MEASUREMENT ERROR & EXPERIMENT

MEASUREMENT :

Measurement is the process of determination the value of a physical quantity experimentally with the help of special technical means called measuring instruments.

ERROR :

The difference between the true value and the measured value of quantity is called error of measurement.

One basic thing on which every branch of science depends is measurement, there are always many factors which influence the measurement. These factors always introduce error. So no measurement is perfect. We can only minimise the error using best methods and techniques, but we cannot eliminate them permanently.

TYPES OF ERROR :

Error is measurement may arise due to several factor errors are broadly classified into two categories.

(I) Cause of Error
(II) Magnitude of Error

1. Cause of Error :

We have three types of Error :

(i) Systematic Errors
(ii) Random Errors
(iii) Gross Errors

(i) Systematic Errors

Errors whose causes are known are called systematic errors. These errors can be minimised by applying some corrections. These are of various types.

(a) Errors Due to External Factors

These errors are due to fluctuation in atmospheric conditions such as temperature pressure, humidity, etc.

(b) Errors Due to Imperfection

These are introduced due to negligence of facts. For example, error in weighing of a body arising out of buoyancy, which is usually ignored.

(c) Instrumental Errors

These errors are introduced due to improper designing and manufacturing defects of instrument. Often there may be zero error. For example, a meter scale may be worn off at the end of zero mark. Instrumental errors can be reduced by using more accurate instrumentals and applying zero correction, when required.

(d) Personal Errors

These errors are introduced due to the lack of proper care of the observer. For example, lack of proper setting of the apparatus, recording the reading the reading without applying proper precautions, and so on.

(ii) Random Errors

The cause of such errors are not known precisely. Hence, it is not possible to eliminate the random errors. For example, same person repeating the same experiment may get different readings each time. These errors are also known as chance errors. These are minimized by repeating the experiment and taking the arithmetic mean of all the observations. The mean value should be close to the accurate value.

(iii) Gross Errors

These errors arise on the account of shear carelessness of the observer. For example:

1. Reading an instrument without setting it properly.
2. Taking the observations wrongly without caring for the sources of errors.
3. Recording the observations wrongly.
4. Using wrong values of the observations in calculations.

These errors can be minimised only if the observer is sincere and mentally alert.

2. Magnitude of Errors :

(i) Absolute Error
(ii) Relative Error
(iii) Percentage Error

(i) Absolute Error

It is the magnitude of the difference between the true value and the measured value of a physical quantity.
Let $x_1, x_2, x_3, ...x_n$ observation recorded corresponding to a physical quantity x, then the mean value x_m is given by $x_m = \frac{x_1 + x_2 + x_3 + ...x_n}{n}$.

As the correct value of x is not known, x_m is taken to be the absolute value or correct value.
The absolute errors in various observations are given by

$$\Delta x_1 = |x_m - x_1|,\ \Delta x_2 = |x_m - x_2|,\ ....,\ \Delta x_n = |x_m - x_n|$$

The arithmetic mean of these absolute errors is called mean absolute error. Let us donate it by Δx_m.

Then $\overline{\Delta x_m} = \frac{\Delta x_1 + \Delta x_2 + ... + \Delta x_n}{n} = \frac{1}{n}\sum_{i=1}^{n}|\Delta x_i|$

Now the measurement is likely to be between $\Delta x_m - \overline{\Delta x_m}$ and $\Delta x_m + \overline{\Delta x_m}$. Therefore, the final result of the measured physical quantity can be written as $x = xm \pm \overline{\Delta x_m}$.

(ii) Relative Error :

Relative error is defined as the ratio of the mean absolute error and the mean value of the quantity measured.

$$\text{Relative error} = \frac{\text{Mean absolute error}}{\text{Real value (mean value)}} = \frac{\overline{\Delta x_m}}{x_m}$$

(iii) Percentage Error or Functional Error :

When the relative error is expressed in percentage, it is called the percentage error. Thus,

$$\text{Percentage error} = \frac{\overline{\Delta x_m}}{x_m} \times 100$$

Ex. Repeated measurements of a certain quantity in an experiment gave the following values :1.29, 1.33, 1.34, 1.35, 1.32, 1.36, 1.30, and 1.33. Calculate the mean value, mean absolute error, the relative error and the percentage error $\left(\frac{\Delta v}{v} \times 100\right)$

Sol. Here mean value

$$x_m = \frac{1.29 + 1.33 + 1.34 + 1.35 + 1.32 + 1.36 + 1.30 + 1.33}{8}$$

1.3275 = 1.33 (rounded off to two places of decimal)

Absolute errors in measurement are

$\Delta x_1 = |1.33 - 1.29| = 0.04$; $\Delta x_2 = |1.33 - 1.33| = 00.0$;

$\Delta x_3 = |1.33 - 1.34| = 0.01$; $\Delta x_4 = |1.33 - 1.35| = 00.2$;

$\Delta x_5 = |1.33 - 1.32| = 0.01$; $\Delta x_6 = |1.33 - 1.36| = 00.3$;

$\Delta x_7 = |1.33 - 1.30| = 0.03$; $\Delta x_8 = |1.33 - 1.33| = 00.0$;

Mean absolute error :

$$\overline{\Delta x_m} = \frac{0.04 + 0.00 + 0.01 + 0.02 + 0.01 + 0.03 + 0.03 + 0.00}{8}$$

$= 0.0175$

$= 0.02$

(rounded off to two places of decimal)

$$\text{Relative error} = \pm\frac{\overline{\Delta x_m}}{x_m} = \pm\frac{0.02}{1.33} = \pm 0.01503 = \pm 0.02\ \%$$

(rounded off to two places of decimal)

Percentage error $= \pm\ 0.01503 \times 100 = \pm\ 1.503 = \pm\ 1.5\,\%$

COMBINATION OF ERRORS :

When we do calculations using measured values, which themselves contain error, definitely error will be present in the final result. To calculate the net error in the final result, we should know how errors propagate in different mathematical operations.

(i) Error in Summation

Maximum absolute error in the sum of two quantities is equal to the sum of the absolute errors in the calculation of x, i.e., sum of a and b.

Thus, value of the maximum absolute error in x is given by

$$\Delta x = \pm(\Delta a + \Delta b).$$

(ii) Error in Difference

Maximum absolute error in the difference of two quantities is equal to the sum of the absolute errors in the individual quantities. Let x = a – b.

Let Δa = absolute error in measurement of a, Δb = absolute error in measurement of b, and Δx = absolute in calculation of x, i.e. difference of a and b.

Thus, the value of the maximum absolute error in x is given by $\Delta x = \pm(\Delta a + \Delta b)$.

(iii) Error in Product

Maximum fractional error of relative in the product of quantities is equal to the sum of the fractional or relative errors in the individual quantities. Let $x = a \times b$

Let Δa = absolute error in measurement of a , Δb = absolute error in measurement of b, and Δx = absolute error in calculation of x, i.e., product of a and b.

So the maximum possible value of fractional error in the product of quantities is given by

$$\frac{\Delta x}{x} = \pm\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$$

(iv) Error in Division

The maximum value of fractional or relative error in the division of quantities is equal to the sum of the fractional or relative errors in the individual quantities. Let $x = \frac{a}{b}$

Let Δa = absolute error in measurement of a , Δb = absolute error in measurement of b, and Δx = absolute error in calculation of x, i.e., product of a and b.

So the maximum possible value of fractional error in the product of quantities is given by

$$\frac{\Delta x}{x} = \pm\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$$

(v) Error in Power of a Quantity

Fractional error or relative error in a quantity is equal to the sum of fractional or relative error of the individual quantities multiplied by their powers.

As the error multiplies n times, in any formula, the quantity with maximum power should be measured with the highest degree of accuracy, i.e., with least error.

Consider a quantity, $x = \frac{a^m}{b_n}$

Let Δa = absolute error in measurement of a , Δb = absolute error in measurement of b, and Δx = absolute error in calculation of x, then fractional or relative error in x is given by

$$\frac{\Delta x}{x} = \pm\left[m\left(\frac{\Delta a}{a}\right) + n\left(\frac{\Delta b}{b}\right)\right]$$

Ex. The initial and final temperatures of water as recorded by an observer are $(40.6 \pm 0.2)^{o}C$ and $(78.3 \pm 0.3)^{o}C$. Calculate the rise in temperature with proper error limits.

Sol. Here, $\theta_1 = (40.6 \pm 0.2)^{o}C$, $\theta_2 = (78.3 \pm 0.3)^{o}C$

Rise in temperature : $\theta = \theta_2 - \theta_1 = 78.3 - 40.6 = 37.7\ ^{o}C$

Error in $\theta : \Delta\theta = \pm(\Delta\theta_1 + \Delta\theta_2) = \pm(0.2 + 0.3) = \pm 0.5\ ^{o}C$

Hence, rise in temperature $= (37.7 \pm 0.5)\ ^{o}C$.

Ex. The length and breadth of a rectangle are (5.7 ± 0.1) cm and (3.4 ± 0.2) cm. Calculate the area of rectangle with error limits.

Sol. Here, $l = (5.7 \pm 0.1)$ cm, $b = (3.4 \pm 0.2)$ cm

Area : $A = l \times b = 5.7 \pm 3.4 = 19.38\ cm^2 = 19\ cm^2$

(rounding off to two significant figures)

$$\therefore \frac{\Delta A}{A} = \pm\left(\frac{\Delta l}{l} + \frac{\Delta b}{b}\right) = \pm\left(\frac{0.1}{5.7} + \frac{0.2}{3.4}\right)$$

$$= \pm\left(\frac{0.34 + 1.14}{5.7 \times 3.4}\right) = \pm\frac{1.48}{19.38}$$

$$\Rightarrow \Delta A = \pm\frac{1.48}{19.38} \times A$$

$$= \pm\frac{1.48}{19.38} \times 19.38 = \pm 1.48 = \pm 1.5$$

(rounding off to two significant figures)

So, Area $= (19.0 \pm 1.5)\ cm^2$

Ex. The distance covered by a body in time (5.0 ± 0.6) s is (40.0 ± 0.4) m. Calculate the speed of the body . Also determine the percentage in the speed.

Sol. Here, $s = 40.0 \pm 0.4$ m and $t = 5.0 \pm 0.6$ s

$\therefore$ speed $v = \dfrac{s}{t} = \dfrac{40.0}{5.0} = 8.0$ ms–1 As $v = \dfrac{s}{t}$

$$\therefore \frac{\Delta v}{v} = \frac{\Delta s}{s} + \frac{\Delta t}{t}$$

Here $\Delta s = 0.4$ m, $s = 40.0$ m, $\Delta t = 0.6$s, $t = 5.0$ s

$$\therefore \frac{\Delta v}{v} = \frac{0.4}{40.0} + \frac{0.6}{5.0} = 0.13$$

$\therefore \Delta v = 0.13 \times 8.0 = 1.04$

Hence, $v = (8.0 \pm 1.04)\ ms^{-1}$

$$\therefore \text{ Percentage error} = \left(\frac{\Delta v}{v} \times 100\right) = 0.13 \times 100 = 13\%$$

Accuracy and Precision

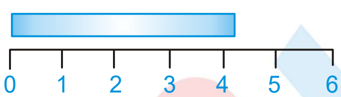
Accuracy tells us how close the measured value is to the true value. Precision indicates the instrument with which the measurement is taken.

LEAST COUNT & SIGNIFICANT FIGURES :

Errors in measurement

To get some overview of error, least count and significant figures, lets consider the example given below. Suppose we have to measure the length of a rod. How can we!

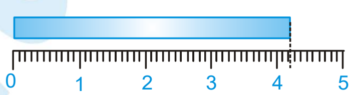
(a) Lets use a cm. scale: (a scale on which only cm. marks are there)



We will measure length = 4 cm.
Although the length will be a bit more than 4 , but we cannot say its length to be 4.1 cm or 4.2 cm., as the scale can measure upto cms only , not closer than that.
* It (this scale) can measure upto cms accuracy only.
* so we'll say that its least count is 1 cm.

To get a closer measurement,
We have to use a more minute scale, that is mm scale

(b) Lets use an mm scale : (a scale on which mm. marks are there)



We will measure length ● = 4.2 cm. , which is a more closer measurement. Here also if we observe closely, we'll find that the length is a bit more than 4.2, but we cannot say its length to be 4.21, or 4.22, or 4.20 as this scale can measure upto 0.1 cms (1 mm) only, not closer than that.

* It (this scale) can measure upto 0.1 cm accuracy
Its least count is 0.1 cm.

Max uncertainty in ● can be = 0.1cm
Max possible error in ● can be = 0.1cm

Measurement of length = 4.2 cm. has two significant figures ; 4 and 2 , in which 4 is absolutely correct, and 2 is reasonably correct (Doubtful) because uncertainty of 0.1 cm is there.

To get more closer measurement
We have to use a more minute scale, that is mm scale

(c) We can use Vernier callipers : (which can measure more closely , upto 0.01 cm)

Then we'll measure length ● = 4.23 cm which is more closer measurement.

* It can measure upto 0.01 cm accuracy

Least count = 0.01 cm

Max uncertainty in l can be = 0.01cm
Max possible error in l can be = 0.01cm

Measurement of length = 4.23 cm. has three significant figures ; 4 , 2 and 3, in which 4 and 2 are absolutely correct ,

and 3 is reasonable correct (Doubtful) because uncertainty of 0.01 cm is there.

To get further more closer measurement :-

(d) We can use Screw Gauge : (which can measure more closely , upto 0.001 cm)

we'll measure length ● = 4.234 cm.
* **Max possible uncertainty (error)** in l can be = 0.001 cm
* length = 4.234 cm. has four **significant figures** ;

4	2	3	4
absolutely correct	absolutely correct	absolutely correct	reasonably correct

To get further more closer measurement

(e) We can Use microscope :

we'll measure length l = 4.2342 cm.
* **Max possible uncertainty (error)** in l can be = 0.0001cm
* length = 4.2342cm. has five **significant figures** ; 4 , 2 , 3 ,4 and 2

LEAST COUNT :

We have studied (from page 1) that no measurement is perfect. Every instrument can measure upto a certain accuracy; called least count.

Least count : Smallest quantity an instument can measure

mm. scale	Vernier	Screw gauge	stop watch	Temp thermometer
L.C. = 1 mm	L.C. =0.1 mm	L.C. = 0.01 mm	L.C. = 0.1 sec	L.C. = 1ºC

PERMISSIBLE ERROR

Error in measurement due to the limitation (least count) of the instrument, is called permissible error.

From mm scale $\rightarrow$ we can measure upto 1 mm accuracy (least count = 1mm). From this we will get measurement like ● = 34 mm

$\downarrow$

Max uncertainty can be 1 mm.

Max permissible error (Δ●) = 1 mm.

But if from any other instrument, we get

● = 34.5 mm then max permissible error (Δ●) = 0.1 mm

and if from a more accurate instrument, we get

● = 34.527 mm then max permissible error (Δ●) = 0.001 mm = place value of last number

Note. : Max permissible error in a measured quantity is = least count of the measuring instrument and if nothing is given about least count then Max permissible error = place value of the last number

Max. Permissible Error in result due to error in each measurable quantity :

Let Result $f(x, y)$ contains two measurable quantity x and y

Let error in x is $= \pm \Delta x$ i.e. $x \in (x - \Delta x, x + \Delta x)$

error in y is $= \pm \Delta y$ i.e. $y \in (y - \Delta y, y + \Delta y)$

Case - I If $f(x, y) = x + y \Rightarrow df = dx + dy$

error in f $= \Delta f = \pm \Delta x \pm \Delta y$

max possible error in $f = (\Delta f)_{max} = \text{max of } (\pm \Delta x \pm \Delta y)$

$(\Delta f)_{max} = \Delta x + \Delta y$

Case - II If $f = x - y$

$df = dx - dy$

$(\Delta f) = \pm \Delta x$ ❍ Δy

max possible error in $f = (\Delta f)_{max} = $ max of ($\pm \Delta x$ ❍ Δy) $\Rightarrow (\Delta f)_{max} = \Delta x + \Delta y$

For getting maximum permissible error , sign should be adjusted, so that errors get added up to give maximum effect

i.e. $f = 2x - 3y - z \Rightarrow (\Delta f)_{max} = 2\Delta x + 3\Delta y + \Delta z$

Ex. In resonance tube exp. we find $●_1 = 25.0$ cm and $●_2 = 75.0$ cm. The least count of the scale used to measure ● is 0.1 cm. If there is no error in frequency What will be max permissible error in speed of sound (take $f_0 = 325$ Hz.)

Sol. $V = 2f_0 (●_2 - ●_1)$

$(dV) = 2f_0 (d●_2 - d●_1)$

$(\Delta V)_{max} = $ max of $[2f_0(\pm \Delta ●_2$ ❍ $\Delta ●_2] = 2f_0 (\Delta ●_2 + \Delta ●_1)$

$\Delta ●_1$ = least count of the scale = 0.1 cm

$\Delta ●_2$ = least count of the scale = 0.1 cm

So max permissible error in speed of sound $(\Delta V)_{max} = 2(325\text{Hz})(0.1 \text{ cm} + 0.1 \text{ cm}) = 1.3$ m/s

Value of $V = 2f_0 (●_2 - ●_1) = 2(325\text{Hz})(75.0 \text{ cm} - 25.0 \text{ cm}) = 325$ m/s

So $V = (325 \pm 1.3)$ m/s

Ex. In resonance tube exp. we find $\ell_1 = 25.0$ cm and $\ell_2 = 75.0$ cm. If there is no error in frequency. What will be max permissible error in speed of sound (take $f_0 = 325$ Hz.)

Sol. $V = 2f_0(\ell_2 - \ell_1)$

$(dV) = 2f_0(d\ell_2 - d\ell_1)$

$(\Delta V)_{max} = \text{max of}[2f_0(\pm\Delta\ell_2 \mp \Delta\ell_2] = 2f_0(\Delta\ell_2 + \Delta\ell_1)$

here no information of least count is given so maximum permissible error in ℓ = place value of last number.

$\ell_1 = 25.0 \text{ cm} \Rightarrow \Delta\ell_1 = 0.1 \text{ cm}$ (place value of last number)

$\ell_1 = 75.0 \text{ cm} \Rightarrow \Delta\ell_2 = 0.1 \text{ cm}$ (place value of last number)

So max permissible error in speed of sound $(\Delta V)_{max} = 2(325\text{Hz})(0.1 \text{ cm} + 0.1 \text{ cm}) = 1.3$ m/s

Value of $V = 2f_0(\ell_2 - \ell_1) = 2(325\text{Hz})(75.0 \text{ cm} - 25.0 \text{ cm}) = 325$ m/s

So $V = (325 \pm 1.3)$ m/s

Case -III If $f(x, y, z) = (\text{constant})\, x^a y^b z^c$

to scatter all the terms, Lets take log on both sides

$\ln f = \ln(\text{constant}) + a\ln x + b\ln y + c\ln z$

$\downarrow$ Differentiating both sides

$$\frac{df}{f} = 0 + a\frac{dx}{x} + b\frac{dy}{y} + c\frac{dz}{z}$$

$$\frac{\Delta f}{f} = \pm a\frac{\Delta x}{x} \pm b\frac{\Delta y}{y} \pm c\frac{\Delta z}{z}$$

$$\left(\frac{\Delta f}{f}\right)_{max} = \text{max of}\left(\pm a\frac{\Delta x}{x} \pm b\frac{\Delta y}{y} \pm c\frac{\Delta z}{z}\right)$$

i.e. $f = 15\, x^2 y^{-3/2} z^{-5}$

$$\frac{df}{f} = 0 + 2\frac{dx}{x} - \frac{3}{2}\frac{dy}{y} - 5\frac{dz}{z}$$

$$\frac{\Delta f}{f} = \pm 2\frac{\Delta x}{x} \mp \frac{3}{2}\frac{\Delta y}{y} \mp 5\frac{\Delta z}{z}$$

$$\left(\frac{\Delta f}{f}\right)_{max} = \text{max of}\left(\pm 2\frac{\Delta x}{x} \mp \frac{3}{2}\frac{\Delta y}{y} \mp 5\frac{\Delta z}{z}\right)$$

$$\left(\frac{\Delta f}{f}\right)_{max} = 2\frac{\Delta x}{x} + \frac{3}{2}\frac{\Delta y}{y} + 5\frac{\Delta z}{z}$$

✪ **sign should be adjusted, so that errors get added up**

Ex. If measured value of resistance $R = 1.05\ \Omega$, wire diameter $d = 0.60$ mm, and length $\ell = 75.3$ cm. If maximum error in resistance measurement is $0.01\ \Omega$ and least count of diameter and length measuring device are 0.01 mm and 0.1 cm respectively, then find max. permissible error in resistivity $\rho = \dfrac{R\left(\dfrac{\pi d^2}{4}\right)}{l}$

Sol. $\left(\frac{\Delta\rho}{\rho}\right)_{max} = \frac{\Delta R}{R} + 2\frac{\Delta d}{d} + \frac{\Delta \lambda}{\lambda}$

$\Delta R = 0.01\ \Omega$

$\Delta d = 0.01$ mm (least count)

Δ● = 0.1 cm (least count)

$$\left(\frac{\Delta\rho}{\rho}\right)_{max} = \left(\frac{0.01\ \Omega}{1.05\ \Omega} + 2\frac{0.01\text{mm}}{0.60\text{mm}} + \frac{0.1\text{cm}}{75.3\text{cm}}\right) \times 100 = 4\ \%.$$

Ex. In ohm's law experiment, potential drop across a resistance was measured as v = 5.0 volt and current was measured as i = 2.0 amp. If least count of the voltmeter and ammeter are 0.01 V and 0.01 A respectively then find the maximum permissible error in resistance.

Sol. $R = \frac{v}{i} = v \times i^{-1}$

$$\left(\frac{\Delta R}{R}\right)_{max} = \frac{\Delta v}{v} + \frac{\Delta i}{i}$$

$\Delta v = 0.1$ volt (least count)

$\Delta i = 0.01$ amp (least count)

$$\%\left(\frac{\Delta R}{R}\right)_{max} = \left(\frac{0.1}{5.0} + \frac{0.01}{2.00}\right) \times 100\ \% = 2.5\ \%$$

value of R from the observation $R = \frac{v}{i} = \frac{5.0}{2.00} = 2.5\ \Omega$

So we can write $R = (2.5 \pm 2.5\%)\ \Omega$

Ex. In Searle's exp to find Young's modulus, the diameter of wire is measured as D = 0.05 cm, length of wire is L = 125 cm, and when a weight, m = 20.0 kg is put, extension in wire was found to be 0.100 cm. Find maximum permissible error in
young's modulus (Y).

Sol. $\frac{mg}{\pi d^2/4} = Y\left(\frac{x}{\lambda}\right) \Rightarrow Y = \frac{mg\lambda}{(\pi/4)\,d^2 x}$

$$\left(\frac{\Delta Y}{Y}\right)_{max} = \frac{\Delta m}{m} + \frac{\Delta \lambda}{\lambda} + 2\frac{\Delta d}{d} + \frac{\Delta x}{x}$$

here no information of least count is given so maximum permissible error in ● = place value of last number.

m = 20.0 kg ⇒ $\Delta m = 0.1$ kg (place value of last number)

● = 125 cm ⇒ Δ● = 1 cm (place value of last number)

d = 0.050 cm ⇒ $\Delta d = 0.001$ cm (place value of last number)

x = 0.100 cm ⇒ $\Delta x = 0.001$ cm (place value of last number)

$$\left(\frac{\Delta Y}{Y}\right)_{max} = \left(\frac{0.1\text{kg}}{20.0\text{kg}} + \frac{1\text{cm}}{125\text{cm}} + \frac{0.001\text{cm}}{0.05\text{cm}} + \frac{0.001\text{cm}}{0.100\text{cm}}\right) \times 100\ \% = 4.3\ \%$$

Ex. To find the value of 'g' using simple pendulum. T = 2.00 sec; ● = 1.00 m was measured. Estimate maximum permissible error in 'g'. Also find the value of 'g'. (use $\pi^2 = 10$)

Sol. $T = 2\pi\sqrt{\frac{\lambda}{g}} \quad \Rightarrow \quad g = \frac{4\pi^2\lambda}{T^2}$

$$\left(\frac{\Delta g}{g}\right)_{max} = \frac{\Delta\lambda}{\lambda} + 2\frac{\Delta T}{T} = \left(\frac{0.01}{1.00} + 2\frac{0.01}{2.00}\right) \times 100\,\%. = 2\,\%$$

$$\text{value of } g = \frac{4\pi^2\lambda}{T^2} = \frac{4 \times 10 \times 1.00}{(2.00)^2} = 10.0 \text{ m/s}^2$$

$$\left(\frac{\Delta g}{g}\right)_{max} = 2/100 \quad \text{so} \quad \frac{\Delta g_{max}}{10.0} = \frac{2}{100} \quad \text{so} \quad (\Delta g)_{max} = 0.2 = \text{max error in 'g'}$$

So 'g' = (10.0 ± 0.2) m/s²

SIGNIFICANT FIGURES

From the above example , we can conclude that ,in a measured quantity,

Significant figures are = Figures which are absolutely correct + The first uncertain figure

Common rules of counting significant figures :

Rule 1 :

All non-zero digits are significant

i.e. 123.56 has five S.F.

Rule 2 :

All zeros occurring between two non-zeros digits are significant (obviously)

i.e. 1230.05 has six S.F.

Rule 3 :

λ = 3.5 cm	λ = 3.50 cm	λ = 3.500 cm
↓	↓ closer!	↓ more closer!
0.1 cm accuracy	0.01 cm accuracy	0.001 cm accuracy
↓	↓	↓
λ lies between(3.4 – 3.6)	λ lies between (3.49 – 3.51)	λ lies between (3.499 – 3.501)

So trailing zeroes after decimal place are significant (Shows the further accuracy)

3.5 cm	3.50 cm	3.500 cm
↓	↓	↓ more closer!
Two S.F	Three S.F	Four S.F

Once a measurement is done, significant figures will be decided according to closeness of measurement. Now if we want to display the measurement in some different units, the S.F. shouldn't change (S.F. depends only on accuracy of measurement)

Number of S.F. is always conserved, change of units cannot change S.F.

Suppose measurement was done using mm scale, and we get ● = 85 mm (Two S. F.)

If we want to display it in other units.

85 mm → 8.5 cm → 0.085 m

85 mm ↓ 85000 mm $= 8.5 \times 10^4$ mm

0.085 m ↓ 0.000085 km $= 8.5 \times 10^{-5}$ km

All should have two S.F.

The following rules support the conservation of S.F.

Rule 4:

From the previous example, we have seen that,

0.000085 km → also should has two S.F.; 8 and 5, So leading Zeros are not significant.

Not significant

In the number less than one, all zeros after decimal point and to the left of first non-zero digit are insignificant (arises only due to change of unit)

0.000305 has three S.F.

⇒ 3.05×10^{-4} has three S.F.

Rule 5 :

From the previous example, we have also seen that

85000 μm → should also has two S.F., 8 and 5. So the trailing zeros are also not significant.

Not significant

The terminal or trailing zeros in a number without a decimal point are not significant. (Also arises only due to change of unit)

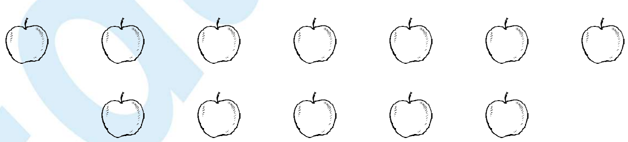
154 m = 15400 cm = 15400 mm

$= 154 \times 10^9$ nm

all has only three S.F. all trailing zeros are insignificant

Rule 6 :

There are certain measurement, which are exact i.e.



Number of apples are = 12 (exactly) = 12.000000........... ∞

This type of measurement is infinitely accurate so, it has ∞ S.F.

* Numbers of students in class = 125 (exact)
* Speed of light in the vacuum = 299,792,458 m/s (exact)

Ex. Count total number of S.F. in 3.0800
Sol. S.F. = Five , as trailing zeros after decimal place are significant.

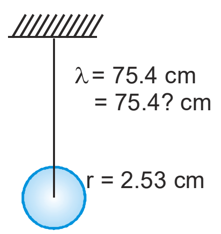
Ex. Count total number of S.F. in 0.00418
Sol. S.F. = Three, as leading zeros are not significant.

Ex. Count total number of S.F. in 3500
Sol. S.F. = Two, the trailing zeros are not significant.
Ex. Count total number of S.F. in 300.00
Sol. S.F. = Five, trailing zeros after decimal point are significant.
Ex. Count total number of S.F. in 5.003020
Sol. S.F. = Seven, the trailing zeros after decimal place are significant.
Ex. Count total number of S.F. in 6.020×10^{23}
Sol. S.F. = Four ; 6, 0, 2, 0 ; remaining 23 zeros are not significant.
Ex. Count total number of S.F. in 1.60×10^{-19}
Sol. S.F. = Three ; 1, 6, 0 ; remaining 19 zeros are not significant.

Operations according to significant figures:

Now lets see how to do arithmetic operations i.e.. addition, subtraction, multiplication and division according to significant figures

(a) Addition $\longleftrightarrow$ subtraction



For this, lets consider the example given below.

In a simple pendulum, length of the thread is measured (from mm scale) as 75.4 cm. and the radius of the bob is measured (from vernier) as 2.53 cm.

Find $\bullet_{eq} = \bullet + r$

$\bullet$ is known upto 0.1 cm(first decimal place) only. We don't know what is at the next decimal place. So we can write $\bullet = 75.4 \text{ cm} = 75.4? \text{ cm}$ and the radius $r = 2.53$ cm .

If we add $\bullet$ and r, we don't know which number will be added with 3. So we have to leave that position.

$$\bullet_{eq} = 75.4? + 2.53 = 77.9? \text{ cm} = 77.9 \text{ cm}$$

Rules for Addition $\longleftrightarrow$ subtraction : (based on the previous example)

* First do the addition/subtraction in normal manner.
* Then round off all quantities to the decimal place of least accurate quantity.

i.e.

423.5 + 20.23 + 10.15

$$\begin{array}{r} 423.5 \\ +\ 20.23 \\ +\ 10.15 \\ \hline 453.88 \end{array} \xrightarrow[\text{decimal place}]{\text{Round off to one}} 453.9$$

486.2 – 35.18

$$\begin{array}{r} 486.2 \\ -\ 35.18 \\ \hline 451.02 \end{array} \xrightarrow[\text{decimal place}]{\text{Round off to one}} 451.0$$

Rules for Multiply ⟵⟶ Division

Suppose we have to multiply 2.11 x 1.2 = 2.11 ? x 1.2 ?

```
      2 . 11 ?
      x 1.2 ?
      ?  ? ? ?
   4 2 2 ? x
 2 1 1 ? x x
 2.5 ? ? ? ?  = 2.5
```

So answer will come in least significant figures out of the two numbers.

(i) Multiply divide in normal manner.

(ii) Round off the answer to the weakest link (number having least S.F.)

312.65 × 26.4 = 8253.960

5 S.F.   3 S.F

$\downarrow$ round off to three S.F.

8250

Ex. A cube has a side ● = 1.2×10^{-2} m. Calculate its volume

Sol. ● = 1.2×10^{-2}

V = ●^{3} = (1.2×10^{-2}) (1.2×10^{-2}) (1.2×10^{-2})

Two S.F. Two S.F. Two S.F.

$= 1.728 \times 10^{-6}\ m^3$

$\downarrow$ Round off to 2 S.F.

$= 1.7 \times 10^{-6}\ m^3$ **Ans.**

Rules of Rounding off

(i) If removable digit is less than 5 (50%) ; drop it.

$$47.833 \xrightarrow[\text{till one decimal place}]{\text{Round off}} 47.8$$

(ii) If removable digit is greater than 5(50%), increase the last digit by 1.

$$47.862 \xrightarrow[\text{till one decimal place}]{\text{Round off}} 47.9$$

(iii) If removable number is exactly 5(50%)

- If last number is even drop 5

  20.65 $\downarrow$ 20.6

- If last number is odd, increase the last digit by 1

  20.75 $\downarrow$ 20.8

Ex. In ohm's law exp., reading of voltmeter across the resistor is 12.5 V and reading of current $i = 0.20$ Amp. Estimate the resistance in correct S.F.

Sol. $R = \dfrac{V}{i} = \dfrac{12.5 \rightarrow 3\text{ SF}}{0.20 \rightarrow 2\text{ SF}} = 62.5\ \Omega \xrightarrow[\text{to 2 S.F.}]{\text{round off}} 62\ \Omega$

Ex. Using screw gauge radius of wire was found to be 2.50 mm. The length of wire found by mm. scale is 50.0 cm. If mass of wire was measured as 25 gm, the density of the wire in correct S.F. will be (use $\pi = 3.14$ exactly)

Sol. $\rho = \dfrac{m}{\pi r^2 \lambda} = \dfrac{\overset{\text{(two S.F.)}}{25}}{\pi \underbrace{(0.250)^2}_{\text{three S.F}} \underbrace{(50.0)}_{\text{three S.F.}}}$

$= \underline{2.5}465 \xrightarrow[\text{S.F.}]{\text{two}} 2.5\ \text{gm/cm}^3$