FRICTION

Friction

Whenever surfaces in contact are pressing each other slide or tend to slide over each other, opposing forces are generated tangentially to the surfaces in contact. These tangential forces, which oppose sliding or tendency of sliding between two surfaces are called frictional forces. Frictional forces on both bodies constitute third law action-reaction pair.

TYPES OF FRICTION

All types of frictional phenomenon can be categorized are

Friction
↓ Dry Friction ↓ Fluid Friction ↓ Internal Friction

1. Dry friction:

It exists when two solid un-lubricated surfaces are in contact under the condition of sliding or tendency of sliding. It is also known as Coulomb friction.

2. Fluid friction:

Fluid friction is developed when adjacent layers of a fluid move at different velocities and gives birth to phenomena, which we call viscosity of the fluid. Resistance offered to motion of a solid body in a fluid also comes in this category and commonly known as viscous drag. We will study this kind of friction in fluid mechanics

3. Internal friction:

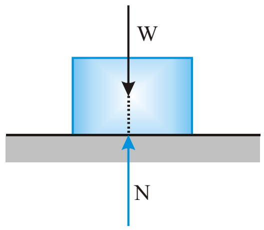
When solid materials are subjected to deformation, internal resistive forces developed because of relative movement of different parts of the solid. These internal resistive forces constitute a system of force, which is defined as internal friction. They always cause loss of energy.

Frictional forces exist everywhere in nature and result in loss of energy is primarily dissipated in form of heat. Wear and tear of moving bodies in another unwanted result of friction. Therefore, sometimes, we try to reduce their effect - such as in bearings of all types, between piston and the inner walls of the cylinder of an IC engine, flow of liquid in pipes, and aircraft and missile propulsion through air. Though these examples create a negative picture of frictional forces, yet there are other situations where frictional forces becomes essential and we try to maximize the effects. It is the friction between our feet and the earth surface, which enables us to walk and run. Both the traction and braking of wheeled vehicles depends on friction.

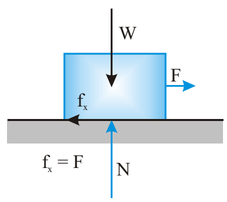
TYPES OF DRY FRICTION

In mechanics of non- deformable bodies, we are always concerned with the dry friction. Therefore, we often drop the word "dry" and simply call it friction.

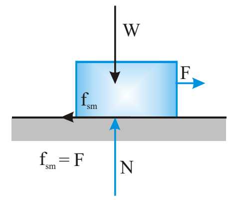
To understand nature of friction let us consider a box of weight W placed on a horizontal rough surface. The forces acting on the box are its weight and reaction from the horizontal surface. They are shown in the figure. The weight does not have any horizontal component, so the reaction of the horizontal surface on the box is normal to the surface. It is represented by N in the figure. The box is in equilibrium therefore both W and N are equal in magnitude, opposite in direction, and collinear..



Now suppose the box is being pulled by a gradually increasing horizontal force F to slide the box. Initially when the force F is small enough, the box does not slide. This can be explained if we assume a frictional force, which is equal in magnitude and opposite in direction to the applied force F acts on the box. The force F produce in the box a tendency of sliding and the friction force is opposing this tendency of sliding. The frictional force developed before sliding initiates is defined as static friction. It opposes tendency of sliding.

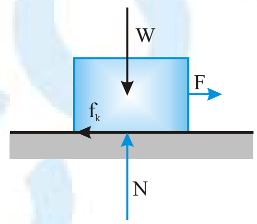


Static friction



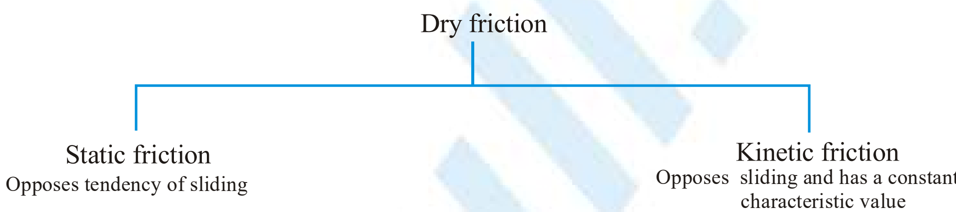
Limiting friction: The maximum static friction

As we increase F, the box remains stationary until a value of F is reached when the box starts sliding. Before the box starts sliding, the static friction increases with F and counterbalances F until the static friction reaches its maximum value known as limiting friction or maximum static friction f_{sm}.



Kinetic friction

When the box starts sliding, to maintain it sliding still a force F is needed to over come frictional force is known as kinetic friction (f_k). It always opposes sliding.



LAWS OF FRICTION

When a normal force N exists between two surfaces, and we try to slide them over each other, the force of static friction (f_s) acts before sliding initiates. It can have a value maximum up to the limiting friction (f_{sm}).

$$fs \leq fsm$$

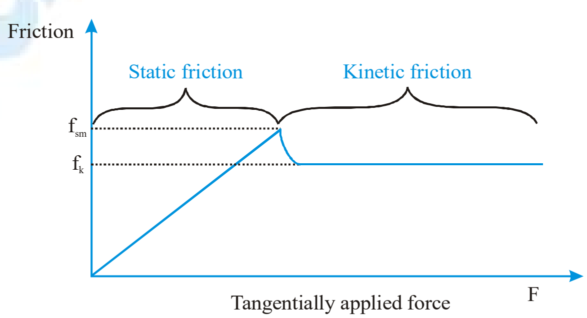
The limiting friction is experimentally observed proportional to the normal reaction between surfaces in contact.

$$f_{sm} = \mu_s N$$

Here μ_s is the constant of proportionally. It is known as the coefficient of static friction for the two surfaces involved.

When sliding starts between the surface, the frictional force rapidly drops to a characteristic value, which always oppose the sliding. This characteristic frictional force is known as kinetic friction (f_k). Kinetic friction is experimentally found proportional to the normal reaction between surfaces in contact.

$$f_k = \mu_k N$$



Here μ_k is the constant of proportionally. It is known as the coefficient of kinetic friction for the two surfaces involved.

The frictional forces between any pair of surfaces are decided by the respective coefficient of friction. The coefficient of friction are dimensionless constants and have no units. The coefficient of static friction (μ_s) is generally larger than the coefficient of kinetic friction (μ_k) but never becomes smaller; at the most both of them may be equal. Therefore, the magnitude of kinetic friction is usually smaller than the limiting static friction (f_{sm}) and sometimes kinetic friction becomes equal to the limiting static friction but it can never exceed the limiting friction.

The limiting static friction and the kinetic friction between any pair of solid surfaces follow these two empirical laws.

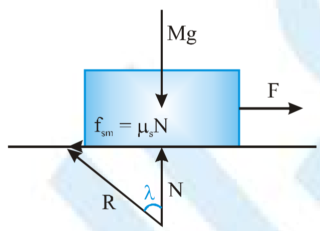
- Frictional forces are independent of measured area of contact.
- Both the limiting static friction and kinetic friction are proportional to the normal force pressing the surfaces in constant.

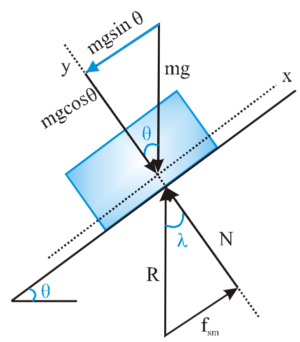
Angle of Friction

The angle of friction is the angle between resultant contact force of normal reaction N, when sliding is initiating. It is denoted by λ

$$\tan\lambda = \frac{f_{sm}}{N} = \frac{\mu_s N}{N} = \mu_s$$

For smooth surface $\lambda = 0$





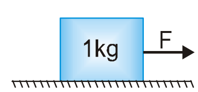
Angle of Repose (θ)

A body is placed on an inclined plane and the angle of inclination is gradually increased. At some angle of inclination θ the body starts sliding down the plane due to gravity. This angle of inclination is called angle of repose (θ).

Angle of repose is that minimum angle of inclination at which a body placed on the inclined starts sliding down due to its own weight.

Thus, angle of repose = angle of friction.

Ex. A block of mass 1 kg is at rest on a rough horizontal surface, where coefficients of static and kinetic friction are 0.2 and 0.15. Find the frictional forces if a horizontal force **(a)** F = 1N **(b)** F = 1.96 N **(c)** F = 2.5 N is applied on a block



Sol.

Maximum force of friction is the limiting friction $f_{sm} = 0.2 \times 1 \times 9.8\ N = 1.96\ N$

(a) For $F = 1\ N,\ F < f_{sm}$

So, body is in rest means static friction is present and hence $f_s = F = 1\ N$

(b) For $F = 1.96\ N$, $F = f_{sm} = 1.96\ N$. The block is about to slide, therefore $f = 1.96\ N$

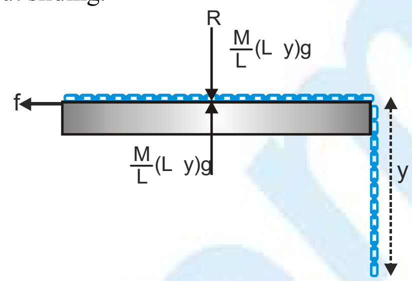
(c) For $F = 2.5\ N$, So $F > f_{sm}$

Now body is sliding and kinetic friction acts.

Therefore $f = f_k = \mu_k N = \mu_k\, mg = 0.15 \times 1 \times 9.8 = 1.47\ N$

Ex. Length of a uniform chain is L and coefficient of static friction is μ between the chain and the table top. Calculate the maximum length of the chain which can hang from the table without sliding.

Sol. Let y be the maximum length of the chain that can hang without causing the portion of chain on table to slide.



Length of chain on the table = (L – y)

Weight of part of the chain on table $= \frac{M}{L}(L-y)\,g$

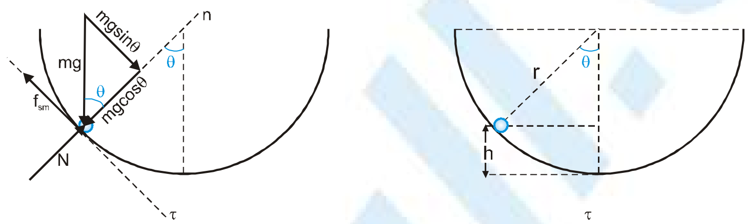
Weight of hanging part of the chain $= \frac{M}{L}yg$

For equilibrium with maximum portion hanging, limiting friction = weight of hanging part of the chain

$$\mu\frac{M}{L}(L-y)g = \frac{M}{L}yg \;\Rightarrow\; y = \frac{\mu L}{1+\mu}$$

Ex. An insect crawls on the inner surface of hemispherical bowl of radius r. If the coefficient of friction between an insect and bowl is μ and the radius of the bowl is r, find the maximum height to which the insect can crawl up.

Sol. The insect can crawl up, the bowl till the component of its weight tangent to the bowl is balanced by limiting frictional force.



$\sum F_n = 0 \rightarrow \qquad N = mg\cos\theta$ ...**(i)**

$\sum F_\tau = 0 \rightarrow \qquad f_{sm} = mgin\theta$ ...**(ii)**

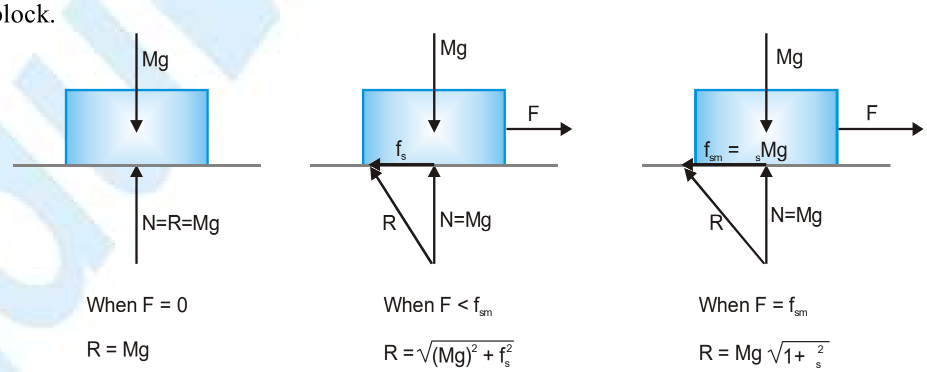
Force of limiting friction $f_{sm} = \mu N$ ...**(iii)**

From equation **(i)**, **(ii)** and **(iii)**, $\tan\theta = \mu$ ...**(iv)**

$$h = r - r\cos\theta = r\left[1 - \frac{1}{\sqrt{1+\mu^2}}\right]$$

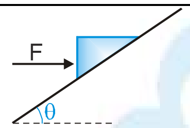
Ex. A body of mass M is kept on a rough horizontal ground (static friction coefficient = μ_s). A person is trying to pull the body by applying a horizontal force F, but the body is not moving. What is the contact force between the ground and the block.

Sol.



Therefore $\qquad Mg \le R \le Mg\sqrt{1+\mu_s^2}$

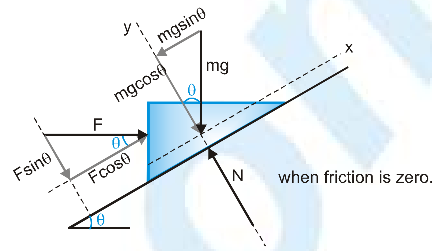
Ex. A block rest on a rough inclined plane as shown in fig. A horizontal force F is applied to it **(a)** Find the force of normal reaction, **(b)** Can the force of friction be zero, if yes when? and **(c)** Assuming that friction is not zero find its magnitude and direction of its limiting value.



Sol. **(a)** $\sum F_y = 0 \rightarrow \quad N = mg\cos\theta + F\sin\theta$

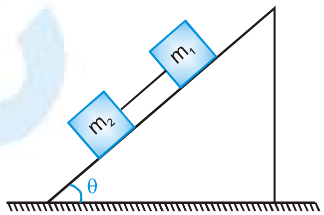
(b) $\sum F_x = 0 \rightarrow \quad F\cos\theta = mg\sin\theta \Rightarrow F = mg\tan\theta$

(c) Limiting friction $f_{sm} = \mu N = \mu\,(mg\cos\theta + F\sin\theta)$;



It acts down the plane if body has tendency to slide up and acts up the plane if body has tendency to slide down.

Ex. Two blocks with masses m_1=1 kg and m_2 = 2 kg are connectedly a string and slide down a plane inclined at an angle $\theta = 45°$ with the horizontal. The coefficient of sliding friction between m_1 and plane is μ_1=0.4 and that between m_2 and plane is μ_2=0.2. Calculate the common acceleration of the two blocks and the tension in the string.



Sol.

As $\mu_2 < \mu_1$, block m_2 has greater acceleration than m_1 if we separately consider the motion of blocks. But they are connected so they move together as a system with common acceleration.

So acceleration of the blocks :

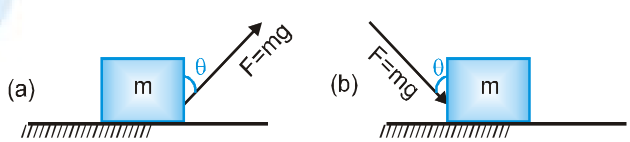
$$a = \frac{(m_1 + m_2)g\sin\theta - \mu_1 m_1 g\cos\theta - \mu_2 m_2 g\cos\theta}{m_1 + m_2}$$

$$= \frac{(1+2)(10)\left(\frac{1}{\sqrt{2}}\right) - 0.4\times1\times10\times\frac{1}{\sqrt{2}} - 0.2\times2\times10\times\frac{1}{\sqrt{2}}}{1+2} = \frac{22}{3\sqrt{2}}\,\text{ms}^{-2}$$

For block m_2 : $m_2 g\sin\theta - \mu_2 m_2 g\cos\theta - T = m_2 a \Rightarrow T = m_2 g\sin\theta - \mu_2 m_2 g\cos\theta - m_2 a$

$$= 2\times10\times\frac{1}{\sqrt{2}} - 0.2\times2\times10\times\frac{1}{\sqrt{2}} - 2\times\frac{22}{3\sqrt{2}} = \frac{2}{3\sqrt{2}}\,\text{N}$$

Ex. A block of mass m rests on a rough horizontal surface as shown in figure (a) and (b). Coefficient of friction between block and surface is μ. A force F = mg acting at an angle θ with the vertical side of the block. Find the condition for which block will move along the surface.

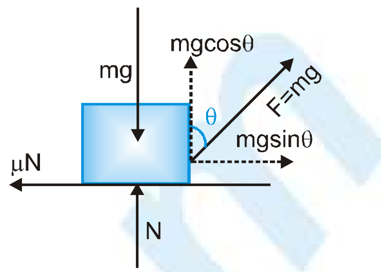


Sol. For **(a)** : normal reaction $N = mg - mg\cos\theta$

frictional force $= \mu N = \mu(mg - mg\cos\theta)$

Now block can be pulled when : Horizontal component of force $\geq$ frictional force

i.e. $mg\sin\theta \geq \mu(mg - mg\cos\theta)$



or $\quad 2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2} \geq \mu(1 - \cos\theta)$

or $\quad 2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2} \geq 2\mu\sin^2\dfrac{\theta}{2}$ or $\cot\dfrac{\theta}{2} \geq \mu$

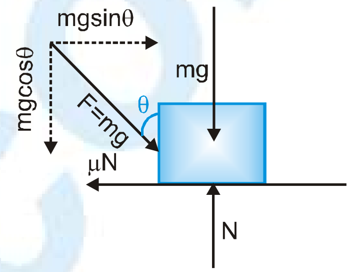
For (b) : Normal reaction $N = mg + mg\cos\theta = mg(1 + \cos\theta)$

Hence, block can be pushed along the horizontal surface when

horizontal component of force $\geq$ frictional force

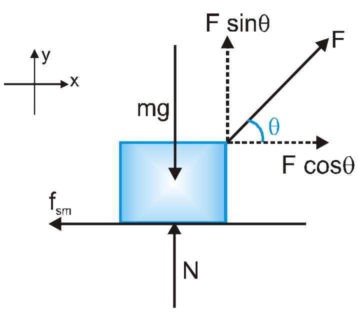
i.e. $\quad mg\sin\theta \geq \mu\, mg(1 + \cos\theta)$

or $\quad 2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2} \geq \mu \times 2\cos^2\dfrac{\theta}{2} \Rightarrow \tan\dfrac{\theta}{2} \geq \mu$



Ex. A body of mass m rests on a horizontal floor with which it has a coefficient of static friction μ. It is desired to make the body move by applying the minimum possible force F. Find the magnitude of F and the direction in which it has to be applied.

Sol. Let the force F be applied at an angle θ with the horizontal as shown in figure.



$\sum F_y = 0 \rightarrow \quad N = mg - F\sin\theta$ ...**(i)** $\quad N = mg - F\sin\theta$ ...**(i)**

$\sum F_x = 0 \rightarrow \quad F\cos\theta \geq f_{sm} \Rightarrow F\cos\theta \geq \mu N \quad [\text{as } f_{sm} = \mu N]$ ...**(ii)**

Substituting value of N from equation **(i)** in **(ii)**,

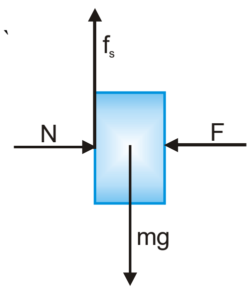
$$F \geq \frac{\mu mg}{(\cos\theta + \mu\sin\theta)} \qquad \text{...(iii)}$$

For the force F to be minimum $(\cos\theta + \mu\sin\theta)$ must be maximum,

maximum value of $\cos\theta + \mu\sin\theta$ is $\sqrt{1+\mu^2}$ so that $F_{min} = \dfrac{\mu mg}{\sqrt{1+\mu^2}}$ with $\theta = \tan^{-1}(\mu)$

Ex. A book of 1 kg is held against a wall by applying a force F perpendicular to the wall. If $\mu_S = 0.2$, what is the minimum value of F ?

Sol. The situation is shown in fig. The forces acting on the book are–

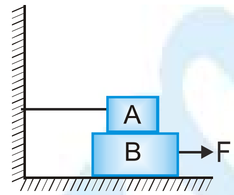


For book to be at rest it is essential that $Mg = f_s$

But $f_{s\,max} = \mu_S N$ and $N = F$

$\therefore \quad Mg = \mu_S F \Rightarrow F = \dfrac{Mg}{\mu_s} = \dfrac{1 \times 9.8}{0.2} = 49\ N$

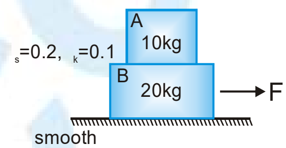
Ex. A is a 100 kg block and B is a 200 kg block. As shown in fig., the block A is attached to a string tied to a wall. The coefficient of friction between A and B is 0.2 and the coefficient of friction between B and floor is 0.3. Then calculate the minimum force required to move the block B. ($g = 10\ m/s^2$).



Sol. When B is tied to move, by applying a force F, then the frictional forces acting on the block B are f_1 and f_2 with limiting values, $f_1 = (\mu_S)_A m_A g$ and $f_2 = (\mu_S)_B (m_A + m_B)g$
Then minimum value of F should be (for just tending to move),

$F = f_1 + f_2 = 0.2 \times 100\ g + 0.3 \times 300\ g = 110\ g = 1100\ N$

Ex. In the given figure block A is placed on block B and both are placed on a smooth horizontal plane. Assume lower block to be sufficiently long. The force F pulling the block B horizontally is increased according to law F = 10t N



(a) When does block A start slipping on block B? What will be force F and acceleration just before slipping starts?

(b) When F is increased beyond the value obtained in part (a), what will be acceleration of A ?

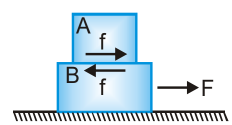
(c) Draw acceleration-time graph.

Sol.

Direction of friction forces

Block A moves forward always, due to friction, therefore friction on it must be in forward direction.

Friction between two adjacent surfaces are equal and opposite because they make Newton's is third law action reaction pair.



Range of Value of friction

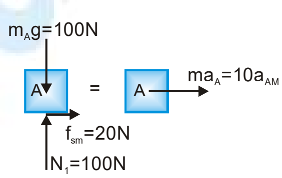
Before slipping starts, friction is static $f_s \leq 20$ N

After slipping starts, friction is kinetic $f_k = 10$ N

Maximum possible acceleration

A can accelerate only due to friction, its maximum possible acceleration is a_{AM} (when $f_s = f_{sm} = 20$ N)

Block A

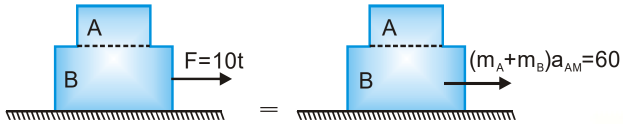


So $20 = 10a_{AM} \Rightarrow a_{AM} = 2\ m/s^2$

Sequence of slipping :

Since ground is smooth, block B first starts slipping on the ground and carries A together with it. When acceleration of A & B becomes equal to a_{AM}, Block A starts slipping on B.

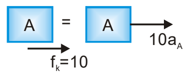
(a) Just before the moment A starts slipping, both were moving together with acceleration a_{AM}. Considering them as a one body.



(On a smooth stationary surface we will not show the normal forces i.e. FBD of combined block showing horizontal forces only).

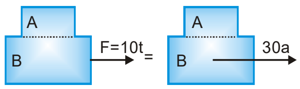
Value of F F = 60 N and Time $10t = 60 \Rightarrow t = 6s$

(b) If F is increased beyond 60 N, A slides and kinetic friction acts on it. Now acceleration of A



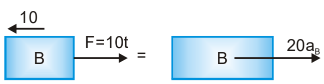
$\Rightarrow 10 = 10a_A \Rightarrow a_A = 1\ m/s^2$

(c) When $F \leq 60$ N, both are moving with same acceleration a. We treat them as one body.



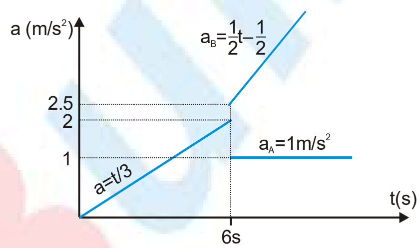
So $10t = 30a \Rightarrow a = \frac{1}{3}t$

This acceleration increases to $a_{AM} = 2\ m/s^2$, when F = 60 N at t = 6 s. Thereafter A starts slipping and its acceleration provided by kinetic friction, drops to a constant value $a_A = 1\ m/s^2$. However acceleration of B keeps on increasing according to equation



$10t - 10 = 20a_B \Rightarrow a_B = \frac{1}{2}t - \frac{1}{2}$

Graph between acceleration and time



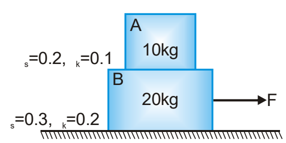
Ex. Block A is placed on another block B, which rests on a rough horizontal ground. Horizontal force F pulling the block B is increased gradually.

(a) Find the maximum value of F so that no motion occurs.

(b) Find maximum F so that A does not slide on B.

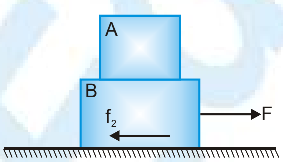
(c) If F is increased beyond the value obtained in part (b) what are acceleration of both the blocks ? Explain your answer in terms of F.

(d) If F is increased according to law F =10t N draw a–t graph

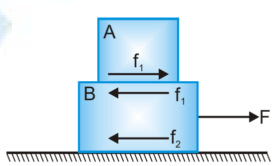


Sol.

Directions of friction forces



Before B starts slipping



After B starts slipping

Range of values of frictional forces

f_{1k} = 10 N (A is slipping)

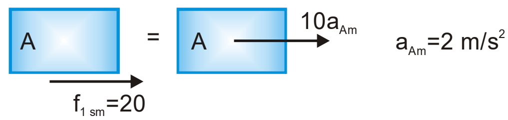
f_{1s} = 20 N (A is not slipping)

f_{2k} = 60 N (B is slipping)

$f_{2s} \leq 90$ N (B is at rest)

Maximum possible acceleration : A can move only due to friction. Its maximum possible acceleration is

Block A

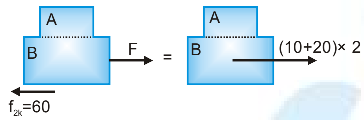


Sequence of slipping : When $F \geq f_{2s}$, block B starts slipping on ground and carries block A together with it till its acceleration reaches value a_{AM}. Thereafter A also starts slipping on B.

(a) F = 90 N

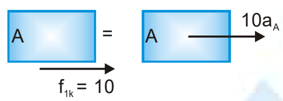
(b) When A does not slide on B, both move with the same acceleration (a_{Am}) and can be treated as one body, which can have maximum acceleration $a_{AM} = 2\ m/s^2$.

$F - 60 = 60 \Rightarrow F = 120$ N



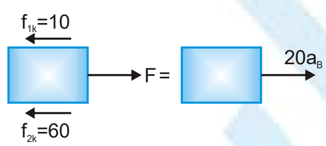
(c) When F is increased beyond F = 120 N, block A starts sliding and friction between A & B drops to $f_{1K} = 10$ N. Both the blocks now move with different acceleration so we treat them separate bodies. Now acceleration A also drops to a constant value a_A.

Acceleration of A :



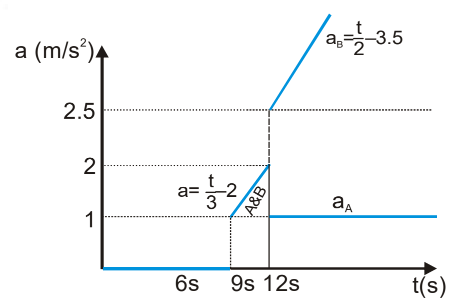
$\Rightarrow 10 = 10\,a_A \Rightarrow a_A = 1\ m/s^2$

Acceleration of B :



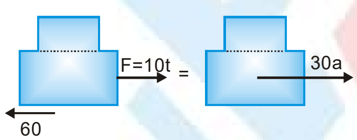
So $F - 70 = 20a_B \Rightarrow a_B = \dfrac{F-70}{20}$

(d) If F = 10t, values of acceleration of both the blocks in different time intervals are as under:



- $F \leq 90\ N \Rightarrow \quad t \leq 9\ s \quad a_A = a_B = 0$
- $90\ N < F \leq 120\ N\ \ 9\ s < t \leq 12\ s \quad a_A = a_B = \dfrac{t}{3} - 2$

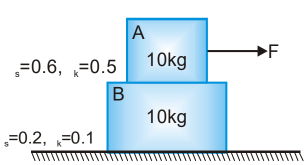
  In the interval both the blocks move as one body



$a = \dfrac{10t - 60}{30} = \dfrac{t}{3} - 2$

- $F > 120\ N \quad \Rightarrow t > 12s \quad a_A = 1;\ a_B = \dfrac{t}{2} - 3.5$

Ex. Block A is placed on another block B, which rests on a rough horizontal ground. Horizontal force pulling A is increased gradually

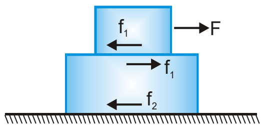


(a) Find maximum F so that none of the blocks move. Which block starts sliding first ?

(b) Express acceleration of each block as function of F for all positive values of F.

(c) If F=10t draw a-t graph

Sol. **Directions of friction forces**



Range of values of friction forces

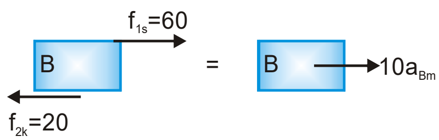
$f_{1s} \le 60$ N

$f_{1k} = 50$ N

$f_{2s} \le 40$ N

$f_{2k} = 20$ N

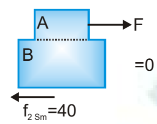
Maximum possible acceleration of B : Block B acceleration due to friction only. Its maximum acceleration is



$60–20 = 10a_{Bm} \Rightarrow a_{Bm} = 4$ m/s^2

Sequence of slipping : Smaller, limiting friction is between B and ground so it will start sliding first. Then both will move together till acceleration B reaches its maximum possible values 4 m/s^2. Thereafter A starts sliding on B

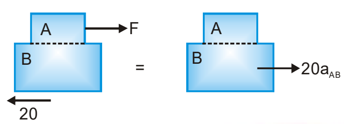
(a) Till the F reaches the limiting friction between block B and the ground none of the blocks move.



$\Rightarrow F = 40$ N

(b) If $F \le 40 \Rightarrow a_A = a_B = 0$ ...(i)

If F > 40 N, block B starts sliding and carries A together with it with the same acceleration till acceleration reach to 4 m/s^2. At this moment A starts slipping. Before this moment we may treat both of them as single body.



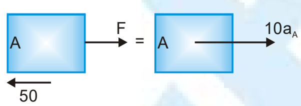
$\Rightarrow F – 20 = 20a_{AB} \Rightarrow a_{AB} = a_A = a_B = \dfrac{F-20}{20}$

When A starts sliding on B, $a_A = a_B = 4$, from the above equation, we have F = 100 N.

When $F \ge 100$ N block A also starts slipping on B and friction between A & B drops to value 50 N.

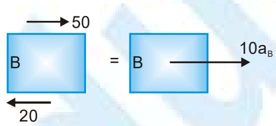
Now since they move with different acceleration we treat them separately.

Block A



$\Rightarrow a_A = \dfrac{F-50}{10}$

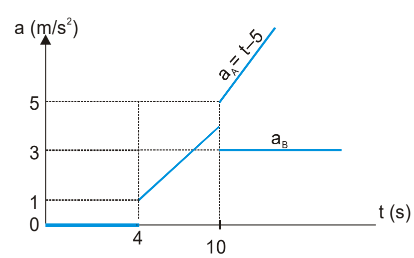
Block B



$\Rightarrow a_B = \dfrac{50-20}{10} = 3$ m/s^2

(c)

$F \le 40$ N $t \le 4$ s	$a_A = a_B = 0$	
$40 \le F \le 100$	$4 < t \le 10$	$a_A = a_B = \dfrac{t}{2} - 1$
$100 < F \Rightarrow t > 10$		$a_A = t - 5$, $a_B = 3$ m/s^2

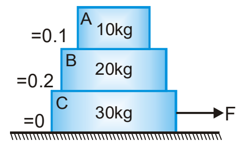


Ex. Block A is placed on B and B is placed on block C, which rests on smooth horizontal ground as shown in the figure. Block A is pulled horizontally by a force F which increases gradually.

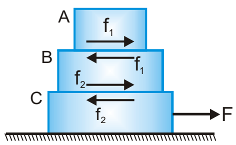
(a) Decide sequence of slipping.

(b) If F is increased gradually find acceleration of each block for all values of F.

(c) If F = 15t N, draw a–t graph.



Sol. **Direction of friction forces :**



Range of values of friction forces

$f_1 \le 10$ N (A does not slides on B)

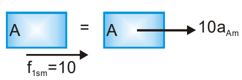
$f_1 = 10$ N (A slides on B)

$f_2 \le 60$ N (B does not slides on C)

$f_2 = 60$ N (B slides on C)

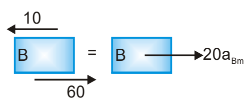
Maximum possible acceleration : Blocks A and B move due to friction forces only, we find their maximum possible acceleration.

Block A



$a_{Am} = 1 \text{ m/s}^2$

Block B



$$a_{Bm} = \frac{60-10}{20} = 2.5 \text{ m/s}^2$$

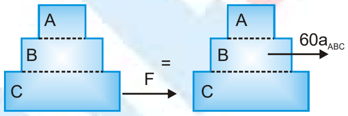
(a) **Sequence of slipping**

Since ground is smooth the block C starts sliding first

A starts slipping on B secondly till that moment all the three blocks move with same acceleration, which can achieve maximum value of $a_{AM} = 1 \text{ m/s}^2$.

Thirdly B starts sliding on C, till that moment B & C move with the same acceleration $a_{Bm} = 2.5 \text{ m/s}^2$

(b) Before A starts slipping, all the three were moving with the same acceleration $a_{AM} = 1 \text{ m/s}^2$. We therefore treat then as a single body.

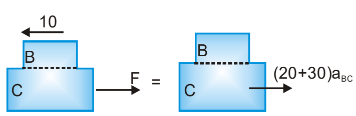


$$a_{ABC} = \frac{F}{60}$$

When A starts sliding $\quad a_{ABC} \le a_{AM} \Rightarrow \dfrac{F}{60} \le 1 \Rightarrow F \le 60$ N

When $F \ge 60$ N, block A starts slipping on B and its acceleration decided by friction f_1, achieves a constant value $a_A = 1 \text{ m/s}^2$.

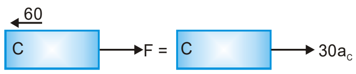
Now, F is increased beyond 60 N and B and C will continue to move together till their acceleration a_{BC} becomes $a_{Bm} = 2.5 \text{ m/s}^2$, when slipping between B and C starts. Till this moment, we treat B and C as one body.



$$\Rightarrow a_{BC} = \frac{F-10}{50}$$

When slipping between B & C starts : $a_{BC} = a_{Bm} \Rightarrow \dfrac{F-10}{50} = 2.5 \Rightarrow F \le 135$ N

When F > 135 N, block B also starts slipping on C. Now acceleration of A & B achieves the maximum value $a_{Bm} = 2.5$ m/s^2 and acceleration of block C is decided by F.



$\Rightarrow F - 60 = 30\, a_C \Rightarrow a_C = \dfrac{F-60}{30}$

Acceleration of blocks for different values of force.

- $F \le 60$ N $\qquad a_A = a_B = a_C = a_{ABC} = \dfrac{F}{60}$
- $60 < F \le 135$ N $\qquad a_A = a_{Am} = 1$ m/s^2, $a_B = a_C = a_{BC} = \dfrac{F-10}{50}$
- $135 < F \quad a_A = a_{Am} = 1$, $a_B = a_{Bm} = 2.5$, and $a_C = \dfrac{F-60}{30}$

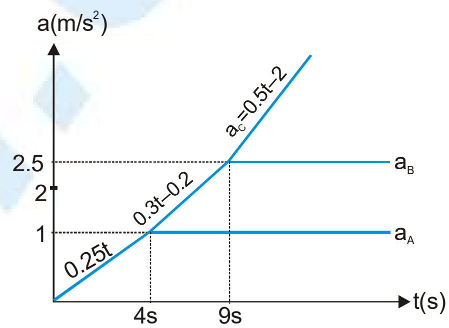
(c) If F = 15t

- $F \le 60 \quad t \le 4$ s $\quad a_A = a_B = a_C = \dfrac{F}{60} = \dfrac{t}{4} = 0.25t$
- $60 < F \le 135 \qquad 4 < t \le 9 \; a_A = 1$ m/s^2

  $a_B = a_C = \dfrac{F-10}{50} = 0.3t - 0.2$
- $135 < F \quad 9 < t \qquad a_A = 1$ m/s^2

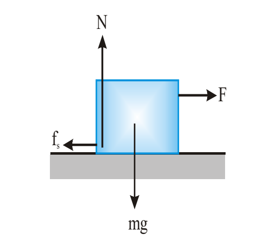
  $a_B = 2.5$ m/s^2

  $a_C = \dfrac{F-60}{30} = 0.5t - 2$



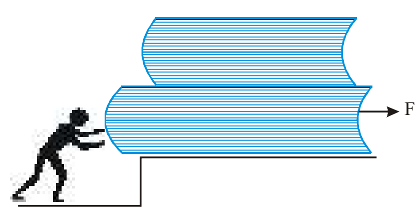
Static Friction

Static friction acts when two contact surface are not moving relative to each other. For example, consider a block on a horizontal table, as in figure. If we apply an external horizontal force F to the block, acting to the right, the block remains stationary if F is not too large. The force that counteracts F and keeps the block from moving acts to the left and is the frictional force f. As long as the block is not moving, f = F. Since the block is stationary ,we call this frictional force the force of static friction, f_s.



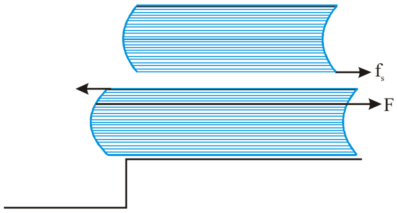
$F=F_s$ As long as the block is not moving

If we keep two books one on top the other and now we slowly push the lower book, both the book move together, the force moving the upper book is friction. Since the book are moving with respect to ground but they are not moving with respect to each other, this force of friction between two books is of static nature.

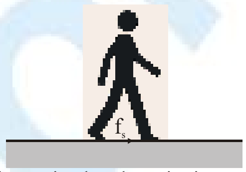


Two books kept on top of each other
& the lower book being pushed slowly

Similarly when we walk on ground the friction force acting on the foot is of static nature. You may be surprised but the foot in contact with ground is not moving while the other foot is moving. During this time friction acting between foot and ground is static friction.



Free body diagram of two books



Man is moving but the point in contact of the foot is not moving i.e., Static friction is acting on the man person is walking due to static friction

If we are increasing our speed then static friction is acting in forward direction. This happens because we try to pull our leg backward while walking in forward direction. This means foot is trying to move in back ward direction with respect to ground and ground applied friction in forward direction.

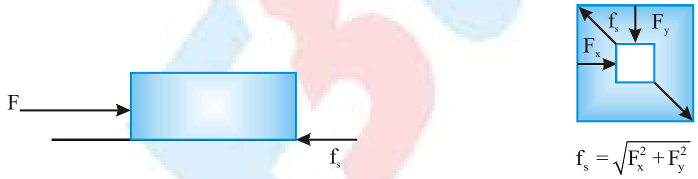
Static friction is self-adjusting and it opposes the tendency of relative motion

In fact, in the previous case of books also, we can see that static friction acting in forward direction on upper book has given it some motion so it can move with the lower book. Another point to be denoted here is that even by third law, pair of static friction is opposing relative motion by trying to slow down the lower book (which is being accelerated externally).

For example in figure we assume that the block is stationary and we can see that static friction is acting in such a direction so as to oppose the relative motion. F represents external force and f_s represents friction.

The magnitude of the static friction between any two surfaces in contact can have the values

$f_s \leq \mu F_N$.



$f_s = \sqrt{F_x^2 + F_y^2}$

Block is stationary due to static friction

where the dimensionless constant μ_s is known as the coefficient of static friction and F_N is the magnitude of the normal contact force exerted by one surface on the other. The equation holds when the surfaces are the normal contact force exerted by one surface on the other. The equality in equation holds when the surfaces are on the verge of slipping, that is when $f_s = f_{s,max} = \mu_s F_N$, This maximum value of fs is called limiting friction. This situation is called impending motion. The inequality hold when the surfaces are not on the verge of slipping. Maximum strength of the joints formed is directly proportional to the normal contact force, that is $f_{s,max} \propto F_N$.

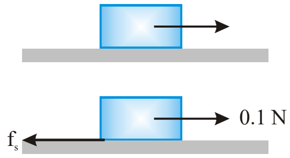
Maximum strength also depends on the roughness of contact surface fs,max (also called $f_{limiting}$) = $\mu_s N$. Magnitude of static friction is self-adjusting such that relative motion does not start (but still it has maximum value).

Let us say we are applying force F on a block kept on horizontal rough surface with coefficient of static friction $\mu_s = 0.1$ & mass of block is 5 kg. When applied F is less than 5 N the value of static friction is equal to the applied force, not 5 N. It is the maximum value of friction. But when applied force F is equal to 5 N, the value of static friction is 5 N

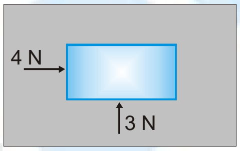
Ex. A block lying on a horizontal surface is pulled by a force of 0.1 N but the block does not move i.e., remains at rest. Find friction force

Sol. To analyse the frictional force on the block we proceed as follows.

As the block remains at rest, the force of static friction is balancing the 0.1 N force (the applied force). So the frictional force – 0.1 N

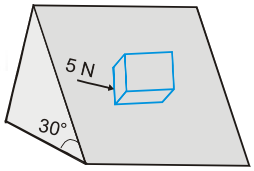


Ex. A block of lying on a horizontal surface is pushed by forces as shown in figure but the block does not move i.e. it remains at rest. Find the frictional force on the block.



Sol. Net force acting on the block is $\sqrt{3^2+4^2} = 5N$

As the block remains at rest, the force of static friction is balancing the applied force (5 N)



Ex. A block lying on an inclined surface at an angle 30° with the horizontal is pushed by force as shown in figure but the block does not move i.e., it remains at rest. Find the frictional force on the block.

Sol. Two forces are acting on the block along the surface of inclined, first one is mg sin 30° down the inclined and second 5 N shown in figure.

So the net force acting on the block along the surface of inclined is $\sqrt{12^2+5^2} = 13N$

As the block remains at rest, the force of static friction is balancing the applied force (13 N)

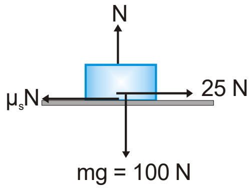
Ex. A block of weight 100 N lying on a horizontal surface just begins to more when a horizontal force of 25 N acts on it. Determine the coefficient of static friction.

Sol. At the 25 N force brings the block to the point of sliding i.e. limiting friction is 25 N.

limiting frictional force = $\mu_s N$

From force diagram : $N = 100$ N

$\mu_s N = 25 \quad \Rightarrow \quad \mu_s = 0.25$



Ex. A block of weight 100 N lying on a horizontal surface is pushed by a force F acting at an angle 30° with horizontal. For what value of F will the block begin to move if $\mu_s = 0.25$?

Sol. Consider the force diagram of the block at the moment when it is just to start moving.

Balancing force :

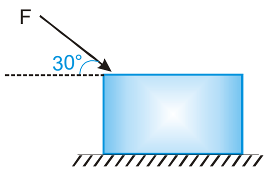
$N = mg + F \sin 30°$

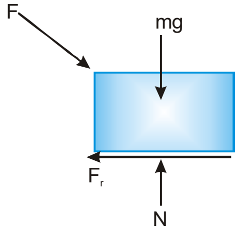
$F = \cos 30° = \mu_s N$

$F = \cos 30° = \mu_s (mg + F \sin 30°)$

$$F = \frac{\mu_s mg}{\cos 30° - \mu_s \sin 30°} = \frac{0.25(100)^2}{\sqrt{3} - 0.25}$$

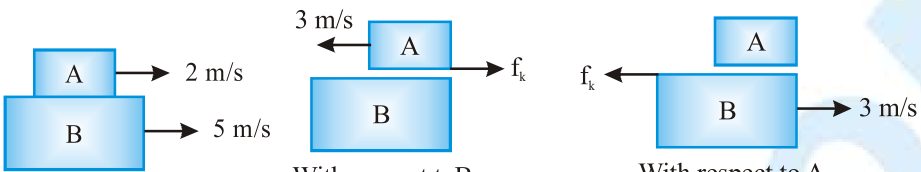
$F = 33.74$ N





Kinetic Friction

Kinetic friction acts when there is relative motion between two surfaces in contact. It acts always opposite to the relative velocity as we see in figure. The magnitude is not self-adjusting as in static friction. It is always is equal to $\mu_k F_N$.



With respect toB

With respect to A

Experimentally, we find that to a good approximation, both f_{smax} and f_k are proportional to the magnitude of the normal force. The following empirical laws of friction summarize the experimental observations:

1. The magnitude of the force of kinetic friction acting between two surface is

$f_k = \mu_k N$ where μ_k is the coefficient of kinetic friction. Although the coefficient of kinetic friction can vary with speed, we shall usually neglect any such variations in problems.

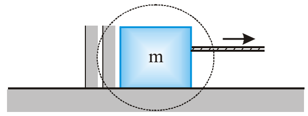
2. The value of μ_k and μ_s depend on the nature of the surface.

3. μ_k is generally less than μ_s.

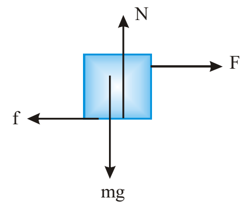
The actual value depends on the degree of smoothness and other environmental factors. For example, wood may be prepared at various degrees of smoothness and the friction coefficient will vary. Dust impurities, surface oxidation etc. have a great role in determining the friction coefficient. Suppose we take two blocks of pure copper, clean them carefully to remove any oxide or dust layer at the surfaces, heat them to push out any dissolved gases and keep them in contact with each other in an evacuated chamber at a very low pressure of air. The blocks stick to each other and a large force is needed to slide on over the other. The friction coefficient as defined above, becomes much larger than one this is called cold welding. If a small amount of air is allowed to go into the chamber so that some oxidation takes place at the surface, the friction coefficient reduces to usual values.

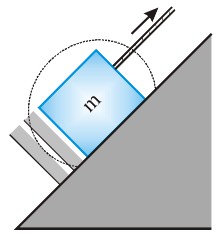
4. The direction of the friction force on an object is parallel to the surface with which the object is in contact and opposite to the motion (kinetic friction) or the tendency of motion (static friction) of the object relative to the surface.

5. The coefficient of friction are nearly independent of the area of contact between the surfaces. We might expect that placing an object on the side having the more area might increase the friction force. While this provides more points in contact, the weight of the object is spread out over a larger area, so that the individual points are not pressed so tightly together. These effects approximately compensate for each other, so that the friction force is independent of the area. So those extra wide tires you see on some cars provide no more friction than narrower tires. The wider tire simply spreads the weight of the car over more surface area to reduce heating and wear. similarly, the friction between a truck and the ground is the same whether the truck has four tires. or eighteen! More tires spread the load over more ground area and reduces the pressure per tire. Interestingly, stopping distance when brakes are applied is not affected by the number of tires. But the wear that tires experience, very much depends on the number of tires.

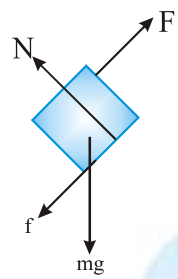


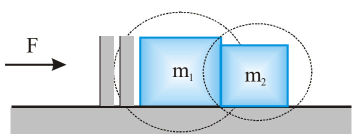
A block is pulled by force F to right on a rough horizontal surface



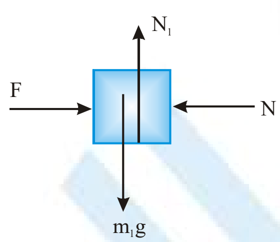


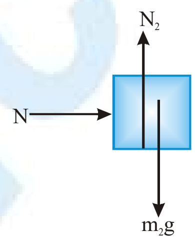
A block is pulled up the rough incline

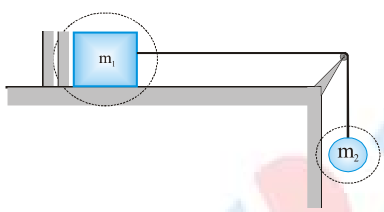




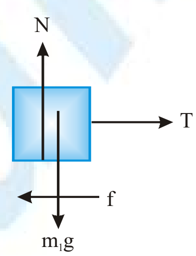
Two block in contact pushed by the force F to the right on a frictionless surface.

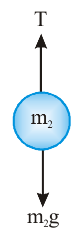






Two block connected by a light cord.
The surface is rough and the pulley is frictionless

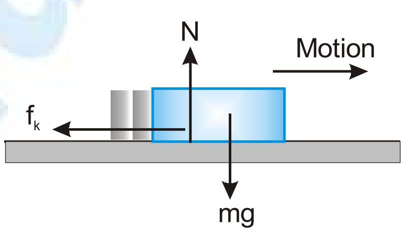




Ex. A hockey puck on a frozen pond is given an initial speed of 20.0 m/s. If puck always remains on the ice and slides 115 m before coming to rest, determine the coefficient of kinetic friction between the puck and ice.

Sol. Imagine that the puck in figure slides to the right and eventually comes to rest. The forces acting on the puck after it is in motion are shown in figure first, we find the acceleration in terms of the coefficient of kinetic friction, using Newton's second law. Knowing the acceleration of the puck and the distance it travels, we can then use the equation of kinematics to find the numerical value of the coefficient of kinetic friction.

Defining rightward and upward as our positive directions, we apply Newton's second law in component from to the puck and obtain.



$\Sigma F_x = f_k = ma_x$ .... **(1)**

$\Sigma F_y = N - mg = 0$ $\quad (a_y = 0)$ .... **(2)**

But $F_k = \mu_k N$ and from **(2)** we see that N = mg, Therefore,

(1) becomes

$$-\mu_k N = -\mu_k mg = ma_x$$

$$a_x = \mu_k g$$

The negative sign means the acceleration is to the left in figure because the velocity of the puck is to the right, this means that the puck is slowing down. The acceleration is independent of the mass of the puck and is constant because we assume that μ_k remains constant.

Because the acceleration is constant, we can use equation $v^2 = u^2 + 2as$

$$0 = u^2 + 2a\,S = u^2 - 2\mu_k gS \quad \Rightarrow \quad v = 0 \quad \Rightarrow \quad \mu_k = \frac{u^2}{2gS}$$

$$\mu_k = \frac{\left(20.0m/s^2\right)^2}{2(9.80m/s^2)(115m)} = 0.136$$

Ex. A 5 kg block slides down a plane inclined at 30° to the horizontal. Find

(a) The acceleration of the block if the plane is frictionless.

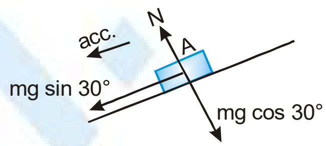
(b) The acceleration if the coefficient of kinetic friction is 0.2.

Sol. **(a)** N = mg cos 30°

mg sin 30° = ma

down the plane if plane is smooth.

$a = g / 2 = 4.9$ m/s^2

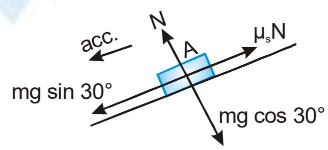


(b) N = mg cos 30°

mg sin 30° – μ_kN = ma

a = g sin30° – μ_kg cos30°

a = 3.20 m/s^2



Ex. 5 kg block projected upwards with an initial speed of 10 m/s from the bottom of a plane inclined at 30° with horizontal. The coefficient of kinetic friction between the block and the plane is 0.2.

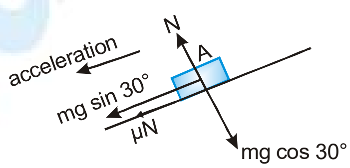
(a) How far does the block move up the plane ?

(b) How long it move up the plane ?

Sol. While the block is moving up the frictional force acts downward.

As the block is slowing downs, the velocity and acceleration must be in opposite direction.

Velocity in this case is upwards, so acceleration is in downward direction.



the magnitude of acceleration $= \dfrac{\text{mgsin30°} + \mu\text{mgcos30°}}{\text{m}} = \text{g(sin30°} + \mu\text{mgcos30°)}$

$\Rightarrow$ a = –g(sin 30° + μ mg cos 30°) - -6.6 m/s^2

For the motion of block from the bottom to up the plane :

$u = +10$ m/s $\Rightarrow$ $v^2 = u^2 + 2as$, we get

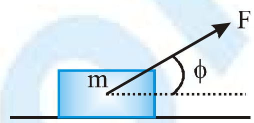
$0^2 = 10^2 + 2(-6.6)(s)$ $\Rightarrow$ $s = 7.58$ m

$v = u + at$ $\Rightarrow$ $0 = 10 - 6.6 \times t$ $\Rightarrow$ $t = 1.5$ second

Hence the block moves up the plane for 1.5 sec covering 7.58 m.

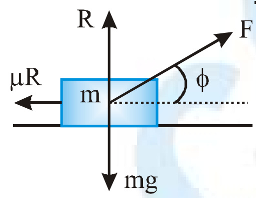
Varieties of Questions in Friction

1. Minimum force required to move the particle



A body of a mass m rests on a horizontal floor with which it has a coefficient of static friction μ. It is desired to make the body slide by applying the minimum possible force F.

Let the applied force F be at angle ϕ with the horizontal



For vertical equilibrium,

$R + F\sin\phi = mg$ or, $R = (mg - F\sin\phi)$ ....(i)

$F\cos\phi = \mu R$ ....(ii)

Substituting for R,

$$F\cos\phi = \mu(mg - F\sin\phi) \qquad \text{or} \qquad F = \mu mg/\cos\phi + \mu\sin\phi)$$

for minimum F $(\cos\phi + \mu\sin\phi)$ is maximum,

$\Rightarrow$ Let $x = \cos\phi + \mu\sin\phi)$

$$\frac{dx}{d\phi} = -\sin\phi + \mu\cos\phi$$

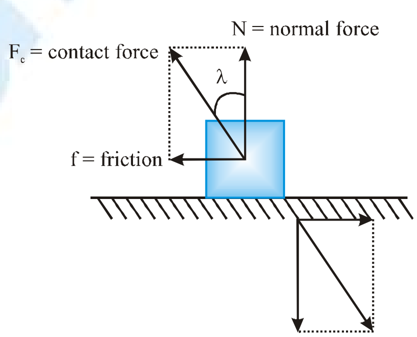
for maximum of x, $\dfrac{dx}{d\phi} = 0$

$\tan\phi = \mu$ and at this value of ϕ

$$F_{min} = \frac{\mu mg}{\sqrt{1+\mu^2}}$$

2. Friction as the component of contact force

When two bodies are kept in contact, electromagnetic forces act between the charged particles at the surfaces of the bodies. As a result, each body exerts a contact force on other. The magnitudes of the contact forces acting on the two bodies are equal but their directions are opposite and hence the contact forces obey Newton`s third law.



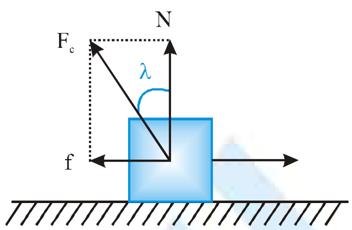
The direction of the contact force acting on a particular body is not necessarily perpendicular to the contact surface. We can resolve this contact force into two components, one perpendicular to the contact surface and the other parallel to it. The perpendicular component is called the normal contact force or normal force and parallel component is called friction.

Contact force $= \sqrt{f^2 + N^2} \Rightarrow F_{C\,min} = N\{\text{when } f_{min} = 0\}$

$F_{C\,max} = \sqrt{\mu^2 N^2 = N^2}$ {when $f_{max} = \mu N$} $\Rightarrow N \le F_c \le \sqrt{(\mu^2 + 1)}N \Rightarrow 0 \le \lambda \le \tan^{-1}\mu$

Ex. A body of mass 400 g slide on a rough horizontal surface. If the frictional force is 3.0 N, find **(a)** the angle made by the contact force on the body with the vertical and **(b)** the magnitude of the contact force. Take g = 10 m/s^2.

Sol. Let the contact force on the block by the surface be F_c which makes an angle λ with the vertical (shown figure)



The component of Fc perpendicular to the contact surface is the normal force N and the component of r parallel to the surface is the friction f. As the surface is horizontal, N is vertically upward. For vertical equilibrium,

N = Mg = (0.400 kg) (10 m/s^2) = 4.0 N

The frictional force is f = 3.0 N

(a) $\tan\lambda \frac{f}{N} = \frac{3}{4}$ or, $\lambda = \tan^{-1}(3/4) = 37^{\circ}$

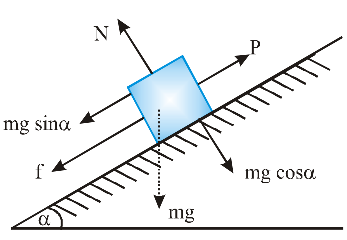
(b) The magnitude of the contact force is

$$F = \sqrt{N^2 + f^2} = \sqrt{(4.0N)^2 + (3.0N)^2} = 5.0N$$

3. Motion on a rough inclined plane

Suppose a motion up the plane takes place under the action of pull P acting parallel to the plane

N = mg cosα



Frictional force acting down the plane

F = μN = μmg cosα

Applying Newton`s second law for motion up the plane

P – (mg sinα + f) = ma

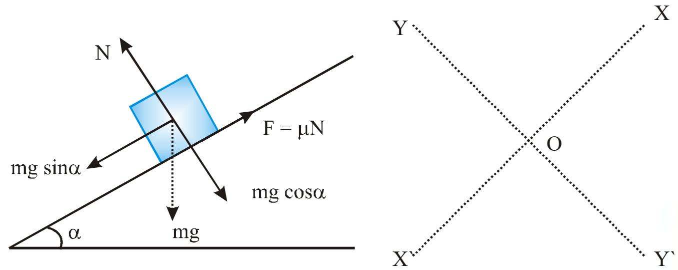
P – mg sinα – μ mg cosα = ma

If P = 0 the block may slide downwards with an acceleration a. The frictional force would then act up the plane

mg sinα – F = ma

or, mg sinα – μ mg cosα = ma

Ex. A 20 kg box is gently placed on a rough inclined plane of inclination 30o with horizontal. The coefficient of sliding friction between the box and the plane is 0.4. Find the acceleration of the box down the incline.



Sol. In solving inclined plane problems, the X - Y directions along which the forces are to be considered, may be taken as shown. The components of weight of the box are

(i) mg sina acting down the plane and

(ii) mg cos a acting perpendicular to the plane.

$N = mg\cos\alpha$

$mg\sin\alpha - m\,N = ma \Rightarrow mg\sin\alpha - m\,mg\cos\alpha = ma$

$a = g\sin\alpha - mg\cos\alpha = g(\sin\alpha - m\cos\alpha)$

$$= 9.8\left(\frac{1}{2} - 0.4\times\frac{\sqrt{3}}{2}\right) = 4.9\times 0.3072 = 1.505\ \text{m/s}^2$$

The box accelerates down the plane at 1.505 m/s^2

Ex. A force of 400 N acting horizontal pusher up a 20 kg block placed on a rough inclined plane which makes an angle of 45o with the horizontal. The acceleration experienced by the block is 0.6 m/s^2. Find the coefficient of sliding friction between the box and incline.

Sol. The horizontal directed force 400 N and weight 20 kg of the block are resolved into two mutually perpendicular components, and parallel and perpendicular to the plane as shown.

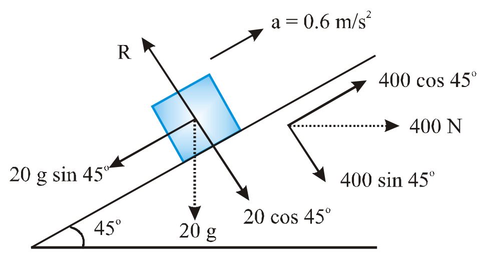
$N = 20\,g\cos 45^\circ + 400\sin 45^\circ = 421.4\,N$

The frictional force experienced by the block

$F = \mu N = \mu\times 421.4 = 421.4\,\mu N.$

As the accelerated motion is taking placed up the plane

$400\cos 45^\circ - 20\,g\sin 45^\circ - f = 20a$



$$\frac{400}{\sqrt{2}} - \frac{20\times 9.8}{\sqrt{2}} - 421.4\mu = 20a = 20\times 0.6 = 12$$

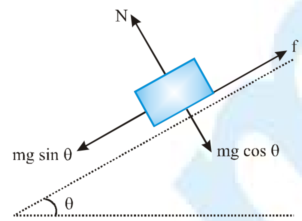
$$\mu = \left(\frac{400}{\sqrt{2}} = -\frac{196}{\sqrt{2}} - 12\right) - \frac{1}{421.4} = \frac{282.8 - 138.6 - 12}{421.4} = 0.3137$$

The coefficient of sliding friction between the block and the incline = 0.3137

4. Angle of repose

Consider a rough inclined plane whose angle of inclination θ with ground can be changed. A block of mass m is resting on the plane. Coefficient of (static) friction between the block and plane is μ.

For a given angle θ, the FBD (Free body diagram) of the block is



Where f is force of static friction on the block. For normal direction to the plane, we have N = mg cosθ. As θ increases, the force of gravity down the plane, mg sinθ, increases. Friction force resists the slide till it attains its maximum value.

$f_{max} = \mu N = \mu\, mg \cos\theta$

which decreases with θ (because cosθ decreases as θ increases)

Hence, beyond a critical value $\theta = \theta_c$, the blocks starts to slide down the plane. The critical angle is the one when mg sinθ is just equal of f_{max}, i.e. when

$mg \sin\theta_c = \mu\, mg \cos\theta_c$

or $\tan\theta_c = \mu$

where θ_c is called angle of repose

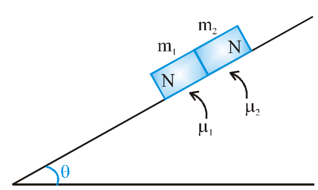
If $\theta > \theta_c$, block will slide down. For $\theta < \theta_c$ the block stays at rest on the incline.

5. Two blocks on an inclined plane

Consider two blocks having masses m_1 & m_2 respectively. If N is the normal force between the contact surface of m_1 & m_2.

Now three condition arises.

(i) If $\mu_1 = \mu_2 = \mu$ then



N = 0 because, Both the blocks are in contact but does not press each other.

$a_1 = a_2 = g \sin\theta - \mu\, mg \cos q$

(a_1, a_2 are acceleration of block μ_1 & μ_2 respectively)

(ii) If $\mu_1 < \mu_2 =$ then

N = 0 because, there is no contact between the blocks.

$a_1 = g \sin\theta - \mu_1 g \cos\theta$

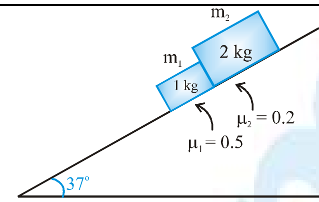
$a_2 = g \sin\theta - \mu_2 g \cos\theta$

$\Rightarrow a_1 > a_2$

(iii) If $\mu_1 > \mu_2 =$ then $N \neq 0$

$a_1 = a_2$

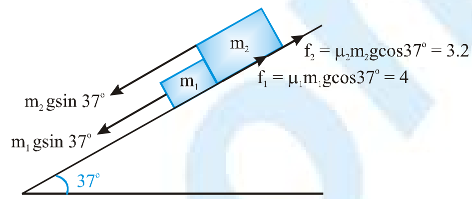
Ex. Mass m_1 & m_2 are place on a rough inclined plane as shown in figure. Find out the acceleration of the blocks and contact force in between these surface.



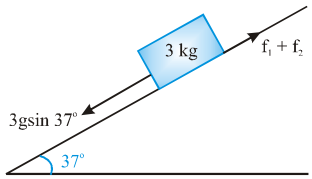
Sol. As we known if $\mu 1 > \mu 2$ both will travel together so

$a_1 = a_2 = a$

F.B.D

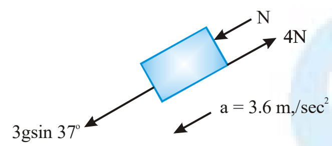


which is equivalent to



$$a = \frac{3g\sin 37° - (f_1 + f_2)}{3} \qquad a = \frac{18 - 7.2}{2}\ 3.6\ \text{m/sec}^2$$

Now F.B.D of 1 kg block is

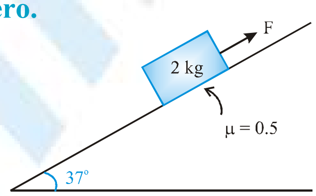


$g\sin 37° + N - 4 = (1)\,a$

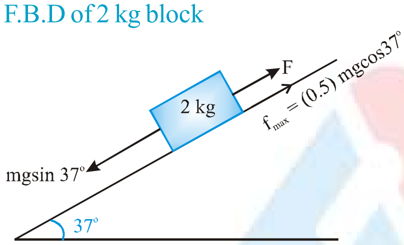
$N = 3.6 + 4 - 6 = 1.6$ Newton

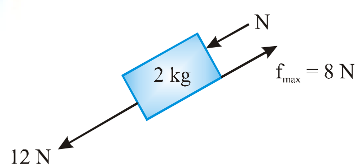
6. Range of force f for which acceleration of body is zero.

Ex. Find out the range of force in the above situation for which 2 kg block does not move on the incline.

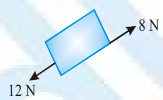
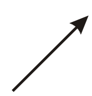
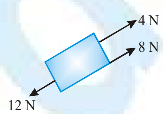
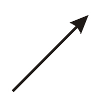
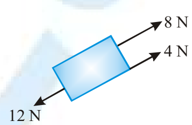
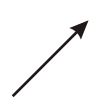


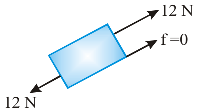
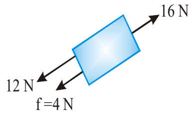
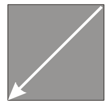
Sol. F.B.D of 2 kg block



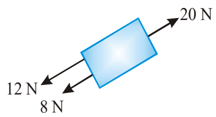
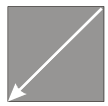
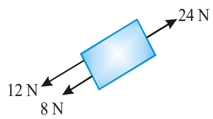
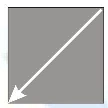


Now take different value of F

	Force (F)	F.B.D	Acceleration	Direction & magnitude	Friction Type
			$a = \frac{F_{net}}{m}$		
(i)	$F = 0$ N		2 m/s²	 8 N	Kinetic
(ii)	$F = 4$ N		0 m/s²	 8 N	Static
(iii)	$F = 8$ N		0 m/s²	 4 N	Static

(iv)	F = 12 N		0 m/s^2	0	Static
(v)	F = 16 N		0 m/s^2	 4 N	Static

(in this condition friction change its direction to stop relative slipping)

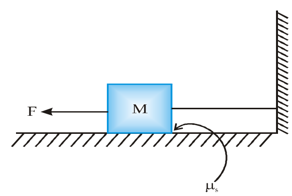
(vi)	F = 20 N		0 m/s^2	 8 N	Static
(vii)	F = 24 N		2 m/s^2	 8 N	Kinetic

From the above table block doesn`t move from F = 4N ($mg\sin\theta - \mu mg\cos\theta$) to F = 20 N ($mg\sin\theta + \mu mg\cos\theta$). So friction develope a range of force for which block doesn`t move

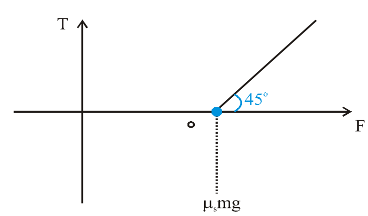
Note: If Friction is not present then only for F = 12 N the block will not move but friction develop a range of body is zero.

$mg\sin\theta - \mu mg\cos\theta \;\; F \;\; mg\sin\theta + \mu\, mg\cos\theta$.

Ex. In the following figure force F is gradually increase from zero. Draw the graph between applied force F and tension T in the string. The coefficient of static friction between the block and the ground is μ_s.

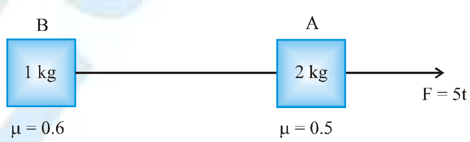


Sol. As the external force F is gradually increased from zero it is compensated by the friction and the string bears no tension. When limiting friction is achieved by increasing force F to a value till μ_s mg, the further increase in F is transferred to the string.

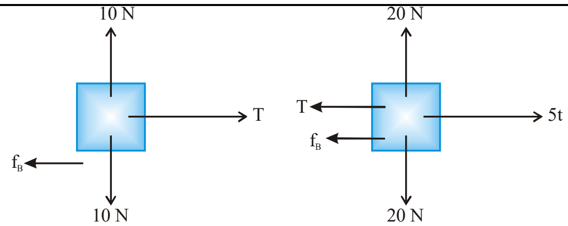


Ex. Fig. shows two blocks tied by a string. A variable force F = 5t is applied on the block. The coefficient of friction for the blocks are 0.6 and 0.5 respectively. Find the frictional force between blocks and ground as well as tension in the string at

(A) t = 1s **(B)** t = 2s **(C)** t = 3s



Sol.



(a) At t = 1s, F = 5 × 1 = 5 N

Maximum value of friction force

f_A = mN = 0.5 × 20 = 10 N

To keep the block stationary the magnitude of frictional force should be 5 N. So

f_A = 5 N

Now from the figure it become clear that if

f_A = 5N & F = 5 N, Tension T = 0

Since tension is not in application so frictional force on the block B is 0 i.e.,

f_B = 0

(b) At t = 2s, F = 5 × 2 = 10 N

Maximum value of friction force

f = μN = 0.5 × 20 = 10 N

To keep the block stationary the magnitude of friction force should be 10 N. So

f_A = 10 N

From the figure it is clear that if

f_A = 10 N and F = 10 N

Tension T = 0

Hence friction force on block B is f_B = 0

(c) At t = 3s, F = 5 × 3 = 15 N

Maximum value of friction force

f = μN = 0.5 × 20 = 10 Newton

Again applying the same analogy f_A = 10 N

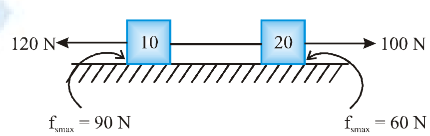
From the figure it is clear that if

f_A = 10 N and F = 15 N

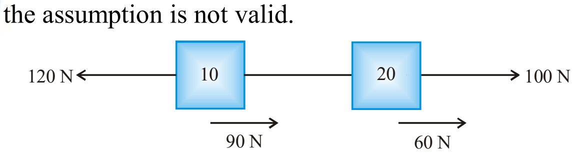
Tension T = 5 N

So frictional force on block B is f_B = 5 Newton

Ex. Find the tension in the string in situation as shown in the figure below. Forces 120 N and 100 N start acting when the system is at rest.

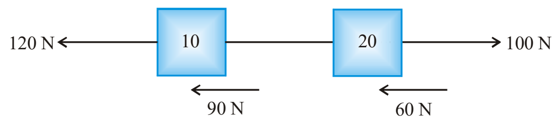


Sol. **(i)** Let us assume that system moves towards left then as it is clear from FBD, net force in horizontal direction is towards right. Therefore the assumption is not valid.



Above assumption is not possible as net force on system comes towards right. Hence system is not moving towards left.

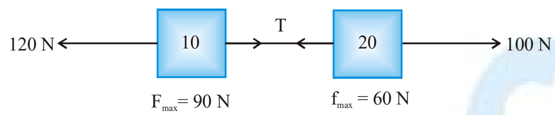
(ii) Similarly let us assume that system moves towards right.



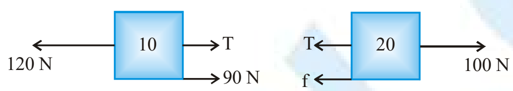
Above assumption is also not possible as net force on the system is towards left in this situation.

Hence assumption is again not valid.

Therefore it can be concluded that the system is stationary



Assuming that the 10 kg block reaches limiting friction first then using FBD`s



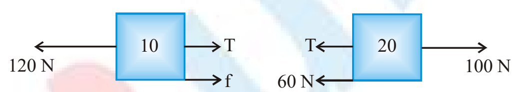
$120 = T + 90 \quad \Rightarrow \quad T = 30\text{ N}$

Also $\quad T + f = 100$

$\therefore \quad 30 + f = 100 \quad \Rightarrow \quad f = 70\text{ N}$

which is not possible as the limiting value is 60 N for this surface of block.

$\therefore$ Our assumption is wrong and now taking the 20 kg surface to be limiting we have



$T + 60 = 100\text{ N} \quad \Rightarrow \quad T = 40\text{ N}$

Also $\quad f + T = 120\text{ N} \quad \Rightarrow \quad f = 80\text{ N}$

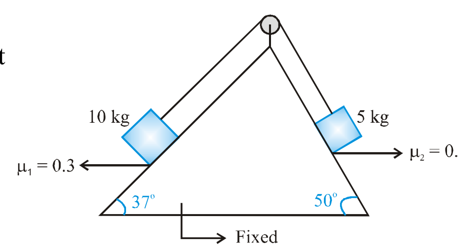
This is acceptable as static friction at this surface should be less than 90 N.

Hence the tension in the string is T = 40 N

7. Pully block system involving friction

If friction force is acting and value of acceleration of a particle is negative, then it means direction of friction force is opposite to that what we assumed and acceleration would be having a different numerical value.

Ex. Two blocks of masses 5 kg and 10 kg are attached with the help of light string and placed on a rough incline as shown in the figure. Coefficient of friction are as marked in the figure. The system is released from rest. Determine the acceleration of the two blocks.



Sol. Let 10 kg block is sliding down, then acceleration of both the blocks are given by,

$$a = \frac{10g\sin 37° - \mu_1 \times 10g\cos 37° - 5g\sin 53° - \mu_2 \times 5g\cos 53°}{15} = -ve$$

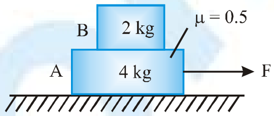
It means our assumed direction of motion is wrong and 5 kg block is going to slide down, if this would be the case, the direction of friction force will reverse and acceleration of blocks would be given by

$$a_1 = \frac{5g\sin 53° - \mu_2 \times 5g\cos 53° - \mu_1 \times 10g\cos 37° - 10g\sin 37°}{15} = -ve$$

It means in this direction also there is no motion. So we can conclude that the system remains at rest and friction force is static in nature.

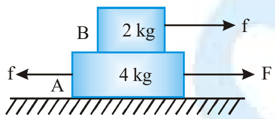
8. Two Block System

Ex. Find out the maximum value of F for which both the blocks will move together



Sol. In the given situation 2 kg block will move only due to friction force exerted by the 4 kg block

F.B.D

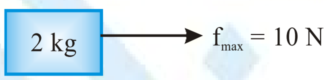


The maximum friction force exerted on the block B is

$f_{max} = \mu N$

$f_{max} = (0.5)(20) = 10$ N

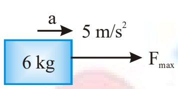
So the maximum acceleration of 2 kg block is



$a_{2max} = \frac{10}{2} = 5$ m/s^2

a_{max} is the maximum acceleration for which both the block will move together, i.e., for $a \le 5$ ms^{-2} acceleration of both blocks will be same and we can take both the blocks as a system.

F.B.D



$F_{max} = 6 \times 5 = 30$ N

for $0 < F < 30$

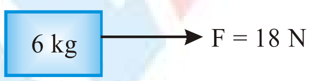
Both the block move together.

Ex. In the above question find the acceleration of both the block when

(i) F = 18 N **(ii)** F = 36 N

Sol. **(i)** Since F < 30 both the blocks will move together

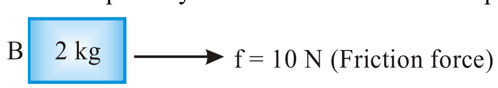
F.B.D



$a = \frac{18}{6} = 3$ m/s^2

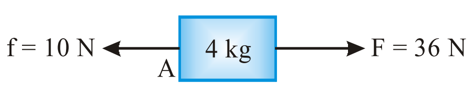
(ii) When F > 30 both the blocks will move separately so we treat each block independently

F.B.D of 2 kg block



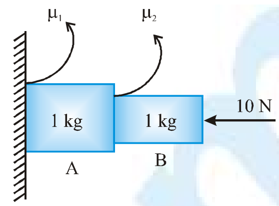
$a_B = 5$ m/s^2

F.B.D of 4 kg block



$aA = \frac{36-10}{4} = \frac{26}{4}$ m/s2

Ex. Find the acceleration of blocks A and B for the following cases.

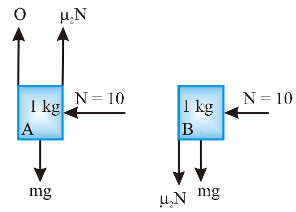


(A) $\mu_1 = 0$ and $\mu_2 = 0.1$	**(P)** $a_A = a_B = 9.5\ m/s^2$
(B) $\mu_2 = 0$ and $\mu_1 = 0.1$	**(Q)** $a_A = 9\ m/s^2\ a_B = 10\ m/s^2$
(C) $\mu_1 = 0.1$ and $\mu_2 = 1.0$	**(R)** $a_A = a_B = g = 10\ m/s^2$
(D) $\mu_1 = 1.0$ and $\mu_2 = 0.1$	**(S)** $a_A = 1,\ a_B = 9\ m/s^2$

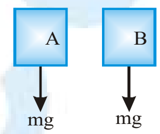
Sol. **(a)** R, **(b)** Q, **(c)** P, **(d)** S

(i) FBD in (case **(i)**)

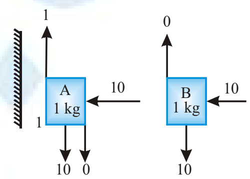
$\{\mu_1 = 0, \mu_2 = 0.1\}$



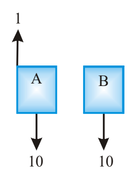
While friction`s work is to oppose the relative motion and here if relative motion will start then friction comes and without relative motion there is no friction so both the block move together with same acceleration and friction will not come.



(ii) Friction between wall and block A oppose relative motion since wall is stationary so friction wants to slop block A also and maximum friction will act between wall and block while there is no friction between block.

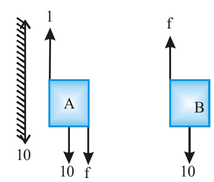


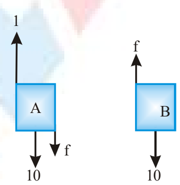
Note: Friction between wall and block will oppose relative motion between wall and block only it will not do anything for two block motion.



$a_A = 9\ m/s^2$; $a_B = 10\ m/s^2$

(iii) Friction between wall and block will be applied maximum equal to 1 N but maximum friction available between block A and B is 10 N but if this will there then relative motion will increase while friction is to oppose relative motion. So friction will come less than 10 so friction will be f that will be static.

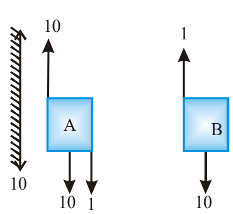




by system $(20 - 1) = 2 \times a \Rightarrow a = \dfrac{19}{2} = 9.5$ m/s2

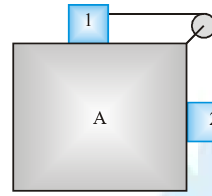
(iv) $a_A = \dfrac{11-10}{1} = 1\ m/s^2$

$a_B = \dfrac{10-1}{1} = 9\ m/s^2$



9. Friction involving Pseudo Concept

Ex. What is the minimum acceleration with which bar A should be shifted horizontally to keep the bodies 1 and 2 stationary relative to the bar? The masses of the bodies are equal and the coefficient of friction between the bar and the bodies equal to μ. The masses of the pulley is absent.



Sol. Let us place the observer on A.

Since we have non-inertial frame we have pseudo forces.

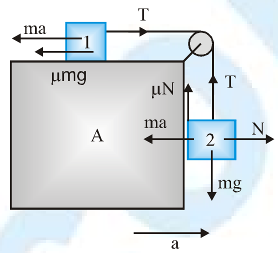
For body '1' we have,

$T = ma + \mu mg$ ....**(1)**

For body '2' we have,

$N = ma$

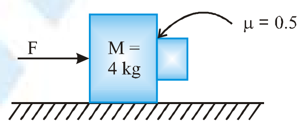
$\therefore \quad mg - T - \mu ma$ ....**(2)**



From (1) and (2) $\quad a_{min} = g\left(\dfrac{1-\mu}{1+\mu}\right)$

Ex. Find out the range of force for which smaller block is at rest with respect to bigger block. DIAGRAM

Sol. Smaller block is at rest w.r.t. the bigger block. Let both the block travels together with acceleration a F.B.D of smaller block w.r.t. to the bigger block.

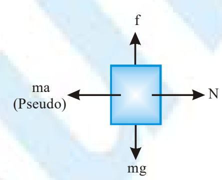


$f_{max} = m \times N$

$N = ma$

$f = \mu\, ma$ ....**(1)**

$\Rightarrow \quad f = mg$ ....**(2)**



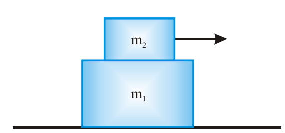
from (1) & (2)

$a = g/\mu = 20\ m/s^2$

So $F = 20\,(M + m) = 20\,(5) = 100\ N$

If $F \geq 100\ N$ Both will travel together

Ex. Mass m_2 placed on a plank of mass m_1 lying on a smooth horizontal plane. A horizontal force $F = a_0 t$ (a_0 is a constant) is applied to a bar. If acceleration of the plank and bar are a_1 and a_2 respectively and the coefficient of friction between m_1 and m_2 is μ. Then find acceleration a with time t.



Sol. If $F < \mu\, m_2 g$ then both blocks move with common acceleration, i.e., $a_1 = a_2$

When $F > \mu\, m_2 g$, then

Equation for block of mass m

$F - \mu m_2 g = m_2 a_2$ ....**(1)**

and $\quad \mu m_2 g = m_1 a_1$ ....**(2)**

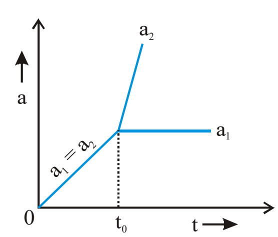
From equation (1)

$a0t - \mu m_2 g = m_2 a_2$

i.e., acceleration a_2 varies with time linearly, its slope positive and intercept negative.

From equation (2) a_1 is independent of time.

So, the graph between a & t is as follow



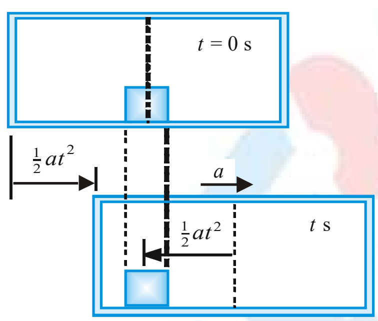
Inertial and Non-inertial Reference Frames

A body is observed in motion, when it changes its position or orientation relative to another body or another set of bodies. A frame of reference consists of a set of coordinate axes fixed with the body relative to which the motion is observed and a clock. The coordinate axes are required to measure position of the moving body and the clock is required to measure time.

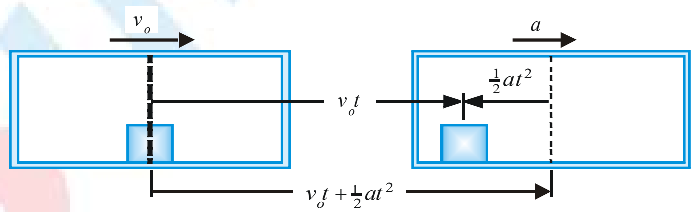
All the kinematical variables position, velocity, and acceleration are measured relative to a reference frame; therefore depend on the state of motion of the reference frame and we say that motion is essentially a relative concept. When the reference frame and a body both are stationary or move identically, the body is observed stationary relative to the reference frame. It is only when the reference frame and the body move in different manner, the body is observed moving relative the reference frame.

Now think about the whole unive rse where the planets, stars, galaxies and other celestial bodies all are in motion relative to each other. If any one of them can be assumed in state of rest, we can attach a reference frame to it and define motion of all other bodies relative to it. Such a body, which we assume in state of rest with respect to all other bodies in the universe, is known in absolute rest and the reference frame attached to it as most preferred reference frame. Unfortunately, the very notion of the reference frame and the idea motion as a relative concept, make it impossible to find a body anywhere in the universe at absolute rest. Therefore, the idea of absolute rest and a preferred reference frame become essentially meaningless. Now we can have only two categories of reference frames. In one category, we can have reference frames that move with uniform velocities and in the other category; we can have reference frames that are in accelerated motion.

To understand the above ideas let us think an experiment. Consider a closed container on a goods train either at rest or moving with constant velocity v_o on a level track. The floor of the container is smooth and a block is placed in the center of the container. We observe the situation relative to two reference frames, one fixed with the ground and other fixed with the container. Relative to the ground frame both the container and the block are at rest or move together with the same velocity and relative to the container frame the block is at rest as shown in the figure.



Container accelerates from rest



Uniformly moving container accelerates

Now let the driver of the train accelerates the container at uniform rate a. If the train were initially at rest, relative to the ground, the block remains at rest and the container moves forward. Relative to the container the block moves backwards with the same magnitude of acceleration as with the container moves forward. If the train were initially moving uniformly, relative to the ground the block continues to move with the same original velocity and train accelerates and becomes ahead in space. Relative to the container the block appears moving backward with acceleration that is equal in magnitude to the acceleration of the container.

Now consider a man sitting on a fixed chair in the container. He is always stationary relative to the container. If he does not look outs side, in no way he can know whether the container is at rest or moving uniformly. However, he can definitely say whether the container accelerates or not, by observing motion of the block relative to the container. Because net forces acting on the block are still zero, therefore observed acceleration of the block can only be due to acceleration of the container as per Newton's laws of motion.

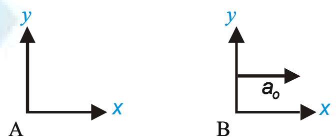
Now we can conclude that there can be only two kinds of reference frames either non-accelerated or accelerated. The reference frames that are non-accelerated i.e. at rest or moving with uniform velocities are known as inertial reference frames and those in accelerated motion as non-inertial reference frames.

Inertial Reference Frames and Newton's laws of motion

In Newton's laws of motion, force is conceived as two-body interaction that can be the only agent producing acceleration in a body. As far as we observe motion of a body from an inertial frame, any acceleration observed in a body can only be due to some forces acting on the body. All the three laws are in perfect agreement with the observed facts and we say that all the laws holds true in inertial reference frames.

Non-Inertial Reference Frames and Newton's laws of motion

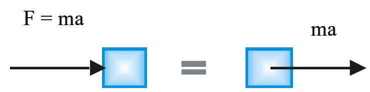
A body if at rest or in uniform velocity motion relative to some inertial frame net forces acting on it must be zero. Now if motion of the same body is observed relative to a non-inertial frame, it will be observed moving with acceleration that is equal in magnitude and opposite in direction to the acceleration of the non-inertial frame. This observed acceleration of the body is purely a kinematical effect. But to explain this observed acceleration relative to the non-inertial frame according to Newton's laws of motion, we have assume a force must be acting on the body. This force has to be taken equal to product of mass of the body and opposite of acceleration vector of the non-inertial frame. Since this force is purely an assumption and not a result of interaction of the body with any other body, it is a fictitious force. This fictitious force is known as pseudo force or inertial force.



Reference frames A is an inertial frame and B is a non inertial frame.

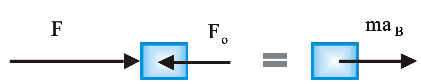
Until now, we have learn the idea that how we can apply Newton's laws of motion in non-inertial frame to a body in equilibrium in inertial frame. Now it is turn to discuss how we can apply Newton's laws of motion to analyze motion in non-inertial frame of a body, which is in accelerated motion relative to an inertial frame.

Consider a net physical force $\vec{F}$ in positive x-direction applied on the box. Here by the term physical force, we refer forces produced by two body interactions. Relative to inertial frame A, the box is observed to have an acceleration $\vec{a} = \vec{F}/m$ defined by the second law of motion and a force equal in magnitude and opposite in direction to $\vec{F}$ can be assigned to the body exerting $\vec{F}$ on the box constituting Newton's third law pair. All the three laws of motion hold equally well in inertial frame A.



A net physical force imparts acceleration to the box in inertial frame A.

Relative to non-inertial frame B, the box is observed moving in x-direction with acceleration $\vec{a}_B = \vec{a} - \vec{a}_o$, which can satisfy Newton's second law, only if the fictitious force $\vec{F}_o = -m\vec{a}_o$ is assumed acting together with the net physical force $\vec{F}$ as shown in the figure. Now we can write modified equation of Newton's second law in non-inertial frame.



A net physical force and pseudo force imparts acceleration to the box in non-inertial frame B.

$$\vec{F} + \vec{F}_o = m\vec{a}_B \rightarrow F - F_o = ma_B$$

From the above discussion, we can conclude that in non-inertial frames Newton's second law is made applicable by introducing pseudo force in addition to physical forces. The pseudo force equals to the product of mass of the concerned body and the acceleration of the frame of reference in a direction opposite to the acceleration of the frame of reference.

Practical inertial frame of reference

The definition of an inertial frame of reference is based on the concept of a free body in uniform velocity motion or absolute rest. It is impossible to locate a body anywhere in the universe, where forces from all other bodies exactly balance themselves and lead to a situation of uniform velocity motion according to the first law. Thus, we cannot find anywhere in the universe a body, to which we attach a frame of reference and say that it is a perfect inertial frame of reference. It is the degree of accuracy, required in analyzing a particular physical situation that decides which body in the universe is to be selected a preferred inertial frame of reference.

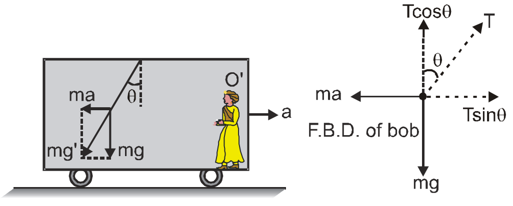
The earth and other planets of the sun are rotating about their own axis and revolving around the sun and the sun is moving too. In fact, all celestial bodies in the universe the sun, its planets, other stars, our galaxy the Milky Way, and other galaxies are in accelerated motion whose nature is not known exactly. Therefore, none of them can be used as a perfect inertial frame of reference. However, when the acceleration of any one of the above-mentioned celestial bodies becomes negligible as compared to the accelerations involved in a physical situation, a frame of reference attached with the corresponding celestial body may be approximated as an inertial frame of reference and the physical situation under consideration may be analyzed with acceptable degree of accuracy.

The centripetal acceleration due to rotation of the earth at any point on its surface varies from zero at the poles to a maximum value of approximately 0.034 m/s^2 at the equator. The physical phenomena, which we usually observe describe motion of a body on or near the earth surface such as motion of transport vehicles, short-range missiles, an oscillating pendulum etc. In these phenomena, the acceleration due to rotation of the earth may be neglected and a frame of reference attached at any point on the earth surface may be considered as a preferred inertial frame of reference. If the moving body is at considerable distance from the earth as in the case of satellites, long-range missiles etc., the effect of rotation of the earth become significant. For these situations or like ones we can attach the frame of reference at the earth's center and consider it as an inertial frame of reference. In astronomical field and space exploration programs, we require very high accuracy. Therefore, a frame of reference attached to distant stars is used as an inertial frame of reference. These stars are situated at such a vast distance from the earth that they appear as a motionless point source of light thus closely approach to the condition of absolute rest.

Ex. A pendulum of mass m is suspended from the ceiling of a train moving with an acceleration 'a' as shown in figure. Find the angle θ in equilibrium position.

Sol.

Non–inertial frame of reference (Train)

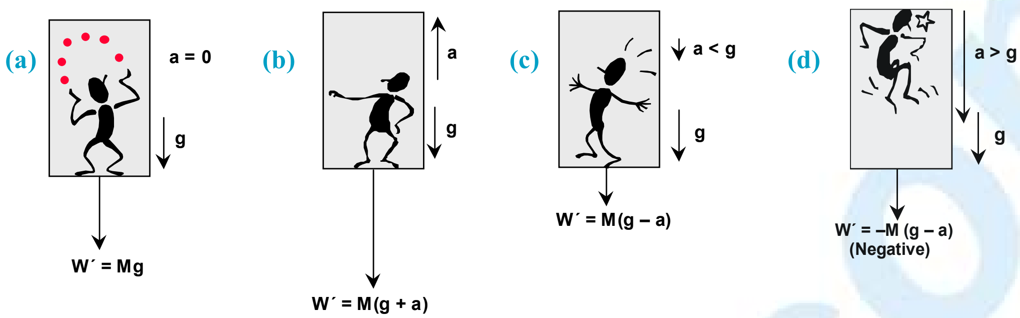


F.B.D. of bob w.r.t. train. (physical forces + pseudo force) : with respect to train, bob is in equilibrium

$\therefore \Sigma F_y = 0 \Rightarrow T\cos\theta = mg$ and $\Sigma F_x = 0 \Rightarrow T\sin\theta = ma \Rightarrow \tan\theta = \dfrac{a}{g} \Rightarrow \theta = \tan^{-1}\left(\dfrac{a}{g}\right)$

Ex. The weight of a body is simply the force exerted by earth on the body. If body is on an accelerated platform, the body experiences fictitious force, so the weight of the body appears changed and this new weight is called apparent weight. Let a man of weight W = Mg be standing in a lift.

We consider the following cases :



Case

(a) If the lift moving with constant velocity v upwards or downwards. In this case there is no accelerated motion hence no pseudo force experienced by observer inside the lift. So apparent weight W′ = Mg Actual weight.

(b) If the lift is accelerated upward with constant acceleration a. Then net forces acting on the man are **(i)** weight W = Mg downward **(ii)** fictitious force F_0=Ma downward.

So apparent weight $W' = W + F_0 = Mg + Ma = M(g + a)$

(c) If the lift is accelerated downward with acceleration a < g . Then fictitious force F_0 = Ma acts upward while weight of man W = Mg always acts downward. So apparent weight $W'=W+F_0 = Mg - Ma = M(g-a)$

Special Case :

If a = g then W′ = 0 (condition of weightlessness).Thus, in a freely falling lift the man will experience weightlessness.

(d) If lift accelerates downward with acceleration a > g . Then as in Case c . Apparent weight W′ =M(g–a) is negative, i.e., the man will be accelerated upward and will stay at the ceiling of the lift.

Ex. A spring weighing machine inside a stationary lift reads 50 kg when a man stands on it. What would happen to the scale reading if the lift is moving upward with **(i)** constant velocity, and **(ii)** constant acceleration?

Sol.

(i) In the case of constant velocity of lift, there is no fictitious force; therefore the apparent weight = actual weight. Hence the reading of machine is 50 kgwt.

(ii) In this case the acceleration is upward, the fictitious force ma acts downward, therefore apparent weight is more than actual weight i.e. W′= m (g + a).

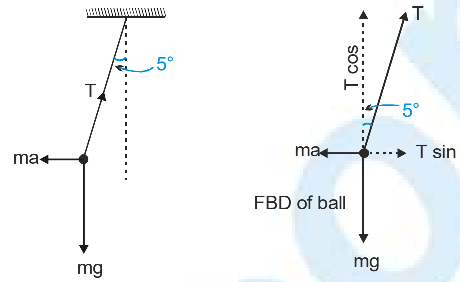
Hence scale shows a reading = m (g + a) = $\dfrac{mg\left(1+\dfrac{a}{g}\right)}{g} = \left(50 + \dfrac{50a}{g}\right)$ kg wt.

Ex. Two objects of equal mass rest on the opposite pans of an arm balance. Does the scale remain balanced when it is accelerated up or down in a lift?

Sol. Yes, since both masses experience equal fictitious forces in magnitude as well as direction.

Ex. A passenger on a large ship sailing in a quiet sea hangs a ball from the ceiling of her cabin by means of a long thread. Whenever the ship accelerates, she notes that the pendulum ball lags behind the point of suspension and so the pendulum no longer hangs vertically. How large is the ship's acceleration when the pendulum stands at an angle of 5° to the vertical?

Sol. The ball is accelerated by the force T sin5°.

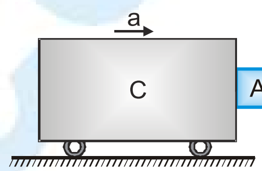


Therefore T sin5° = ma

Vertical component ΣF = 0, so T cos5° = mg

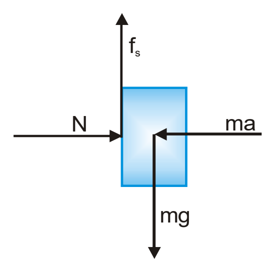
By solving a = g tan 5° = 0.0875 g = 0.86 m/s²

Ex. Consider the figure shown here of a moving cart C. If the coefficient of friction between the block A and the cart is μ, then calculate the minimum acceleration a of the cart C so that the block A does not fall.



Sol.

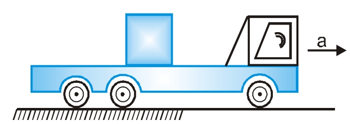
The forces acting on the block A (in block A's frame (i.e. non inertial frame) are :



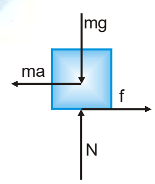
For A to be at rest in block A's frame i.e. no fall,

We require W = f_s ⇒ mg = μ(ma) Thus $a = \dfrac{g}{\mu}$

Ex. A block of mass 1kg lies on a horizontal surface in a truck, the coefficient of static friction between the block and the surface is 0.6, What is the force of friction on the block. If the acceleration of the truck is 5 m/s².



Sol. Fictitious force on the block F = ma = 1 × 5 = 5N



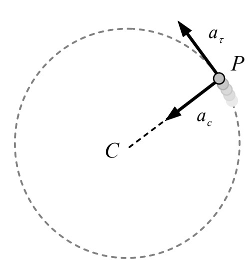
While the limiting friction force

$F = \mu_s N = \mu_s mg = 0.6 \times 1 \times 9.8 = 5.88$ N

As applied force F lesser than limiting friction force. The block will remain at rest in the truck and the force of friction will be equal to 5N and in the direction of acceleration of the truck.

Dynamics of Circular Motion

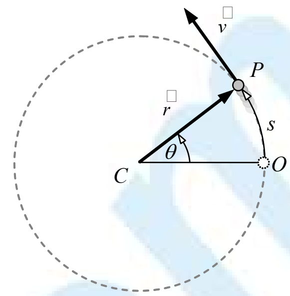
Velocity vector points always tangent to the path and continuously change its direction, as a particle moves on a circular path even with constant speed and give rise to normal component of acceleration, which always points toward the center of the circular path. This component of acceleration is known as centripetal (center seeking) acceleration and denoted by a_c. Moreover, if speed also changes the particle will have an additional acceleration component along the tangent to the path. This component of acceleration is known as tangential acceleration and denoted by a_τ.



The centripetal and tangential accelerations

The centripetal acceleration accounts only for continuous change in the direction of motion whereas the tangential acceleration accounts only for change in speed.

Consider a particle moving on circular path of radius r. It passes point O with velocity v_o at the instant t = 0 and point P with velocity v at the instant t traveling distance s along the path and angular displacement θ as shown in the figure. Kinematics of this circular motion is described in terms of linear variables as well as angular variables.



Kinematics of Circular Motion

Angular Variables	Linear Variables
Angular displacement θ	Distance traveled s
Angular velocity $\omega = \dfrac{d\theta}{dt}$	Speed $v = \dfrac{ds}{dt}$
Angular acceleration $\alpha = \dfrac{d\omega}{dt} = \dfrac{d^2\theta}{dt^2} = \omega\dfrac{d\omega}{d\theta}$	Tangential acceleration $a_\tau = \dfrac{dv}{dt} = \dfrac{d^2s}{dt^2} = v\dfrac{dv}{ds}$
	Centripetal acceleration $a_C = \dfrac{v^2}{r}$

Relations between angular and linear variable in circular motion

Distance traveled $s = \theta r$

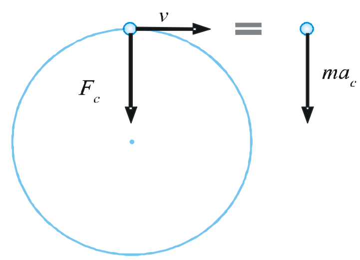
Speed $v = \omega r$

Tangential acceleration $a_\tau = ar$

Centripetal acceleration $a_C = \dfrac{v^2}{r} = \omega^2 r = \omega v$

Application of Newton's law in Circular Motion

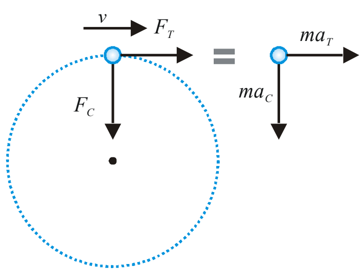
Consider a particle of mass m moving with uniform speed v in a circle of radius r as shown in figure. It necessarily posses a centripetal acceleration and hence there must be a net force $\left(\sum \vec{F} = \vec{F}_C\right)$ acting always towards the center according to the second law. This net force F_C acting towards the center is known as centripetal force.



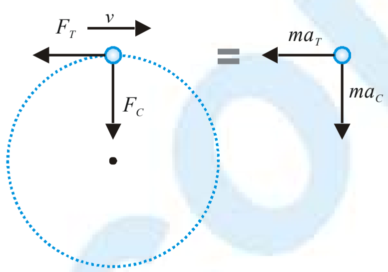
$$\sum \vec{F} = m\vec{a} \rightarrow F_C = ma_C$$

When a particle is whirled with the help of a string in a horizontal circle, the required centripetal force is the tension in the string. The gravitational attraction between a satellite and the earth, between moon and the earth, between the sun and its planets, and the electrostatic attraction between the nucleus and electrons are the centripetal forces and provide the necessary centripetal acceleration.

Now consider a particle moving on circular path with varying speed. The net acceleration has two components, the tangential acceleration and the centripetal acceleration. Therefore, the net force must also have two components, one component in tangential direction to provide the tangential acceleration and the other component towards the center to provide the centripetal acceleration. The former one is known as tangential force and the latter one as centripetal force.



Particle moving with increasing speed



Particle moving with decreasing speed

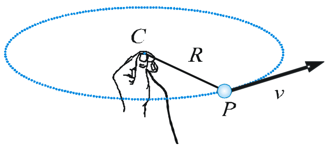
When the particle moves with increasing speed the tangential force acts in the direction of motion and when the particle moves with decreasing speed the tangential force acts in direction opposite to direction of motion.

To write equations according to the second law, we consider the tangential and the radial directions as two mutually perpendicular axes. The components along the tangential and the radial directions are designated by subscripts T and C.

$$F_C = ma_C$$

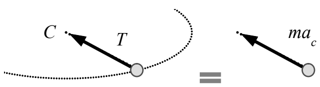
$$F_T = ma_T$$

Ex. In free space, a man whirls a small stone P of mass m with the help of a light string in a circle of radius R as shown in the figure. Establish the relation between the speed of the stone and the tension developed in the string. Also, find the force applied on the string by the man.



Sol. The system is in free space therefore no force other than the tension acts on the stone to provide necessary centripetal force. The tension does not have any component in tangential direction therefore tangential component of acceleration is zero. In the adjoining figure it is shown that how tension (T) in the string produces necessary centripetal force.

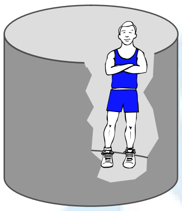
Applying Newton's second law of motion to the stone, we have $F_C = ma_C \rightarrow T = ma_C = \dfrac{mv^2}{R}$



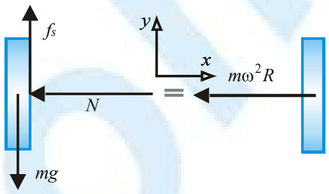
The end where man holds the string is stationary and tension applied by the string to this end is T towards the stone, therefore the man must apply a force equal to the tension in magnitude but in a direction away from the stone.

$$T = \frac{mv^2}{R}$$

Ex. A boy stands on a horizontal platform inside a cylindrical container of radius R resting his back on the inner surface of the container. The container can be rotated about the vertical axis of symmetry. The coefficient of static friction between his back and the inner surface of the container is μ_s. The angular speed of the container is gradually increased. Find the minimum angular speed at which if the platform below his feet removed, the boy should not fall.



Sol. As the container rotates at angular speed ω the boy moves in a circular path of radius R with a speed v = ωR. Since the angular speed is increased gradually the angular acceleration can be ignored and hence the tangential acceleration of the boy too. Thus, the boy has a centripetal acceleration of ω^2R, provided by the normal reaction N applied by the wall of the container. The weight of the boy is balanced by the force of static friction. All these force are shown in the adjoining figure where the boy is shown schematically by a rectangular box of mass m.



$$\sum F_x = ma_x \rightarrow N = m\omega^2 R \qquad \text{...(i)}$$

$$\sum F_y = ma_y \rightarrow f_s = mg \qquad \text{...(ii)}$$

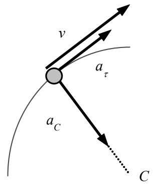
Since force of static friction cannot be greater than the limiting friction $\mu_S N$,

we have $f_s \leq \mu_s N$ ...(iii)

From the above equations, the minimum angular speed is $\omega_{min} = \sqrt{\dfrac{g}{\mu_s R}}$

Ex. A motorcyclist wishes to travel in circle of radius R on horizontal ground and increases speed at constant rate a. The coefficient of static and kinetic frictions between the wheels and the ground are μ_s and μ_k. What maximum speed can he achieve without slipping?

Sol. The motorcyclist and the motorcycle always move together hence they can be assumed to behave as a single rigid body of mass equal to that of the motorcyclist and the motorcycle. Let the mass of this body is m. The external forces acting on it are its weight (mg), the normal reaction N on wheels from ground, and the force of static friction f_s. The body has no acceleration in vertical direction therefore; the normal reaction N balances the weight (mg).

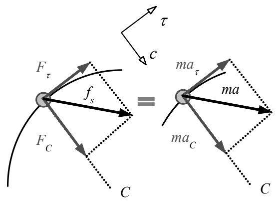


$$N = mg \qquad \text{...(i)}$$

The frictional force cannot exceed the limiting friction.

$$f_{sm} \leq \mu_s N \qquad \text{...(ii)}$$

During its motion on circular path, the only external force in horizontal plane is the force of static friction, which is responsible to provide the body necessary centripetal and tangential acceleration. These conditions are shown in the adjoining figure where forces in vertical direction are not shown.



$$F_\tau = ma_\tau \qquad \text{...(iii)}$$

$$F_C = ma_C = \frac{mv^2}{r} \qquad \text{...(iv)}$$

The above two forces are components of the frictional force in tangential and normal directions. Therefore, we have

$$f_s = \sqrt{F_\tau^2 + F_C^2} = m\sqrt{a_\tau^2 + a_C^2} \qquad \text{...(v)}$$

The centripetal acceleration increases with increase in speed and the tangential acceleration remains constant. Therefore, their resultant increases with speed. At maximum speed the frictional force achieves its maximum value (limiting friction f_{sm}), therefore from eq. (i), (ii), (iii), (iv), and (v), we have

$$v = \left[r^2\left\{(\mu_s g)^2 - a^2\right\}\right]^{\frac{1}{4}}$$

Circular Turning of Roads

When vehicles go through turnings, they travel along a nearly circular arc. There must be some force which will produce the required centripetal acceleration. If the vehicles travel in a horizontal circular path, this resultant force is also horizontal. The necessary centripetal force is being provided to the vehicles by following three ways:

• By friction only. • By banking of roads only. • By friction and banking of roads both.

In real life the necessary centripetal force is provided by friction and banking of roads both.

(i) By Friction only

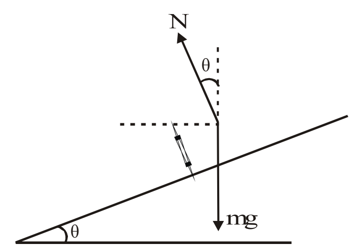
Suppose a car of mass m is moving at a speed v in a horizontal circular arc of radius r. In this case, the necessary centripetal force to the car will be provided by force of friction f acting towards centre.

Thus, $f = \dfrac{mv^2}{r} \rightarrow f_{max} = \mu N = \mu mg$

Therefore, for a safe turn without sliding $\dfrac{mv^2}{r} \le f_{max} \Rightarrow \dfrac{mv^2}{r} \le \mu mg \Rightarrow v \le \sqrt{\mu rg}$

(ii) By Banking of Roads only

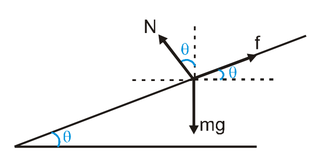
Friction is not always reliable at circular turns if high speeds and sharp turns are involved. To avoid dependence on friction, the roads are banked at the turn so that the outer part of the road is some what lifted compared to the inner part.



$$N\sin\theta = \frac{mv^2}{r} \text{ and } N\cos\theta = mg \Rightarrow \tan\theta = \frac{v^2}{rg} \Rightarrow v = \sqrt{rg\tan\theta}$$

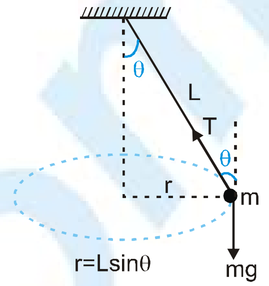
(iii) Friction and Banking of Road both

If a vehicle is moving on a circular road which is rough and banked also, then three forces may act on the vehicle, of these the first force, i.e., weight (mg) is fixed both in magnitude and direction. The direction of second force, i.e., normal reaction N is also fixed (perpendicular to road) while the direction of the third force, i.e., friction f can be either inwards or outwards while its magnitude can be varied upto a maximum limit ($f_{max} = \mu N$). So, the magnitude of normal reaction N and direction plus magnitude of friction f are so adjusted that the resultant of the three forces mentioned above is $\dfrac{mv^2}{r}$ towards the centre.



Conical Pendulum

If a small particle of mass m tied to a string is whirled in a horizontal circle, as shown in figure. The arrangement is called the 'conical pendulum'. In case of conical pendulum the vertical component of tension balances the weight while its horizontal component provides the necessary centripetal force. Thus,,



$$T\sin\theta = \frac{mv^2}{r} \text{ and } T\cos\theta = mg \Rightarrow v = \sqrt{rg\tan\theta}$$

$$\therefore \text{ Angular speed } \omega = \frac{v}{r} = \sqrt{\frac{g\tan\theta}{r}}$$

So, the time period of pendulum is $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{r}{g\tan\theta}} = 2\pi\sqrt{\frac{L\cos\theta}{g}}$

Ex. Find the maximum speed at which a car can turn round a curve of 30 m radius on a level road if the coefficient of friction between the tyres and the road is 0.4 [acceleration due to gravity = 10 m/s^2]

Sol. Here centripetal force is provided by friction so

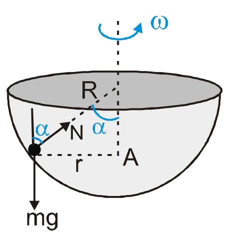
$$\frac{mv^2}{r} \le \mu mg \Rightarrow v_{max} = \sqrt{\mu rg} = \sqrt{120} \approx 11 \text{ ms}^{-1}$$

Ex. For traffic moving at 60 km/hr, if the radius of the curve is 0.1 km, what is the correct angle of banking of the road ? (g = 10 m/s^2)

Sol. In case of banking $\tan\theta = \frac{v^2}{rg}$ Here v= 60 km/hr = $60 \times \frac{5}{18}$ ms^{-1} = $\frac{50}{3}$ ms^{-1} r = 0.1 km = 100 m

So $\tan\theta = \frac{50/3 \times 50/3}{100 \times 10} = \frac{5}{18} \Rightarrow \theta = \tan^{-1}\left(\frac{5}{18}\right)$

Ex. A hemispherical bowl of radius R is rotating about its axis of symmetry which is kept vertical. A small ball kept in the bowl rotates with the bowl without slipping on its surface. If the surface of the bowl is smooth and the angle made by the radius through the ball with the vertical is α. Find the angular speed at which the bowl is rotating.



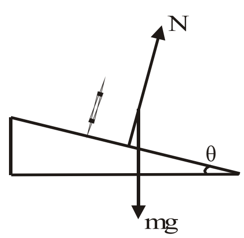
Sol.

Ncosα=mg and Nsinα = mrω^2 but r =R sinα

$\Rightarrow$ Nsinα = mRsinαω^2 $\Rightarrow$ N=mRω^2

$\Rightarrow (mR\omega^2)\cos\alpha = mg \Rightarrow \omega = \sqrt{\frac{g}{R\cos\alpha}}$

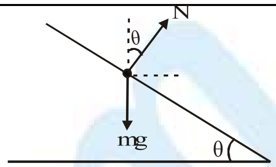
Ex. A car is moving along a banked road laid out as a circle of radius r.



(a) What should be the banking angle θ so that the car travelling at speed v needs no frictional force from the tyres to negotiate the turn?

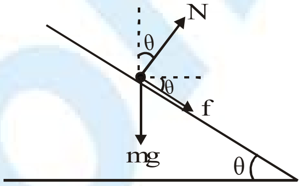
(b) The coefficients of friction between tyres and road are μ_s = 0.9 and μ_k = 0.8. At what maximum speed can a car enter the curve without sliding towards the top edge of the banked turn?

Sol. **(a)** $N\sin\theta = \dfrac{mv^2}{r}$ and $N\cos\theta = mg \Rightarrow \tan\theta = \dfrac{v^2}{rg}$



Note : In above case friction does not play any role in negotiating the turn.

(b) If the driver moves faster than the speed mentioned above, a friction force must act parallel to the road, inward towards centre of the turn.



$\Rightarrow F\cos\theta + N\sin\theta = \dfrac{mv^2}{r}$ and $N\cos\theta = mg + f\sin\theta$

For maximum speed of $f = \mu N$

$$\Rightarrow N(\mu\cos\theta + \sin\theta) = \frac{mv^2}{r} \text{ and } N(\cos\theta - \mu\sin\theta) = mg$$

$$\Rightarrow \frac{v^2}{rg} = \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} \Rightarrow v = \sqrt{\left(\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right)rg}$$

• Etoos Tips & Formulas •

NEWTON'S LAWS OF MOTION

1. Force

A push or pull that one object exerts on another.

2. Forces in nature

There are four fundamental forces in nature :

1. Gravitational force
2. Electromagnetic force
3. Strong nuclear force
4. Weak force

3. Types of forces on macroscopic objects

(a) Field Forces or Range Forces :

There are the forces in which contact between two object is not necessary touching each other.

Ex. **(i)** Gravitational force between two bodies

(ii) Electrostatic force between two charges.

(b) Contact Forces :

Contact forces exist only as long as the objects are touching each other.

Ex. **(i)** Normal forces. **(ii)** Frictional force

(c) Attachment to Another Body :

Tension (T) in a string and spring force (F = kx) comes in this group.

4. Newton`s first law of motion (or Galileo`s law of Inertia)

Every body continues in its state of rest or uniform motion in a straight line unless compelled by an external force to change that state.

Inertia : Inertia is the property of the body due to which body opposes the change of it`s state. Inertia of a body measured by mass of the body.

$$\boxed{\text{inertia} \propto \text{mass}}$$

5. Newton`s second law

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt} \quad \text{(Linear momentum } \vec{p} = m\vec{v}\text{)}$$

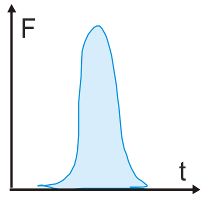
For constant mass system $\vec{F} = m\vec{a}$

Momentum : It is the product of the mass and velocity of a body i.e. momentum $\vec{P} = m\vec{v}$.

SI Unit : kg m s^{-1} **Dimensions** : $[M L T^{-1}]$

Impulse : Impulse = product of force with time

For a finite interval of time from t_1 to t_2 then the impulse = $\int_{t_1}^{t_2} \vec{F}dt$



impulse= area under curve

If constant force acts for an interval Δt then : Impulse = $\vec{F}\Delta t$

Impulse - Momentum theorem

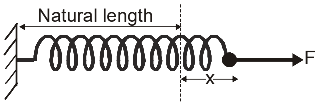
Impulse of a force is equal to the change of momentum $\vec{F}\Delta t = \Delta p$

6. Newton`s third law of motion :

Whenever a particle A exerts a force on another particle B, B simultaneously exerts a force on A with the same magnitude in the opposite direction.

7. Spring Force (According to Hooke`s law) :

In equilibrium F = kx (k is spring constant)



Note : Spring force is non impulsive in nature.

8. Motion of bodies in contact

When two bodies of masses m_1 and m_2 are kept on the frictionless surface and a force F is applied on one body, then the force with which one body presses the other at the point of contact is called force of contact. These two bodies will move with same acceleration a.

(a) When the force F acts on the body with mass m_1 as shown in fig (i)

$$F = (m_1 + m_2)a$$

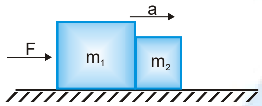


Fig. (1) : When the force F acts on mass m_1

(b) If the force exerted by m_2 on m_1 is f_1 (force of contact) then for body m_1 : $(F - f_1) = m_1a$

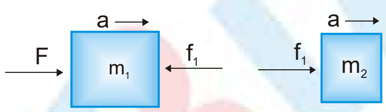


Fig. 1(a) : F.B.D. representation of action and reaction forces

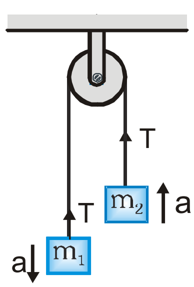
For body m_2 : $f_1 = m_2a \Rightarrow$ action of m_1 on m_2 : $\dfrac{m_2F}{m_1 + m_2}$

9. Pulley system

A single fixed pulley changes the direction of force only and in general, assumed to be massless and frictionless.

10. SOME CASES OF PULLEY

Case I



Let $m_1 > m_2$

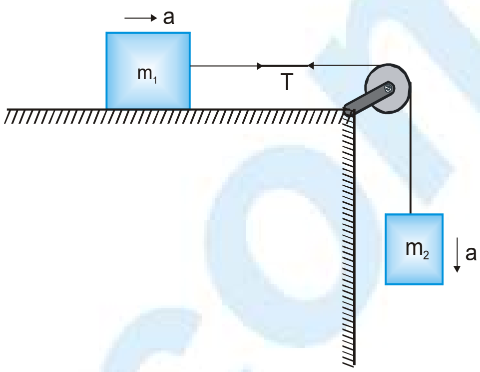
now for mass m_1, $m_1g - T = m_1a$

for mass m_2, $T - m_2g = m_2a$

$$\text{Acceleration} = a = \frac{(m_1 - m_2)}{(m_1 + m_2)}g = \frac{\text{net pulling force}}{\text{total mass to be pulled}}$$

Tension $= T = \dfrac{2m_1m_2}{(m_1+m_2)}g = \dfrac{2\times \text{Product of masses}}{\text{Sum of two masses}}g$

Reaction at the suspension of pulley $R = 2T = \dfrac{4m_1m_2g}{(m_1+m_2)}$



Case II

For mass m_1 : $T = m_1 a$

For mass m_2 : $m_2g - T = m_2a$

Acceleration $a = \dfrac{m_2g}{(m_1+m_2)}$ and $T = \dfrac{m_1m_2}{(m_1+m_2)}g$

11. FRAME OF REFERENCE

Inertial frames of reference : A reference frame which is either at rest or in uniform motion along the straight line. A non-accelerating frame reference is called an inertial frame of reference.

All the fundamental laws of physics have been formulates in respect of inertial frame of reference.

Non-inertial frame of reference : An accelerating frame of reference is called a non-inertial frame of reference. Newton`s laws of motion are not directly applicable in such frames, before application we must add pseudo force.

Pseudo force : The force on a body due to acceleration of non- inertial frame is called fictitious or apparent or pseudo force and is given by $\vec{F} = -m\vec{a}_0$ where $\vec{a}_0$ is acceleration of non-inertial frame with respect to an inertial frame and m is mass of the particle or body. The direction of pesudo force must be opposite to the direction of acceleration of the non-inertial frame

When we draw the free body diagram of a mass, with respect to an inertial frame of reference we apply only the real forces (forces which are actually acting on the mass). But when the free body diagram is drawn from a non-inertial frame of reference a pseudo force (in additional to all real forces) has to be applied to make the equation $\vec{F} = m\vec{a}$ to be valid in this frame also .

12. Man in a Lift

(a) If the lift moving with constant velocity v upwards or downwards this case there is no accelerated motion hence no pseudo force experienced by observer inside the lift.

So apparent weight W` = Mg = Actual weight.

(b) If the lift is accelerated upward with constant acceleration a. Then forces acting on the man w.r.t. observed inside the lift are

(i) Weight W = Mg downward

(ii) Fictitious force F_0 = Ma downward.

So apparent weight W` $= W + F_0 = Mg + Ma = M(g + a)$

(c) If the lift accelerated downward with acceleration a < g.

Then w.r.t. observer inside the lift fictitious force $F_0 = Ma$ acts upward while weightof man $W = Mg$ always acts downwards.

So apparent weight $W` = W + F_0 = Mg - Ma = M(g - a)$

Special Case :

If $a = g$ then $W` = 0$ (condition of weightlessness).

Thus, in a freely falling lift the man will experience weightlessness.

(d) If lift accelerates downward with acceleration $a > g$. Then as in Case (c). Apparent weight $W` = M(g - a)$ is negative, i.e. the man will be accelerated upward and will stay at the ceiling of the lift.

• Etoos Tips & Formula •

FRICTION

Friction is the force of two surfaces in contact, or the force of a medium acting on a moving object. i.e. air on aircraft. Friction forces arise due to molecular intractions. In some cases friction acts as a supporting force and in some cases it acts as opposing force.

1. Cause of Friction

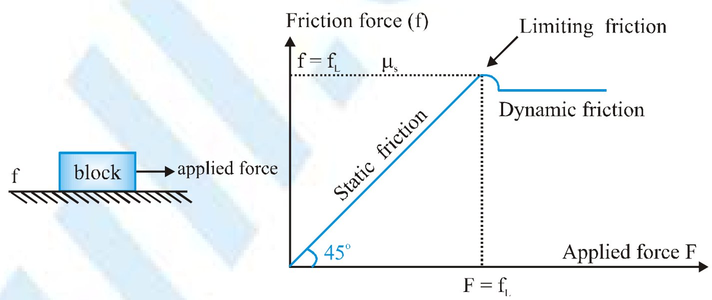
Friction is arises on account of strong atomic or molercular forces of attraction between the two surfaces at the point of actula contact.

2. Types of friction

Friction

- Static friction (No relative motion between objects)
- Dynamic friction (Kinetic friction) (There is relative motion between objects)

3. Graph between applied force and forces of friction



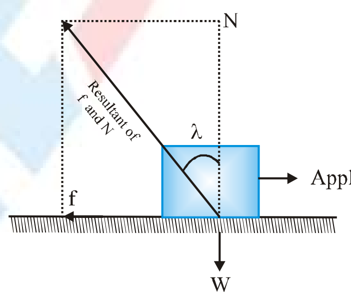
4. Static friction coefficient

$\mu_s = \frac{f_s}{N}, 0 \le f_s \le \mu_s N, \vec{f}_s = -\vec{F}_{applied}$; $(f_s)_{max} = \mu_s N =$ limiting friction

5. Sliding friction coefficient

$\mu_k = \frac{f_k}{N}, \vec{f}_k = -(\mu_k N)\hat{v}_{relative}$

6. Angle of Friction (λ)

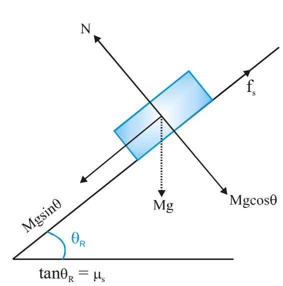


$\tan\lambda = \frac{f_s}{N} = \frac{\mu_s N}{N} = \mu_s$

7. Angle of repose :

The maximum angle of an inclined plane for which block remains stationary on the plane.

For smooth surface $\theta_R = 0$



8. **Dependent Motion of Connected Bodies**

Method I : Methos of constraint equations

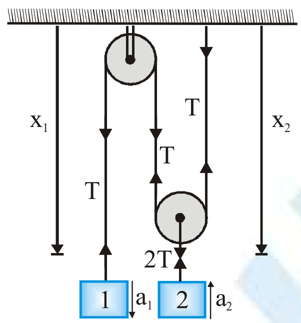
$$\sum x_i = \text{constant} \Rightarrow \sum \dot{x}_i = 0 \Rightarrow \sum \ddot{x}_i = 0$$

For n moving bodies we have $x_1, x_2, ...x_n$

No. of constraint equations = no. of strings

Method II : Method of virtual work : The sum of scalar products of forces applied by connecting links of constant length snd displacement of corresponding contact point equal to zero.

$$\sum \vec{F}_i.\delta\vec{r}_i = 0 \Rightarrow \sum \vec{F}_i.\vec{v}_i = 0 \Rightarrow \sum \vec{F}_i.\vec{a}_i = 0$$



Here $a_2 = 2a_1$

9. Aeroplane always fly at low altitudes becuase according to Newton`s II law of motion as aeroplane displaces air & at low altitude density of air is high.

10. Rockets move by pushing the exhaust gases out so they can fly at low & high altitude.

11. Pulling a lawn roller is easier than pushing it beacuse pushing increases the apparent weight and hence friction.

12. A moonghphaliwala sells his moongphali using a weighing machine in an elevator.He gain more profit if the elevator is accelerating up because the apparent weight of an object increases is an elevator while accelerating upward.

13. Pulling (figure I) is easier than pushing (figure II) on a rough horizontal surface beacuse normal reaction is less in pulling than in pushing.

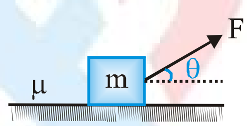


Fig - I

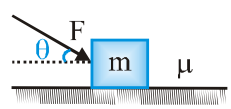


Fig - II

14. While walking on ice, one should take small steps to avoid slipping. This is beacuse smaller step increases the normal reaction and that ensure smaller friction.

15. A man in a closed cabin (lift) falling freely does not experience gravity as inertial and gravitational mass have equivalence.