen as basic quantities.

In fact any set of independent quantities can be chosen as basic quantities by which all other physical quantities can be derived.





PHYSICS FOR JEE MAIN & ADVANCED

Can be chosen as basic quantities (on some other planet, these might also be used as basic quantities).

But Area Velocity Length

Cannot be used as basic quantities as

Area = $(Lenght)^2$ so they are not independent.

(2) Derived Quantities:

The quantities which can be expressed in term of the fundamental quantities (M, L, T.....) are known as derived quantities.

i.e., Speed =
$$\frac{\text{distance}}{\text{time}}$$

Volume, acceleration, force, pressure etc.

(3) Supplementary Quantities:

Besides seven fundamental quantities two supplementary quantities are also defined.

(a) Plane angle (radian)

The radian is the angle subtended at the centre of a circle by an arc on its circumference equal in the length to the radius of the circle.

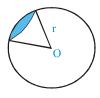


As the circumference of the circle is equal to 2π times the radius there will be 2π radian in a complete circle

$$\therefore$$
 1 rad = $\frac{360}{2\pi} = 57.3^{\circ}$

(b) Solid angle (steradian)

The steradian is the angle subtended at the centre of a sphere by a spherical area on the surface of the sphere, where the spherical area has a circular boundary and an area equal to the square of the radius of the sphere.



As the surface area of sphere is equal to 4π times the radius squared, the total solid angle at the centre of the sphere is equal to 4π steradian

- Ex. Classify the quantities displacement, mass, force, time, speed, velocity, acceleration, moment of inertia, pressure and work under the following categories:
 - (a) base and scalar
- (b) base and vector
- (c) derived and scalar
- (d) derived and vector

- Sol. (a) mass, time
- (b) displacement
- (c) speed, pressure, work (d) force, velocity, acceleration

UNITS

All physical quantities are measured w.r.t. standard magnitude of the same physical quantity and these standards are called units e.g. meter, kilogram, second etc.

The requisites for fundamental quantities and their properties of unit:

- (i) They are well defined and suitable size.
- (ii) They should be easily available and reproducible at all places.
- (iii) They are independent of other fundamental quantities



- (iv) They are not subject to changes with time
- (v) They should not vary with physical conditions like temperature, pressure etc.
- (si) It should be possible to express any other physical quantity in term of fundamental quantities.
- (vii) They should be accepted to all

Classification of units:

Fundamental or base units;

The unit of fundamental quantities are called base units. In SI there are seven base unit.

2. Derived unit:

The unit of derived quantities or the units that ear be expressed in term of the base units are called derived units.

Some derived units are named in honour of great scientist.

Unit of force - newton (N)

Unit of frequency - hortz (Hz)

Set of Fundamental Quantities

A set of physical quantities which are completely independent of each other and all other physical quantities can be expressed in terms of fundamental quantities.

System of Units

(a) FPS or British Engineering system :

In this system length, muss and time are taken as fundamental quantities and their base units are foot (ft.), pound (fb) and second (s).

(b) CGS or Gaussian system :

In this system the fundamental quantities are length, mass and time but their respective units are confineter (cm), grum (g) and second (s).

(c) MKS system :

In this system also the fundamental quantities are length, mass and time but their fundamental units are metre (iii), alinguam (kg) and second (s).

Units of some physical quantities in Different systems

| Type of Physical Quantity | Physical Quantity | CGS (Originated in France) | VIKS (Originated in France) | (Originated in France) |
|------------------------------|----------------------|-------------------------------|--------------------------------|---------------------------|
| Fundamental | Length | gn gn | ж | ï, |
| | Vlass | g . | kŁ | Lo |
| | line | 5 | 3 | 5 |
| Derived | Force | SUVO | newton (N) | Poundat |
| | Work or Energy | erg | jou'e (Ti | fi-perinda" |
| | Power | eigés | wattr/W) | .l- poundats |

INTERNATIONAL SYSTEM OF UNITS (SI)

At it various meeting the international body CGPM selected a set of seven quantities as base units which is now known as International system of unit. It is abbreviated as SI from the french name Le systeme International distributions. This system is widely used throughout the world.

[&]quot;This system is modification over the MKS system and so it is also known as Rationalised MKS system"



PHYSICS FOR JEE MAIN & ADVANCED

Ex. Find the SI unit of speed and acceleration.

Sol. Speed = $\frac{\text{distance}}{\text{time}} = \frac{\text{meter}(m)}{\text{seconds}(s)} = m/s \text{ (called as mater per second)}$

acceleration $\frac{\text{velocity}}{\text{time}} = \frac{\text{displacement/time}}{\text{time}} = \frac{\text{displacement}}{(\text{time})^2} = \frac{\text{meter}}{\text{second}^2} = \frac{\text{nn/s}^2(\text{meter per second square})}{\text{nn/s}^2(\text{meter per second square})}$

Ex. Find the SI unit of (i) area (ii) density (iii) momentum

Sol. (i) n# (ii) \g/m² (iii) kg/m/s.

SI Base Quantities and Units

| | St Units | | | | |
|--------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|--|
| Base Quantity | Name | Symbol | Definition | | |
| Length | meter | in | The motor is the length of the path traveled by light in vaccum during a time interval of 1/299, 792, 458 of a second (1987) | | |
| Mass | kilogram | kg | The kilegram is equal to the riess of the international prototype of the kilogram (a platinum-iridium alloy by inder) kept at International Bureau of Weight and Measures, at Sevies, near Paris, France. (1889) | | |
| Lime | seamd | 8 | The second in the duration of 9, 192, 631, 770 periods of the radiation corresponding to the transition between the two byperfine levels of the ground state of the perfun-133tom (1967) | | |
| Liectrio Current | Enicere | | The ampere is that constant current which, if maintained in two straight percilet conductor of infinite length, of negfigible discular cross-section, placed I meter agant in years in, would produce between these conductors a force equal to 2 x 10 ¹² . Newton permanent length, (1948) | | |
| Facimo dynamic Foniperature | kely în | K | The kelvin, is the itaction 1/273, in of the thermodynamic temperature of the triple point of water. (1967) | | |
| Amount of Substance | mole | mol | The mole is the amount of substance of daystem, which contains as many elementry critics as there are atoms in 0.012 kilogram of earbon2. (1971) | | |
| Laminous Intensity | The candela is the luminous intensity, in a given direction, of a source that emits monochromatics candela. Cd radiation of frequency 5-0 x 10 ¹⁷ hortz and that has a radiant intensity in that direction of 1/685 watt per stending (1979). | | | | |

Supplementary units

In SI, two supplementary units are also defined viz.

- (i) radian (rad) for plane angle and
- (ii) steradian (sr) for solid angle.

| Ownering | 611.mm | | |
|-------------|------------|--------|--|
| Quantity | Name | Symbol | |
| Plane angla | radia y | toc: | |
| Selet ang c | a terachar | £1 | |

- (i) Radian: Radian is the angle substended at the centre of a circle by an arc equal in length to the radius of the circle.
- (ii) Steradian: Steradian is the solid angle subtended at the centre of a sphere, by that surface of the sphere which is equal in area to the square of the radius of the sphere.

Practical units:

Due to the fixed sizes of SI units, some practical units are also defined for both fundamental and derived quantities. e.g. light year (ly) is a practical unit of astronomical distance (a fundamental quantity) and horse power (hp) is a practical unit of power (a derived quantity). Practical units may or may not belong to a particular system of units but can be expressed in any system of units. e.g. 1 mile = $1.6 \text{ km} 1.6 \times 10^3 \text{ m} = 1.6 \times 10^5 \text{ cm}$.

Improper units:

These are the units which are not of the same nature as that of the physical quantities for which they are used. e.g. kg-wt is an improper unit of weight. Here kg is a unit of mass but it is used to measure the weight (force).

DIMENSIONS & DIMENSIONAL FORMULA

1. Dimensions

Dimension of a physical quantity are the power (or exponents) to which the base quantities are raised to represent that quantity.

Its is written by enclosing the symbols for base quantities with appropriate power in square brackets i.e. [] to make it clear, consider the physical quantity "force"

Force $= mass \times acceleration$

$$= \max_{mass} \times \frac{length/time}{time}$$

= mass \times length \times (time)²

So the dimensions of force are 1 in mass, 1 in length and -2 in time. Thus

$$[Force] = MLT^{-2}$$

Similarly energy has dimensional formula given by

$$[Energy] = ML^2T^{-2}$$

i.e., energy has dimensions, 1 in mass, 2 in length, and -2 in time.

Such an expression for a physical quantity in terms of base quantities is called dimensional formula.

2. Dimensional equation

The equation obtained by equating a physical quantity with its dimensional formula is called a dimensional equation.

e.g.
$$[v] = [M^0L^1T^{-1}]$$

For example $[F] = [MLT^{-2}]$ is a dimensional equation, $[MLT^{-2}]$ is the dimensional formula of the force and the dimensions of force are 1 in mass, 1 in length and -2 in time

3. Principle of Homogeneity

According to this principle, we can multiply physical quantities with same or different dimensional formulae at our convenience, however no such rule applies to additional and substraction, where only like physical quantities can only be added or substracted.

e.g. If $P + Q \implies P \& Q$ both represent same physical quantity.



PHYSICS FOR JEE MAIN & ADVANCED

- Calculate the dimensional formula of energy from the equation $E = \frac{1}{2}mv^2$. Ex.
- Sol. Dimensionally, $E = mass \times (velocity)^2$.

Since $\frac{1}{2}$ is a number and has no dimension.

or [E]=
$$M\left(\frac{L}{T}\right)^2 = \left[ML^2T^{-2}\right]$$
.

- Ex. Kinetic energy of a particle moving along elliptical trajectory is given by $K = \alpha s^2$ where s is the distance travelled by the particle. Determine dimensions of α
- $K = \alpha s^2$ Sol.

$$\left[\alpha\right] = \frac{\left[ML^2T^{-2}\right]}{\left[L^2\right]} \qquad \Rightarrow \qquad \left[\alpha\right] = \left[M^1L^0T^{-2}\right] \qquad \Rightarrow \qquad \left[\alpha\right] = \left[M^1T^{-2}\right]$$

- The position of a particle at time t, is given by the equation, $x(t) = \frac{v_0}{\alpha} (1 e^{\alpha t})$, where v_0 is a constant and $\alpha > 0$. Ex. The dimensions of v_0 & α respectively.
 - (A) $M^0L^1T^0 & T^{-1}$
- **(B)** $M^0L^1T^{-1} & T$
- (C) $M^0L^1T^{-1} \& T^{-1}$ (D) $M^1L^1T^{-1} \& LT^{-2}$

- Sol.
- $[V_0] = [x][\alpha] \qquad & [\alpha][t] = M^0 L^0 T^0 \qquad \Rightarrow \qquad [V_0] = M^0 L^1 T^{-1}, \quad [\alpha] = M^0 L^0 T^{-1}$
- Ex. The distance covered by a particle in time t is going by $x = a + bt + ct^2 + dt^3$; find the dimensions of a, b, c and d.
- Sol. The equation contains five terms. All of them should have the same dimensions. Since [x] = length, each of the remaining four must have the dimension of length.

- $[dt^3] = L$, or $[d] = LT^{-3}$ and
- 4. **Application of Dimensional Analysis:** To convert units of a physical quantity from one system of units to another. **(i)**

It is based on the fact that,

Numerical value \times unit = constant

So on changing unit, numerical value will also gets changed. If n_1 and n_2 are the numerical values of a given physical quantity and u_1 and u_2 be the units respectively in two different systems of units, then

$$n_1 u_1 = n_2 u_2$$

$$n_2 = n_1 \left[\frac{M_1}{M_2} \right]^a \left[\frac{L_1}{L_2} \right]^b \left[\frac{T_1}{T_2} \right]^c$$

Here,

 n_{j} numerical value in I system numerical value in II system Munit of mass in I system unit of mass in II system L, unit of length in I system unit of length in II system L_{i} unit of time in II system unit of time in I system T T_{i}

This is based on a fact that magnitude of a physical quantity remains same whatever system is used for measurement i.e. magnitude = numeric value (n) \times unit (u) = constant or $n_1u_1 = n_2u_2$

So if a quantity is represented by $M^a L^b T^c$

Then

$$n_2 = n_1 \left(\frac{u_1}{u_2}\right) = n_1 \left(\frac{M_1}{M_2}\right)^a \left(\frac{L_1}{L_2}\right)^b \left(\frac{T_1}{T_2}\right)^c$$

Ex. The dimensional formula for viscosity of fluids is,

$$\eta = M^{1}L^{-1}T^{-1}$$

Find how many poise (CGS unit of viscosity) is equal to 1 poiseuille (SI unit of viscosity).

Sol. $\eta = M^1 L^{-1} T^{-1}$

1 CGS units =
$$g_{Cm}^{-1}s^{-1}$$

1 SI units =
$$kg m^{-1} s^{-1}$$

=
$$1000 \text{ g} (100 \text{ cm})^{-1} \text{ s}^{-1} = 10 \text{ g} \text{ cm}^{-1} \text{ s}^{-1}$$

Thus, 1 poiseuilli = 10 poise

- **Ex.** Convert 1 newton (SI unit of force) into dyne (CGS unit of force)
- **Sol.** The dimensional equation of force is $[F] = [M^1L^1T^{-2}]$

Therefore if n_1 , u_1 and n_2 , u_2 corresponds to SI & CGS units respectively, then

$$n_2 = n_1 \left[\frac{M_1}{M_2} \right]^1 \left[\frac{L_1}{L_2} \right]^1 \left[\frac{T_1}{T_2} \right]^{-2} = 1 \left[\frac{kg}{g} \right] \left[\frac{m}{cm} \right] \left[\frac{s}{s} \right]^{-2} = 1 \times 1000 \times 100 \times 1 = 10^5$$

- \therefore 1 newton = 10^5 dyne
- **Ex.** The acceleration due to gravity is 9.8 m/s^{-2} . Give its value in ft/ s^{-2}
- **Sol.** As 1 m = 3.2 ft

$$\therefore$$
 9.8 × 3.28 ft / s^2 = 32.14 ft / $s^2 \approx 32$ ft / s^2

(ii) To check the dimensional correctness of a given physical relation:

If in a given relation, the term on both the sides have the same dimensions, then the relation is dimensionally correct. This is known as the principle of homogeneity of dimensions.

- Ex. Check the accuracy of the relation $T = 2\pi \sqrt{\frac{L}{g}}$ for a simple pendulum using dimensional analysis.
- **Sol.** The dimensions of LHS = the dimension of $T = [M^0L^0T^1]$

The dimension of RHS = $\left(\frac{\text{dimension of length}}{\text{dimension of acceleration}}\right)^{1/2}$ (Q 2π is a dimensionless constant)

$$= \left[\frac{L}{LT^{-2}} \right]^{1/2} = \left[T^2 \right]^{1/2} = \left[T \right] = \left[M^0 L^0 T^1 \right]$$

Since the dimensions are same on both the sides, the relation is correct.

(iii) To derive relationship between different physical quantities:

Using the same principle of homogeneity of dimensions new relations among physical quantities can be derived if the dependent quantities are known.

Ex. It is known that the time of revolution T of a satellite around the earth depends on the universal gravitational constant G, the mass of the earth M, and the radius of the circular orbit R. Obtain an expression for T using dimensional analysis.

Sol. We have
$$[T] = [G]^r[M]^s[R]^r \Rightarrow [T] = [M]^r[L]^r[T] = [M]^r[L]^r \cdot [L]^r \cdot [M]^r \cdot [L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]^{r-1}[L]$$

$$\mathsf{For}[\mathsf{TI}]: 1 = 2a \Rightarrow a = \frac{1}{2} \; , \; \mathsf{For}[\mathsf{MI}]: 0 = b + a \Rightarrow b = a + \frac{1}{2} \; , \qquad \mathsf{For}[\mathsf{LI}]: 0 = e - 3a \to e = -3a \frac{3}{2} \; .$$

Putting the values we get
$$|T| < G^{-22}M^{-12}R^{3/2} \approx \sqrt{\frac{R^3}{GM}}$$
. The actual expression is $T = -2\pi \sqrt{\frac{R^3}{GM}}$.

Unit and Dimensions of Some Physical Quantities

| Quantity | SIUnit | Dimensional | |
|--------------------------|--------------------|-------------------|--|
| Density | kg/m | M/L | |
| force | Newton (N) | ML/T ² | |
| Work | Joule (J) (= N -m) | ML/T | |
| Energy | Joule (J) | ML^2/T^2 | |
| Prover | watt (W) (= 1/s) | ML/T | |
| Momentum | sg-m/s | MIZT. | |
| Gravitational constant | N-m /kg | LAMIT | |
| Angular velocity | raidia.m/s | T" | |
| Angular acceleration | rnd inn/a* | T-2 | |
| Angular momentum | kg-m²/s | ML27I | |
| Moment of inertia | kg-m* | ML ² | |
| Torque | N-In | MUZE | |
| Angular frequency | radian/s | 1 | |
| Progrency | Hortz (Hz) | lo, | |
| Period | 4 | T | |
| Surface Leasion | N/m | M/Lå | |
| Coefficient of viscosity | N a/m² | MALT | |
| Wavelength | m | f. | |
| Intensity of wave | Wam ² | M21* | |

Dimensions of some Mathematical Function :

Dimensions of different queffic ent and integrals

In General
$$\begin{bmatrix} d^2v \\ dv^2 \end{bmatrix} \begin{bmatrix} v \\ v^2 \end{bmatrix}$$
 and $\left[\int f^{*}\dot{w}\right] = \left[v_f\right]$

Ex. Pind dimensional formula:

(i)
$$\frac{dx}{dt}$$
 (ii) $\frac{d^2y}{dt}$ (iii) $\int vdt$ (iv) $\int adt$

where x → displacement, c → time, v → velocity and a → acceleration

(i)
$$\left[\frac{\partial x}{\partial t}\right] = \left[\frac{x}{t}\right] = \left[\frac{t}{T}\right] = \left[M^T L T\right]$$

(i)
$$\left| \frac{dx}{dt} \right| = \frac{x}{t} = \frac{f}{T} = \left[M^T L T \right]$$
 (ii) $\left| m \frac{d^T x}{dt^2} \right| = \left| m \frac{x}{t^2} \right| = \left[ML \right] = \left[ML \right]$

(iii)
$$\left[vdv \right] = \left[vt \right] = \left[LT^{-1} \times T \right] = \left[M^{-1}LT^{0} \right]$$
 (iv) $\left[\int adt \right] \left[ut \right] \left[LT^{-2} \times T \right] \left[M^{2}LT^{-1} \right]$

(iv)
$$\int a dt = [at] = LT^{-1} \times T = [M^{2}LT]$$

Dimensions of trigonometric, exponential and logarithmic functions and their arguments are dimensionless.

Note: Trigonometric timetion via 0 are dimensionless.

Ex. If $\alpha = \frac{F}{v^2} \sin \beta t$, find dimensions of α and β . Here v – velocity, F – force and t – time.

Sol. Here sinßt and fit most be dimensionless

So,
$$[\beta t] + 1 \Rightarrow [\beta] + \begin{vmatrix} 1 \\ t \end{vmatrix} = \begin{vmatrix} T^{+} & t[\alpha] + \begin{vmatrix} F \\ e^{t} \end{vmatrix} \sin \beta t = \begin{vmatrix} F \\ t^{2} \end{vmatrix} = \begin{bmatrix} MLT^{-2} \\ t^{2}T^{2} \end{bmatrix} = MT$$

Limitations of Dimensional Analysis :-

- Concerness of the constant appearing in an equation connut be verified.
- (ii) While deriving an equation, the value of constant of proportionality cannot be obtained.
- (iii) Equation involving trigonometric and exponent at function cannot be verified or derived.
- (iv) An equation can be derived only if it is of product type.
- (v) We equate the power of M_s L-and T hence we get only three equations, so we can have only three variable (only three dependent quantities)

SI - PREFIXES :

The magnitudes of physical quantities vary over a wide range. The CGPM recommended standard prefixes for magnitude too large or too small to be expressed more contractly for declain power of 10.

Prefixes used for different power of 10

| Power of 10 | Prefix | Symbol | Power of 10 | Prefix | Symbol |
|------------------|--------|--------|-------------------|--------|--------|
| 10 ¹⁸ | exa | L | 10 | deci | ď |
| 10 5 | peta | P | 10 | centi | c |
| 1012 | tera | 1 | 103 | milli | m |
| 10 | giga | G | 10 € | micra | μ |
| 10 | mega | M | 10 ⁻⁵ | nano | n |
| 10 ⁵ | kilo | k | 10 ¹² | pico | р |
| 10 ² | hecto | h | 10 15 | femto | f |
| 10, | deca | ca | 10 ⁻¹⁸ | atto | а |

Some Physical Quantities with their symbols, units and dimensions

| Mik | Physical Quantity | Symbol | Definition/Papression | Unit in S1 | Dimension |
|-----|--------------------------|-----------------------------|---------------------------------------------|------------------------------------|----------------------------------------------|
| 1 | Vocania | V | length speath sheight | 'n' | 1,7 |
| 2 | Density | ρ | arass/volume | kg m² | All a |
| 1 | Volseity | 9.3 | displacement per unit time | ans/ | EZ. |
| de | Appelention | (a) | charge in velocity her unit time | ens x | IT' |
| 5 | Parage | ar . | mass vacceleration | newton (N) | MLT ² |
| 6 | Work | 11. | force x displacement | joule (1) | MLT" |
| 7 | Energy | E.U. & | capacity to de work | jeule (I) | MICTO |
| 8 | Power | Р | work done per unit time | watt (W) | ML L |
| 9 | Momentum | Р | mass x velochy | ិវញ្ញ កាន | MLT |
| 10 | Garitational constant | G | $\sigma = G \frac{m_1 m_2}{g^2}$ | Nri ² kg ² | M ³ L ³ L ³ |
| 11 | Angle | # <u>/</u> U | are length radius | radian | dimensionless |
| 12 | Angular velocity | 11. | angle described per unit time | cod si | т.' |
| 132 | Angular ness cretion | ı. | change in angular volvoiry perunditing | rad a " | 1,2 |
| 14 | Augularnymentum | 1. | linear momentur i x ner sembleu ar distance | kg m s | MLT ^d |
| 15 | Moment of inertia | P | mass x (radius or gyration) ² | kg m² | ML ² |
| 16. | Torque | ē:: | force a perpendicular Jistance | Иm | ML ² T |
| 17 | Street | | Firmerarea | Nm ² | MUT? |
| 18 | Shum | | change in length/origing/length | No unit | dimensionless |
| 19 | Young's modules Y | | stress/s min | Nn" | MILT? |
| 'n | Surface tension | l.n | force/length | Net | M12 |
| 21 | Pressure | Р | forço/crea | Nat Pa | ML'1 |
| 22 | Intensity of a wave | 1 | оленду Делен жтігас) | 9c ar 2 | M1, |
| 23 | Specific hear capacity | e e | heal/ (mass sichange in termerature) | J K ⁻¹ kp ⁻¹ | LTK |
| 24 | Heal | Q | a form of energy | 1 | V0 21.5 |
| 25 | Charge | Q. t. | current x frie | contomb (C) | AT |
| 26 | Current density | nt density I current/area | | A ri ² | A .7 |
| 27 | Electrical cipole inomer | p | elarga xaista tea | (' n | 131 |
| 28 | Electrical Potentia, (90 | vo v electric energy/ehargo | | volt (V) | MEA ^A I ^A |
| 29 | Flacuric field | F. | electric perential/distance | Var | MLA T |



• Etoos Tips & Formulas •

1. Fundamental or base quantities :

The quantities which do not depend upon other quantities for their complete definition are known as fundamental or base quantities. e.g.: length, mass, time, etc.

2. Derived quantities :

The quantities which can be expressed in terms of the fundamental quantities are known as derived quantities. e.g. Speed (=distance/time), volume, acceleration, force, pressure, etc.

3. Units of physical quantities

The chosen reference standard of measurement in multiples of which, a physical quantity is expressed is called the unit of that quantity.

Physical Quantity = Numerical Value × Unit

4. Supplementary Units

(a) Radian (rad) – for measurement of plane angle (b) Steradian (sr) – for measurement of solid angle

5. Dimensional formula

Physical quantity which express physical quantities in terms of appropriate powers of fundamental units.

6. Use of dimensional analysis

- (a) To check the dimensional correctness of a given physical relation
- (b) To derive relationship between different physical quantities
- (c) To convert units of a physical quantity from one system to the other

$$\mathbf{n}_{1}\mathbf{u}_{1} = \mathbf{n}_{2}\mathbf{u}_{2} \implies n_{2} = n_{1} \left(\frac{M_{1}}{M_{2}}\right)^{a} \left(\frac{L_{1}}{L_{2}}\right)^{b} \left(\frac{T_{1}}{T_{2}}\right)^{c} \text{ where } \mathbf{u} = \mathbf{M}^{a} \, \mathbf{L}^{b} \, \mathbf{T}^{c}$$

7. Limitations of this method

- (a) In Mechanics the formula for a physical quantity depending on more than three physical quantities cannot be derived. It can only be checked.
- (b) This method can be used only if the dependency is of multiplication type. The formulae containing exponential, trigonometrical and logarithmic functions can't be derived using this method. Formulae containing more than one term which are added or subtracted like $s = ut + \frac{1}{2}at^2$ also can't be derived.
- (iii) The relation derived from this method gives no information about the dimensionless constants.
- (iv) If dimensions are given, physical quantity may not be unique as many physical quantities have the same dimensions
- (v) It gives no information whether a physical quantity is a scalar or a vector.

8. SI PREFIXES

The magnitudes of physical quantities vary over a wide range. The CGPM recommended standard prefixes for magnitude too large or too small to be expressed more compactly for certain powers of 10.

- 9. Trigonometric functions $\sin \theta$, $\cos \theta$, $\tan \theta$ etc. and their arrangements θ are dimensionless.
- 10. Dimensions of differential coefficients $\left[\frac{d^n y}{dx^n}\right] = \left[\frac{y}{x^n}\right]$
- 11. Dimensions of integrals $\left[\int y dx \right] = \left[yx \right]$
- 12. We can't add or subtract two physical quantities of different dimensions.
- 13. Independent quantities may be taken as fundamental quantities in a new system of units.