ELASTICITY

A body is said to be rigid if the relative positions of its constituent particles remains unchanged when external deforming forces are applied to it. The nearest approach to a rigid body is diamond or carborundum.

Actually no body is perfectly rigid and every body can be deformed more or less by the application of suitable forces. All these deformed bodies however regain their original shape or size, when the deforming forces are removed.

The property of matter by virtue of which a body tends to regain its original shape and size after the removal of deforming forces is called elasticity.

Some terms related to elasticity :

1. **Deforming Force**

   External force which try to change in the length, volume or shape of the body is called deforming force.

2. **Perfectly Elastic Body**

   The body which perfectly regains its original form on removing the external deforming force, is defined as a perfectly elastic body. Ex. : quartz – Very nearly a perfect elastic body.

3. **Plastic Body**

   (a) The body which does not have the property of opposing the deforming force, is known as a plastic body.

   (b) The bodies which remain in deformed state even after removed of the deforming force are defined as plastic bodies.

4. **Internal restoring force**

   When a external force acts at any substance then due to the intermolecular force there is a internal resistance produced into the substance called internal restoring force.

   At equilibrium the numerical value of internal restoring force is equal to the external force.

STRESS

The internal restoring force acting per unit area of cross–section of the deformed body is called stress.

$$\text{Stress} = \frac{\text{Internal restoring force}}{\text{Area of cross section}} = \frac{F_{internal}}{A} = \frac{F_{external}}{A}$$

Stress depends on direction of force as well as direction of area of application so it is tensor.

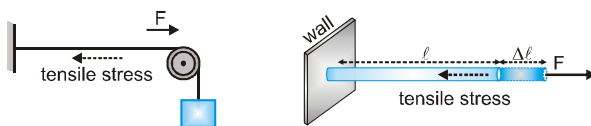**SI Unit :** $N\text{–}m^{-2}$

Dimensions : $M^1\ L^{-1}\ T^{-2}$

There are three types of stress :-

(a) **Longitudinal Stress :** When the stress is normal to the surface of body, then it is known as longitudinal stress. There are two types of longitudinal stress

(i) **Tensile Stress :** The longitudinal stress, produced due to increase in length of a body, is defined as tensile stress.

(ii)   **Compressive stress :** The longitudinal stress, produced due to decrease in length of a body, is defined as compressive stress.
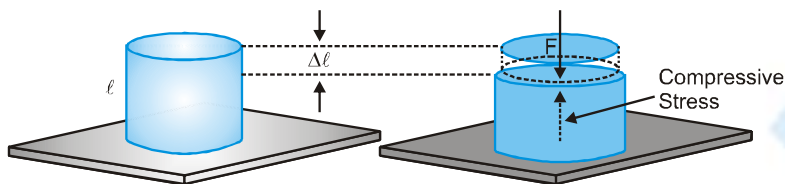

(b)   **Volume Stress**

If equal normal forces are applied every one surface of a body, then it undergoes change in volume. The force opposing this change in volume per unit area is defined as volume stress.

(c)   **Tangential Stress** or **Shear Stress**

When the stress is tangential or parallel to the surface of a body then it is known as shear stress. Due to this stress, the shape of the body changes or it gets twisted.
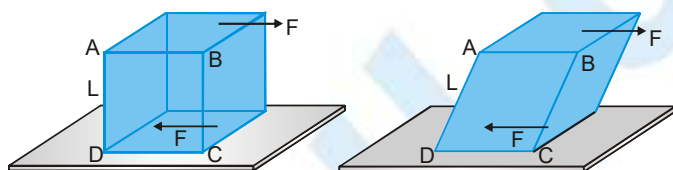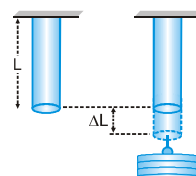

STRAIN

The ratio of change of any dimension to its original dimension is called strain.

$$\text{Strain} = \frac{\text{change in size of the body}}{\text{original size of the body}}$$
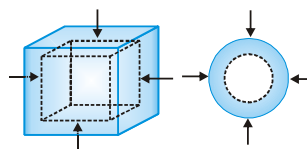
It is a unitless and dimensionless quantity.

There are three types of strain : Type of strain depends upon the directions of applied force.

(a)   **Longitudinal strain** $= \dfrac{\text{change in length of the body}}{\text{initial length of the body}} = \dfrac{\Delta L}{L}$


(b)   **Volume strain** $= \dfrac{\text{change in volume of the body}}{\text{original volume of the body}} = \dfrac{\Delta V}{V}$

(c)   **Shear strain**

When a deforming force is applied to a body parallel to its surface then its shape (not size) changes. The strain produced in this way is known as shear strain. The strain produced due to change of shape of the body is known as shear strain. $\tan \phi = \dfrac{l}{L}$ or $\phi = \dfrac{l}{L} = \dfrac{\text{displacement of upper face}}{\text{distance between two faces}}$
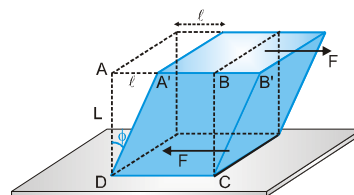
## Relation Between angle of twist and Angle of shear

When a cylinder of length '●' and radius 'r' is fixed at one end and tangential force is applied at the other end, then the cylinder gets twisted. Figure shows the angle of shear ABA' and angle of twist AOA'. Arc AA' = r θ and Arc AA' = ●φ so rθ = ●φ $\Rightarrow$ φ = $\dfrac{r\theta}{l}$ where θ = angle of twist, φ = angle of shear
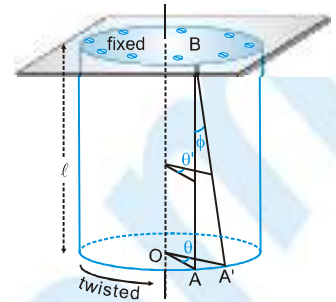
ETOOS KEY POINTS

(i) When a material is under tensile stress restoring force are caused by intermolecular attraction while under compressive stress, the restoring force are due to intermolecular repulsion.

(ii) If the deforming force is inclined to the surface at an angle θ such that θ ≠ 0 and θ ≠ 90° then both tangential and normal stress are developed.

(iii) Linear strain in the direction of force is called **longitudinal** strain while in a direction perpendicular to force **lateral strain**.

Stress – Strain Graph
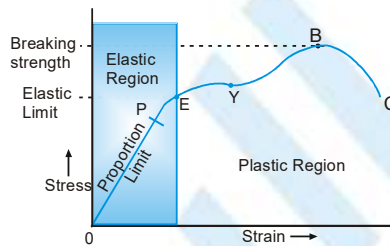
Proportion Limit

The limit in which Hook's law is valid and stress is directly proportional to strain is called proportion limit.

Stress ∝ Strain

Elastic limit

That maximum stress which on removing the deforming force makes the body to recover completely its original state.

Yield Point

The point beyond elastic limit, at which the length of wire starts increasing without increasing stress, is defined as the yield point.

Breaking Point

The position when the strain becomes so large that the wire breaks down at last, is called breaking point. At this position the stress acting in that wire is called breaking stress and strain is called breaking strain.

Elastic after effect

We know that some material bodies take some time to regain their original configuration when the deforming force is removed. The delay in regaining the original configuration by the bodies on the removal of deforming force is called elastic after effect. The elastic after effect is negligibly small for quartz fibre and phosphor bronze. For this reason, the suspensions made from quartz and phosphor-bronze are used in galvanometers and electrometers. For glass fibre elastic after effect is very large. It takes hours for glass fibre to return to its original state on removal of deforming force.

Elastic Fatigue :

The loss of strength of the material due to repeated strains on the material is called elastic fatigue. That is why bridges are declared unsafe after a long time of their use.

Creep :

If a small force is applied for a long time then it causes breaking of metal. For example A fan is hung for 200 years then the shaft will break.

Elastic Hysteresis :

The strain persists even when the stress is removed. This lagging behind of strain is called elastic hysteresis. This is the reason why the values of strain for same stress are different while increasing the load and while decreasing the load.
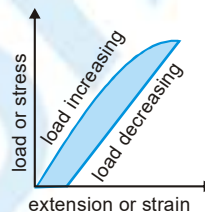
Breaking Stress :

The stress required to cause actual facture of a material is called the breaking stress Breaking stress = F/A

ETOOS KEY POINTS

(i) Breaking stress also measures the tensile strength.

(ii) Metals with small plastic deformation are called brittle.

(iii) Metals with large plastic deformation are called ductile.

(iv) Elasticity restoring forces are strictly conservative only when the elastic hysteresis is zero. i.e. the loading and unloading stress – strain curves are identical.

(v) The material which have low elastic hysteresis have also low elastic relaxation time.

Ex. Find out longitudinal stress and tangential stress on a fixed block.

Sol. Longitudinal or normal stress $\sigma_1 = \dfrac{100\sin 30^0}{5 \times 2} = 5 \text{ N/m}^2$

Tangential stress $\sigma_2 = \dfrac{100\cos 30^0}{5 \times 2} = 5\sqrt{3}\,\text{N}/\,\text{m}^2$
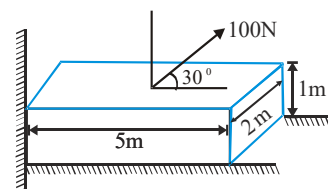
Ex. The breaking stress of aluminium is 7.5×10^8 dyne cm^{-2}. Find the greatest length of aluminium wire that can hang vertically without breaking. Density of aluminium is 2.7 g cm^{-3}. [Given : g = 980 cm s^{-2}]

Sol. Let ● be the greatest length of the wire that can hang vertically without breaking.
Mass of wire m = cross–sectional area **(A)** × length (●) × density (ρ), Weight of wire = mg = A●ρg
This is equal to the maximum force that the wire can withstand.

∴ Breaking stress $= \dfrac{lA\rho g}{A} = l\rho g \Rightarrow 7.5 \times 10^8 = ● \times 2.7 \times 980$

$\Rightarrow l = \dfrac{7.5 \times 10^8}{2.7 \times 980}$ cm $= 2.834 \times 10^5$cm$= 2.834$ km

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Hooke's Law

If the deformation is small, the stress in a body is proportional to the corresponding strain, this fact is known as Hooke's Law. Within elastic limit : stress $\propto$ strain $\Rightarrow \dfrac{\text{stress}}{\text{strain}} =$ constant

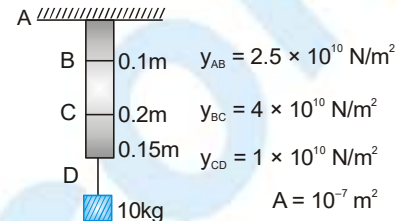This constant is known as modulus of elasticity or coefficient of elasticity.

The modulus of elasticity depends only on the type of material used. It does not depend upon the value of stress and strain.

Ex. Find out the shift in point B, C and D

Sol. $\Delta L_B = \Delta L_{AB} = \dfrac{FL}{AY} = \dfrac{MgL}{AY} = \dfrac{10 \times 10 \times 0.1}{10^{-7} \times 2.5 \times 10^{10}} = 4 \times 10^{-3} \, m = 4mm$

$\Delta L_C = \Delta L_B + \Delta L_{BC} = 4 \times 10^{-3} + \dfrac{100 \times 0.2}{10^{-7} \times 4 \times 10^{10}} = 4 \times 10^{-3} + 5 \times 10^{-3} = 9mm$

$\Delta L_D = \Delta L_C + \Delta L_{CD} = 9 \times 10^{-3} + \dfrac{100 \times 0.15}{10^{-7} \times 1 \times 10^{10}} = 9 \times 10^{-3} + 15 \times 10^{-3} = 24 \, mm$

A ////////////////

B   0.1m $y_{AB} = 2.5 \times 10^{10}$ N/m²

C   0.2m $y_{BC} = 4 \times 10^{10}$ N/m²

  0.15m $y_{CD} = 1 \times 10^{10}$ N/m²

D

10kg $A = 10^{-7}$ m²

Young's Modulus of Elasticity 'Y'

Within elastic limit the ratio of longitudinal stress and longitudinal strain is called Young's modulus of elasticity.

$$Y = \frac{\text{longitudinal stress}}{\text{longitudinal strain}} = \frac{F/A}{1/L} = \frac{FL}{1\,A}$$

Within elastic limit the force acting upon a unit area of a wire by which the length of a wire becomes double, is equivalent to the Young's modulus of elasticity of material of a wire. If L is the length of wire, r is radius and ● is the increase in length of the wire by suspending a weight Mg at its one end then Young's modulus

of elasticity of the material of wire $Y = \dfrac{\left(Mg/\pi r^2\right)}{\left(1/L\right)} = \dfrac{MgL}{\pi r^2 1}$

Unit of Y : N/m²   Dimensions of Y : $M^1 L^{-1} T^{-2}$

Increment of length due to own weight

Let a rope of mass M and length L is hanged vertically. As the tension of different point on the rope is different. Stress as well as strain will be different at different point.

(i) maximum stress at hanging point   **(ii)** minimum stress at lower point

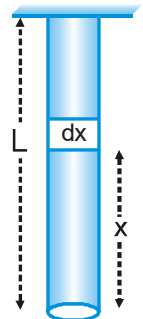Consider a dx element of rope at x distance from lower end then tension $T = \left(\dfrac{M}{L}\right) x \, g$

So stress $= \dfrac{T}{A} = \left(\dfrac{M}{L}\right)\dfrac{xg}{A}$

Let increase in length of dx is dy then strain $= \dfrac{dy}{dx}$

So Young modulus of elasticity $Y = \dfrac{\text{stress}}{\text{strain}} = \dfrac{\dfrac{M}{L}\dfrac{xg}{A}}{dy/dx} \Rightarrow \left(\dfrac{M}{L}\right)\dfrac{xg}{A}dx = Y\,dy$

For full length of rope $\dfrac{Mg}{LA}\displaystyle\int_0^L x\,dx = Y\displaystyle\int_0^{\Delta l} dy \Rightarrow \dfrac{Mg}{LA}\dfrac{L^2}{2} = Y\Delta● \Rightarrow \Delta● = \dfrac{MgL}{2\,AY}$

[Since the stress is varying linearly we may apply average method to evaluate strain.]

Ex. A thin uniform metallic rod of length 0.5 m and radius 0.1 m rotates with an angular velocity 400 rad/s in a horizontal plane about a vertical axis passing through one of its ends. Calculate **(a)** tension in the rod and **(b)** the elongation of the rod. The density of material of the rod is 10^4 kg/m³ and the Young's modulus is 2×10^{11} N/m².

Sol.

(a) Consider an element of length dr at a distance r from the axis of rotation as shown in figure. The centripetal force acting on this element will be $dT = dmr\omega^2 = (\rho A dr) r\omega^2$. As this force is provided by tension in the rod (due to elasticity), so the tension in the rod at a distance r from the axis of rotation will be due to the centripetal force due to all elements between x = r to x = L i.e.,
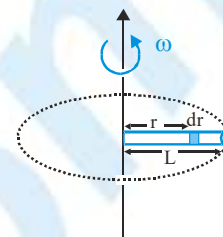
$$T = \int_r^L \rho A \omega^2 r dr = \frac{1}{2} \rho A \omega^2 \left[L^2 - r^2 \right] \qquad ...(i)$$

So here $T = \frac{1}{2} \times 10^4 \times \pi \times 10^{-2} \times (400)^2 \left[\left(\frac{1}{2}\right)^2 - r^2 \right] = 8\pi \times 10^6 \left[\frac{1}{4} - r^2 \right] N$

(b) Now if dy is the elongation in the element of length dr at position r then strain
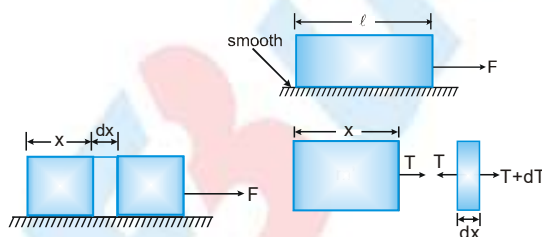
$$\frac{dy}{dr} = \frac{stress}{Y} = \frac{T}{AY} = \frac{1}{2} \frac{\rho\omega^2}{Y} [L^2 - r^2]$$

So the elongation of the whole rod

$$\Delta L = \frac{\rho\omega^2}{2Y} \int_0^L (L^2 - r^2) \, dr = \frac{1}{3} \frac{\rho\omega^2 L^3}{Y} = \frac{1}{3} \times \frac{10^4 \times (400)^2 (0.5)^3}{2 \times 10^{11}} = \frac{1}{3} \times 10^{-3} m$$

Ex. Find out the elongation in block. If mass, area of cross-section and young modulus of block are m, A and y respectively.

Sol.

Acceleration, $a = \frac{F}{m}$ then T = m'a where $\Rightarrow m' = \frac{m}{l} x$; $T = \frac{m}{l} x \frac{F}{m} = \frac{Fx}{l}$

Elongation in element 'dx' = $\frac{Tdx}{Ay}$, total elongation, $\delta = \int_0^l \frac{Tdx}{Ay} = \int_0^l \frac{Fxdx}{Aly} = \frac{Fl}{2Ay}$

Bulk modulus of elasticity 'K' or 'B'

Within elastic limit the ratio of the volume stress and the volume strain is called bulk modulus of elasticity.

$$K \text{ or } B = \frac{volume \ stress}{volume \ strain} = \frac{F/A}{\frac{-\Delta V}{V}} = \frac{\Delta P}{\frac{-\Delta V}{V}}$$

The minus sign indicates a decrease in volume with an increase in stress.

Unit of K : N m⁻² or pascal

Bulk modulus of an ideal gas is process dependence.

For isothermal process $PV = $ constant $\Rightarrow PdV + VdP = 0 \Rightarrow P = \dfrac{-dP}{dV / V}$ So bulk modulus $= P$

For adiabatic process $PV^\gamma = $ constant $\Rightarrow \gamma PV^{\gamma-1}dV + V^\gamma dP = 0 \Rightarrow \gamma PdV + VdP = 0 \Rightarrow \gamma P = \dfrac{-dP}{dV / V}$

So bulk modulus $= \gamma P$

For any polytropic process $PV^n = $ constant $\Rightarrow nPV^{n-1}dV | V^n dP = 0 \Rightarrow PdV + VdP = 0 \Rightarrow nP = \dfrac{-dP}{dV / V}$

So bulk modulus $= nP$

Compressibility : The reciprocal of bulk modulus of elasticity is defined as compressibility. $C = \dfrac{1}{K}$
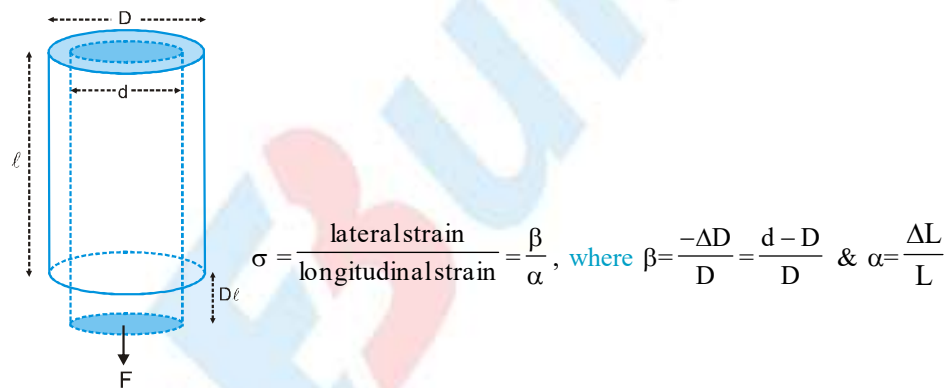
Modulus of Rigidity 'η'

Within elastic limit the ratio of shearing stress and shearing strain is called modulus of rigidity of a material.

$$\eta = \frac{\text{shearing stress}}{\text{shearing strain}} = \left(\frac{\dfrac{F_{\text{tan gential}}}{A}}{\phi} \right) = \frac{F_{\text{tan gential}}}{A\phi}$$

Note : Angle of shear 'ϕ' always take in radian

Poisson's Ratio (σ)

In elastic limit, the ratio of lateral strain and longitudinal strain is called Poisson's ratio.

$$\sigma = \frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{\beta}{\alpha} , \text{ where } \beta = \frac{-\Delta D}{D} = \frac{d - D}{D} \text{ \& } \alpha = \frac{\Delta L}{L}$$

## Work done in stretching a wire (Potential energy of a stretched wire)

When a wire is stretched, work is done against the interatomic forces, which is stored in the form of elastic potential energy.

For a wire of length L_o stretched by a distance x, the restoring elastic force is :

$$F = \text{stress} \times \text{area} = Y\left[\frac{x}{L_o}\right]A$$

The work has to be done against the elastic restoring forces in stretching dx

$$dW = F\,dx = \frac{YA}{L_0}x\,dx$$
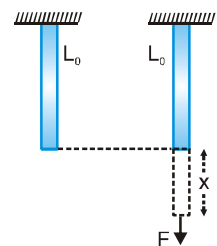
The total work done in stretching the wire from x = 0 to x = Δ● is, then

$$W = \int_0^{\Delta l} \frac{YA}{L_o} x dx = \frac{YA}{L_o} \left[\frac{x^2}{2} \right]_0^{\Delta l} = \frac{YA(\Delta l)^2}{2L_o}$$

$W = \frac{1}{2} \times Y \times (strain)^2 \times$ original volume $= \frac{1}{2}$ (stress) (strain) (volume)

$$\boxed{\text{ETOOS KEY POINTS}}$$

(i)   The value of K is maximum for solids and minimum for gases.

(ii)   For any ideal rigid body all three elastic modulus are infinite .

(iii) η is the characteristic of solid material only as the fluids do not have fixed shape.

(iv)   Potential energy density = area under the stress–strain curve.

(v)   Young's modulus = Slope of the stress–strain curve
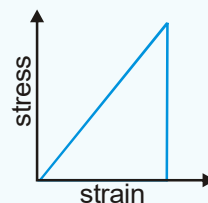
Ex.   A steel wire of 4.0 m in length is stretched through 2.00 mm. The cross–sectional area of the wire is 2.0 mm². If Young's modulus of steel is 2.0×10^{11} N/m² find **(i)** the energy density of wire **(ii)** the elastic potential energy stored in the wire.

Sol.   **(i)** The energy density of stretched wire

$$= \frac{1}{2} \times stress \times strain = \frac{1}{2} \times Y \times (strain)^2 = \frac{1}{2} \times 2.0 \times 10^{11} \times \left[\frac{2 \times 10^{-3}}{4} \right]^2 = 2.5 \times 10^4 \text{ J/m}^3$$

(ii) Elastic potential energy = energy density × volume = $2.5 \times 10^4 \times (2.0 \times 10^{-6}) \times 4.0$ J $= 20 \times 10^{-2} = 0.20$J

Ex.   Find the depth of lake at which density of water is 1% greater than at the surface. Given compressibility $K = 50 \times 10^{-6}$ /atm.
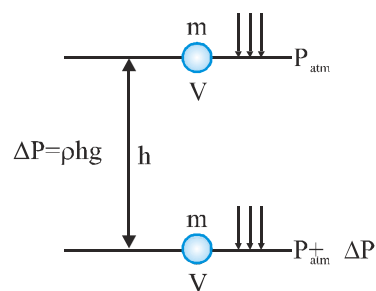
Sol. $B = \frac{\Delta P}{-\Delta V / V} \Rightarrow \frac{\Delta V}{V} = -\frac{\Delta P}{B}$

We know $P = P_{atm} + h\rho g$ and $m = \rho V = $ constant $\Delta P = \rho hg$ h

$d\rho V + dV\rho = 0 \Rightarrow \frac{d\rho}{\rho} = -\frac{dV}{V}$ i.e. $\frac{\Delta\rho}{\rho} = \frac{\Delta P}{B} \Rightarrow \frac{\Delta\rho}{\rho} = \frac{1}{100} = \frac{h\rho g}{B}$

[assuming $\rho = $ constant]; $h\rho g = \frac{B}{100} = \frac{1}{100K} \Rightarrow h\rho g = \frac{1 \times 1 \times 10^5}{100 \times 50 \times 10^{-6}}$

$h = \frac{10^5}{5000 \times 10^{-6} \times 1000 \times 10} = \frac{100 \times 10^3}{50}$ m $= 2$km

Ex.   A rubber cube of side 5 cm has one side fixed while a tangential force equal to 1800 N is applied to opposite face. Find the shearing strain and the lateral displacement of the strained face. Modulus of rigidity for rubber is 2.4×10^6 N/m².

Sol. Stress $= \dfrac{F}{A} = \eta \dfrac{x}{L}$

Strain $= \theta = \dfrac{F}{A\eta} = \dfrac{1800}{25 \times 10^{-4} \times 2.4 \times 10^{6}} = \dfrac{180}{25 \times 24} = \dfrac{3}{10} = 0.3$ rad

as $\dfrac{x}{L} = 0.3 \Rightarrow x = 0.3 \times 5 \times 10^{-2} = 1.5 \times 10^{-2}$ m $= 1.5$ mm

Analogy of Rod as a spring

$Y = \dfrac{\text{stress}}{\text{strain}} \Rightarrow Y = \dfrac{Fl}{A\Delta l}$ or $F = \dfrac{AY}{l} \Delta \bullet$

$\dfrac{AY}{l} =$ constant, depends on type of material and geometry of rod.

$F = k\Delta \bullet$ where $k = \dfrac{AY}{l} =$ equivalent spring constant.

For the system of rods shown in figure **(a)**, the replaced spring system is shown in figure **(b)** [two spring in series]. Figure **(c)** represents equivalent spring system.
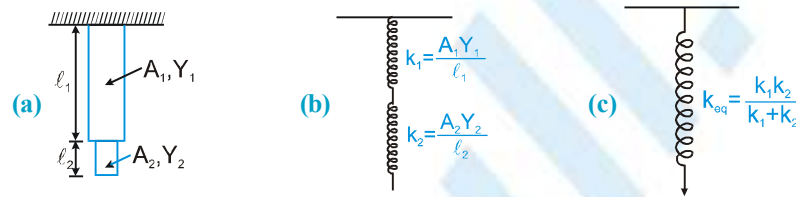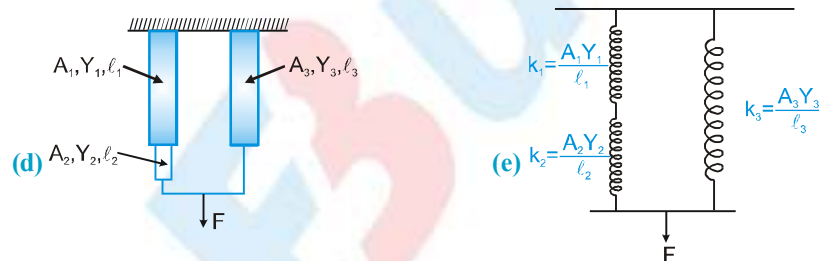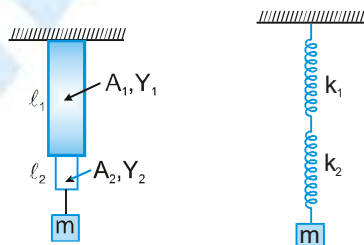
Figure **(d)** represents another combination of rods and their replaced spring system.

Ex. A mass 'm' is attached with rods as shown in figure. This mass is slightly stretched and released whether the motion of mass is S.H.M., if yes then find out the time period.

Sol. Here $k_{eq} = \dfrac{k_1 k_2}{k_1 + k_2}$ and $T = 2\pi \sqrt{\dfrac{m}{k_{eq}}} = 2\pi \sqrt{\dfrac{m(k_1 + k_2)}{k_1 k_2}}$ where $k_1 = \dfrac{A_1 Y_1}{l_1}$ and $k_2 = \dfrac{A_2 Y_2}{l_2}$

Ex. Hanger is mass-less. A ball of mass 'm' drops from a height 'h', which sticks to hanger after striking. Neglect overturning, find out the maximum extension in rod. Assuming rod is massless. Let maximum extension be x_{max}.
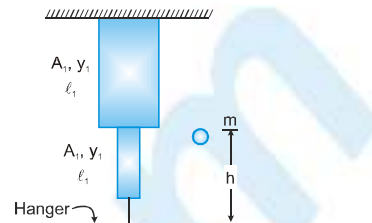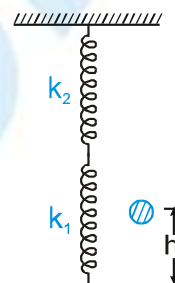
Sol. Applying energy conservation $mg (h + x_{max}) = \dfrac{1}{2} \dfrac{k_1 k_2}{k_1 + k_2} x^2$

where $\quad k_1 = \dfrac{A_1 y_1}{l_1}$; $k_2 = \dfrac{A_2 y_2}{l_2}$ & $K_{eq} = \dfrac{A_1 A_2 y_1 y_2}{A_1 y_1 l_2 + A_2 y_1 l_1}$

therefore $\quad k_{eq} x^2_{max} - 2mgx_{max} - 2mgh = 0$

$$x_{max} = \frac{2mg \pm \sqrt{4m^2 g^2 + 8mghk_{eq}}}{2k_{eq}} = \frac{mg}{k_{eq}} + \sqrt{\frac{m^2 g^2}{k_{eq}^2} + \frac{2mgh}{k_{eq}}}$$
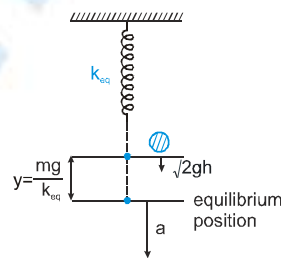
Otherway by S.H.M.

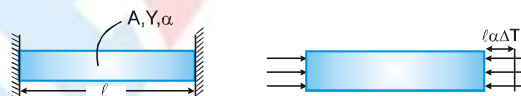$\omega = \sqrt{\dfrac{k_{eq}}{m}}$ and $v = \omega \sqrt{a^2 - y^2}$

$\Rightarrow \sqrt{2gh} = \sqrt{\dfrac{keq}{m}} \sqrt{a^2 - y^2} \Rightarrow \sqrt{\dfrac{2mgh}{k_{eq}} + \dfrac{m^2 g^2}{k_{eq}^2}} = a$

Maximum extension $= a + y = \dfrac{mg}{k_{eq}} + \sqrt{\dfrac{m^2 g^2}{k_{eq}^2} + \dfrac{2mgh}{k_{eq}}}$

Thermal Stress :

If temperature of rod is increased by ΔT, then change in length $\Delta \bullet = \bullet \ \alpha \Delta T$; strain $= \dfrac{\Delta \lambda}{\lambda} = \alpha \Delta T$

But due to rigid support, there is no strain. Supports provide force on stresses to keep the length of rod same

$Y = \dfrac{stress}{strain}$ \qquad If $\Delta T = (+)$ tive

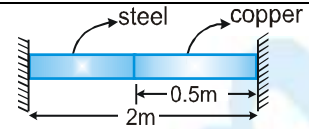Thermal stress $= y$ strain $= Y\alpha \Delta T$ \qquad If $\Delta T = (-)$ tive

$\dfrac{F}{A} = Y\alpha \Delta T$ \qquad\qquad $F = AY\alpha \Delta T$

Ex. When composite rod is free, then composite length increases to 2.002 m for temperature 20°C to 120°C. When composite rod is fixed between the support, there is no change in component length find y and α of steel, if $y_c = 1.5 \times 10^{13}$ N/m² $\alpha_c = 1.6 \times 10^{-5}$/°C.

Sol. $\Delta\ell = \ell_s \alpha_s \Delta T + \ell_c \alpha_c \Delta T \Rightarrow 0.002 = [1.5\,\alpha_s + 0.5 \times 1.6 \times 10^{-5}] \times 100 \Rightarrow \alpha_s = \dfrac{1.2 \times 10^{-5}}{1.5} = 8 \times 10^{-6}$/°C
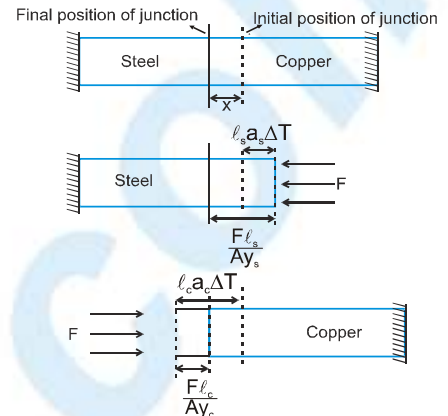
there is no change in component length

For steel $\qquad x = \ell_s \alpha_s \Delta T - \dfrac{Fl_s}{AY_s} = 0 \Rightarrow \dfrac{F}{AY_s} = \alpha_s \Delta T \qquad$ ....**(i)**

For copper $\qquad x = \dfrac{Fl_c}{AY_c} - \ell_c \alpha_c \Delta T = 0 \Rightarrow \dfrac{F}{Ay_c} = \alpha_c \Delta T \qquad$ ...**(ii)**

(ii)/(i) $\Rightarrow Y_s = Y_c \dfrac{\alpha_c}{\alpha_s} = \dfrac{1.5 \times 10^{13} \times 16 \times 10^{-5}}{8 \times 10^{-6}}$

$Y_s = 3 \times 10^{13}$ N/m²

Applications of Elasticity :

Some of the important applications of the elasticity of the materials are discussed as follows :

The material used in bridges lose its elastic strength with time bridges are declared unsafe after long use.

To estimate the maximum height of a mountain :

The pressure at the base of the mountain = hρg = stress.

The elastic limit of a typical rock is 3×10^8 N m^{-2}

The stress must be less than the elastic limits, otherwise the rock begins to flow.

$h < \dfrac{3 \times 10^8}{\rho g} < \dfrac{3 \times 10^8}{3 \times 10^3 \times 10} < 10^4\,\text{m}\,(\because \rho = 3 \times 10^3\,\text{kg m}^{-3}\,;\,g = 10\,\text{ms}^{-2}) \quad \text{or} \quad h = 10\,\text{km}$

It may be noted that the height of Mount Everest is nearly 9 km.

Torsion constant of a wire

$C = \dfrac{\pi \eta r^4}{2l}$ Where η is modulus of rigidity r and ℓ is radius and length of wire respectively.

(a) Toque required for twisting by angle θ, $\tau = C\theta$.

(b) Work done in twisting by angle θ, $W = \dfrac{1}{2} C\theta^2$.

Effect of Temperature on elasticity

When temperature is increased then due to weakness of inter molecular force the elastic properties in general decreases i.e. elastic constant decreases. Plasticity increases with temperature. For example, at ordinary room temperature, carbon is elastic but at high temperature, carbon becomes plastic. Lead is not much elastic at room temperature but when cooled in liquid nitrogen exhibit highly elastic behaviour.

For a special kind of steel, elastic constants do not vary appreciably temperature. This steel is called 'INVAR steel'.

Effect of Impurity on elasticity

Y is slightly increase by impurity. The inter molecular attraction force inside wire effectively increase by impurity due to this external force can be easily opposed.

SURFACE TENSION

Surface tension is basically a property of liquid. The liquid surface behaves like a stretched elastic membrane which has a natural tendency to contract and tends to have a minimum possible area. This property of liquid is called *surface tension*.

INTERMOLECULAR FORCES

l. The force which acts between the atoms or the molecules of different substances is called intermolecular force. This force is of two types.

(a) Cohesive force – The force acting between the molecules of one type of molecules of same substance is called cohesive force.

(b) Adhesive force – The force acting between different types of molecules or molecules of different substance is called adhesive force.

2. Intermolecular forces are different from the gravitational forces not obey the inverse–square law

3. The distance upto which these forces effective, is called molecular range. This distance is nearly 10^{-9} m. Within this limit this increases very rapidly as the distance decreases.

4. Molecular range depends on the nature of the substance

Examples :

(a) **Water Glass :** Water wets glass surface but mercury does not. Because when water comes in contact with glass the adhesive force acts between water and glass molecules. As adhesive force is greater than the cohesive force of water molecules, the water molecules, cling with glass surface and surface become wet. In case of mercury adhesive force is less than that of cohesive force and Hg–molecules do not cling with glass surface and surface does not wet with Hg–molecules.

(b) **Oil–water :** Since Cohesive force of water >Adhesive force oil–water> Cohesive force of oil.

(i) If water drop is poured on the surface of oil, it contracts in the form of globule.

(ii) If oil drop is poured on the surface of water it spreads to a larger area in the form of thin film.

(c) **Ink–paper** : Since adhesive force between ink–paper > cohesive force on ink, the ink sticks to the paper.

Explanation of Surface Tension (Molecular theory of surface Tension)

Laplace firstly explained the phenomenon of surface tension on the basis of intermolecular forces. According to him surface tension is a molecular phenomenon and its root causes are electromagnetic forces. He explained the cause of surface tension as described under. If the distance between two molecules is less than the molecular range C ($\approx 10^{-9}$ m) then they attract each other, but if the distance is more than this then the attraction becomes negligible. If a sphere of radius C with a molecule at centre is drawn, then only those molecules which are enclosed within this sphere can attract or be attracted by the molecule at the centre of the sphere. This sphere is called sphere of molecular activity or sphere of influence. In order to understand the tension acting at the free surface of liquid, let us consider four liquid molecules like A, B, C and D along with their spheres of molecular activity.

(a) According to figure D sphere is completely inside liquid. So molecule is attracted equally in all directions and hence resultant force is equal to zero.

(b) According to figure sphere of molecule C is just below the liquid surface. So resultant force is equal to zero.

(c) The molecule B which is a little below the liquid surface is more attracted downwards due to excess of molecules downwards. Hence the resultant force is acting downwards.

(d) Molecule A is situated at surface so that its sphere of molecular activity is half outside the liquid and half inside. Only down portion has liquid molecules. Hence it experiences a maximum downward force. Thus all the molecules situated between the surface and a plane XY, distant C below the surface, experience a resultant downward cohesive force. When the surface area of liquid is increased molecules from the interior of the liquid rise to the surface.
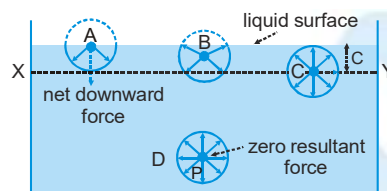
As these molecules reach near the surface, work is done against the downward cohesive force. This work is stored in the molecules in the form of potential energy. Thus the potential energy of the molecules lying in the surface is greater than that of the molecules in the interior of the liquid. A system is in stable equilibrium when its potential energy is minimum. Hence in order to have minimum potential energy the liquid surface tends to have minimum number of molecules in it. In other words the surfaces tends to contract to a minimum possible area. This tendency is exhibited as surface tension.

ETOOS KEY POINTS

(i) Surface tension is a scalar quantity.

(ii) It acts tangential to liquid surface.

(iii) Surface tension is always produced due to cohesive force.

(iv) More is the cohesive force, more is the surface tension.

(v) When surface area of liquid is increased, molecules from the interior of the liquid rise to the surface. For this, work is done against the downward cohesive force.

As a result, its potential energy increases and internal energy decreases. So on increase in surface area cooling occurs. If liquid temperature remains same, then extra energy may be given by external agency. So the molecules in the surface have some additional energy due to their position. This additional energy per unit area of the surface is called Surface Energy, free surface energy or surface energy density.

DEPENDENCY OF SURFACE TENSION

1   On Cohesive Force

Those factors which increase the cohesive force between molecules increase the surface tension and those which decrease the cohesive force between molecules decrease the surface tension.

2   On Impurities

If the impurity is completely soluble then on mixing it in the liquid, its surface tension increases. e.g., on dissolving ionic salts in small quantities in a liquid, its surface tension increases. If the impurity is partially soluble in a liquid then its surface tension decreases because adhesive force between insoluble impurity molecules and liquid molecules decreases cohesive force effectively, e.g.

(a) On mixing detergent in water its surface tension decreases.

(b) Surface tension of water is more than (alcohol + water) mixture.

3   On Temperature

On increasing temperature surface tension decreases. At critical temperature and boiling point it becomes zero.

Note : Surface tension of water is maximum at 4°C

4   On Contamination

The dust particles or lubricating materials on the liquid surface decreases its surface tension.

5   On Electrification

The surface tension of a liquid decreases due to electrification because a force starts acting due to it in the outward direction normal to the free surface of liquid.

DEFINITION OF SURFACE TENSION

Surface tension can be defined in the form of an imaginary line on the surface or by relating it to the work done. The force acting per unit length of an imaginary line drawn on the free liquid surface at right angles to the line and in the plane of liquid surface, is defined as surface tension. Let an imaginary line AB be drawn in any direction on a liquid surface. The surface on either side of this line exerts a pulling force, which is perpendicular to line AB. If force is F and length of AB is L then $T = \dfrac{F}{L}$

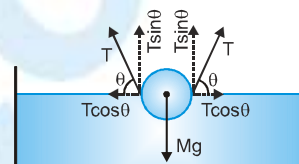**SI Units :** N/m and J/m²   **CGS Units :** dyne/cm and erg/cm²   **Dimensions :** $M^1L^0T^{-2}$
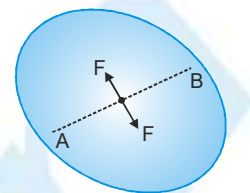
Examples :

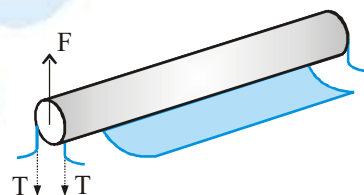**(i)** When any needle floats on the liquid surface then $2T \bullet \sin\theta = mg$
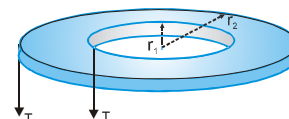
Ex. A mosquito sitting on a liquid surface.

(ii) If the needle is lifted from the liquid surface then required excess

force will be $F_{excess} = 2T \bullet$

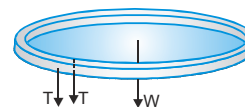Minimum force required $F_{min} = mg + 2T \bullet$

(iii) Required excess force for a circular thick ring (or hollow disc) having internal and external radii r_1 and r_2 is dipped in and taken out from liquid $F_{excess} = F_1 + F_2 = T(2\pi r_1) + T(2\pi r_2) = 2\pi T(r_1 + r_2)$
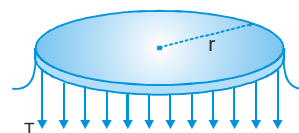
(iv) Required excess force for a circular ring $(r_1 = r_2 = r)$

$F_{excess} = 2\pi T(r + r) = 4\pi r T$

(v) Required excess force for a circular disc $(r_1 = 0, r_2 = r)$

$F_{excess} = 2\pi rT$

Surface Energy

According to molecular theory of surface tension the molecule in the surface have some additional energy due to their position. This additional energy per unit area of the surface is called ' Surface energy". Let a liquid film be formed on a wire frame and a straight wire of length $\bullet$ can slide on this wire frame as shown in figure. The film has two surfaces and both the surfaces are in contact with the sliding wire and hence, exert forces of surface tension on it. If T be the surface tension of the solution, each surface will pull the wire parallel to itself with a force $T \bullet$.
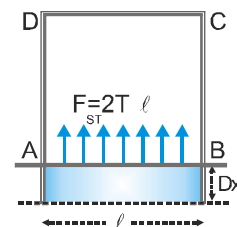
Thus, net force on the wire due to both the surfaces is 2T●. Apply an external force F equal and opposite to it to keep the wire is equilibrium. Thus, F=2T● Now, suppose the wire is moved through a small distance dx, the work done by the force is, dW = F dx = (2T●) dx But (2●) (dx) is the total increase in area of both the surfaces of the film. Let it be dA.   Then, dW = T dA $\Rightarrow$ T = dW/dA

Thus, the surface tension T can also be defined as the work done in increasing the surface area by unity. Further, since there is no change in kinetic energy, the work done by the external force is stored as the potential energy of the

new surface. $\qquad T = \dfrac{dU}{dA}$ [as dW = dU]

Special Cases

(i)   Work done (surface energy) in formation of a drop of radius r = Work done against surface tension

$\qquad$ W = Surface tension T × change in area ΔA = T × $4\pi r^2 = 4\pi r^2 T$

(ii)   Work done (surface energy) in formation of a soap bubble of radius r :

$\qquad$ W = T × ΔA or W = T × 2 × $4\pi r^2 = 8\pi r^2 T$ [➤ soap bubble has two surfaces]

Ex.   The length of a needle floating on water is 2.5 cm. Calculate the added force required to pull the needle out of water. [T = 7.2 × 10^{-2} N/m]

Sol.   The force of surface tension F = T× 2● (➤ Two free surfaces are there)

$\qquad \Rightarrow$ F = 7.2 × 10^{-2} × 2 × 2.5 × 10^{-2} = 3.6 × 10^{-3} N

Ex.   A paper disc of radius R from which a hole of radius r is cut out, is floating in a liquid of surface tension, T. What will be force on the disc due to surface tension?

Sol. $T = \dfrac{F}{L} = \dfrac{F}{2\pi(R+r)}$ $\qquad \therefore$ F = 2π (R + r)T

Ex.   Calculate the work done against surface tension in blowing a soap bubble from a radius 10 cm to 20 cm if the surface tension of soap solution is 25 × 10^{-3} N/m. Then compare it with liquid drop of same radius.

Sol.   **(i)**   For soap bubble : Extension in area = 2 × $4\pi r_2^2$ – 2 × $4\pi r_1^2$ = 8π [(0.2)² – (0.1)²] = 0.24 π m²

$\qquad$ Work done W_1 = surface tension × extension in area = 25 × 10^{-3} × 0.24 π = 6π × 10^{-3} J

(ii) For Liquid Drop : in case of liquid drop only one free surface, so extension in area will be half of soap bubble

$\qquad \therefore W_2 = \dfrac{W_1}{2}$ = 3π × 10^{-3} J

SPLITTING OF BIGGER DROP INTO SMALLER DROPLETS

If bigger drop is spitted into smaller droplets then in this process volume of liquid always remain conserved. Let bigger drop has radius R. It is splitted into n smaller drops of radius r then by conservation of volume

(i) $\dfrac{4}{3}\pi R^3 = n\left[\dfrac{4}{3}\pi r^3\right] \Rightarrow n = \left[\dfrac{R}{r}\right]^3 \Rightarrow r = \dfrac{R}{n^{1/3}}$

(ii)   Initial surface area = $4\pi R^2$ and final surface area = n($4\pi r^2$)

$\qquad$ Therefore initial surface energy E_i = $4\pi R^2 T$ and final surface energy E_f = n($4\pi r^2 T$)

$\qquad$ Change in area ΔA = n$4\pi r^2$ – $4\pi R^2$ = 4π (nr² – R²)

$\qquad$ Therefore the amount of surface energy absorbed i.e. ΔE = E_f – E_i = $4\pi T$ (nr² – R²)

$\qquad \therefore$ Magnitude of work done against surface tension i.e. W = 4π (nr² – R²)T

$\qquad$ W = $4\pi T$ (nr² – R²) = $4\pi R^2 T$ (n^{1/3} – 1) = $4\pi R^2 T\left[\dfrac{R}{r} - 1\right] \Rightarrow$ W = $4\pi R^3 T\left[\dfrac{1}{r} - \dfrac{1}{R}\right]$

In this process temperature of system decreases as energy gets absorbed in increasing surface area.

$$W = J \, m \, s \, \Delta\theta = 4\pi R^3 T \left[\frac{1}{r} - \frac{1}{R}\right] \Rightarrow \Delta\theta = \frac{4\pi R^3 \, T}{\frac{4}{3}\pi R^3 J \rho s}\left[\frac{1}{r} - \frac{1}{R}\right] = \frac{3 \, T}{J \rho s}\left[\frac{1}{r} - \frac{1}{R}\right]$$

Where ρ = liquid density, s = liquid's specific heat

Thus in this process area increasing, surface energy increasing, internal energy decreasing, temperature decreasing, and energy is absorbed.

Ex. A big drop is formed by coalescing 1000 small droplets of water. What will be the change in surface energy. What will be the ratio between the total surface energy of the droplets and the surface energy of the big drop ?

Sol. By conservation of volume $\frac{4}{3}\pi R^3 = 1000 \times \frac{4}{3}\pi r^3 \Rightarrow r = \frac{R}{10}$

Surface energy of 1000 droplets $= 1000 \times T \times 4\pi \left[\frac{R}{10}\right]^2 = 10 \, (T \times 4\pi R^2)$

Surface energy of the big drop $= T \times 4\pi R^2$

Surface energy will decrease in the process of formation of bigger drop, hence energy is released and temperature increases . $\therefore \dfrac{\text{total surface energy of 1000 droplets}}{\text{surface energy of big drop}} = \dfrac{10(T \times 4\pi R^2)}{T \times 4\pi R^2} = \dfrac{10}{1}$

Ex. A water drop of radius 1mm is broken into 10^6 identical drops. Surface tension of water is 72 dynes/cm. Find the energy spent in this process.

Sol. As volume of water remains constant, so $\frac{4}{3}\pi R^3 = n\frac{4}{3}\pi r^3 \Rightarrow r = \frac{R}{n^{1/3}}$

Increase in surface area $\Delta A = n \, (4\pi r^2) - 4\pi R^2 = 4\pi(n^{1/3} - 1) R^2 = 4\pi(100 - 1)10^{-6}$

$\therefore$ Energy spent $= T\Delta A = 4\pi \times 99 \times 10^{-6} \times 72 \times 10^{-3} = 89.5 \times 10^{-6} \, J$

EXCESS PRESSURE INSIDE A CURVED LIQUID SURFACE

The pressure on the concave side of curved liquid surface is greater than that on the convex side. Therefore pressure difference exists across two sides of a curved surface. This pressure difference is called *excess pressure*.

Excess pressure inside the drop

Let a drop of radius r having internal and external pressure P_i and P_0 respectively, so that excess pressure

$P_{ex} = (P_i - P_0)$.

If the radius of the drop is changed from r to (r+dr) then

Work done $= F.dr = (PA) \, dr = P \, 4\pi r^2 dr$
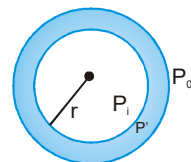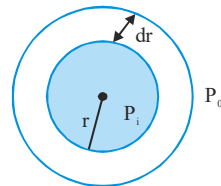
Change in surface area $= r\pi(r + dr)^2 - 4\pi r^2 = 8\pi r dr$

So by definition of surface energy $T = \dfrac{W}{\Delta A} = \dfrac{4\pi r^2 P dr}{8\pi r dr} \Rightarrow P_{ex} = \left(P_i - P_0\right) = \dfrac{2T}{r}$
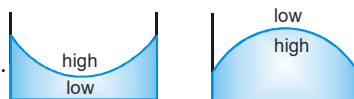
Excess pressure inside soap bubble :

Since the soap bubble has two surfaces. The excess pressure will get doubled as compared to a drop

$P_i - P' = \dfrac{2T}{r}$, $P' - P_0 = \dfrac{2T}{r} \Rightarrow$ excess pressure $= P_i - P_0 = \dfrac{4T}{r}$

(i)   For liquid surface, pressure on concave side is always higher than convex side

(ii)   If a bubble is formed inside a liquid, the pressure inside the bubble is more than the pressure outside the bubble.

(iii)   A soap film is formed in a wire frame. A loop of thread is lying on the film. If the film inside the loop is broken then the

tension in the thread will be 2Tr.
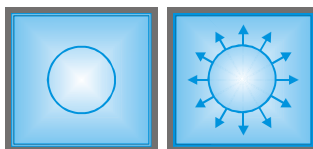
(iv)   In the following arrangement, air will flow from bubble A to B if T_2 and T_3 are opened, because pressure in A is greater

than in B.
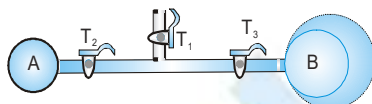
(v)   The force required to separate two plates of Area A is given by $F = \dfrac{2\,AT}{d}$

Ex.   Prove that If two bubbles of radii r_1 and r_2 coalesce isothermally in vacuum then the radius of new bubble will be

$$r = \sqrt{r_1^2 + r_2^2}$$

Sol.   When two bubbles coalesce then number of molecules of air will remain constant and temperature also constant so

$$n_1 + n_2 = n \Rightarrow P_1 V_1 + P_2 V_2 = PV \Rightarrow \frac{4\,T}{r_1}\left(\frac{4}{3}\pi r_1^3\right) + \frac{4\,T}{r_2}\left(\frac{4}{3}\pi r_2^3\right) = \frac{4\,T}{r}\left(\frac{4}{3}\pi r^3\right) \Rightarrow r = \sqrt{r_1^2 + r_2^2}$$

Ex.   Prove that If two bubbles of radii r_1 and r_2 $(r_1 < r_2)$ come in contact with each other then the radius of curvature of the

common surface $r = \dfrac{r_1 r_2}{r_2 - r_1}$.

Sol.   → $r_1 < r_2$ ∴ $P_1 > P_2$ Small part of bubbles is in equilibrium
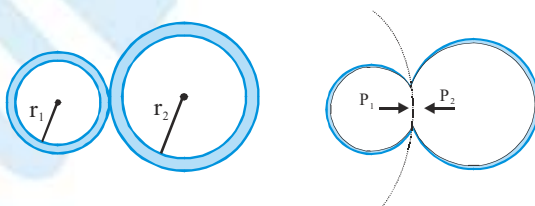
$$\Rightarrow P_1(\Delta A) - P_2(\Delta A) = \frac{4\,T}{r}\Delta A \Rightarrow \frac{4\,T}{r_1} - \frac{4\,T}{r_2} = \frac{4\,T}{r} \Rightarrow r = \frac{r_1 r_2}{r_2 - r_1}$$

Ex.   Calculate the excess pressure within a bubble of air of radius 0.1 mm in water. If the bubble had been formed 10 cm below the water surface on a day when the atmospheric pressure was 1.013×10^5 Pa, then what would have been the total pressure inside the bubble? Surface tension of water = 73×10^{-3} N/m

Sol. Excess pressure $P_{excess} = \dfrac{2T}{r} = \dfrac{2 \times 73 \times 10^{-3}}{0.1 \times 10^{-3}} = 1460 \, Pa$

The pressure at a depth d, in liquid is P = hdg. Therefore, the total pressure inside the air bubble is

$$P_{in} = P_{atm} + hdg + \dfrac{2T}{r} = (1.013 \times 10^5) + (10 \times 10^{-2} \times 10^3 \times 9.8) + 1460$$

$$= 101300 + 980 + 1460 = 103740 = 1.037 \times 10^5 \, Pa$$

Ex. The limbs of a manometer consist of uniform capillary tubes of radii are 10^{-3} & 7.2×10^{-4} m. Find out the correct pressure difference if the level of the liquid in narrower tube stands 0.2 m above that in the broader tube.(Density of liquid $=10^3 \, kg/m^3$, Surface tension $= 72 \times 10^{-3} \, N/m$)

Sol. If P_1 and P_2 are the pressures in the broader and narrower tubes of radii r_1 and r_2 respectively, the pressure just below

the meniscus in the respective tubes will be $P_1 - \dfrac{2T}{r_1}$ and $P_2 - \dfrac{2T}{r_2}$

So that $\left[P_1 - \dfrac{2T}{r_1}\right] - \left[P_2 - \dfrac{2T}{r_2}\right] = h\rho g$ or $P_1 - P_2 = h\rho g - 2T\left[\dfrac{1}{r_2} - \dfrac{1}{r_1}\right]$

$P_1 - P_2 = 0.2 \times 10^3 \times 9.8 - 2 \times 72 \times 10^{-3}\left[\dfrac{1}{7.2 \times 10^{-4}} - \dfrac{1}{14 \times 10^{-4}}\right] = 1960 - 97 = 1863 \, Pa$

ANGLE OF CONTACT (θ_C)

The angle enclosed between the tangent plane at the liquid surface and the tangent plane at the solid surface at the point of contact inside the liquid is defined as the *angle of contact*. The angle of contact depends the nature of the solid and liquid in contact.

1   Effect of Temperature on angle of contact

On increasing temperature surface tension decreases, thus $\cos\theta_c$ increases $\left[Q \; \cos\theta_c \propto \dfrac{1}{T}\right]$ and θ_c decrease.

So on increasing temperature, θ_c decreases.

2   Effect of Impurities on angle of contact

(a) Solute impurities increase surface tension, so $\cos\theta_c$ decreases and angle of contact θ_c increases.

(b) Partially solute impurities decrease surface tension, so angle of contact θ_c decreases.

3   Effect of Water Proofing Agent

Angle of contact increases due to water proofing agent. It gets converted acute to obtuse angle.

Table of angle of contact of various solid–liquid pairs

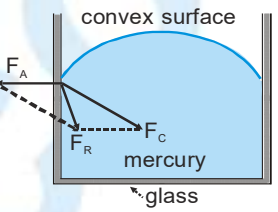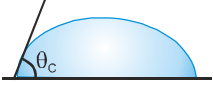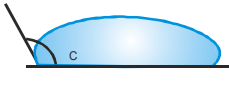| Solid - Liquid Pair	$\theta° \, C$
Glass -Normal water	8°
Glass -Distilled water	0°
Glass - Alcohol	0°
Glass - Mercury	135°
Paraffin wax - Water	108°
Silver - Water	90°
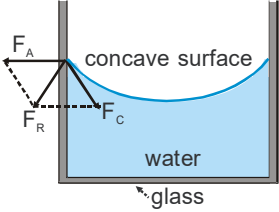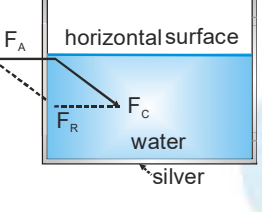
Shape of Liquid Surface

When a liquid is brought in contact with a solid surface, the surface of the liquid becomes curved near the place of contact. The shape of the surface (concave or convex) depends upon the relative magnitudes of the cohesive force between the liquid molecules and the adhesive force between the molecules of the liquid and the solid.

The free surface of a liquid which is near the walls of a vessel and which is curved because of surface tension is known as *meniscus*. The cohesive force acts at an angle 45° from liquid surface whereas the adhesive force acts at right angles to the solid surface. The relation between the shape of liquid surface, cohesive/adhesive forces, angle of contact, etc are summarised in the table below :

Relation Between Cohesive and Adhesive Force

	$F_A > \dfrac{F_C}{\sqrt{2}}$	$F_A = \dfrac{F_C}{\sqrt{2}}$	$F_A < \dfrac{F_C}{\sqrt{2}}$
Shape of meniscus	Concave	Plane	Convex
Angle of contact	$\theta_C < 90°$ (Acute angle)	$\theta_C = 90°$ (Right angle)	$\theta_C > 90°$ (Obtuse angle)
Shape of liquid drop			
Level of liquid	Liquid rises up nor falls	Liquid neither rises	Liquid falls
Wetting property	Liquid wets the solid surface	Liquid does not wet the solid surface	Liquid does not wet the solid surface
Example	Glass – Water	Silver – Water	Glass – Mercury

CAPILLARY TUBE AND CAPILLARITY

A glass tube with fine bore and open at both ends is known as capillary tube. The property by virtue of which a liquid rise or depress in a capillary tube is known as capillarity. Rise or fall of liquid in tubes of narrow bore (capillary tube) is called capillary action.

Calculation of Capillary Rise

R= Radius of the meniscus

Enlarged view

$R = \dfrac{r}{\cos\theta}$

r= radius of the capillary tube
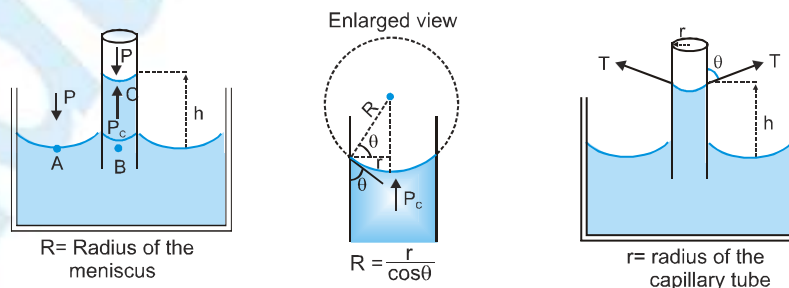
When a capillary tube is first dipped in a liquid as shown in the figure, the liquid climbs up the walls curving the surface. The liquid continues to rise in the capillary tube until the weight of the liquid column becomes equal to force due to surface tension. Let the radius of the meniscus is R and the radius of the capillary tube is r. The angle of contact is θ, surface tension is T, density of liquid is ρ and the liquid rises to a height h.

Now let us consider two points A and B at the same horizontal level as shown. By Pascal's law

$$P_A = P_B \implies P_A = P_C + \rho\,gh \implies P_A - P_C = \rho\,gh \qquad (\rightarrow P_B = P_C + \rho gh)$$

Now, the point C is on the curved meniscus which has P_A and P_C as the two pressures on its concave and convex sides respectively.

$$\therefore \quad P_A - P_C = \frac{2T}{R} = \frac{2T}{r/\cos\theta} \implies \frac{2T}{r/\cos\theta} = \rho\,gh \implies 2T\cos\theta = r\rho\,gh \implies h = \frac{2T\cos\theta}{r\rho g}$$
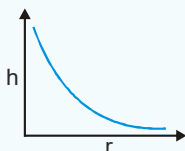
Zurin's Law :

The height of rise of liquid in a capillary tube is inversely proportional to the radius of the capillary tube, if T, θ, ρ

and g are constant $h \propto \dfrac{1}{r}$ or rh = constant. It implies that liquid will rise more in capillary tube of less radius and vice versa.


ETOOS KEY POINTS

(i) For water and clean glass capillary $\theta_c \approx 0°$

(ii) If angle of contact θ_c is acute then $\cos\theta_c$ is positive, so h is positive and liquid rises. If θ_c is obtuse then $\cos\theta_c$ is negative, so h is negative, therefore liquid depresses.

(iii) Rise of liquid in a capillary tube does not obey the law of conservation of energy.

(iv) For two capillaries of radii r_1 and r_2, the capillary rise h_1 and h_2 are such that $\dfrac{h_2}{h_1} = \dfrac{r_1}{r_2}$. The relation between h and r is graphically represented as

(v) Inside a satellite, water will rise upto top level but will not come outside. Radius of curvature(R') increases in such a way that final height h' is reduced and given by $h' = \dfrac{hR}{R'}$. (It is in accordance with Zurin's law).

(vi) If a capillary tube is dipped into a liquid and tilted at an angle α from vertical then the vertical height of liquid column remains same whereas the length of liquid column in the capillary tube increases.
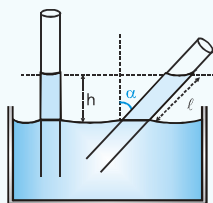
$$h = \ell\cos\alpha \implies 1 = \frac{h}{\cos\alpha}$$

(v) The height 'h' is measured from the bottom of the meniscus. However, there exist some liquid above this line also. If correction of this is applied then the formula will be $T = \dfrac{r\rho g\left[h + \dfrac{1}{3}r\right]}{2\cos\theta}$

(vi) If a hollow sphere of radius r which has a fine hole, drowned in a vessel upto h depth, then liquid will not enter upto critical height h, given by $h\rho g = \dfrac{2T\cos\theta}{r}$ [normally $\theta \approx 0°$ therefore $\cos\theta \approx 1$]

Ex. Calculate the height to which water will rise in a capillary tube of diameter 1×10^{-3} m. [Given : surface tension of water is 0.072 N m^{-1}, angle of contact is 0°, g = 9.8 m s^{-2} and density of water = 1000 kg m^{-3}]

Sol. Height of capillary rise $h = \dfrac{2\,T\cos\theta}{r\rho g} = \dfrac{2 \times 0.072 \times \cos 0^0}{5 \times 10^{-4} \times 1000 \times 9.8} m = 2.94 \times 10^{-2}\,m$

Ex. Water rises to a height of 20 mm in a capillary. If the radius of the capillary is made one third of its previous value then what is the new value of the capillary rise?

Sol. Since $h = \dfrac{2\,T\cos\theta}{rdg}$ and for the same liquid and capillaries of difference radii $h_1 r_1 = h_2 r_2$

$\therefore \dfrac{h_2}{h_1} = \dfrac{r_1}{r_2} = \dfrac{1}{(1/3)} = 3$ hence $h_2 = 3h_1 = 3 \times 20$ mm = 60 mm

FLUID STATICS

Matter exists in three states solid, liquid and gas. Liquids and gases are referred to as fluids. Any state of matter that can flow is a fluid. Study of a fluid at rest is called fluid statics or hydrostatics and the study of fluid in motion is called fluid dynamics of hydrodynamics. Both combined are called fluid mechanics.

The intermolecular force in liquids are comparatively weaker than in solids. Therefore, their shape can be changed easily. Thus liquids assume the shape of the container. But their volume (or density) cannot be changed so easily. Liquids are incompressible and have free surface of their own.

The intermolecular forces are weakest in gases, so their shape and size can be changed easily. Gases are compressible and occupy all the space of the container.

Density (ρ)

Mass per unit volume is defined as density. So density at a point of a fluid is represented as

$$\rho = \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V} = \frac{dm}{dV} \qquad \text{Density is a positive scalar quantity.}$$

SI Unit : kg/m^3 **CGS Unit :** g/cc **Dimensions :** $[ML^{-3}]$

Relative Density

It is defined as the ratio of the density of the given fluid to the density of pure water at 4°C.

$$\text{Relative density (R.D.)} = \frac{\text{density of given liquid}}{\text{density of pure water at } 4\,°C}$$

Relative density or specific gravity is a unitless and dimensionless positive scalar physical quantity.

Being a dimensionless/unitless quantity R.D. of a substance is same in SI and CGS system.

Specific Gravity

It is defined as the ratio of the specific weight of the given fluid to the specific weight of pure water at 4°C.

$$\text{Specific gravity} = \frac{\text{specific weight of given liquid}}{\text{specific weight of pure water at } 4\,°C \ (9.81 \ kN/m^3)} = \frac{\rho_l \times g}{\rho_w \times g} = \frac{\rho_l}{\rho_w} = \text{R.D. of the liquid}$$

Thus specific gravity of a liquid is numerically equal to the relative density of that liquid and for calculation purposes they are used interchangeably.

Ex.   A hollow metallic sphere has inner and outer radii, respectively, as 5 cm and 10 cm. If the mass of the sphere is 2.5 kg, find **(a)** density of the material, **(b)** relative density of the material of the sphere.

Sol.

The volume of the material of the sphere is

$$V = \left(\frac{4}{3}\right)\pi\left(r_2^3 - r_1^3\right) = \frac{4}{3} \times 3.14 \times \left[\left(\frac{10}{100}\right)^3 - \left(\frac{5}{100}\right)^3\right] = \frac{4}{3} \times 3.14 \times [0.001 - 0.000125]$$

$$= \frac{4}{3} \times 3.14 \times 0.000875 \, m^3 = 0.00367 \, m^3$$

(a) Therefore, density of the material of the sphere is $\rho = \dfrac{M}{V} = \dfrac{2.5}{0.00367} \ kg/m^3 = 681.2 \ kg/m^3$

(b) Relative density of the material of the sphere $\rho_r = \dfrac{681.2}{1000} = 0.6812$

Density of a Mixture of substance in the proportion of mass

Let a number of substances of masses M_1, M_2, M_3 etc., and densities ρ_1, ρ_2, ρ_3 etc. respectively are mixed together. The total mass of the mixture = $M_1 + M_2 + M_3 + ...$

The total volume = $\dfrac{M_1}{\rho_1} + \dfrac{M_2}{\rho_2} + \dfrac{M_3}{\rho_3} + ...$ therefore, the density of the mixture is $\rho = \dfrac{M_1 + M_2 + M_3 ....}{\dfrac{M_1}{\rho_1} + \dfrac{M_2}{\rho_2} + \dfrac{M_3}{\rho_3} + ...}$

For two substances the density of the mixture $\rho = \dfrac{\rho_1 \rho_2 (M_1 + M_2)}{\rho_1 M_2 + \rho_2 M_1}$

Ex. Two immiscible of densities 2.5 g/cm³ and 0.8 g/cm³ are taken in the ratio of their masses as 2:3 respectively. Find the average density of the liquid combination.

Sol. Let masses be 2m g & 3m g, then $V = V_1 + V_2 = \left(\dfrac{2m}{2.5} + \dfrac{3m}{0.8}\right)$ cm³

Total mass = $2m + 3m = 5m$ g.

Therefore, the average density $\rho_{av} = \dfrac{5m}{V} = \dfrac{5m}{\dfrac{2m}{2.5} + \dfrac{3m}{0.8}} = \dfrac{5}{\dfrac{2}{2.5} + \dfrac{3}{0.8}} = \dfrac{10}{9.1}$ gm/cm³ = 1.09 gm/cm³

Density of a mixture of substance in the proportion of volume

Suppose that a number of substances of volume V_1, V_2, V_3 etc. and densities ρ_1, ρ_2, ρ_3 etc. respectively are mixed.

The total mass of the mixture is $= \rho_1 V_1 + \rho_2 V_2 + \rho_3 V_3 + ...$

The total volume of the mixture is $= V_1 + V_2 + V_3 + ...$

Therefore, the density of the mixture is $\rho = \dfrac{\rho_1 V_1 + \rho_2 V_2 + \rho_3 V_3}{V_1 + V_2 + V_3 + ...}$

Therefore, for two substances we can write $\rho = \dfrac{\rho_1 V_1 + \rho_2 V_2}{V_1 + V_2}$
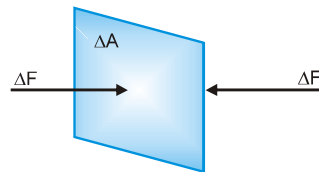
Ex. Two miscible liquids of densities 1.2 gm/cc and 1.4 gm/cc are mixed with a proportion ratio of their volumes equal to 3:5: What is the density of resulting liquid?

Sol. $\rho_1 = \dfrac{\rho_1 V_1 + \rho_2 V_2}{V_1 + V_2} \Rightarrow \rho = \dfrac{\rho_1 \dfrac{V_1}{V_2} + \rho_2}{V_1 / V_2 + 1} \Rightarrow \rho = \dfrac{(1.2)(3/5) + 1.4}{(3/5) + 1} = \dfrac{3.6 + 7}{8} \Rightarrow \rho = 1.325$

PRESSURE

The pressure P is defined as the magnitude of the normal force acting on a unit surface area.

$P = \dfrac{\Delta F}{\Delta A}$ here ΔF = normal force on a surface area ΔA

The pressure is a scalar quantity. This is because hydrostatic pressure is transmitted equally in all directions when force is applied, which shows that a definite direction is not associated with pressure.

CONSEQUENCES OF PRESSURE

(i) Railway tracks are laid on large sized wooden or iron sleepers. This is because the weight (force) of the train is spread over a large area of the sleeper. This reduces the pressure acting on the ground and hence prevents the yielding of ground under the weight of the train.

(ii) A sharp knife is more effective in cutting the objects than a blunt knife. The pressure exerted = Force / area. The sharp knife transmits force over a small area as compared to the blunt knife. Hence the pressure exerted in case of sharp knife is more than in case of blunt knife.

(iii) A camel walks easily on sand but a man cannot inspite of the fact that a camel is much heavier than man. This is because the area of camel's feet is large as compared to man's feet. So the pressure exerted by camel on the sand is very small as compared to the pressure exerted by man. Due to large pressure, sand under the feet of man yields and hence he cannot walk easily on sand.

TYPES OF PRESSURE

There are three types of pressure

(i) Atmospheric pressure (P_0) **(ii)** Gauge pressure (P_{gauge}) **(iii)** Absolute pressure ($P_{abs.}$)
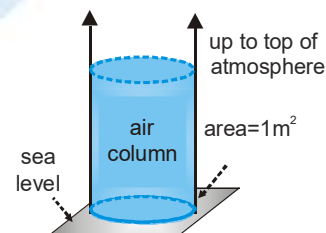
1. **Atmospheric pressure :**

Force exerted by air column on unit cross–section area of sea level called

atmospheric pressure (P_o)

$$P_o = \frac{F}{A} = 101.3 \text{ kN/m}^2 \qquad \therefore P_o = 1.013 \times 10^5 \text{ N/m}^2$$

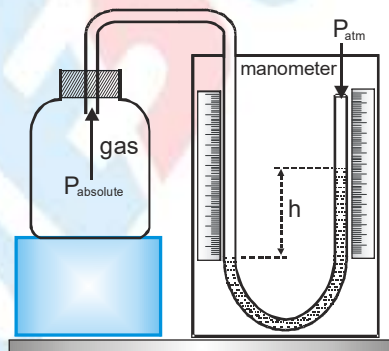Barometer is used to measure atmospheric pressure.

Which was discovered by **Torricelli**.

Atmospheric pressure varies from place to place and at a particular place from time to time.

2. **Gauge Pressure :**

Excess Pressure ($P - P_{atm}$) measured with the help of pressure measuring instrument called Gauge pressure.

$$P_{gauge} = h\rho g \quad \text{or} \quad P_{gauge} \propto h$$

Gauge pressure is always measured with help of **"manometer"**

3. **Absolute Pressure :**

Sum of atmospheric and Gauge pressure is called absolute pressure.

$$P_{abs} = P_{atm} + P_{gauge} \implies P_{abs} = P_o + h\rho g$$

The pressure which we measure in our automobile tyres is gauge pressure.
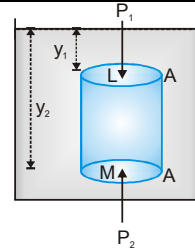
VARIATION OF PRESSURE WITH DEPTH

(i) Let pressure at L is P_1 and pressure at M is P_2

Then, $P_2 A = P_1 A + \rho g A (y_2 - y_1) \Rightarrow P_2 = P_1 + \rho g(y_2 - y_1)$

Here pressure gradient $\dfrac{dP}{dy} = \rho g$

(ii) Pressure is same at two points in the same horizontal level.

As body is in equilibrium, $P_1 A = P_2 A \Rightarrow P_1 = P_2$

Note : Pressure P is independent of shape of container

Ex. Assuming that the atmosphere has a uniform density of (1.3 kg/m^3) and an effective height of 10 km, find the force exerted on an area of dimensions 100 m × 80 m at the bottom of the atmosphere .

Sol. $F = PA = \rho g h \, A = (1.3)(9.8)(10^4)(100 \times 80) = 12.74 \times 8 \times 10^7 \, N = 1.0192 \times 10^9 \, N$

PRESSURE EXERTED BY A LIQUID (EFFECT OF GRAVITY) :

Consider a vessel containing liquid. As the liquid is in equilibrium, so every volume element of the fluid is also in equilibrium. Consider one volume element in the form of a cylindrical column of liquid of height h and of area of cross section A. The various forces acting on the cylindrical column of liquid are :

(i) Force, $F_1 = P_1 A$ acting vertically downward on the top face of the column. P_1 is the pressure of the liquid on the top face of the column and is known as atmospheric pressure.

(ii) Force, $F_2 = P_2 A$ acting vertically upward at the bottom face of the cylindrical column. P_2 is the pressure of the liquid on the bottom face of the column.

(iii) Weight, $W = mg$ of the cylindrical column of the liquid acting vertically downward. Since the cylindrical column of the liquid is in equilibrium, so the net force acting on the column is zero. i.e. $F_1 + W - F_2 = 0$

$\Rightarrow P_1 A + mg - P_2 A = 0 \Rightarrow P_1 A + mg = P_2 A \quad \therefore P_2 = P_1 + \dfrac{mg}{A} \qquad ...\textbf{(i)}$

Now, mass of the cylindrical column of the liquid is,

$m = $ volume × density of the liquid = Area of cross section × height × density $= A h \rho$

$\therefore$ equation **(i)** becomes $P_2 = P_1 + \dfrac{A h \rho g}{A}$, $P_2 = P_1 + h \rho g \qquad ...\textbf{(ii)}$
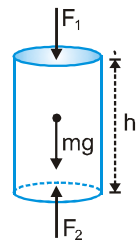
P_2 is the absolute pressure at depth h below the free surface of the liquid. Equation **(ii)**, shows that the absolute pressure at depth h is greater than the atmospheric pressure (P_1) by an amount equal to $h\rho g$.
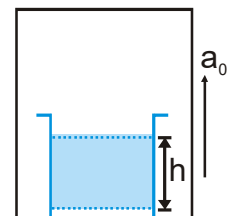
Equation **(ii)** can also be written as $(P_2 - P_1) = h \rho g$ which is the difference of pressure between two points separated by a depth h.
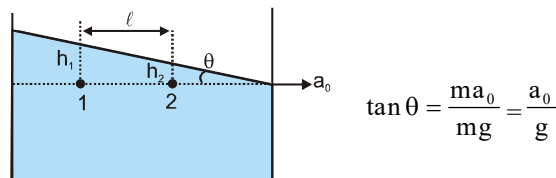
PRESSURE IN CASE OF ACCELERATING FLUID

(i) **Liquid placed in elevator :**

When elevator accelerates upward with acceleration a_0 then pressure in the fluid, at depth 'h' may be given by,

$$P = h\rho \left[g + a_0 \right]$$

(ii) **Free surface of liquid in case of horizontal acceleration :**

$$\tan\theta = \frac{ma_0}{mg} = \frac{a_0}{g}$$

If P_1 and P_2 are pressures at point 1 & 2 then $P_1 - P_2 = \rho g\,(h_1 - h_2) = \rho g \cdot \ell \tan\theta = \rho \cdot \ell \cdot a_0$

(iii) **Rotating Vessel**

Consider a cylindrical vessel, rotating at constant angular velocity about its axis. If it contains fluid then after an initial irregular shape, it will rotate with the tank as a rigid body. The acceleration of fluid particles located at a distance r from the axis of rotation will be equal to $\omega^2 r$, and the direction of the acceleration is toward the axis of rotation as shown in the figure. The fluid particles will be undergoing circular motion.

Lets consider a small horizontal cylinder of length dr and cross-sectional area A located y below the free surface of the fluid and r from the axis. This cylinder is accelerating in ground frame with acceleration $\omega^2 r$ towards the axis hence the net horizontal force acting on it should be equal to the product of mass (dm) and acceleration.

$$dm = Adr\rho$$

$$P_2 A - P_1 A = (Adr\rho)\omega^2 r$$

If we say that the left face of the cylinder is y below the free surface of the fluid then the right surface is y + dy below the surface of liquid. Thus $P_2 - P_1 = \rho g dy$

Thus solving we get $$\frac{dy}{dr} = \frac{r\omega^2}{g}$$

and, therefore, the equation for surfaces of constant pressure is $y = \dfrac{\omega^2 r^2}{2g} + \text{constant}$

This equation means that these surface of constant pressure are parabolic as shown in figure.
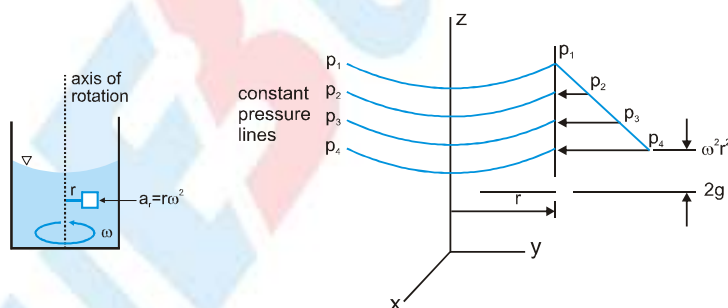
The pressure varies with the distance from the axis of rotation, but at a fixed radius, the pressure varies hydrostatically in the vertical direction as shown in figure.

Ex.   An open water tanker moving on a horizontal straight road has a cubical block of cork floating over its surface. If the tanker has an acceleration of 'a' as shown, the acceleration of the cork w.r.t. container is (ignore viscosity)
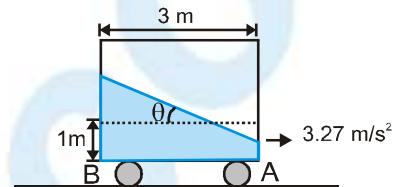
Sol.
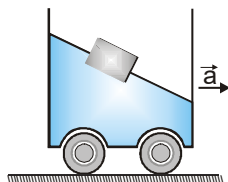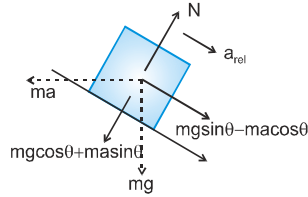
$ma_{rel} = mgsin\theta - macos\theta$ but for water surface $\tan\theta = a/g \Rightarrow a_{rel} = 0$

Ex. An open rectangular tank 1.5 m wide, 2 m deep and 2m long is half filled with water. It is accelerated horizontally at 3.27 m/s² in the direction of its length. Determine the depth of water at each end of tank. [g = 9.81 m/s²]

Sol. Here $\tan\theta = \dfrac{a}{g} = \dfrac{1}{3}$, depth at corner 'A' = $1 - 1.5 \tan\theta = 0.5$ m

depth at corner 'B' = $1 + 1.5 \tan\theta = 1.5$ m
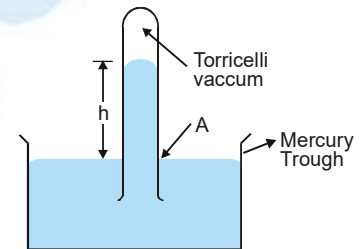

MEASUREMENT OF ATMOSPHERIC PRESSURE

1. Mercury Barometer :

To measure the atmospheric pressure experimentally, Torricelli invented a mercury barometer in 1643.

$p_a = h\rho g$

The pressure exerted by a mercury column of 1mm high is called 1 Torr.

1 Torr = 1 mm of mercury column

OPEN TUBE MANOMETER :

Open-tube manometer is used to measure the pressure gauge. When equilibrium is reached, the pressure at the bottom of left limb is equal to the pressure at the bottom of right limb.
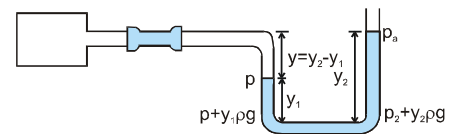
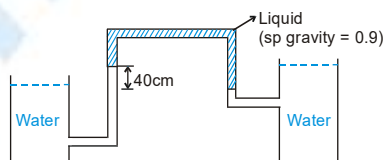i.e. $p + y_1 \rho g = p_a + y_2 \rho g$

$p - p_a = \rho g (y_2 - y_1) = \rho g y$

$p - p_a = \rho g (y_2 - y_1) = \rho g y$

p = absolute pressure, $p - p_a$ = gauge pressure.

Thus, knowing y and ρ (density of liquid), we can measure the gauge pressure.

Ex. The manometer shown below is used to measure the difference in water level between the two tanks. Calculate this difference for the conditions indicated
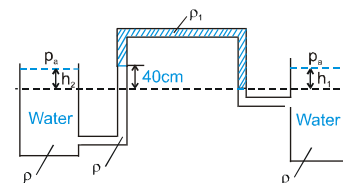

Sol. $p_a + h_1 \rho g - 40\rho_1 g + 40\rho g = p_a + h_2 \rho g$

$h_2 \rho g - h_1 \rho g = 40 \rho g - 40 \rho_1 g$ as $\rho_1 = 0.9\rho$

$(h_2 - h_1) \rho g = 40\rho g - 36\rho g$

$h_2 - h_1 = 4$ cm

WATER BAROMETER

Let us suppose water is used in the barometer instead of mercury. $h\rho g = 1.013 \times 10^5$ or $h = \dfrac{1.013 \times 10^5}{\rho g}$

The height of the water column in the tube will be 10.3 m. Such a long tube cannot be managed easily, thus water barometer is not feasible.

Ex. In a given U-tube (open at one-end) find out relation between p and p_a.

Given $d_2 = 2 \times 13.6$ gm/cm³ $d_1 = 13.6$ gm/cm³

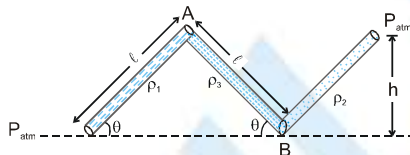**Sol.** Pressure in a liquid at same level is same i.e. at A – A–,

$p_a + d_2 \, yg + xd_1 g = p$

In C.G.S. $p_a + 13.6 \times 2 \times 25 \times g + 13.6 \times 26 \times g = p$

$p_a + 13.6 \times g \, [50 + 26] = p$

$\Rightarrow 2p_a = p$ as $p_a = 13.6 \times g \times 76$

Ex. Find out pressure at points A and B. Also find angle 'θ'.

Sol. Pressure at A – $P_A = P_{atm} - \rho_1 \, g \bullet \sin \theta$

Pressure at B $P_B = P_{atm} + \rho_2 \, gh \, \theta$

But P_B is also equal to $P_B = P_A + \rho_3 \, g \bullet \sin \theta$

Hence - $P_{atm} + \rho_2 \, gh = P_A + \rho_3 \, g \bullet \sin \theta$

$P_{atm} + \rho_2 \, gh = P_{atm} - \rho_1 \, g \bullet \sin \theta + \rho_3 \, g \bullet \sin\theta$
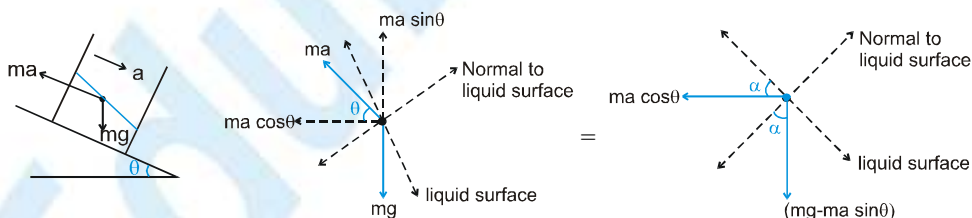
$$\sin \theta = \dfrac{\rho_2 h}{(\rho_3 - \rho_1)l} \,.$$

Ex. In the given figure, the container slides down with acceleration 'a' on an incline of angle 'θ'. Liquid is stationary with respect to container. Find out

(i) Angle made by surface of liquid with horizontal plane.

(ii) Angle if $a = g \sin \theta$.

Sol. Consider a fluid particle on surface. The forces acting on it are shown in figure.

Resultant force acting on liquid surface, will always normal to it $\tan \alpha = \dfrac{ma \cos \theta}{mg - ma \sin \theta} = \dfrac{a \cos \theta}{(g - a \sin \theta)}$

Thus angle of liquid surface with the horizontal is equal to $\alpha = \tan^{-1} \dfrac{a \cos \theta}{(g - a \sin \theta)}$

(ii) If $a = g \sin \theta$, then $\alpha = \tan^{-1}\left(\dfrac{a \cos \theta}{g - g \sin^2 \theta}\right) = \tan^{-1} \dfrac{g \sin \theta \cos \theta}{g \cos^2 \theta}$

$$= \tan^{-1}(\tan \theta)$$

$$\Rightarrow \alpha = \theta$$

Ex. An L shaped glass tube is kept inside a bus that is moving with constant acceleration. During the motion, the level of the liquid in the left arm is at 12 cm whereas in the right arm, it is at 8 cm when the orientation of the tube is as shown. Assuming that the diameter of the tube is much smaller than levels of the liquid and neglecting effect of surface tension, acceleration of the bus find the ($g = 10$ m/s²).
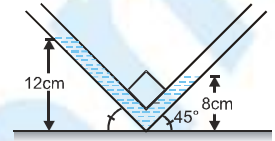
Sol. $\tan \theta = \dfrac{a}{g} = \dfrac{h_2 - h_1}{h_2 \tan 45° + h_1 \tan 45°} = \dfrac{4 \text{ cm}}{20 \text{ cm}} \Rightarrow a = 2 \text{ m/s}^2$

FORCE ON SIDE WALL OF VESSEL

Force on the side wall of the vessel can not be directly determined as a different depths pressures are different. To find this we consider a strip of width dx at a depth x from the surface of the liquid as shown in figure, and on this strip the force due to the liquid is given as : $dF = x\rho g \times b dx$
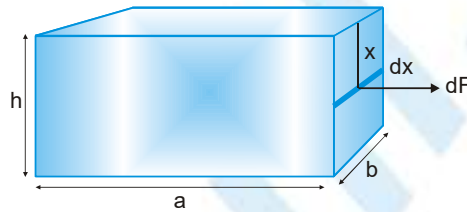
This force is acting in the direction normal to the side wall. Net force can be evaluated by integrating equation

$$F = \int dF = \int_0^h x\rho g b dx \qquad F = \frac{\rho g b h^2}{2}$$

Average pressure on side wall

The absolute pressure on the side wall cannot be evaluated because at different depths on this wall pressure is different. The average pressure on the wall can be given as :

$$\langle P \rangle_{av} = \frac{F}{bh} = \frac{1}{2}\frac{\rho g b h^2}{bh} = \frac{1}{2}\rho g h$$

Equation shows that the average pressure on side vertical wall is half of the net pressure at the bottom of the vessel.

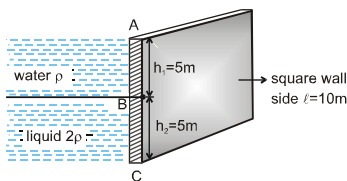## TORQUE ON THE SIDE WALL DUE TO FLUID PRESSURE

As shown in figure, due to the force dF, the side wall experiences a torque about the bottom edge of the side which is given as $d\tau = dF \times (h - x) = x\rho g b \, dx (h - x)$

This net force is $\tau = \int d\tau = \int_0^h \rho g b(hx - x^2) dx \Rightarrow = \rho g b\left[\dfrac{h^3}{2} - \dfrac{h^3}{3}\right] = \dfrac{1}{6}\rho g b h^3$

Ex. Water and liquid is filled up behind a square wall of side ●. Find out

(a) Pressures at A, B and C

(b) Forces in part AB and BC

(c) Total force and point of application of force. (Neglect atmosphere pressure in every calculation)

Sol.

(a) As there is no liquid above 'A'

So pressure at A, $p_A = 0$

Pressure at B, $p_B = \rho g h_1$
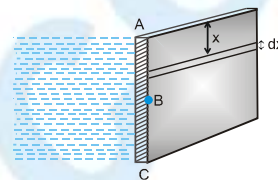
Pressure at C, $p_C = \rho g h_1 + 2\rho g h_2$

(b) Force at A = 0

Take a strip of width 'dx' at a depth 'x' in part AB.

Pressure is equal to $\rho g x$.

Force on strip = pressure × area

$dF = \rho g x ● dx$

Total force upto B : $F = \int_0^{h_1} \rho g x l \, dx = \dfrac{\rho g x l h_1^2}{2} = \dfrac{1000 \times 10 \times 10 \times 5 \times 5}{2} = 1.25 \times 10^6 \, N$

In part BC for force take a elementary strip of width dx in portion BC. Pressure is equal to $= \rho g h_1 + 2\rho g(x - h_1)$

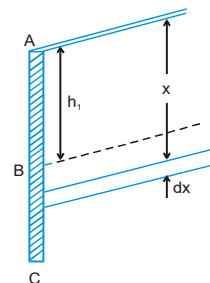Force on elementary strip = pressure × area $dF = [\rho g h_1 + 2\rho g(x - h_1)] ● dx$

Total force on part BC $F = \int_{h_1}^{l} [\rho g h_1 + 2\rho g(x - h_1)] l \, dx$

$= \left[\rho g h_1 x + 2\rho g\left[\dfrac{x^2}{2} - h_1 x \right] \right]_{h_1}^{l} l$

$= \rho g h_1 h_2 ● + 2\rho g ● \left[\dfrac{l^2 - h_1^2}{2} - h_1 l + h_1^2 \right]$

$= \rho g h_1 h_2 ● + \dfrac{2\rho g l}{2} [●^2 + h_1^2 - 2h_1 ●] = \rho g h_1 h_2 ● + \rho g ● (● - h_1)^2$

$= \rho g h_2 ● [h_1 + h_2] = \rho g h_2 ●^2 = 1000 \times 10 \times 5 \times 10 \times 10 = 5 \times 10^6 \, N$

(c) Total force $= 5 \times 10^6 + 1.25 \times 10^6 = 6.25 \times 10^6 \, N$

Taking torque about A

Total torque of force in AB $= \int dF \cdot x = \int_0^{h_1} \rho g x l \, dx . x$

$= \left[\dfrac{\rho g l x^3}{3} \right]_0^{h_1} = \dfrac{\rho g l h_1^3}{3} = \dfrac{1000 \times 10 \times 10 \times 125}{3} = \dfrac{1.25 \times 10^7}{3} N - m$

Total torque of force in BC $= \int dF \cdot x$

On solving we get $= \rho g h_1 h_2 \bullet [h_1 + \dfrac{h_2}{2}] + \rho g h_2^2 \bullet [h_1 + \dfrac{2h_2}{3}]$

$$= 1000 \times 10 \times 5 \times 5 \times 10\,[5 + 2.5] + 1000 \times 10 \times 25 \times 10\,[5 + \dfrac{10}{3}]$$

$$= 2.5 \times 7.5 \times 10^6 + \dfrac{62.5}{3} \times 10^6 = \dfrac{118.75}{3} \times 10^6$$

Total torque $= \dfrac{11.875 \times 10^7}{3} + \dfrac{1.25 \times 10^7}{3} = \dfrac{13.125 \times 10^7}{3}$

Total torque = total force × distance of point of application of force from top = $F.x_p$

$$6.25 \times 10^6 \, x_p = \dfrac{13.125 \times 10^7}{3}$$

$$x_p = 7m$$

PASCAL's LAW

If the pressure in a liquid is changed at a particular point, the change is transmitted to the entire liquid without being diminished in magnitude. Pascal's law is stated in following ways –

(i) The pressure in a fluid at rest is same at all the points if gravity is ignored.

(ii) A liquid exerts equal pressures in all directions.

(iii) If the pressure in an enclosed fluid is changed at a particular point, the change is transmitted to every point of the fluid and to the walls of the container without being diminished in magnitude.
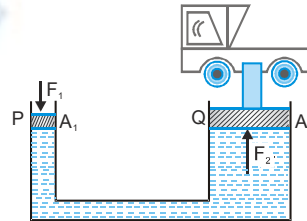
Applications of pascal's law hydraulic jacks, lifts, presses, brakes, etc

For the hydraulic lift

Pressure applied $= \dfrac{F_1}{A_1}$

$\therefore$ Pressure transmitted $= \dfrac{F_2}{A_2}$ ✈ $\dfrac{F_1}{A_1} = \dfrac{F_2}{A_2}$

$\therefore$ Upward force on A_2 is $F_2 = \dfrac{F_1}{A_1} \times A_2 = \dfrac{A_2}{A_1} \times F_1$

Ex. A vertical U–tube of uniform cross–section contains mercury in both arms. A glycerine (relative density = 1.3) column of length 10 cm is introduced into one of the arms. Oil of density 800 kg m^{-3} is poured into the other arm until the upper surface of the oil and glycerine are at the same horizontal level. Find the length of the column. Density of mercury is 13.6×10^3 kgm^{-3}
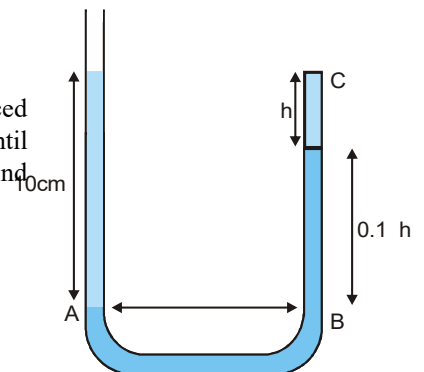
Sol. Pressure at A and B must be same

Pressure at A $= P_0 + 0.1 \times (1.3 \times 1000) \times g$

Pressure at B $= P_0 + h \times 800 \times g + (0.1 - h) \times 13.6 \times 1000 \, g$

$\Rightarrow 0.1 \times 1300 = 800 \, h + (0.1 - h) \times 13600$

$\Rightarrow h = 0.096 \, m = 9.6 \, cm$

BUOYANCY AND ARCHIMEDE'S PRINCIPLE

(i)   **Buoyant force :** If a body is partially or wholly immersed in a fluid, it experiences an upward force due to the fluid surrounding it. This phenomenon of force exerted by fluid on the body is called *buoyancy* and force is called *buoyant force* or *upthrust*.

(ii)   **Archimede's principle :** It states that the buoyant force on a body that is partially or totally immersed in a liquid is equal to the weight of the fluid displaced by it.

Now consider a body immersed in a liquid of density σ.

Top surface of the body experiences a downward force
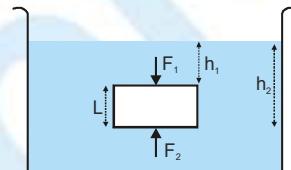
$$F_1 = AP_1 = A[h_1 \sigma g + P_0] \qquad ...(i)$$

Lower face of the body will experiences a upward force

$$F_2 = AP_2 = A[h_2 \sigma g + P_0] \qquad ...(ii)$$

As $h_2 > h_1$.   So F_2 is greater than F_1

So net upward force : $F = F_2 - F_1 = A\sigma g[h_2 - h_1]$

$\therefore$ $F = A\sigma g L = V\sigma g [\Theta \ V = AL]$

FLOATATION

When a body of density (ρ) and volume (V) is completely immersed in a liquid of density (σ), the forces acting on the body are :

(i)   Weight of the body $W = Mg = V\rho g$ (directed vertically downwards through C.G. of the body).

(ii)   Buoyant force or Upthrust $Th = V\sigma g$ (directed vertically upwards through C.B.).

The apparent weight W_{App} is equal to $W - Th$.

The following three cases are possible :

Case I   Density of the body (ρ) is greater than that of liquid (σ)

In this case   if $\rho > \sigma$ then $W > Th$

So the body will sink to the bottom of the liquid.

$$W_{App} = W - Th = V\rho g - V\sigma g = V\rho g (1 - \sigma/\rho) = W (1 - \sigma/\rho).$$

Case II   Density of the body is equal to the density of liquid $(\rho = \sigma)$

In this case   if $\rho = \sigma$ then $W = Th$

So the body will float fully submerged in the liquid. It will be in neutral equilibrium.

$$W_{App} = W - Th = 0$$

Case III Density of the body is lesser than that of liquid $(\rho < \sigma)$

In this case   if $\rho < \sigma$ then $W < Th$

So the body will float partially submerged in the liquid. In this case the body will move up and the volume of liquid displaced by the body (V_{in}) will be less than the volume of body (V). So as to make Th equal to W

$$\therefore W_{App} = W - Th = 0$$

The above three cases constitute the **law of flotation** which states that a body will float in a liquid if weight of the liquid displaced by the immersed part of the body is at least equal to the weight of the body.

Rotatory – Equilibrium in Floatation :

When a floating body is slightly tilted from equilibrium position, the centre of buoyancy B shifts. The vertical line passing through the new centre of buoyancy B' and initial vertical line meet at a point M called meta – centre. If the metacentre M is above the centre of gravity the couple due to forces at G (weight of body W) and at B' (upthrust) tends to bring the body back to its original position (figure) . So for rotational equilibrium of floating body the meta–centre must always be higher than the centre of gravity of the body.
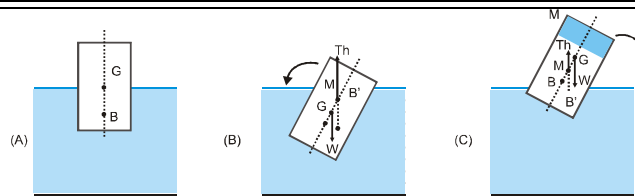
However, if meta–centre goes below centre of gravity, the couple due to forces at G and B' tends to topple the floating body. This is why a wooden log cannot be made to float vertical in water or a boat is likely to capsize if the sitting passengers stand on it. In these situations centre of gravity becomes higher than meta centre and so the body will topple if slightly tilted.

Ex. A cubical block of wood of edge 3 cm floats in water. The lower surface of the cube just touches the free end of a vertical spring fixed at the bottom of the pot. Find the maximum weight that can be put on the block without wetting it. Density of wood = 800 kg/m³ and spring constant of the spring = 50 N/m. Take g = 10 m/s²
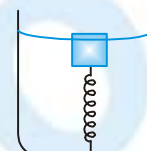
Sol. The specific gravity of the block = 0.8. Hence the height inside water = 3 cm × 0.8 = 2.4 cm. The height outside water = 3 cm – 2.4 = 0.6 cm. Suppose the maximum weight that can be put without wetting it is W. The block in this case is completely immersed in the water.

The volume of the displaced water= volume of the block = 27 × 10⁻⁶ m³.

Hence, the force of buoyancy = (27 × 10⁻⁶ m³) × 1(1000 kg/m3) × (10 m/s²) = 0.27 N.

The spring is compressed by 0.6 cm and hence the upward force exerted by the spring
= 50 N/m × 0.6 cm = 0.3 N.

The force of buoyancy and the spring force taken together balance the weight of the block plus the weight W put on the block. The weight of the block is
W′ = (27 × 10⁻⁶ m) × (800 kg/m³) × (10 m/s²) = 0.22 N.

Thus, W = 0.27 N + 0.3 N – 0.22 N = 0.35 N.

Ex. A wooden plank of length 1 m and uniform cross-section is hinged at one end to the bottom of a tank as shown in figure.
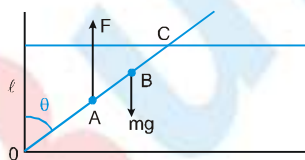
The tank is filled with water up to a height of 0.5 m. The specific gravity of the plank is 0.5. Find the angle θ that the plank makes with the vertical in the equilibrium position. (Exclude the case θ = 0).

Sol. The forces acting on the plank are shown in the figure. The height of water level is ℓ = 0.5 m. The length of the plank is 1.0 m = 2ℓ. The weight of the plank acts through the centre B of the plank. We have OB = ℓ. The buoyant force F acts through the point A which is the middle point of the dipped part OC of the plank.

We have $OA = \dfrac{OC}{2} = \dfrac{1}{2\cos\theta}$.

Let the mass per unit length of the plank be ρ. Its weight mg = 2ℓρg.

The mass of the part OC of the plank = $\left(\dfrac{1}{\cos\theta}\right)\rho$.

The mass of water displaced = $\dfrac{1}{0.5}\dfrac{1}{\cos\theta}\rho = \dfrac{2l\rho}{\cos\theta}$.

The buoyant force F is, therefore, $F = \dfrac{2l\rho g}{\cos\theta}$.

Now, for equilibrium, the torque of mg about O should balance the torque of F about O.

So, $mg\,(OB)\sin\theta = F(OA)\sin\theta$ or, $(2\ell\rho)\ell = \left(\dfrac{2l\rho}{\cos\theta}\right)\left(\dfrac{1}{2\cos\theta}\right)$ or, $\cos^2\theta = \dfrac{1}{2}$ or, $\cos\theta = \dfrac{1}{\sqrt{2}}$, or, θ = 45°.

ETOOS KEY POINTS

(i) Buoyant force act vertically upward through the centre of gravity (C.G.) of the displaced fluid. This point is called centre of buoyancy (C.B.). Thus centre of buoyancy is the point through which the force of buoyancy is supposed to act.

(ii) Buoyant force or upthrust does not depend upon the characteristics of the body such as its mass, size, density, etc. But it depends upon the volume of the body inside the liquid.

(iii) It depends upon the nature of the fluid as it is proportional to the density of the fluid.

This is the reason that upthrust on a fully submerged body is more in sea water than in pure water

(iv) It depends upon the effective acceleration.

If a lift is accelerated downwards with acceleration a (a < g) then $\qquad$ $Th = V_{in} \sigma (g - a)$

If a lift is accelerated downwards with a = g then $\qquad$ $Th = V_{in} \sigma (g - a) = 0$

If a lift is accelerated upward with acceleration a then $\qquad$ $Th = V_{in} \sigma (g + a)$

(v) If a body is weighed in air (W_A), in water (W_w) and in a oil (W_O), then

$$\text{Specific gravity of oil} = \frac{\text{loss of weight in oil}}{\text{loss of weight in water}} = \frac{W_A - W_O}{W_A - W_W}$$

Ex. A body weighs 160 g in air, 130 g in water and 136 g in oil. What is the specific gravity of oil?

Sol. $\text{Specific gravity of oil} = \dfrac{\text{loss of weight in oil}}{\text{loss of weight in water}} = \dfrac{160 - 136}{160 - 130} = \dfrac{24}{30} = \dfrac{8}{10} = 0.8$

Ex. An iceberg is floating partially immersed in sea–water. The density of sea–water is 1.03 gm/cm^3 and that of ice is 0.92 gm/cm^3. What is the fraction of the total volume of the iceberg above the level of sea–water ?

Sol. In case of flotation weight = upthrust i.e.

$$mg = V_{in} \sigma g \Rightarrow V \rho g = V_{in} \sigma g \qquad \therefore V_{in} = \frac{\rho}{\sigma} V \qquad \text{So } V_{out} = V - V_{in} = V\left[1 - \frac{\rho}{\sigma}\right]$$

$$\Rightarrow f_{out} = \frac{V_{out}}{V} = \left[1 - \frac{\rho}{\sigma}\right]\left[1 - \frac{0.92}{1.03}\right] = \frac{0.11}{1.03} = 0.106 \Rightarrow f_{out} = 10.6 \%$$

Ex. A rubber ball of mass 10 gm and volume 15 cm^3 is dipped in water to a depth of 10m. Assuming density of water uniform throughout the depth if it is released from rest. Find (take g = 980 cm/s^2)

(a) the acceleration of the ball, and $\qquad$ **(b)** the time taken by it to reach the surface.

Sol. The maximum buoyant force on the ball is $F_B = V \rho_w g = 15 \times 1 \times 980$ dyne = 14700 dyne.

The weight of the ball is mg = 10 × g = 10 × 980 = 9800 dyne

The net upward force, F = (15 × 980 – 10 × 980) dyne = 5 × 980 dyne = 4900 dyne

(a) Therefore, acceleration of the ball upward $a = \dfrac{F}{m} = \dfrac{5 \times 980}{10} = 490$ cm/s^2 = 4.9 m/s^2

(b) Time taken by it reach the surface is $t = \sqrt{\dfrac{2h}{a}} = \sqrt{\dfrac{2 \times 10}{4.9}}$ s = 2.02 s

FLUID DYNAMICS

When a fluid moves in such a way that there are relative motions among the fluid particles, the fluid is said to be flowing.

TYPES OF FLUID FLOW : Fluid flow can be classified as :

1   Steady and Unsteady Flow

Steady flow is defined as that type of flow in which the fluid characteristics like velocity, pressure and density at a point do not change with time. In an *unsteady flow*, the velocity, pressure and density at a point in the flow varies with time.

2   Streamline Flow

In steady flow all the particles passing through a given point follow the same path and hence a unique line of flow. This line **or** path is called a *streamline*. Streamlines do not intersect each other because if they intersect each other the particle can move in either direction at the point of intersection and flow cannot be steady.

3   Laminar and Turbulent Flow

Laminar flow is the flow in which the fluid particles move along well–defined streamlines which are straight and parallel. In laminar flow the velocities at different points in the fluid may have different magnitudes, but there directions are parallel. Thus the particles move in laminar **or** layers gliding smoothly over the adjacent layer.

Turbulent flow is an irregular flow in which the particles can move in zig–zag way due to which eddies formation take place which are responsible for high energy losses.

**4   Compressible and Incompressible Flow**

In *compressible flow* the density of fluid varies from point to point i.e. the density is not constant for the fluid whereas in *incompressible flow* the density of the fluid remains constant throughout. Liquids are generally incompressible while gases are compressible.

5   Rotational and Irrotational Flow

Rotational flow is the flow in which the fluid particles while flowing along path–lines also rotate about their own axis. In *irrotational flow* particles do not rotate about their axis. So they have no net angular velocity.

EQUATION OF CONTINUITY

The continuity equation is the mathematical expression of the *law of conservation of mass in fluid dynamics*.
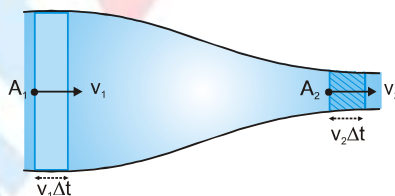
In the steady flow the mass of fluid entering into a tube of flow in a particular time interval is equal to the mass of fluid leaving the tube.

$$\frac{m_1}{\Delta t} = \frac{m_2}{\Delta t} \Rightarrow \rho_1 A_1 v_1 = \rho_2 A_2 v_2 \ (\ \rho_1 = \rho_2) \Rightarrow A_1 v_1 = A_2 v_2 \quad \therefore \ Av = \text{constant}$$

(Here ρ = density of fluid, v = velocity of fluid, A = Area of cross–section of tube)

Therefore the velocity of liquid is smaller in the wider parts of a tube and larger in the narrower parts.

BERNOULLI'S THEOREM

Bernoulli's equation is mathematical expression of the law of mechanical energy conservation in fluid dynamics.

Bernoullis theorem is applied to the ideal fluids.

Characteristics of an ideal fluid are :

(i) The fluid is incompressible.   **(ii)** The fluid is non–viscous.

(iii) The fluid flow is steady.   **(iv)** The fluid flow is irrotational.

Every point in an ideal fluid flow is associated with three kinds of energies :

Kinetic Energy

If a liquid of mass (m) and volume (V) is flowing with velocity (v) then Kinetic Energy $= \dfrac{1}{2}$ mv²and kinetic energy

per unit volume $= \dfrac{\text{Kinetic Energy}}{\text{volume}} = \dfrac{1}{2}\dfrac{m}{V} v^2 = \dfrac{1}{2}\rho v^2$

Potential Energy

If a liquid of mass (m) and volume (V) is at height (h) from the surface of the earth then its

Potential Energy = mgh and potential energy per unit volume $= \dfrac{\text{Potential Energy}}{\text{volume}} = \dfrac{m}{V}$ gh = ρgh

Pressure Energy

If P is the pressure on area A of a liquid and the liquid moves through a distance (●) due to this pressure then

Pressure energy = Work done = force x displacement = pressure x area x displacement = PA● = PV

[✈ A● = volume V]

Pressure energy per unit volume $= \dfrac{\text{Pressure energy}}{\text{volume}}$ =P

THEOREM

According to Bernoulli's Theorem , in case of steady flow of incompressible and non–viscous fluid through a tube of non–uniform cross–section, the sum of the pressure, the potential energy per unit volume and the kinetic

energy per unit volume is same at every point in the tube, i.e., $P + \rho gh + \dfrac{1}{2}\rho v^2 =$ constant.
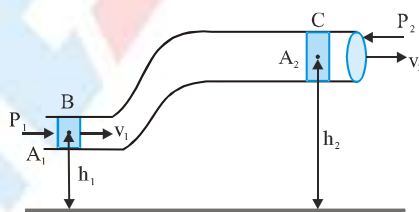
Consider a liquid flowing steadily through a tube of non–uniform area of cross–section as shown in figure. If P₁ and P₂ are the pressures at the two ends of the tube respectively, work done in pushing the volume ΔV of incompressible liquid from point B to C through the tube

$$W = P(\Delta V) = (P_1 - P_2)\Delta V \qquad ...\textbf{(i)}$$

This work is used by the liquid in two ways :

(i)   In changing the potential energy of mass Δm (in the volume ΔV) from

Δmgh_1 to Δmgh_2 i.e., $\Delta U = \Delta mg(h_2 - h_1)$ \qquad ...**(ii)**

(ii) In changing the kinetic energy from $\frac{1}{2}\Delta m v_1^2$ to $\frac{1}{2}\Delta m v_2^2$, i.e. $\Delta K = \frac{1}{2}\Delta m \left(v_2^2 - v_1^2\right)$

Now as the liquid is non–viscous, by conservation of mechanical energy,

$$W = \Delta U + \Delta K \text{ i.e., } \left(P_1 - P_2\right)\Delta V = \Delta m g \left(h_2 - h_1\right) + \frac{1}{2}\Delta m \left(v_2^2 - v_1^2\right)$$
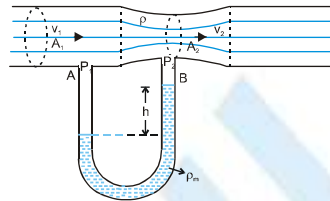
$$P_1 - P_2 = \rho g(h_2 - h_1) + \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) \text{ [as } \rho = \Delta m / \Delta V \text{]}$$

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2 \Rightarrow P + \rho g h + \frac{1}{2}\rho v^2 = \text{constant}$$

This equation is the Bernoulli's equation and represents conservation of mechanical energy in case of moving fluids.

VENTURIMETER

It is a gauge put on a flow pipe to measure the speed of flow of a liquid shown in figure.

Let the liquid of density ρ be flowing through a pipe of area of cross section A_1. Let A_2 be the area of cross section at the throat and a manometer is attached as shown in the figure. Let v_1 and P_1 be the velocity of the flow and pressure at point A, v_2 and P_2 be the corresponding quantities at point B.

Using Bernoulli's theorem :

$$\frac{P_1}{\rho} + g h_1 + \frac{1}{2}v_1^2 = \frac{P_2}{\rho} + g h_2 + \frac{1}{2}v_2^2$$

we get $\quad \frac{P_1}{\rho} + g h + \frac{1}{2}v_1^2 = \frac{P_2}{\rho} + g h + \frac{1}{2}v_2^2$ (Since $h_1 = h_2 = h$) or $(P_1 - P_2) = \frac{1}{2}\rho(v_2^2 - v_1^2)$ ....(i)

According to continuity equation, $A_1 v_1 = A_2 v_2$ or $v_2 = \left(\frac{A_1}{A_2}\right)v_1$

Substituting the value of v_2 in equation **(i)**

we have $\quad \left(P_1 - P_2\right) = \frac{1}{2}\rho = \left[\left(\frac{A_1}{A_2}\right)^2 v_1^2 - v_1^2\right]\frac{1}{2}\rho v_1^2\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]$

Since $A_1 > A_2$, therefore, $\quad P_1 > P_2$ or $v_1^2 = \dfrac{2(P_1 - P_2)}{\rho\left[\left(\dfrac{A_1}{A_2}\right)^2 - 1\right]} = \dfrac{2A_2^2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}$

where $(P_1 - P_2) = \rho_m g h$ and h is the difference in heights of the liquid levels in the two tubes.

$$v_1 = \sqrt{\dfrac{2\rho_m\, g h}{\rho\left[\left(\dfrac{A_1}{A_2}\right)^2 - 1\right]}}$$

The flow rate (R) i.e., the volume of the liquid flowing per second is given by $R = v_1 A_1$.

• **Torricelli's Law of Efflux (Fluid Outflow)**

As shown in the figure since the area of cross–section at A is very large as compared to that at orifice B, speed at A i.e. $v_A \approx 0$. Also the two fluid particles at A and B are at same pressure P_0 (atmospheric pressure). Applying Bernoulli's theorem at A and B.
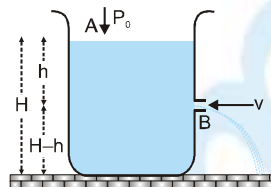
$$P_0 + \rho gH + \frac{1}{2}\rho v_A^2 = P_0 + \rho g(H-h) + \frac{1}{2}\rho v_B^2 \Rightarrow \frac{1}{2}\rho v_B^2 = \rho gh \Rightarrow v_B = \sqrt{2gh}$$

Equation is same as that of freely falling body after falling through h height and is known as Torricelli's law. Writing equation of uniformly accelerated motion in vertical direction

$$H - h = 0 + \frac{1}{2}gt^2 \text{ (from } s_y = u_y t + \frac{1}{2}a_y t^2) \Rightarrow t = \sqrt{\frac{2(H-h)}{g}}$$
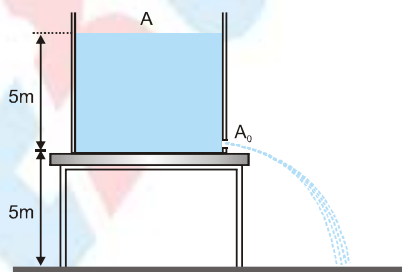
Horizontal range $\quad R = v_x t = \sqrt{2gh} \times \sqrt{\frac{2(H-h)}{g}} = 2\sqrt{h(H-h)}$

Range R will be maximum when R^2 is maximum. i.e.,

$$\frac{d}{dh}\left(R^2\right) = 0 \Rightarrow 4\frac{d}{dh}(Hh - h^2) = 0 \Rightarrow H - 2h = 0, \text{ i.e., } h = \frac{H}{2}$$

$$\therefore R_{maximum} = 2\sqrt{\frac{H}{2}\left[H - \frac{H}{2}\right]} = H$$

Ex. A cylindrical tank 1 m in radius rests on a platform 5 m high. Initially the tank is filled with water upto a height of 5 m. A plug whose area is 10^{-4} m^2 is removed from an orifice on the side of the tank at the bottom. Calculate

(a) initial speed with which the water flows from the orifice

(b) initial speed with which the water strikes the ground

Sol. **(a)** Speed of efflux $v_H = \sqrt{(2gh)} = \sqrt{2 \times 10 \times 5} = 10$ m/s

(b) As initial vertical velocity of water is zero, so its vertical velocity when it hits the ground

$$v_v = \sqrt{2gh} = \sqrt{2 \times 10 \times 5} = 10 \text{ m/s}$$

So the initial speed with which water strikes the ground, $v = \sqrt{v_H^2 + v_V^2} = 10\sqrt{2} = 14.1$ m/s

Ex. In a given arrangement

(a) Find out velocity of water coming out of 'C'

(b) Find out pressure at A, B and C.

Sol.

(a) Applying Bernoulli's equation between liquid surface and point 'C'.

$$p_a + \frac{1}{2}\rho v_1^2 = p_a - \rho g h_3 + \frac{1}{2}\rho v_2^2$$

through continuity equation $Av_1 = av_2$, $v_1 = \dfrac{av_2}{A} \Rightarrow \dfrac{1}{2}\rho \dfrac{a^2}{A^2}v_2^2 = -\rho g h_3 + \dfrac{1}{2}\rho v_2^2$

$$v_2^2 = \frac{2gh_3}{1 - \dfrac{a^2}{A^2}}, \; v_2 = \sqrt{\frac{2gh_3}{1 - \dfrac{a^2}{A^2}}}$$

(b) Pressure at A just outside the tube $p_A = p_{atm} + \rho g h_1$

For pressure at B : $\qquad p_A + 0 + 0 = p_B + \rho g h_2 + \dfrac{1}{2}\rho v_B^2$

$$p_B = P_A - \rho g h_2 - \frac{1}{2}\rho\left(\frac{2gh_3}{1 - \dfrac{a^2}{A^2}}\right)$$

Pressure at C : $\qquad p_C = p_{atm}$

Ex. A tank is filled with a liquid upto a height H. A small hole is made at the bottom of this tank. Let t_1 be the time taken to empty first half of the tank and t_2 time taken to empty rest half of the tank, then find $\dfrac{t_1}{t_2}$.

Sol. Let at some instant of time the level of liquid in the tank is y. Velocity of efflux at this instant of time $v = \sqrt{2gy}$. Now,

at this instant volume of liquid coming out the hole per second is $\left(\dfrac{dV_1}{dt}\right)$

Volume of liquid coming down in the tank per second is $\left(\dfrac{dV_2}{dt}\right)$

$$\frac{dV_1}{dt} = \frac{dV_2}{dt} \Rightarrow av = A\left(\frac{-dy}{dt}\right) \quad \therefore \; a\sqrt{2gy} = A\left(-\frac{dy}{dt}\right) \qquad\qquad ...\textbf{(i)}$$
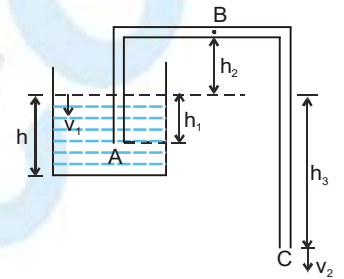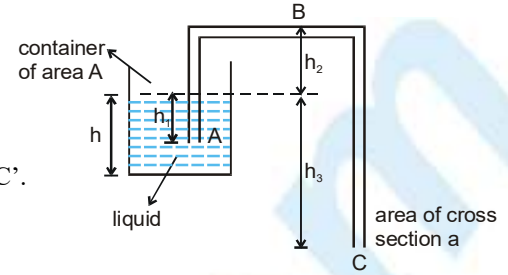
(Here area of cross-section of hole and tank are respectively a and A)

Substituting the proper limits in equation **(i)**,

$$\int_0^{t_1} dt = -\frac{A}{a\sqrt{2g}}\int_H^{H/2} y^{-1/2}dy \Rightarrow t_1 = \frac{2A}{a\sqrt{2g}}\left[\sqrt{y}\right]_{H/2}^{H} = \frac{2A}{a\sqrt{2g}}\left[\sqrt{H} - \sqrt{\frac{H}{2}}\right] = \frac{A}{a}\sqrt{\frac{H}{g}}\left(\sqrt{2}-1\right) ...\textbf{(ii)}$$

Similarly, $\qquad \displaystyle\int_0^{t_2} dt = -\frac{A}{a\sqrt{2g}}\int_{H/2}^{0} y^{-1/2}dy \Rightarrow t_2 = \frac{A}{a}\sqrt{\frac{H}{g}} \qquad\qquad ...\textbf{(iii)}$

From equations **(ii)** and **(iii)**, $\dfrac{t_1}{t_2} = \sqrt{2}-1 = 0.414$

Ex. A fixed container of height 'H' with large cross-sectional area 'A' is completely filled with water. Two small orifice of cross-sectional area 'a' are made, one at the bottom and the other on the vertical side of the container at a distance H/2 from the top of the container. Find the time taken by the water level to reach a height of H/2 from the bottom of the container.

Sol. $v_1 = \sqrt{2g(h - H/2)}$; $v_2 = \sqrt{2gh}$ $\therefore$ By continuity equation

$$A\left(-\frac{dh}{dt}\right) = a(v_1 + v_2) \Rightarrow A\left(-\frac{dh}{dt}\right) = a\left\{\sqrt{2g(h-H/2)} + \sqrt{2gh}\right\}$$

or $-\dfrac{A}{a\sqrt{2g}} \displaystyle\int\limits_{H}^{H/2} \dfrac{dh}{\sqrt{h} + \sqrt{h - H/2}} = \int\limits_0^t dt \Rightarrow t = \dfrac{2A}{3a}\left(\sqrt{2}-1\right)\sqrt{\dfrac{H}{g}}$
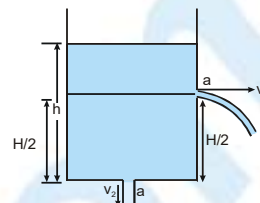
Ex. A cylindrical vessel filled with water upto a height of 2 m stands on a horizontal plane. The side wall of the vessel has a plugged circular hole touching the bottom. Find the minimum diameter of the hole so that the vessel begins to move on the floor if the plug is removed. The coefficient of friction between the bottom of the vessel and the plane is 0.4 and total mass of water plus vessel is 100 kg.

Sol. From Torricelli's theorem, velocity of efflux = $\sqrt{2gh}$

Momentum per second carried by water stream

= density × volume coming out per second × velocity = $\rho \times av \times v = \rho av^2$

Hence force on cylindrical vessel = $\rho a 2gh$

Cylinder starts to move when reaction force is just equal to maximum force of friction.

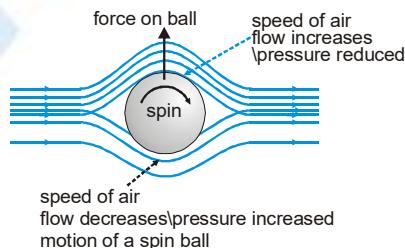i.e., $\mu Mg = \rho a 2gh \Rightarrow a = \dfrac{\mu M}{2\rho h} = \dfrac{0.4 \times 100}{2 \times 10^3 \times 2} = 0.01\ m^2$

Area of circular hole = $\dfrac{\pi d^2}{4} = 0.01\ m^2 \Rightarrow d = \sqrt{\dfrac{0.01 \times 4}{\pi}} = 0.113 m$

EXAMPLES OF BERNOULLI'S THEOREM

l. **Magnus Effect (Spinning Ball)**

Tennis and cricket players usually experience that when a ball is thrown spinning it moves along a curved path. This is called *swing* of the ball. This is due to the air which is being dragged round by the spinning ball. When the ball spins, the layer of the air around it also moves with the ball. So, as shown in figure the resultant velocity of air increases on the upper side and reduces on the lower side.

force on ball
speed of air flow increases \pressure reduced
spin
speed of air flow decreases\pressure increased
motion of a spin ball

Hence according to Bernoulli's theorem the pressure on the upper side becomes lower than that on the lower side. This pressure difference exerts a force on the ball due to which it moves along a curved path. This effect is known as *Magnus–effect*.

2. **Motion of the Ping–Pong Ball**

When a ping–pong ball is placed on a vertical stream of water– fountain, it rises upto a certain height above the nozzle of the fountain and spins about its axis.

The reason for this is that the streams of water rise up from the fountain with very large velocity so that the air–pressure in them decreases. Therefore, whenever the ball goes out from the streams, the outer air which is at atmospheric pressure pushes it back into the streams (in the region of low pressure). Thus the ball remains in *stable equilibrium* on the fountain.
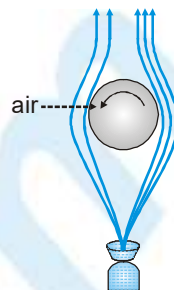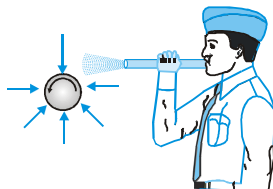
If we blow air at one end of a narrow tube, the air emerges from the other end at high speed and so the pressure falls there. If a ping–pong ball is left free slightly below this end, it does not fall down due to the large pressure (atmospheric) below the ball. Similarly, if we blow air in between two ping–pong balls suspended by light threads near each other, the balls come close to each other due to the decrease of air pressure between them. Same is the reason that when air is blown below a pan of a physical balance the pan is depressed down.

3. **Aerofoil**

This is a device which is shaped in such a way so that the relative motion between it and a fluid produces a force perpendicular to the flow. As shown in the figure the shape of the aerofoil section causes the fluid to flow faster over the top surface then over the bottom i.e. the streamlines are closer above than below the aerofoil. By Bernoullis theorem the pressure above is reduced and that underneath is increased.
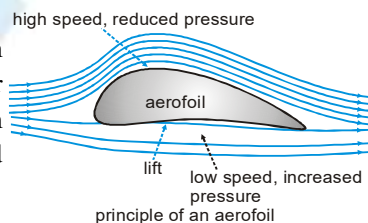
Thus a resultant upward force is created normal to the flow and it is this force which provides most of lift upward force for an aeroplane. Examples of aerofoils are aircraft wings, turbine blades and propellers.

4. **Pull–in or Attraction Force by Fast Moving Trains**

If we are standing on a platform and a train passes through the platform with very high speed we are pulled towards the train. This is because as the train comes at high speed, the pressure between us and the train decreases. Thus the air behind us which is still at atmospheric pressure pushes us towards the train. The reason behind flying–off of small papers, straws and other light objects towards the train is also the same.

5. **Sprayer or Atomizer**

This is an instrument used to spray a liquid in the form of small droplets (fine spray). It consists of a vertical tube whose lower end is dipped in the liquid to be sprayed, filled in a vessel.

The upper end opens in a horizontal tube. At one end of the horizontal tube there is a rubber bulb and at the other end there is a fine bore (hole). When the rubber bulb is squeezed, air rushes out through the horizontal tube with very high velocity and thus the pressure reduces (according to Bernoulli's theorem). So the liquid rises and comes out through narrow end in form of droplets. It is used in spray gun, perfumes, deodorant and etc.
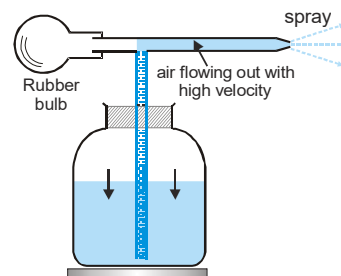
6. Bunsen's Burner (Jet)

It is based on the working of jet. It consists of a long tube having a fine nozzle O at the bottom. The burning gas enters the tube through O and burns in a flame at the top of the tube.

To produce a non–luminous flame, the air of the atmosphere is mixed with the gas. Since the nozzle O is fine, the gas enters with a large velocity and so the pressure inside the tube is lowered than the outer atmospheric pressure. Therefore, air from outside rushes into the tube through a hole and is mixed with the burning gas.
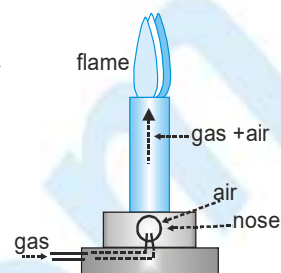
7. Filter Pump (Nozzle)

It is based on the working of nozzle. It consists of a wide tube M N in the upper part of which there is another tube A. The upper end of A is connected to a water tank, while its lower end has a fine bore through which water comes out in the form of a jet. The vessel which is to be evacuated is connected to the tube M N as shown.

The velocity of the emerging water jet is very large. Therefore, the pressure of the air near the jet becomes less than the pressure in the vessel. Hence air from the vessel rushes into the tube M N and is carried out along with the water jet. Thus partial vacuum is created in the vessel.
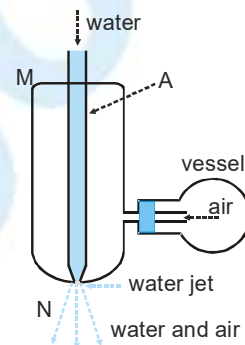
8. Blowing–off of Tin Roof Tops in Wind Storm

When wind blows with a high velocity above a tin roof, it causes lowering of pressure above the roof, while the pressure below the roof is still atmospheric. Due to this pressure–difference the roof is lifted up.
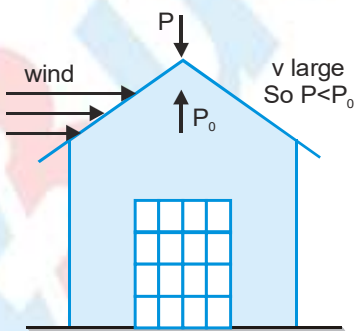
VISCOCITY

Viscosity is the property of the fluid (liquid or gas) by virtue of which it opposes the relative motion between its adjacent layers. It is the fluid friction or internal friction. The internal tangential force which try to retard the relative motion between the layers is called viscous force.

NEWTON'S LAW OF VISCOSITY

Suppose a liquid is flowing in streamlined motion on a horizontal surface OX. The liquid layer in contact with the surface is at rest while the velocity of other layers increases with increasing distance from the surface OX. The highest layer flows with maximum velocity.

horizontal surface
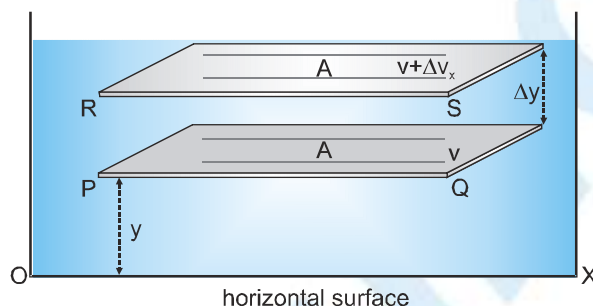
Let us consider two parallel layers PQ and RS at distances y and y + Δy from OX. Thus the change in velocity in a perpendicular distance Δy is Δv_x. The rate of change of velocity with distance perpendicular to the direction of flow

i.e. $\dfrac{\Delta v_x}{\Delta y}$, is called velocity–gradient. According to Newton, the viscous force F acting between two layers of a liquid flowing in streamlined motion depends upon following two factors :

(i) $F \propto$ Contact–area of the layers i.e. $F \propto A$

(ii) $F \propto$ Velocity–gradient $\left[\dfrac{\Delta v_x}{\Delta y}\right]$ between layers i.e. $F \propto \dfrac{\Delta v_x}{\Delta y}$. Therefore $F \propto A \dfrac{\Delta v_x}{\Delta y} \Rightarrow F = \eta A \dfrac{\Delta v_x}{\Delta y}$ where η is a constant called coefficient of viscosity of the liquid.

In above formula if $A = 1$ and $\dfrac{\Delta v_x}{\Delta y} = 1$, then $F = \eta$. i.e. the coefficient of viscosity of a liquid is defined as the viscous force per unit area of contact between two layers having a unit velocity gradient.

• **SI Units** : $\dfrac{N-s}{m^2}$ or deca poise

• **CGS units** : dyne–s/cm^2 or poise (1 decapoise = 10 poise)

• **Dimension** : $M^1 L^{-1} T^{-1}$

Ex. A plate of area 2 m² is made to move horizontally with a speed of 2 m/s by applying a horizontal tangential force over the free surface of a liquid. If the depth of the liquid is 1m and the liquid in contact with the plate is stationary. Coefficient of viscosity of liquid is 0.01 poise. Find the tangential force needed to move the plate.

Sol. Velocity gradient $= \dfrac{\Delta v}{\Delta y} = \dfrac{2-0}{1-0} = 2 \dfrac{m/s}{m} = 2s^{-1}$

From, Newton's law of viscous force,

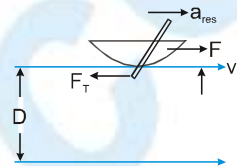$|F| = \eta A \dfrac{\Delta v}{\Delta y} = (0.01 \times 10^{-1})(2)(2) = 4 \times 10^{-3} \, N$
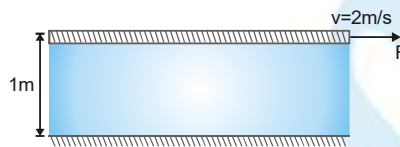
Ex. A man is rowing a boat with a constant velocity v_0 in a river the contact area of boat is 'A' and coefficient of viscosity is η. The depth of river is 'D'. Find the force required to row the boat.

Sol. $F - F_T = m a$

As boat moves with constant velocity a = 0 so $F = F_T$

But $F_T = \eta A \dfrac{dv}{dz}$ but $\dfrac{dv}{dz} = \dfrac{v_0 - 0}{D} = \dfrac{v_0}{D}$
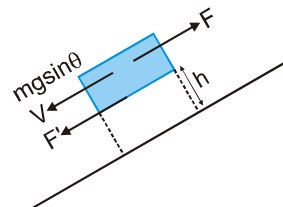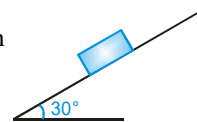
then $F = F_T = \dfrac{\eta A v_0}{D}$

Ex. A cubical block (of side 2m) of mass 20 kg slides on inclined plane lubricated with the oil of viscosity $\eta = 10^{-1}$ poise with constant velocity of 10 m/s. Find out the thickness of layer of liquid. (g = 10 m/s²)
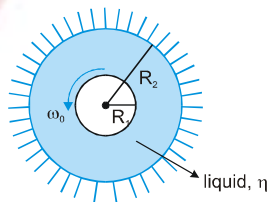
Sol. $F = F' = \eta A \dfrac{dv}{dz} = mg \sin\theta \quad \dfrac{dv}{dz} = \dfrac{v}{h}$

$\Rightarrow 20 \times 10 \times \sin 30° = \eta \times 4 \times \dfrac{10}{h}$

$\Rightarrow h = 4 \times 10^{-3} \, m = 4 \, mm$

Ex. As per the shown figure the central solid cylinder starts with initial angular velocity ω_0. Find out the time after which the angular velocity becomes half.
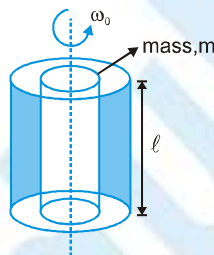
Sol. $F = \eta A \dfrac{dv}{dz}$, where $\dfrac{dv}{dz} = \dfrac{\omega R_1 - 0}{R_2 - R_1}$

$F = \eta \dfrac{2\pi R_1 \, l \, \omega R_1}{R_2 - R_1}$ and $\tau = FR_1 = \dfrac{2\pi \eta R_1^3 \, \omega l}{R_2 - R_1}$

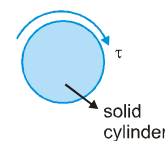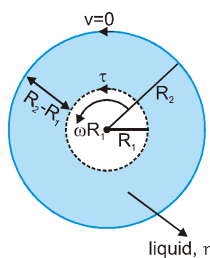$$I\,\alpha = \frac{2\,\pi\,\eta\,R_1^3\,\omega\,l}{R_2 - R_1} \Rightarrow \frac{MR_1^2}{2}\left(-\frac{d\omega}{dt}\right) = \frac{2\,\pi\,\eta\,R_1^3\,\omega l}{R_2 - R_1}$$

or $\quad -\displaystyle\int_{\omega_0}^{\omega_0/2}\frac{d\omega}{\omega} = \frac{4\,\pi\eta R_1\,l}{m\,(R_2 - R_1)}\int_0^t dt \Rightarrow t = \frac{m\,(R_2 - R_1)\ln 2}{4\,\pi\,\eta\,l\,R_1}$

DEPENDENCY OF VISCOSITY OF FLUIDS

1. **On Temperature of Fluid**

    (a) Since cohesive forces decrease with increase in temperature as increase in K.E. Therefore with the rise in temperature, the viscosity of liquids decreases.

    (b) The viscosity of gases is the result of diffusion of gas molecules from one moving layer to other moving layer. Now with increase in temperature, the rate of diffusion increases. So, the viscosity also increases. Thus, the viscosity of gases increases with the rise of temperature.

2. **On Pressure of Fluid**

    (a) The viscosity of liquids increases with the increase of pressure.

    (b) The viscosity of gases is practically independent of pressure.

3. **On Nature of Fluid**

STOKE'S LAW AND TERMINAL VELOCITY

Stoke showed that if a small sphere of radius r is moving with a velocity v through a homogeneous medium (liquid or gas), coefficient of viscosity η then the viscous force acting on the sphere is $F_v = 6\pi\eta rv$. It is called Stoke's Law.
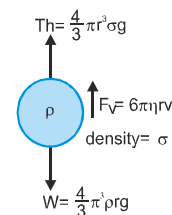
Terminal Velocity

When a solid sphere falls in a liquid then its accelerating velocity is controlled by the viscous force of liquid and hence it attains a constant velocity which is known as terminal velocity (v_T).

As shown in figure when the body moves with constant velocity i.e. terminal velocity (zero acceleration) the net upward force (upthrust Th + viscous force F_v) balances the downward force (weight of body W)

Therefore $\quad Th + F_v = W \Rightarrow \frac{4}{3}\pi r^3 \sigma g + 6\pi\eta rv_T = \frac{4}{3}\pi r^3 \rho g \Rightarrow v_T = \frac{2}{9}\frac{r^2(\rho - \sigma)}{\eta}g$

where r = radius of body, ρ = density of body, σ = density of liquid,

η = coefficient of viscosity

Some applications of Stoke's law :

(a) The velocity of rain drops is very small in comparison to the velocity of a body falling freely from the height of clouds.

(b) Descending by a parachute with lesser velocity.

(c) Determination of electronic charge with the help of Milikan's experiment.

Ex. A spherical ball is moving with terminal velocity inside a liquid. Determine the relationship of rate of heat loss with the radius of ball.

Sol. Rate of heat loss = power = $F \times v = 6\pi\eta \, rv \times v = 6\pi\eta rv^2 = 6p\eta r \left[\dfrac{2}{9} \dfrac{gr^2(\rho_0 - \rho_1)}{\eta} \right]^2$

Therefore rate of heat loss $\propto r^5$

Ex. A drop of water of radius 0.0015 mm is falling in air. If the coefficient of viscosity of air is 1.8×10^{-5} kg m^{-1}s^{-1}. What will be the terminal velocity of the drop. Density of air can be neglected.

Sol. $v_T = \dfrac{2}{9} \dfrac{r^2(\rho - \sigma)g}{\eta} = \dfrac{2 \times \left[\dfrac{15 \times 10^{-4}}{1000} \right]^2 \times 10^3 \times 9.8}{9 \times 1.8 \times 10^{-5}} = 2.72 \times 10^{-4}$ m/s

REYNOLDS NUMBER (Re)

The type of flow pattern (laminar or turbulent) is determined by a non–dimensional number called Reynolds number

(R_e). Which is defined as $R_e = \dfrac{\rho v d}{\eta}$ where ρ is the density of the fluid having viscosity η and flowing with mean

speed v. d denotes the diameter of obstacle or boundary of fluid flow.

Although there is not a perfect demarkation for value of R_e for laminar and turbulent flow but some authentic references take the value as

R_e	< 1000	> 2000	between 1000 to 2000
Type of flow	laminar	often turbulent	may be laminar or turbulent

on gradually increasing the speed of flow at certain speed transition from laminar flow to turbulent flow takes place. This speed is called critical speed. For lower density and higher viscosity fluids laminar flow is more probable.
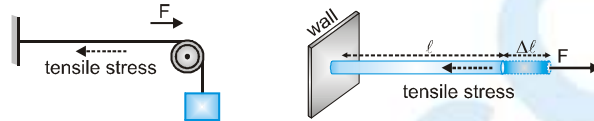
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ELASTICITY

1. $\text{STRESS} = \dfrac{\text{Internal restoring force}}{\text{Area of cross section}} = \dfrac{F_{Res}}{A}$
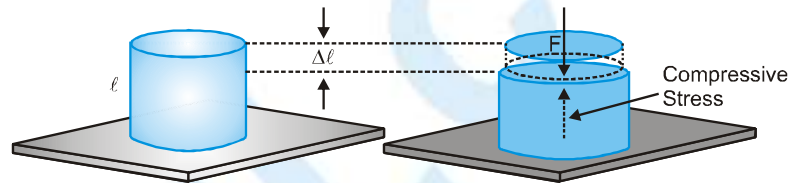
There are three types of stress :

(i) **Longitudinal Stress**

(a) Tensile Stress :

(b) Compressive stress :

(ii) **Volume Stress :**
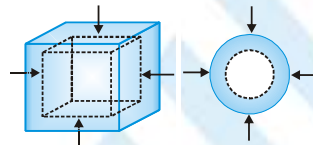
(iii) **Tangential Stress or Shear Stress :**
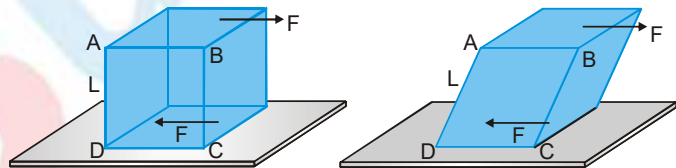
2. **Strain** $= \dfrac{\text{change in size of the body}}{\text{original size of the body}}$

(i) **Longitudinal strain** $= \dfrac{\text{change in length of the body}}{\text{initial length of the body}} = \dfrac{\Delta L}{L}$

(ii) **Volume strain** $= \dfrac{\text{change in volume of the body}}{\text{original volume of the body}} = \dfrac{\Delta V}{V}$

(iii) **Shear strain** $= \tan\phi = \dfrac{l}{L}$ or $\phi = \dfrac{l}{L} = \dfrac{\text{displacement of upper face}}{\text{distance between two faces}}$
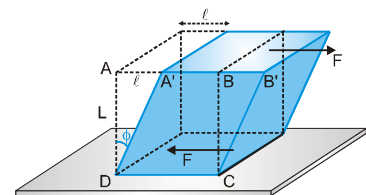
3. **Relation Between angle of twist and Angle of shear**

$AA' = r\theta$ and $Arc\, AA' = \ell\phi$ So $r\theta = \ell\phi \Rightarrow \phi = \dfrac{r\theta}{l}$

where θ = angle of twist, ϕ = angle of shear

4. **Stress – Strain Graph**

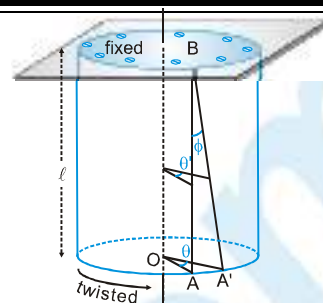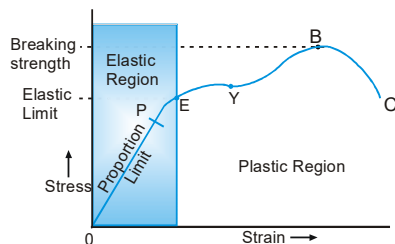**5.** **Hooke's Law** within elastic limit : stress $\propto$ strain

6. **Young's Modulus of Elasticity** $Y = \dfrac{\text{longitudinal stress}}{\text{longitudinal strain}} = \dfrac{Fl}{A\Delta l}$

If L is the length of wire, r is radius and ℓ is the increase in length of the wire by suspending a weight Mg at

its one end then Young's modulus of elasticity of the material of wire $Y = \dfrac{\left(Mg/\pi r^2\right)}{\left(1/L\right)} = \dfrac{MgL}{\pi r^2 l}$

7. **Increment of length due to own weight** $\Delta l = \dfrac{MgL}{2AY} = \dfrac{\rho g L^2}{2Y}$

8. **Bulk modulus of elasticity** $K = \dfrac{\text{volume stress}}{\text{volume strain}} = \dfrac{F/A}{\left(\dfrac{-\Delta V}{V}\right)} = \dfrac{P}{\left(\dfrac{-\Delta V}{V}\right)}$

9. **Bulk modulus of an ideal gas is process dependence**

For isothermal process $PV = $ constant $\Rightarrow PdV + VdP = 0 \Rightarrow P = \dfrac{-dP}{dV/V}$ So bulk modulus = P

For adiabatic process $PV^\gamma = $ constant $\Rightarrow \gamma PV^{\gamma-1}dV + V^\gamma dP = 0 \Rightarrow \gamma PdV + VdP = 0 \Rightarrow \gamma P = \dfrac{-dP}{dV/V}$

So bulk modulus = γP

For any polytropic process $PV^n = $ constant $\Rightarrow nPV^{n-1}dV + V^n dP = 0 \Rightarrow PdV + VdP = 0 \Rightarrow nP = \dfrac{-dP}{dV/V}$

So bulk modulus = nP

10. **Compressibility** $C = \dfrac{1}{\text{Bulk modulus}} = \dfrac{1}{K}$

11. **Modulus of Rigidity** $\eta = \dfrac{\text{shearing stress}}{\text{shearing strain}} = \dfrac{\left(F_{\text{tangential}}\right)/A}{\phi}$

12. **Poisson's Ratio** $\sigma = \dfrac{\text{lateral strain}}{\text{longitudinal strain}}$

13. **Work done in stretching a wire**

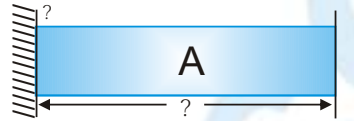$$W = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$$

$$W = \frac{1}{2} \times \frac{F}{A} \times \frac{\Delta l}{l} \times A \times l \times \frac{1}{2} F \times \Delta l$$

14. **Rod is rigidly fixed between walls**

Thermal Strain $= \alpha \Delta \theta$

Thermal stress $= Y \alpha \Delta \theta$

Thermal tension $= Y \alpha A \Delta \theta$

15. **Effect of Temperature on elasticity**

When temperature is increased then due to weakness of inter molecular force the elastic properties in general decreases i.e. elastic constant decreases. Plasticity increases with temperature. For example, at ordinary room temperature, carbon is elastic but at high temperature, carbon becomes plastic. Lead is not much elastic at room temperature but when cooled in liquid nitrogen exhibit highly elastic behaviour.

For a special kind of steel, elastic constants do not vary appreciably temperature. This steel is called 'INVAR steel'.

16. **Effect of Impurity on elasticity**

Y is slightly increase by impurity. The inter molecular attraction force inside wire effectively increase by impurity due to this external force can be easily opposed.

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SURFACE TENSION

Surface tension is basically a property of liquid. The liquid surface behaves like a stretched elastic membrane which has a natural tendency to contract and tends to have a minimum possible area. This property of liquid is called surface tension.

1.   Intermolecular forces

(a)   Cohesive force

The force acting between the molecules of one type of molecules of same substance is called cohesive force.

(b)   Adhesive force

The force acting between different types of molecules or molecules of different substance is called adhesive force.

Intermolecular forces are different from the gravitational forces not obey the inverse-square law.

The distance upto which these forces effective, is called molecular range. This distance is nearly 10^{-9} m. Within this limit this increases very rapidly as the distance decreases.

Molecular range depends on the nature of the substance

2.   Properties of surface tension
   (a) Surface tension is a scalar quantity
   (b) It acts tangential to liquid surface
   (c) Surface tension is always produced due to cohesive force
   (d) More is the cohesive force, more is the surface tension.
   (e) When surface area of liquid is increased, molecules from the interior of the liquid rise to the surface. For this, work is done against the downward cohesive force.

3.   Dependency of Surface Tension

(a)   On Cohesive Force : Those factors which increase the cohesive force between molecules increase the surface tension and those which decrease the cohesive force between molecules decrease the surface tension.

(b)   On Impurities : If the impurity is completely soluble then on mixing in it the liquid, its surface tension increase e.g., on dissolving ionic salts in small quantities in a liquid, its surface tension increases. If the impurity is paratially soluble in a liquid then its surface tension decreases because adhesive force between insoluble impurity molecules and liquid molecules decreases cohesive force effectively. e.g.

   (a) On mixing detergent in water its surface tension decreases.

   (b) Surface tension of water is more than (alcohol + water) mixture.

(c)   On Temperature

On increasing temperature surface tension decreases. At critical temperature and boiling point it becomes zero.

Note : Surface tension of water is maximum at 4 °C

(d)   On Contamination

The dust particles or lubricating materials on the liquid surface decreases its surface tension.

(e) **On Electrification**

The surface tension of a liquid decreases due to electrification because a force starts acting due to it in the outward direction normal to the free surface of liquid

4. **Definition of surface tension**

The force acting per unit length of an imaginary line drawn on the free liquid surface at right angles to the line and in the plane of liquid surface is definedas surface tension.

For floating needle $2Tl \sin\theta = mg$

5. **Required excess force for lift**

- Wire $F_{ex} = 2Tl$
- For ring $F_{ex} = 4\pi rT$
- Square frame $F_{ex} = 8aT$

- Hollow disc $F_{ex} = 2\pi T (r_1 + r_2)$
- Circular disc $F_{ex} = 2\pi rT$
- Square plate $F_{ex} = 4aT$

Work = surface energy $= T\Delta A$

- Liquid drop $W = 4\pi r^2 T$
- Soap bubble $W = 8\pi r^2 T$

Splitting of bigger drop into smaller droples $R = n^{1/3} r$

- Work done = Change in surface energy $= 4\pi R^3 T\left(\dfrac{1}{r} - \dfrac{1}{R}\right) = rR^2 T\left(n^{1/3} - 1\right)$

Excess pressure $P_{ex} = P_{in} - P_{out}$

- In liquid drop $\quad P_{ex} = \dfrac{2T}{R}$

- In soap bubble $\quad P_{ex} = \dfrac{4T}{R}$

6. **ANGLE OF CONTACT (θ_C)**

The angle enclosed between the tangent plane at the liquid surface and the tangent plane at the solid surface at the point of contact inside the liquid is defined as the angle of contact.

The angle of contact depends the nature of the solid and liquid in contact.

Angle of contact $\theta < 90^\circ \Rightarrow$ concave shape, Liquid rise up

Angle of contact $\theta > 90^\circ \Rightarrow$ convex shape, Liquid falls

Angle of contact $\theta = 90^\circ \Rightarrow$ plane shape, Liquid neither rise nor falls

7. **Effect of Temperature on angle of contact**

On increasing temperature surface tension decreases, thus $\cos\theta_c$ increases $\left[Q \ \cos\theta_c \propto \dfrac{1}{T}\right]$ and θ_c decrease.

So on increasing temperature, θ_c decreases.

8. **Effect of Impurities on angle of contact**

(a) Solute impurities increase surface tension, so $\cos\theta_c$ decreases and angle of contact θ_c increases.

(b) Partially solute impurities decrease surface tension, so angle of contact θ_c decreases.

9.   Effect of Water Proofing Agent

Angle of contact increases due to water proofing agent. It gets converted acute to obtuse angle.

Capillary rise : $h = \dfrac{2T\cos\theta}{r\rho g}$

• Zurin's law $h \propto \dfrac{1}{r}$

• Jeager's method $T = \dfrac{rg}{2}(H\rho - hd)$

10.   The height 'h' is measured from the bottom of the meniscuc. However there exist some liquid above this line also.

If correction of this is applied then the formula will be $T = \dfrac{r\rho g\left[h + \dfrac{1}{3}r\right]}{2\cos\theta}$

11.   When two soap bubbles are in contact then radius of curvature of the common surface

$r = \dfrac{r_1 r_2}{r_1 - r_2}(r_1 > r_2)$

12.   When two soap bubbles are combining to form a new bubble then radius of new bubble

$r = \sqrt{r_1^2 + r_2^2}$

13.   Force required to separate two plates $F = \dfrac{2AT}{d}$

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FLUID STATICS

1. **Density** $= \dfrac{\text{mass}}{\text{volume}}$

2. **Specific weight** $= \dfrac{\text{weight}}{\text{volume}} = \rho g$

3. **Relative density** $= \dfrac{\text{density of given liquid}}{\text{density of pure water } 4^\circ C}$

4. **Density of a Mixture of substance in the proportion of mass**

   the density of the mixture is $\rho = \dfrac{M_1 + M_2 + M_3 .....}{\dfrac{M_1}{\rho_1} + \dfrac{M_2}{\rho_2} + \dfrac{M_3}{\rho_3} + ...}$

5. **Density of a mixture of substance in the proportion of volume**

   the density of the mixture is $\rho = \dfrac{\rho_1 V_1 + \rho_2 V_2 + \rho_3 V_3}{V_1 + V_2 + V_3 + ...}$

6. **Pressure** $= \dfrac{\text{normal force}}{\text{area}}$
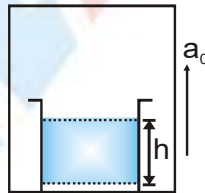
7. **Variation of pressure with depth**

   Pressure is same at two points in the same horizontal level $P_1 = P_2$
   The difference of pressure between two points separated by a depth h
   $$(P_2 - P_1) = h\rho g$$

8. **Pressure in case of acceleration fluid**

(a) **Liquid placed in elevator :** When elevator accelerates upward with acceleration a_0 then pressure in the fluid, at depth h may be given by, $P = h\rho[g + a_0]$
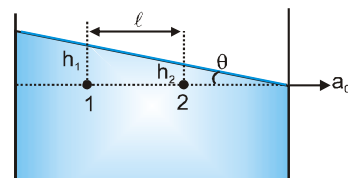

(b) **Free surface of liquid in case of horizontal acceleration :**

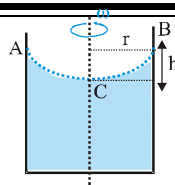 $\tan\theta = \dfrac{ma_0}{mg} = \dfrac{a_0}{g}$

   If P_1 and P_2 are pressure at point 1 & 2 then

   $P_1 - P_2 = \rho g (h_1 - h_2) = \rho g l \tan\theta = \rho l a_0$

(c)   **Free surface of liquid in case of rotating cylinder**

$$h = \frac{v^2}{2g} = \frac{\omega^2 r^2}{2g}$$

9.   **Pascal`s Law**

The pressure in a fluid at rest is same at all the points if gravity is ignored.

A liquid exerts equal pressures in all directions.

If the pressure in an enclosed fluid is changed at a particular point, the charge is transmitted to every point of the fluid and to the walls of the container without being diminished in magnitude. *[for ideal fluids]*

10.   **Types of Pressure**

Pressure is of three types

(a) Atmospheric pressure (P_0)   **(b)** Gauge pressure (P_{gauge})   **(c)** Absolute pressure ($P_{abo.}$)
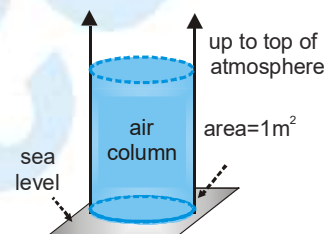
11.   **Atmospheric pressure**

Force exerted by air column on until cross-section area of sea level called atmospheric pressure (P_0)

$$P_o = \frac{F}{A} = 101.3 \text{ kN/m}^2$$

$$P = 1.013 \times 10^5 \text{ N/m}^2$$

Barometer is used to measure atmospheric pressure. Which was discovered by **Torricelli.**
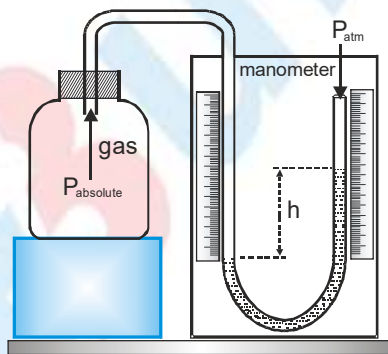
Atmospheric pressure varies from place to place and at a particular plac from time to time.

12.   **Gauge Pressure**

Excess Pressure ($P - P_{atm}$) measured with help of pressure measuring instrument called Gauge pressure.

$$P_{gauge} = h\rho g \text{ or } P_{gauge} \propto h$$

Gauge pressure is always measured with help of **"manometer"**

13.   **Absolute Pressure**

Sum of atmospheric and Gauge pressure is called absolute pressure.

$$P_{abs} = P_{atm} + P_{gauge} \implies P_{abs} = P_o + h\rho g$$

The pressure which we measure in our automobile tyres is gauge pressure.

14.   **Buoyant force** = Weight of displaced fluid = $V\rho g$

15.   **Apparent weight** = weight – Upthrust

16.   **Rotatory – Equilibrium in Floatation :** for rotational equilibrium of floating body the meta-centre must always be higher than the centre of gravity of the body.

17.   **Relative density of body** = $\dfrac{\text{Density of body}}{\text{Density of water}}$

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FLUID DYNAMICS

1. Steady and Unsteady Flow

Steady flow is defined as that type of flow in which the fluid characteristics like velocity, pressure and density at a point do not change with time. In an *unsteady flow*, the velocity, pressure and density at a point in the flow varies with time.

2. Streamline Flow

In steady flow all the particles passing through a given point follow the same path and hence a unique line of flow. This line or path is called a *streamline*.

3. Laminar and Turbulent Flow

Laminar flow is the flow in which the fluid particles move along well–defined streamlines which are straight and parallel.

4. Compressible and Incompressible Flow

In *compressible flow* the density of fluid varies from point to point i.e. the density is not constant for the fluid whereas in *incompressible flow* the density of the fluid remains constant throughout.
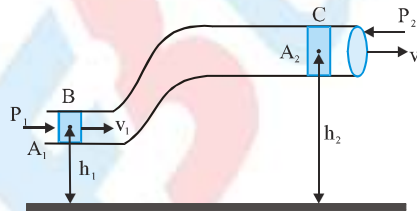
5. Rotational and Irrotational Flow

Rotational flow is the flow in which the fluid particles while flowing along path–lines also rotate about their own axis. In *irrotational flow* particles do not rotate about their axis.

Equation of continuity $A_1 v_1 = A_2 v_2$ Based on conservation of mass

Bernoulli`s theorem : $P + \dfrac{1}{2}\rho v^2 + \rho gh$ = constant

Based on energy conservation

6. Kinetic Energy

Kinetic energy per unit volume = $\dfrac{\text{Kinetic Energy}}{\text{volume}} = \dfrac{1}{2}\dfrac{m}{V}v^2 = \dfrac{1}{2}\rho v^2$

7. Potential Energy

Potential energy per unit volume = $\dfrac{\text{Potential Energy}}{\text{volume}} = \dfrac{m}{V}gh = \rho gh$

8. Pressure Energy

Pressure energy per unit volume = $\dfrac{\text{Pressure energy}}{\text{volume}} = P$
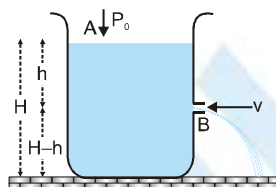
9. **For horizontal flow in venturimeter**

$$P_1 + \frac{1}{2}\rho v^2 = P_2 + \frac{1}{2}\rho v^2 \implies v_1 = A_2 \sqrt{\frac{2gh}{A_1^2 - A_2^2}}$$

10. **Rate of flow**

Volume of water flowing per second $Q = A_1 v_1 = A_1 A_2 \sqrt{\frac{2gh}{A_1^2 - A_2^2}}$

11. **Velocity of efflux** $v = \sqrt{2gh}$

12. **Horizontal range** $R = 2\sqrt{h(H-h)}$

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VISCOSITY THEORY

1. **Newton`s law of viscosity** $F = \eta A \dfrac{\Delta v_x}{\Delta v_y}$

2. **SI UNIT :** $\dfrac{N \times s}{m^2}$ **or deca poise**

3. **CGS UNIT :** dyne-s/cm² or poise (1 decapoise = 10 poise)

4. **Dependency of viscosity of fluids**

(a) **On temperature of fluid**

   (i) Since cohesive forces decrease with increase in temperature as increase in K.E. Therefore with the rise in temperature, the viscosity of liquids decreases.

   (ii) The viscosity of gases is the result of diffusion of gas molecules from one moving layer to other moving layer. Now with increase in temperature, the rate of diffusion increases. So, the viscosity also increases. Thus, the viscosity of gases increases with the rise of temperature.

(b) **On Pressure of fluid**

   (i) The viscosity of liquids increases with the increase of pressure.

   (ii) The viscosity of gases is practically independent of pressure.

(c) **On nature of fluid**

5. **Poiseuillel`s formula** $Q = \dfrac{dV}{dt} = \dfrac{\pi p r^4}{8 \eta L}$

6. **Viscous force** $F_v = 6 \pi \eta r v$

7. **Terminal velocity** $v_T = \dfrac{2}{9} \dfrac{r^2 (\rho - \sigma) g}{\eta} \Rightarrow v_T \propto r^2$

8. **Reynolds number** $R_e = \dfrac{\rho v d}{\eta}$

   $R_e < 1000$ laminar flow, $\qquad R_e > 2000$ turbulent flow