ELECTROSTATICS

(i) **Electrostatics_Electric charges,**
(ii) **Field**
(iii) **Flux**
(iv) **Potential**
(v) **Capacitance and Van de Graff Gen eration**

IMPORTANT DEFINITIONS

1. **Electrostatics :** The branch of Physics which deals with electric charges at rest is called electrostatics (i.e. static electrocity).

2. **Electric Charge:** electric charge is a physical quantity due to which electrical and other related effects are produced in the matter.

3. **Frictional electricity:** It is the electricity produced due to rubbing of two suitable materials.

4. **Electrostatic Induction:** It is the method of charging in which no pysical contact is required between the charged and uncharged conducting bodies.

5. **Quantisaton of Electric Charge:** It is property of an electric charge which statesthat any charged body will have an integral multiple of the basic charge on an electron 'e' i.e. 1.6×10^{-19} C.

6. **Additivity of Electric Charge:** Net charge on an extended body is the algebraic sum of all charges in the body.

7. **Conservation of Electric Charge:** The algebraic sum of positive and negative charges in an isolated system remains same.

8. **Coulomb's Law of Electrostatic Force (Scalar form of Coulomb's Law):** According to the law, the force between any two point charges, at relative rest, is directly proportional to the product of the magnitude of charges and inversely proportional to the square of the distance of separation between them.
   This force acts along the line joining the two charges (It is repulsive if charges are same and attractive if they are of opposite signs).

9. **Relative permittivity ($\in_r$):** It is defined as the ratio of the Coulomb's force F between two point charges placed in free space to the Coulomb's force F', between the same charges having same distance of separation when placed in a medium i.e. $\varepsilon_r = F / F'$.

10. **Dielectric constant (K):** It is the ratio of capacitance of a parallel plate capacitor with dielectric medium between plates to the capacitance C with air between the plates to the capacitance C with air between the plates i.e.
    $K = C' / C$

11. **Coulomb:** One coulomb is that amount of charge which repels equal and similar charge placed at a distance of 1 m from it in free space with a force of 9×10^9 N.

12. **Principle of Superpositon:** According to the principle of superpositon, total force acting on a given charge due to number of charges around it is vector sum of the individual forces acting on that charge due to all charges.

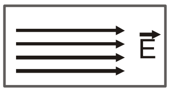
13. **Continuous distribution of charge:** A system of closely spaced electric charges at the macroscopic level is referred to as a continuous charge distribution.

14. Electric field: The space around a charge within which its electrical effect can be felt is called electic field.

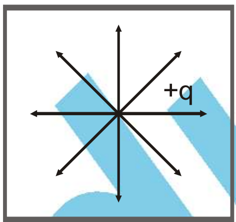
15. Electric field intensity: The electric field intensity at any point in an electric field is defined as the force experienced by a unit positive charge placed at the point.

16. Electric Line: Electric line is defined as the path followed by a unit positive charge when it is free to move in an electric field.

17. Uniform field: A field is uniform if its strenth in the given space is same at every point. A uniform field is represented by straight, equispaced and parallel fieldlines as shown below :



18. Non-uniform field: A field is non-uniform if its strength in the given space is different at different points.



Field of an isolated charge is non-uniform. The field lines are radial and not parallel to one another.

19. Electric Dipole: A pair of two equal and opposite charges separated by some distance is called an electric dipole.

20. Electric dipole moment: Electric dipole moment of an electric dipole is given by the product of the magnitdue of any one of the charges of the dipole and the dipole length

OR

It is equal to the torque acting on a dipole when placed at 90° to a unit uniform electic field.

21. Potential energy of dipole: The work done in rotating the dipole from one position to another is stored in the form of energy called potential energy of the dipole.

22. Work done by Electric field: Work done on a unit positive charge in moving it from one point to another in the electric field is given by the line integral of electric intensity.

23. Electric Potential: Electric potential of a body is a physical quantity which determines the flow of charge from that body to another body.

24. Electric potential at a point in the electric field: It is defined as the work done per unit charge in moving a unit positives test charge from infiniy to that point against the electrostatic force of the field irrespective of the path followed.

25. Electric Potential Difference: between any two points in an electric field is the negative line integral of electric field intensity between these points along any path.

26. Volt: Potential at a point is one volt if one joule of work is done in moving one coulomb of charge from infinity to that point in the electric field.

27. Principle of superposition of potentials: The net potential at any point in the field of a group of charges is given by the algebraic sum of their individual potentials at that very point.

28. Equipotential surface: A surface having same potential at every point due to charged distributon is called equipotential surface.

29. Electric Potential energy: The work done on a charge in bringing it from infinity to a point in an electric field against the electrical force is called electrical potential energy.

30. Electric Flux: Electric flux is defined as the total number of eletric lines of force passing through a surface.

31. Gauss' Theorem in electrostatics: The total electric flux through any closed surface in free space is $1/\epsilon_r$ times the total electric charge enclosed by the surface.

i.e., $\phi = \oint_S \vec{E}.d\vec{S} = \frac{q}{\epsilon_0}$

where q is the total charge enclosed by the surface S and ϵ_0 is the permitivity of free space.

32. Conductors: The materials which easily allow the flow of electric charge through then are called conductors.

33. Insulators: The materials which do not allow the flow of electric charge through them are called insulators.

34. Electrostatic Shielding: The method of protection from the effect of electric field is called electrostatic shielding.

35. Dieletric: The non-conducting material in which charges are easily produced on the application of electric field is called dielectric e.g. Air, H_2 gas, glass, mica, paraffin wax transforme oil etc.

36. Polarization: of a dielectric is the process of getting equal and opposite charges on the two opposite faces of the dielectric on the application of electic field.

37. Dielectric strength: The maximum value of the electric field intenstiy that can be applied to the dielectic material without its electric break down is called the dielectric strength of that material.

38. Electrical Capacitance: The ability of a conductor to store charges is known as electrical capacitance or apacity of a conductor.

39. Farad : It is the S.I. unit of capacitance . Capacitance is should to be 1 farad if 1 coullomb of charge is required to raise the potential through 1 volt.

40. Capacitor: A capacitor has two conductors separated by dilectric medium such that it can store large amount of electric charge in small space.

41. Principle of a parallel plate capacitor: Capacitance of a charged conductor is increased by bringing another uncharged or low potential conductor near it while the two conductors remain separated with some dielectric medium between them.

42. Principle of van de Graaff generator:

(i) Charge remains on the outer surface of a shperical shell.

(ii) Pointed surfaces have larger charge densities.

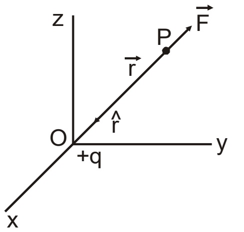
IMPORTANT DERIVATIONS

1. Electric field intenstity at a distance r from a point charge q.

According to *Coulomb's law*, force on a test charge q_0 placed at point P is given by,

$$\vec{F} = \frac{1}{4\pi \in_0} \cdot \frac{qq_0}{r^2} \hat{r} \quad \text{where } \hat{r} \text{ is unit vector.}$$

Let $\vec{E}$ be the electric field intensity field at point P, then



$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi \in_0} \cdot \frac{q}{r^2} \hat{r} = \frac{1}{4\pi \in_0} \cdot \frac{q}{r^3} \vec{r}$$

But $r = |\vec{r}| = |\hat{i}x + \hat{j}y + \hat{k}z| = (x^2 + y^2 + z^2)^{1/2}$

$$\therefore \quad \boxed{\vec{E} = \frac{1}{4\pi \in_0} \cdot \frac{q(\hat{i}x + \hat{j}y + \hat{k}z)}{(x^2 + y^2 + z^2)^{3/2}}}$$

2. Electric field intenstiy at any point on the axial line of a dipole.

Let AX be the axial line of a dipole AB

Electric field due to dipole at any point P on the axial line is given by.

$$\vec{E} = \vec{E}_A + \vec{E}_B$$

But $E_A = \frac{1}{4\pi \in_0} \cdot \frac{q}{(r + a)^2}$

and $E_B = \frac{1}{4\pi \in_0} \cdot \frac{q}{(r - a)^2}$

As $E_B > E_A$ and both act in opposite direction.

so $E = \frac{1}{4\pi \in_0} \cdot \frac{q}{(r - a)^2} - \frac{1}{4\pi \in_0} \cdot \frac{q}{(r + a)^2}$

$= \frac{1}{4\pi \in_0} \cdot \frac{q(4ra)}{(r^2 - a^2)^2}$

But $q \times 2a = p$, the dipole moment

$\therefore \quad E = \frac{1}{4\pi \in_0} \cdot \frac{2pr}{(r^2 - a^2)^2}$ (along PX)

For **short dipole,** $\boxed{\frac{2p}{4\pi \in_0 r^3} \cdot \frac{p}{2\pi \in_0 r^3}}$

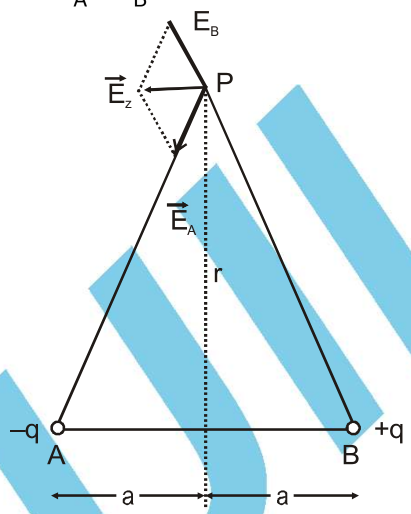
(Result obtained by neglecting a^2 as compared to r^2)

3. Electric field At a point on equatorial line of a dipole

Let a dipole AB have a point P on its equatorial line

Resultant electric field as shown in figure is given by

$$\vec{E} = \vec{E}_A + \vec{E}_B$$



But $E_A = \frac{1}{4\pi \in_0} \cdot \frac{q}{AP^2} = \frac{1}{4\pi \in_0} \cdot \frac{q}{(r^2 + a^2)}$

and $E_B = \frac{1}{4\pi \in_0} \cdot \frac{q}{BP^2} = \frac{1}{4\pi \in_0} \cdot \frac{q}{(r^2 + a^2)}$

By Δ-law of addition of vectors

$$\frac{E}{AB} = \frac{EA}{AP} = \frac{6B}{BP}$$

or $E = \frac{6A}{AP} \times AB$

$= \frac{1}{4\pi \in_0} \frac{q}{(r^2 + a^2)} \cdot \frac{2a}{(r^2 + a^2)^{1/2}}$

or $E = \frac{1}{4\pi \in_0} \frac{q.2a}{(r^2 + a^2)^{3/2}}$

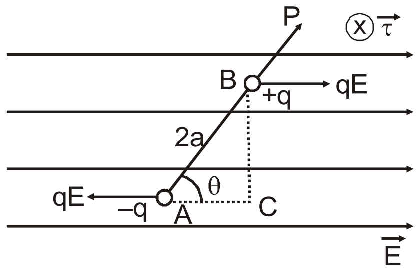
i.e. $\boxed{E = \frac{1}{4\pi \in_0} \frac{p}{(r^2 + a^2)^{3/2}}}$ (along PZ)

where p is dipole moment.

4. Torque on an electric dipole in uniform electric field

Let a dipole AB be placed in uniform electric field E. +q charge will experience a force qE parallel to $\vec{E}$ whereas –q charge experiences a force qE antiparallel to $\vec{E}$. Since these forces are equal and opposite so no net force is experienced.

Forces on A and B constitute couple



$\therefore$ ***Torque*** is given by

$\tau = (qE) \times BC = q\,E\,2a \sin\theta$

$= q \times 2a.E \sin\theta$

$= pE \sin\theta$

where p is the dipole moment

$\Rightarrow \boxed{\vec{\tau} = \vec{P} \times \vec{E}}$

Direction of torque is given by Right handed screw rule.

Direction of torque is $\perp$ to the plane of the paper directed inward.

5. Electric field due to a charged ring

Let a thin ring of radius a be uniformly charged to have a linear charge density as λ. P is any point on the axis of the ring at a distance x from the centre of the ring. Consider a small element of length dl whose distance from point P is r.

Charge on element, $dq = \lambda\, dl$.

then $\quad dE = \dfrac{1}{4\pi \epsilon_0} \dfrac{dq}{r^2} = \dfrac{1}{4\pi \epsilon_0} \dfrac{\lambda dl}{r^2}$

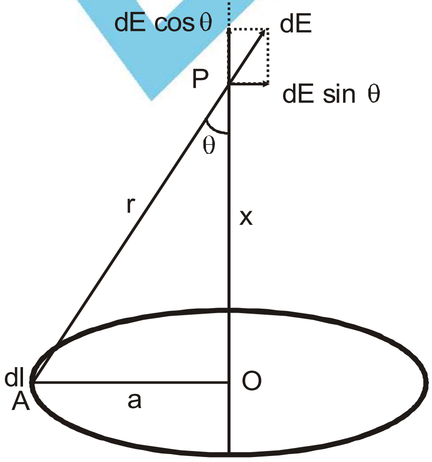
Now, $r^2 = x^2 + a^2$ (from ΔAOP)

$\therefore \quad dE = \dfrac{1}{4\pi \epsilon_0} \dfrac{\lambda dl}{(x^2 + a^2)}$

Resolving dE in two components (i) dE cos θ parallel to axis of the ring (ii) dE sin θ perpendicular to the axis of the ring.

dE cos θ components of all such elements are added up as they are in the same direction and dE sin θ of opposite elements cancel each other as they are equal and opposite.

$\therefore$ Net electric field at P is



$$E = \Sigma\, dE \cos\theta = \int_{\text{whole ring}} dE \cos\theta$$

Using equation (i),

$$E = \int_{\text{whole ring}} \frac{1}{4\pi \epsilon_0} \bullet \frac{\lambda dl}{(x^2 + a^2)} \cos\theta$$

From ΔAOP, $\cos\theta = \dfrac{x}{r} = \dfrac{x}{(x^2 + a^2)^{1/2}}$

$\therefore \quad E =$

$$\int_{\text{whole ring}} \frac{1}{4\pi \epsilon_0} \frac{\lambda dl}{(x^2 + a^2)} \frac{x}{(x^2 + a^2)^{1/2}} = \frac{\lambda}{4\pi \epsilon_0 (x^2 + a^2)} \int_{\text{whole ring}} dl$$

Now $\displaystyle\int_{\text{whole ring}} dl = \text{circumference of ring} = 2\pi a$

By $\lambda\,(2\pi a)$ = total charge of ring = q

$$\therefore \quad \boxed{E = \frac{qx}{4\pi \epsilon_0 (x^2 + a^2)^{3/2}}}$$

Total field intensity E is directed along the axis of the ring.

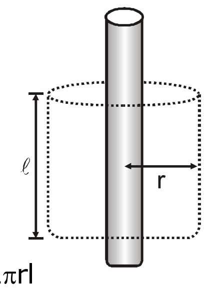
6. Electric field due to a charged cylinder or infinitely long straight conductor.

Consider a cylinder having charge density λ

Electric flux through a Gaussian surface in teh shape of cylinder

of radius r and length *l* is given by

$\phi = E \times$ area of curved surface



i.e. $\phi = E \times 2\pi rl$

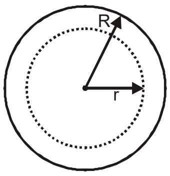
According to Gauss' law, $\phi = \dfrac{q}{\epsilon_0}$

$\therefore \quad E, 2\pi rl = \dfrac{q}{\epsilon_0} = \dfrac{\lambda l}{\epsilon_0}$

i.e. $\boxed{E = \dfrac{\lambda}{2\pi \epsilon_0 r}}$

7. Electric field due to a uniformly charged hollow sphere.

Consider a Gaussian surface at radius r inside the hollow sphere. Here the Gaussian surface will not enclose any charge, therefore from Gauss' theorem

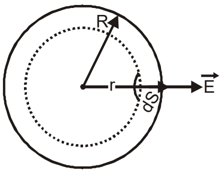


$$E \times (4\pi r^2) = \frac{0}{\epsilon_0} \quad \therefore E = 0$$

Thus, there is no electic field inside a hollow sphere.

Consider a Gaussian surface at radius r outside the hollow sphere having uniform charge. As per Gauss' theorem.

$$\text{flux, } \phi = \int \vec{E}.\vec{ds} = E.4\pi r^2 = \frac{q}{\epsilon_0}$$



i.e. $E = \dfrac{1}{4\pi \epsilon_0} \dfrac{q}{r^2}$

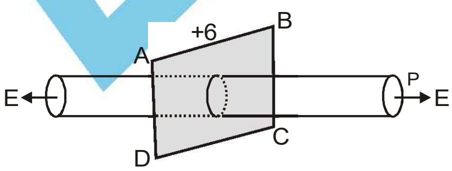
i.e. $\boxed{E = \dfrac{\sigma R^2}{\epsilon_0 r^2}}$

($\because$ Surface charge density $\sigma = \dfrac{q}{4\pi R^2}$)

8. Electric field due to thin sheet of charge

Let ABCD be a plane sheet carrying same charge on both sides. Let σ be the flux density due to charge on the sheet. Select a cylinder of cross-sectional area A through a point P as a Gaussian surface. Magnitude of electric flux crossing through the Gaussian surface is given by

$$\phi = (E \times A) \times 2 = 2\,EA$$



Applying Gauss' theorem,

$$2EA = \frac{q}{\epsilon_0} \qquad \text{But} \qquad q = \sigma A$$

$$\therefore \quad 2EA = \frac{\sigma A}{\epsilon_0} \qquad \text{or} \qquad \boxed{E = \frac{\sigma}{2\,\epsilon_0}}$$

9. Electric field due to a plate of charge

Consider a sheet of charge having charge density σ. E on either side of the sheet, perpendicular to the plane of sheet, has same magnitude at all points equidistant from the sheet.

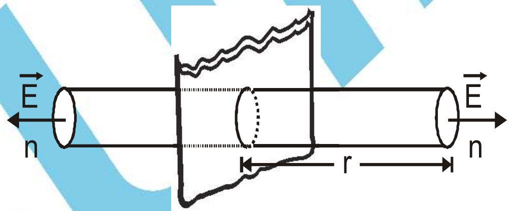
Consider a pill box of length r on each side of the sheet.

$\therefore$ Electric flux over the edges = $2\vec{E}ds\hat{n} = 2Eds$

According to Gauss's laq

$$2E\,ds = \frac{q}{\epsilon_0} = \sigma\,ds\,\epsilon_0$$

i.e. $E = \dfrac{\sigma}{2\,\epsilon_0}$



For a uniform thick plate, electric field intensity at both sides is given by,

$$\text{net intensity} = \frac{\sigma}{2\,\epsilon_0} + \frac{\sigma}{2\,\epsilon_0}$$

i.e. $\boxed{E = \dfrac{\sigma}{\epsilon_0}}$

The electric field tangential to the plate is continuous throughout.

10. Electric potential at a distance r from A point charge.

Let A be the point at a distance r_A from charge q.

Let W be the work done in moving a test charge q_0 from infinity to A, then potential is

given by $\quad V_A = \dfrac{W}{q_0}$

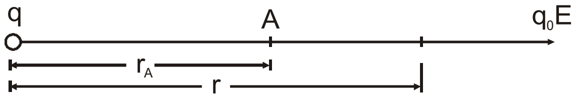
Let the test charge at any instant be at P small work done in moving it through a small distance $d\vec{l}$ is given by

$$dW = \vec{F}.d\vec{l} = (-q_0\vec{E})dl = -q_0 E dl \cos 180^\circ$$

Since distance r decrases in the direction of $d\vec{l}$. Therefore, dl can be taken as – dr.

$\therefore \quad dW = -\,q_0\,E\,dr$

But $E = \dfrac{1}{4\pi\,\epsilon_0} \dfrac{q}{r^2}$



$$\therefore \quad dW = -q_0\left(\frac{1}{4\pi \in_0}\frac{q}{r^2}\right)dr = -\frac{1}{4\pi \in_0}\frac{qq_0}{r^2}dr$$

Work done is moving test charge from ∞ to A is given by

W =

$$\int_{\infty}^{A} dW = \int_{\infty}^{r_A} -\frac{1}{4\pi \in_0}\frac{qq_0}{r^2}dr = -\frac{1}{4\pi \in_0}qq_0\left[\frac{r^{-1}}{-1}\right]$$

$$= \frac{1}{4\pi \in_0}qq_0\left(\frac{1}{r_A}-\frac{1}{\infty}\right)\frac{1}{4\pi \in_0}\frac{qq_0}{r_A}$$

$$V_A = \frac{1}{4\pi \in_0}\frac{q}{r_A}$$

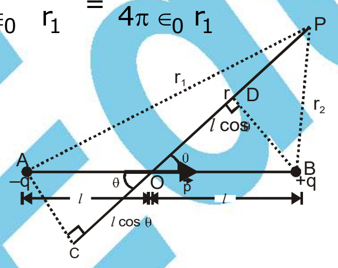
In genral, $\boxed{V = \frac{1}{4\pi \in_0}\frac{q}{r}}$

11. Electric potential at any point due to an elctric dipole-thus-electric potential axial line of a dipole

Consider P to be the point of observation at a distance r from the centre (O) of the electric dipole. Let OP make an angle θ with the dipole moment $\vec{p}$ and r_1, r_2 be the distances of point P from –q charge and +q charge respectively (in figure).

Potential of P due to –q charge,

$$V_1 = \frac{1}{4\pi \in_0}\frac{(-q)}{r_1} = \frac{1}{4\pi \in_0}\frac{q}{r_1}$$



Potential of P due to +q charge,

$$V_2 = \frac{1}{4\pi \in_0}\frac{q}{r_2}$$

$\therefore$ ***Potential at P due to the dipole,***

$V = V_1 + V_2$

(Principle of superposition)

$$\text{or} \quad V = -\frac{1}{4\pi \in_0}\frac{q}{r_1} + \frac{1}{4\pi \in_0}\frac{q}{r_2}$$

$$\text{or} \quad V = \frac{q}{4\pi \in_0}\left[\frac{1}{r_1}-\frac{1}{r_2}\right]$$

Now draw a perpendicular from A which meets the line OP at C when produced backward. Also draw BD $\perp$ on OP>

Then $r_1 = AP \cong CP = OP + OC = r + l\cos\theta$

($\because$ from ΔAOC, $OC = l\cos\theta$)

and $r_2 = BP \cong DP = OP - OD = r - l\cos\theta$

($\because$ from ΔBOD, $OD = l\cos\theta$)

Substituting the values of r_1 and r_2 in equation (i), we get

$$V = \frac{q}{4\pi \in_0}\left[\frac{1}{(r-l\cos\theta)}-\frac{1}{(r+l\cos\theta)}\right]$$

$$= \frac{q}{4\pi \in_0}\left[\frac{r+l\cos\theta-r+l\cos\theta}{(r^2+l^2\cos\theta)}\right]$$

V =

$$\frac{q}{4\pi \in_0}\left(\frac{2l\cos\theta}{r^2-l^2\cos^2\theta}\right) = \frac{q2l\cos\theta}{4\pi \in_0 (r^2-l^2\cos^2\theta)}$$

$$= \frac{p\cos\theta}{4\pi \in_0 (r^2-l^2\cos^2\theta)} \quad ....(ii)$$

($\because$ dipole moment, $p = q \cdot 2l$)

If $r >> l$, then equation (ii) becomes

$$\boxed{V = \frac{p\cos\theta}{4\pi \in_0 r^2}}$$

Since $p\cos\theta = \vec{p}.\hat{r}$, where $\hat{r}$ is unit vector directed along OP.

$$\vec{V} = \frac{\vec{p}\hat{r}}{4\pi \in_0 r^2} \text{ for } r >> 1$$

If point P lies on the axial line of the dipole i.e. $\theta = 0°$

then equaiton (ii) becomes $\boxed{V = \frac{p}{4\pi \in_0 r^2}}$

or $V \propto \frac{1}{r^2}$ ($\because \cos 0° = 1$)

12. Electric potential at equatorial line of a dipole.

Using above result if point P lies on the equatorial line of dipole i.e. $\theta = 90°$,

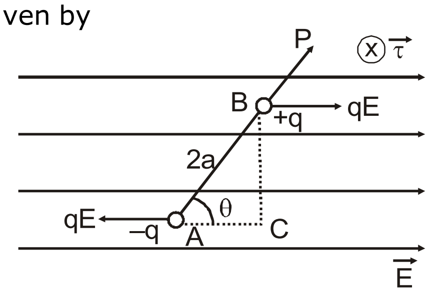
then using equation (ii), we get

$V = 0$ ($\because \cos 90° = 0$)

Potential due to a dipole is zero at a all points on the equatorial line of the dipole.

13. Total work done in rotating a dipole in electric field or potential energy of a dipole in an electric field.

Let the electric dipole having moment p be placed at an angle θ with the direction of electric field E, then torque acting on dipole is given by



$\tau = p\,E \sin\theta$

if dipoole is rotated throug small angle $d\theta$ then small work done is given by

$dW = \tau\, d\theta = -\,pE \sin\theta\, d\theta$

Work done in rotating the dipole to an angle θ from the initial position say perpendicular to the dirction of th electric field is given by, W

$$= \int_{\pi/2}^{\theta} pE\sin\theta\, d\theta = p\,E\,[-\cos\theta]_{\pi/2}^{\theta} = p\,E\left(\cos\frac{\pi}{2} - \cos\theta\right)$$

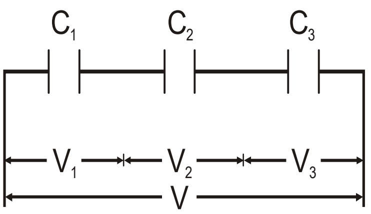
i.e. $\boxed{W = -pE}$]

Note : The initial position of dipole in this expression.

14. Equivalent capacitance of capacitors in series.

Let V_1, V_2 and V_3 be the potential differnece across three capacitors C_1, C_2 and C_3 respectively.

Then total voltage $V = V_1 + V_2 + V_3$



Now $C_1 = \dfrac{q}{V_1}$ or $V_1 = \dfrac{q}{C_1}$

Similarly, $V_2 = \dfrac{q}{C_2}$ and $V_3 = \dfrac{q}{C_3}$ because same charge is induced in all the capacitors w here q is the charge given to first capacitor.

$$V = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3} = q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right]$$

But $V = \dfrac{q}{C}$ where C is equivalent capacitance of the combination

$$\therefore \quad \frac{q}{C} = q = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

$$\text{or} \quad \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

If n capacitors are connected in series then equivalent capacitance of the combination is given by

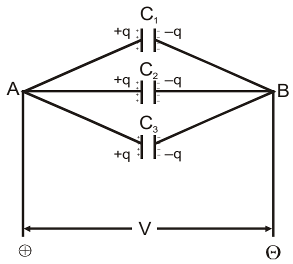
$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots\ldots\ldots\ldots + \frac{1}{C_n}.}$$

15. Equivalent capacitance of capactiors in parallel.

Let three capacitors of capacitances C_1, C_2 and C_3 be connected in parallel as shown in the figure.

All capacitors will have some potential difference across them.

Capactiros C_1, C_2 and C_3 will have differnet amount of charges q_1, q_2 and q_3 respectively. In parallel combination, total charge q is the sum of the charges stored by each capcitor. i.e.



$q = q_1 + q_2 + q_3$

But $q_1 = C_1V$, $q_2 = C_2V$, $q_3 = C_3V$

$\therefore \quad q = C_1V + C_2V + C_3V$

$= (C_1 + C_2 + C_3)\,V$ ...(i)

If C is equivalent capacitance of parallel combination then $q = CV$

Hence, eqn. (i) becomes

$CV = (C_1 + C_2 + C_3)\,V$

or $C = C_1 + C_2 + C_3$

If n capacitors are connected in parallel, the equivalent capacitance of the combination is

$$\boxed{C = C_1 + C_2 + C_3 + \ldots\ldots. + C_n}$$

16. Capacitance of a parallel plate capacitor.

Consider two paralel plates of conducting material separated by distance d.

If a voltage V is applied to the capactior, an electric field E will set up i.e.

$$E = \frac{V}{d} \text{ or } V = Ed.$$

But $E = \frac{\sigma}{\epsilon_0}$ where σ is charge density given

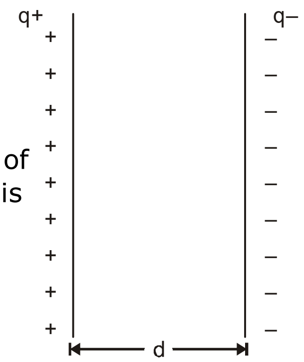
by, $\frac{q}{A}$

$$\therefore \quad V = \frac{\sigma d}{\epsilon_0} = \frac{qd}{A\epsilon_0} \text{ and}$$

Capacitance of a capacitor is given by

$$C = \frac{q}{V} = \frac{q}{(qd / A\,\epsilon_0)}$$

$$= \frac{\epsilon_0 A}{d}\ C = \frac{\epsilon_0 A}{d}$$



If a dielectric medium of dielec tric constant K is kept in between the plates then

$$\boxed{C = \frac{\epsilon_0 KA}{d}}$$

17. Capacitanced with dielectric slab between the plates.

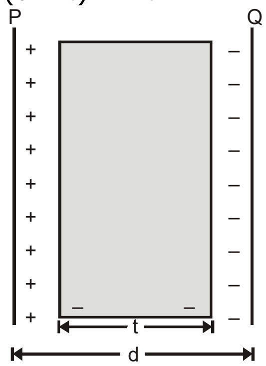
Capacitance of a parallel plate capacitor with air as the medium between two plates of area A lying at a distance d apart is given by,

$$C = \frac{\epsilon_0 A}{d}$$

Let a dielectric slab of thickness t be put between the plates such that t < d. Due to polarisation, electric field will reduce from E_0 to E.

Potential difference across the capacitor is given by

$$V = E_0 (d - t) + Et$$



Dielectric constant, $K = \frac{E_0}{E}$ or $E = \frac{E_0}{K}$

$$\therefore \quad V = E_0 (d - t) + \frac{E_0 t}{K}$$

Now $\quad E = \frac{\sigma}{\epsilon_0} = \frac{q}{A\,\epsilon_0}$

$$\therefore \quad V = \frac{q}{A\,\epsilon_0}\left(d - t + \frac{1}{K}\right)$$

$\therefore$ Capactiance of the arrangement = $C = \frac{q}{V}$

i.e. $\boxed{C = \frac{\epsilon_0 A}{d - t(1 - (1/K))}}$

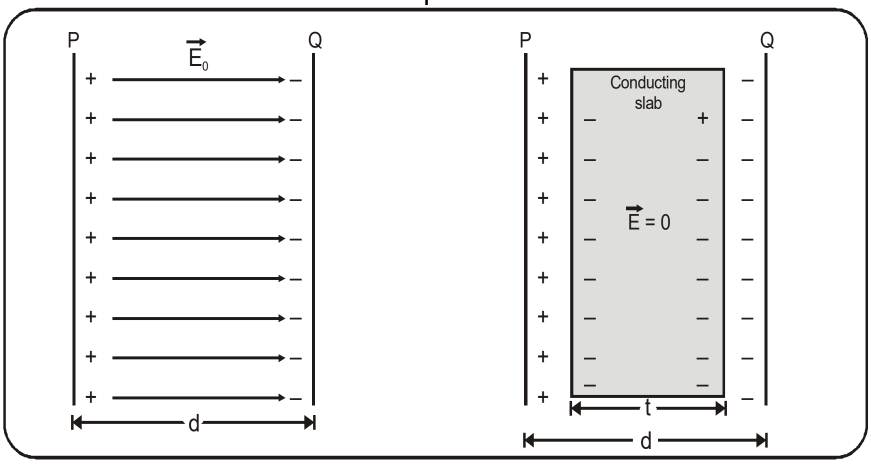
18. Capacitance with condu cting slab between plates.

Capacity of a parallel plate capacitor having vacuum between the plates is given by

$$C_0 = \frac{\epsilon_0 A}{d} \quad ....(1)$$

where A = Area of each plate

d = distance of separation between the plates.



Let a conducting slab of thicknesss 't' be introduced between the plates of the capacitor. Equal and opposite charges appear on the two faces of the conducting slab.

Electric field $(\vec{E})$ inside the conducting slab (i.e. conductor) is zero.

Now the electric field (E_0) exists only in a region of thickness (d – t).

∴ Potential difference between the plates of the capacitor.

$$V = E_0 (d - t)$$

But $E_0 = \dfrac{\sigma}{\epsilon_0}$

where σ is the surface charge density

$$\therefore \quad V = \frac{\sigma(d-t)}{\epsilon_0} \qquad \text{....(2)}$$

If q is the magnitude of the charge on each plate of the capacitor, then

$$\sigma = \frac{q}{A}$$

Hence eqn. (ii) becomes, $V = \dfrac{q(d-t)}{\epsilon_0 A}$ ..(4)

We know, $C = \dfrac{q}{V}$

$$\therefore \quad C = \frac{q\,\epsilon_0 A}{q(d-t)} \text{ or } C = \frac{\epsilon_0 A}{(d-t)}$$

Dividing (iv) by (i), we get

$$\frac{C}{C_0} = \frac{d}{(d-t)} = 1 \Big/ \left(1 - \frac{t}{d}\right)$$

$$\text{or} \quad C = \frac{C_0}{\left(1 - \dfrac{t}{d}\right)} \qquad \text{...(5)}$$

19. Energy stored in a capacitor.

Let a capacitance C be charged to potential V. If q is the charge on the plate then

$$C = \frac{q}{V} \text{ or } V = \frac{q}{C}$$

Small amount of work done by battery in charging the capacitor to small charge dq at constant voltage V is given by

$$dW = V\,dq = \left(\frac{q}{C}\right) dq$$

$$\therefore \quad W = \int_0^q \left(\frac{q}{C}\right) dq = \frac{1}{C}\int_0^q q\,dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^q$$

$$\text{or} \quad W = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}\frac{(CV)^2}{C} = \frac{1}{2}CV^2$$

This work done is stored inside the capacitor as potential energy given by,

$$\boxed{U = \frac{1}{2}CV^2}$$

20. Energy density of a capacitor.

Energy stored in a capacitor, $U = \dfrac{1}{2}CV^2$

But $C = \dfrac{q}{V}$ ∴ $U = \dfrac{1}{2}\dfrac{q^2}{C}$

Again $C = \dfrac{\epsilon_0 A}{d}$

and $q = \sigma A = \epsilon_0 EA$

$$\left[\because E = \frac{\sigma}{\epsilon_0} \text{ or } \sigma = \epsilon_0 E\right]$$

$$\therefore \quad U = \frac{1}{2}\frac{(\epsilon_0 EA)^2}{\epsilon_0 A/d} = \frac{1}{2}\epsilon_0 E^2 Ad$$

But Ad = Volume of the capaction

∴ Energy stored in a capacitor per unit volume of the capacitor

i.e. $\boxed{\textbf{enery density} = \frac{1}{2}\epsilon_0 E^2}$

21. Energy change n two charged capacitors connected in parallel.

Consider two capacitors having capacitance C_1, C_2 charges q_1, q_2 and potentials V_1, V_2 respectively.

Respective charge on the capacitors is given by $q_1 = C_1V_1$ and $q_2 = C_2V_2$.

Therefore, total charge on capacitors (before sharing) = $q_1 + q_2 = C_1V_1 + C_2V_2$.

- **Step 1**

Common Potential

Let these capacitors be connected in parallel by thin metallic wire.

Now the charge will flow from a capacitor having higher potential to the capacitor at lower potential. This flow continues till potential of both the caacitors becomes equal. This equal potential is called common potential (V).

If q_1' and q_2' be the charges on capacitors C_1 and C_2 respectively, after the redistribution of charges, i.e. sharing takes place, then

$q_1' = C_1V$ and $q_2' = C_2V$

$$\therefore \frac{q_1'}{q_2'} = \frac{C_1}{C_2}$$

Now, total charge on the capacitors after connecting them together is given by

$q = q_1' + q_2' = C_1V + C_2V = (C_1 + C_2)V$.

According to the law of conservation of charges,

Total charge after sharing = Total charge before sharing $(C_1 + C_2)V = C_1V_1 + C_2V_2$

$$\text{or} \quad V = \boxed{\frac{C_1V_1 + C_2V_2}{C_1 + C_2}}$$

- **Step 2**

Potential energies

Electrostatic potential energy of two capacitors before sharing

$$U = U_1 + U_2 = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2$$

Electrostatic potential energy of two capacitors after sharing

$$U = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2$$

(After sharing both capacitors attain some potential difference V)

$$= \frac{1}{2}(C_1 + C_2)V^2$$

$$\text{But } V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

$$U = \frac{1}{2}(C_1 + C_2)\left(\frac{C_1V_1 + C_2V_2}{C_1 + C_2}\right)^2 = \frac{1}{2}\frac{(C_1V_1 + C_2V_2)^2}{(C_1 + C_2)}$$

- **Step 3**

Law of energy

On sharing of charges some energy is dissipated in the form of heat.

$$U - U' = \frac{1}{2}C_1V_1^2 + C_2V_2^2 - \frac{(C_1V_1 + C_2V_2)^2}{2(C_1 + C_2)}$$

$$= \frac{1}{2(C_1 + C_2)}\left[(C_1V_1^2 + C_2V_2^2)(C_1 + C_1) - (C_1V_1^2 + C_2V_2^2)\right]$$

$$= \frac{1}{2(C_1 + C_2)}$$

$$[C_1^2V_1^2 + C_1C_2^2V_1^2 + C_1C_2^2V_2^2 + C_2^2V_2^2 - C_1^2V^2 - C_2^2V_2^2 - 2C_1C_2V_1V_2]$$

$$U - U' = \frac{C_1C_2(V_1^2 + V_2^2 - 2V_1V_2)}{2(C_1 + C_2)}$$

$$\text{or} \quad \boxed{U - U' = \frac{C_1C_2(V_1 - V_2)^2}{2(C_1 + C_2)}}$$

SOLVED QUESTION [CBSE]

1. A glass rod when rubbed with silk acquires a charge $+1.6 \times 10^{-12}$C. What is the charge on silk ?

Ans. Silk cloth with acquire equal and opposite charge i.e. -1.6×10^{-12} C.

2. What are the factors on which capacitance of a parallel plate consdenser depends ? Also give the formula ?

Ans. Capacitance depends upon : ϵ_r the relatives perimittivity of the dielectric. A the area of the plate and d the distance between the plates.

Capacitance of a parallel plate capacitor is given by

$$C = \frac{\epsilon_0 \epsilon_r A}{d}$$ with usual notations.

3. If Coulomb's law involved $1/r^3$ dependence (instead of $1/r^2$) would Gauss's law still hold ?

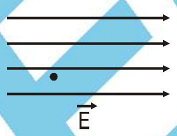
Ans. No, Gauss'law can be obtained from Coulomb's lw only if it has the same $1/r^2$ dependence.

4. What is quantization of electric charge

Ans. Quantization of charge means that the charge on a body is the integral multiple of the smallest elementary quantity of charge (charge on an electron or a proton). $\Rightarrow q = \pm ne$, where n is equal to 1, 2, 3, ...

5. Draw lines of force to represent a uniform electric field.

Ans.



Lines should be parallel and equidistance to represent a uniform electric field.

6. Give the S.I. unit of electric moment of a dipole.

Ans. S.I. unit of dipole moment is coulomb metre i.e. Cm.

7. Define electric potential. Is it a vector or a scalar quantity ?

Ans. Electric potential at a point in the electric field is defined as the work done in moving a unit positive test charge from infinity to that point. It is a scalar quantity.

8. Define the term electric dipole-moment.

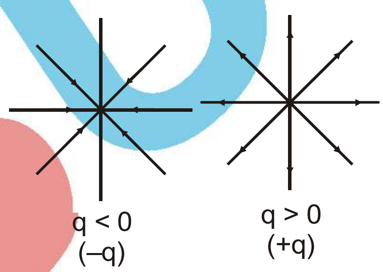
Ans. Electric dipole moment of an electric dipole is the product of the magnitude of either charge and the dipole length.

9. Write down the relation between electric field and electric potential at a point.

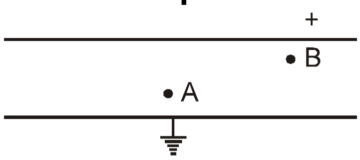
Ans. $E = -dV/dr$ where E is the electric field intensity at a point, V is the potential at that point and r is the distance. dV/dr is known as potential gradient at that point.

10. Sketch the electric lines of force due to point charges (i) $q < 0$ and (ii) $q > 0$.

Ans.



11. In a parallel plate capacitor the potential difference is maintained between the plates. What will be the electric field at point A & B ?



Ans. Electric field due to uniform potential is uniform throughout the plates.

12. Name any two basic properties of electric charge.

Ans. (i) The electric charge is quantized.

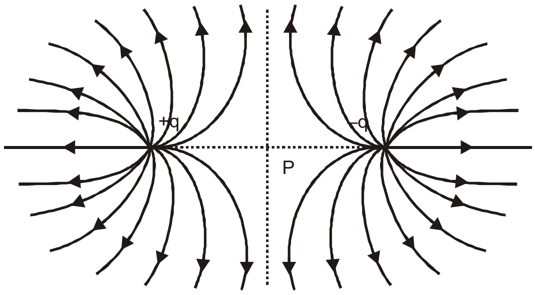
(ii) The electric charge of a system is conserved.

13. Whast is the work done in moving a charge of 10 nC between two points on n equipotential surface ?

Ans. No work is done to move a charge between two points on an equipotential surface.

14. Sketch the lines of force due to two equal positive point charges placed near each other.

Ans.



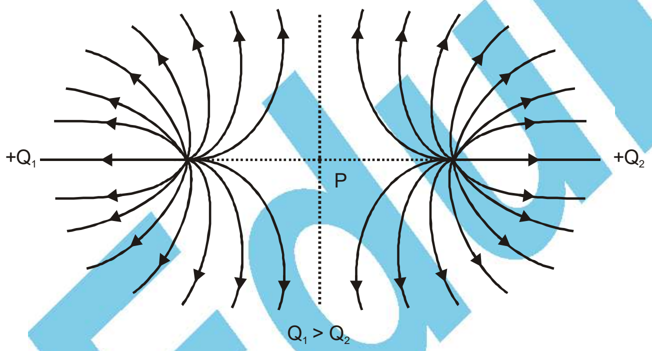
Neutral point P is to be shown in exact middle of the two charges.

15. Force between two point electric charges kept at a distance d apart in air is F. If these charges are kept at the same distance in water, how does the force between them change ?

Ans. If air is replaced by water as a medium then force F will be decreased to F/K, whree K is the dielectric constant of water. Since dielectric constant of water is 80 so the new force will be F/80.

16. Sketch the electric lines of forces for two positive charge Q_1 and Q_2 ($Q_1 > Q_2$) separated by a distance d.

Ans.

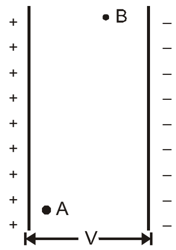


Neutral point P lies closer to the smaller of two charges.

17. Name the physical quantity whose S.I. unit is (i) coulomb/volt (ii) newton/ coulomb (iii) joule/coulomb.

Ans. (i) Capacitance (ii) Electric field intensity (iii) Electric potential.

18. Two protons A and B are placed between two parallel plates having a potential difference V as shown in the figure. Will these protons experience equal or unequal force ?



Ans. Both will experience same force because electric field is uniform throughout the plates.

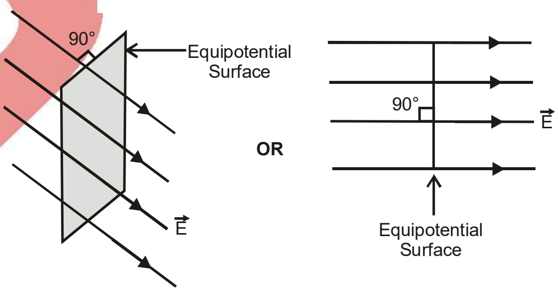
19. How does the force between two point charges if the dielectric constant of the medium in hich they are kept, increases.

Ans. Force between two point charges will decrease if the dielectric constant of the medium increases.

Coulomb's force $F = \frac{1}{K}\frac{q_1 q_2}{4\pi \epsilon_0 r^2}$ where K is the dielectric constant of the medium.

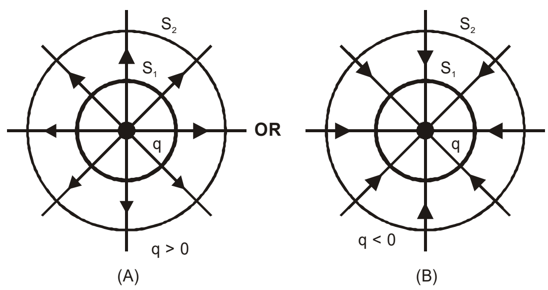
20. Draw an equipotential surface in a uniform electric field.

Ans.



21. Sketch two equipotential surfaces for a point charge

Ans. S_1 and S_2 are two desired surfaces for positive and negative point charges.



22. In an electric field an electron is kept freely. If the electron is replaced by a proton, what will be the relationship between the forces experienced by them ?

Ans. Magnitude of the force will be same.

23. Show that volt and JC^{-1} are the units of the same physical quantity.

Ans. Electric potential (V) = $\frac{\text{Work done (J)}}{\text{Charge (C)}}$.

Clearly unit of V is J C^{-1}

$\therefore$ Volt (V) and joule/coulomb (J C^{-1}) are the units of the same physical quantity i.e. electric potential.

24. Is electric field intensity a scalar or a vector quantity ? Give its S.I. unit.

Ans. Electric intensity is a vector quantity. Its unit in S.I. is newton/coulomb i.e. NC^{-1}.

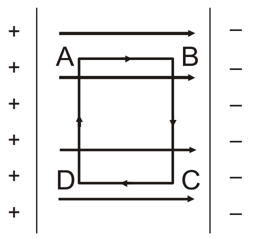
25. Define dielectric constant of a medium in terms of force between electric charges.

Ans. Dielectric constant of a medium is defined as the ratio of the Coulomb's force betwween two point charges placed in air to the Coulomb's force between the same chares placed in the medium at the same distance from each other.

26. What orientation of electric dipole in a uniform electric field corresponds to its stable equilibrium ?

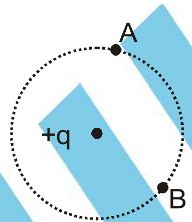
Ans. When angle θ between dipole moment $\vec{p}$ and electric field $\vec{E}$ is zero then potential energy of dipole, U = – pE cos θ = – pE (i.e. minimum) and torque τ = pE sin θ = pE sin 0 = 0; which means that the electric dipole is in stable equilibrium.

27. A uniform electric field E exists between two charged plates as shown in the figure. What would be the work done in moving a charge q along the closed rectangular path ABCDA ?



Ans. Zero, because work done to move a charge in a closed path in a uniform electric field is nil.

28. What would be the work done if a point charge +q is taken from a point A to the point B on the circumference of a circle with another point charge +q at the centre ?



Ans. Zero, because direction of motion is perpendicular to the centripetal electrical force on charges.

29. What does $(q_1 + q_2) = 0$ signify in electrostatics ?

Ans. it signifies an important property of electric charges i.e. the additivity of charges. Here q_1 and q_2 have same magnitudes but opposite signs.

30. A and B are two conducting sphere of the same radii. A being solid and B hollow. Both are charged to same potential. What will be the relation between the charges on the two spheres ?

Ans. Both will have same charges because capacitances in these case depend upon their radii.

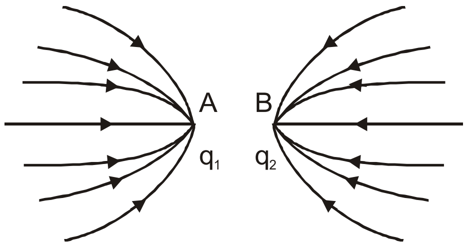
31. Why deos the electric field inside a dielectric decrease when it is placed in as external field ?

Ans. Electric field inside the plates decreases because of polarisation of dielectric which sets up an electric field opposite to the applied field.

32. What is an equipotential surface ?

Ans. Equipotential surface is that surface which has same potential at its every points.

33. Figure shows electric lines of force due to two point charges q_1 and q_2 placed at points A and B respectively. Write the nature of charge on them.



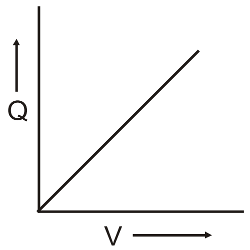
Ans. q_1 and q_2 are of same sign i.e. negative.

34. Two point charges q_1 and q_2 are placed close to each other. What is the nature of force between the charges when $q_1 q_2 < 0$?

Ans. The force is negative i.e. attractive.

35. Sketch a graph to show how the charge Q acquired by a capacitor of capacitance C varies with increase in potential difference between its plates.

Ans.



36. An electric dipole of moment 20×10^{-6} Cm is kept in an enclosed surface. What is the net flux coming out of surface.

Ans. Net flux is zero in this case.

37. Define term dielectric constant of a medium.

Ans. Dielectric constant of a medium is the ratio of permittivity of the medium ($\in$) to the absolute permittiity of free space ($\in_0$) i.e. $K = \in/\in_0$.

38. An electrostatic field line cannot be dicontinuous Why ?

Ans. In case of single charge the electrostatic lines extend from the charge to infinity or infinity to charge.

39. How does the colomb force between two point charges depend upon dielectric constant of the intervening medium ?

Ans. Coulomb's force, $F \propto 1/K$ where K is the dielectric constant of the intervening medium.

40. Two electric field lines never cross each other. Why ?

Ans. Electric field lines of a single charge can only indicate electric field in one direction at one point. Intersection of two lines will result into two directions of the electric field at one point which is not possible.

41. Define the term dielectric constant of a medium in terms of capacitance of a capacitor.

Ans. Dielectric constnt (K) of a medium is given by the ratio of capacitance of a capacitor with the dielectric meidum C_m to the capacitance of same capacitor with air as the medium, C_a i.e. $K = \frac{C_m}{C_a}$.

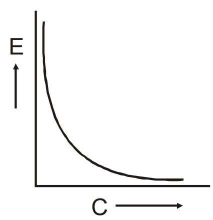
42. Define electric flux. Write its SI unit.

Ans. Electric flux is total number of electric lines passing through a surface. SI unit of electric flux is N m^2 C^{-1}.

43. What is the angle between the direction of electric field at any (i) axial point and (ii) equatorial point due to an electric dipole ?

Ans. 180°, the directions of electric field at any axial point and that at equatorial point are antiparallel to each other.

44. The graph shown here, shows the variation of the total energy (E) stored in a capacitor against the value of the capacitance (C) itself. Which of the two the charge on the capacitor or the potential used to charge it is kept constant for this graph ?



Ans. Charge is ketp constant.

As $E = \frac{1}{2}CV^2$. If V is kept constant then

$E \propto C$ and graph between E and C should be a straight line.

45. (i) Define a coulomb. (ii) Calculate the charge carried by 12.5×10^{18} electrons.

Ans. (i) A coulomb is defined as a charge which repels similar equal charge with a force 9×10^9 N when placed in free space at a distance of 1 m apart.

(ii) Charge q = ne

Here $n = 12.5 \times 10^{18}$ and

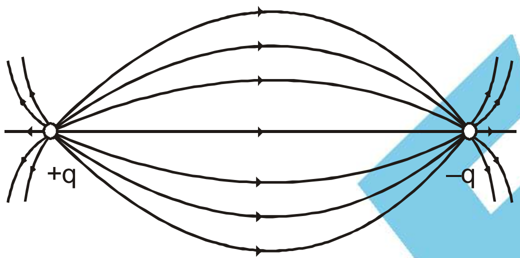
$e = 1.6 \times 10^{-19}$ C Clearly 6.25×10^{18} electrons will be present in 1 coulomb of charge.

46. What is an ideal electric dipole ?

Ans. An ideal dipole is a system of two equal and opposite charges of magnitude very large separated by a negligibly small distance.

47. Draw a diagram to show lines of force of a plane containing two equal point charges of opposite sign separated by a small distance.

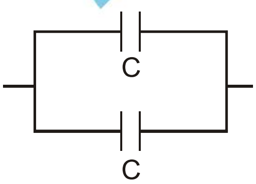
Ans.



48. Define capacitance. Give its unit. Two capacitors of equal capacitance when connected in series have net capacitance C_1 and when connected in parallel have net capacitance C_2. What is the value of C_1/C_2 ?

Ans. Capacitance is defined as the ratio of the electric charge to the electric potential due to this charge.

(Series Combination)



(Parallel Combination)

In SI, unit of capacitance is farad i.e. F.

Let two capacitors of capacitance C each be connected in seris, then net capacitance is given by

$$C_1 = \frac{C \times C}{C + C} = \frac{C^2}{2C} = \frac{C}{2}$$

Again the same capacitors are connected in parallel, the net capacitance is given by

$C_2 = C + C = 2C$

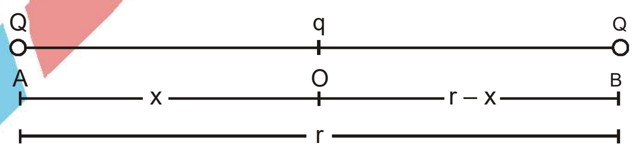
From (i) and (ii),

$$\frac{C_1}{C_2} = \frac{C}{2.2C} = \frac{1}{4}$$

49. Two identical point charges Q are kept at a distance r from each other. A third point charge q is placed on the line joining the above two charges, such that all the three charges are in equilibrium. What is the sign, magnitude and position of the third charge.

Ans. Let the third charge be placed at O such that AO = x

For the system to be in equilibrium, net force on each charge should be zero.



* **Step 1.** If charge q is taken as negative only then forces will act on it in opposite directions. Therefore, the third charge is negative in its sign.

* **Step 2.** Forces on charge at O due to charge at A and charge at B should be equal and opposite i.e.

$$\frac{1}{4\pi \in_0} \frac{Qq}{x^2} = \frac{1}{4\pi \in_0} \frac{Qq}{(r-x)^2}$$

i.e. $x^2 = (r - x)^2 \Rightarrow x = \pm (r - x) \Rightarrow x = \frac{r}{2}$

* **Step 3.** Again forces on charge at A due to charge at O and charge at B should be equal and opposite i.e.,

$$\frac{1}{4\pi \in_0} \frac{Qq}{x^2} = \frac{1}{4\pi \in_0} \frac{Qq}{r^2} \text{ i.e. } \frac{q}{x^2} = \frac{Q}{r^2}$$

But $x = \frac{r}{2}$ $\quad \therefore \frac{a}{(r/2)^2} = \frac{Q}{r^2}$ i.e. q

$= \frac{Q}{4}$

50. Define intensity of electric field at a point. At what points is the electric dipole field intesnsity parallel to the line joining the charges ?

Ans. Intensity of electric field at a point is defined as the electrostatic force per unit positive charges acting on a vanishing positive test charge placed at that point i.e.

$$\vec{E} = \underset{q \to 0}{Lt} \frac{\vec{F}}{q_0}$$

Electric dipole field intensity is parallel to the line joining the charges when consider at points lying on the equatorial line of the charges i.e. perpendicular bisector to the axial line.

51. Two point electric charges of unknown magnitude and sign are placed at a distance d apart. The electric field intensity is zero at a point, not between te charges but on the line joining them. Write two essential conditions for this to happen.

Ans. (i) Both charges are not of same sign.

(ii) The point where electric field intensity is zero on the axial line (not between the charges) lie nearer to the smaller of the two charges.

52. An electric dipole is held in uniform electric field.

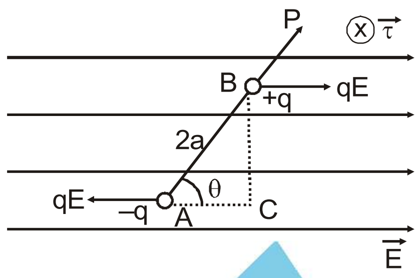
(i) Show that no translatory force acts on it.

(ii) Derive an expression for the torque acting on it.

Ans. Let a dipole AB be placed in a uniform electric field E.

(i) +q charge will experience a force qE parallel to $\vec{E}$ whereas –q charge experiences a force qE antiparallel to $\vec{E}$. Since these forces are equal and opposite so no net force is experienced.

(ii) Forces on A and B constitute couple.



$\therefore$ Torque is given by

$\tau = (qE) \times BC = q\,E\,2a \sin\theta$

$= q \times 2a\,E \sin\theta = pE\sin\theta$, when p is the dipole moment.

$\Rightarrow \vec{\tau} = \vec{P} \times \vec{E}$

Direction of torque is given by Right handed screw rule.

Direction of torque is $\perp$ to the plane of the paper directed inward.

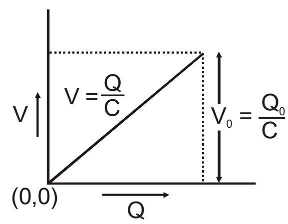
53. Defined the term 'electric field intensity'. Electric field inside a conductor is zero. Explain.

Ans. The electric field intensity at any point in the space around a charged body is defined as the force per unit charge exerted on any charge placed at that point.

Electric field intensity inside a conductor is zero. When a conductor is placed in electrostatic field $(\vec{E}_0)$, free electrons move and collect towards positive of the applied field in the surface of the conductor itself. This redistribution of charges will create its own additional field $\vec{E}_i$ known as induced electric field which cancels the applied field $\vec{E}_0$ inside the conductor. Therefore net field inside a conductor is zero.

54. Sketch a graph to show how charge Q given to a capacitor of capacity C varies with the potential difference V.

Ans.

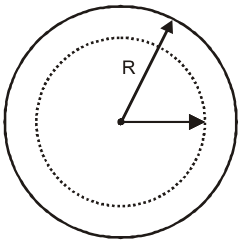


55. State Gauss' thorem. Using Gauss' theorem, derive an expression for electric field intensity at any point inside a hollow charge conducting sphere.

Ans. Gauss' theorem states that the total electric flux through a closed surface enclosing a charge is $1/\in_0$ times the charge enclosed by the surface i.e.

$$\phi = \int_S \vec{E}.d\vec{s} = \frac{q}{\in_0}$$

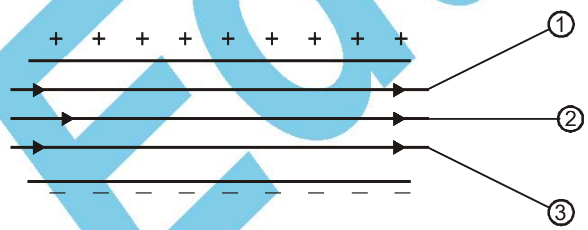
Consider a Gaussian surface at radius r inside the hollow sphere of radius R. hEre the Gaussian surface will not enclose any charge, therefore from Gauss' therorem.



$$E \times (4\pi r^2) = \frac{0}{\in_0} \quad \therefore E = 0$$

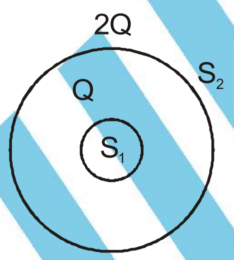
Thus, there is no electric field inside a hollow sphere.

56. Figure beow shown tracks of three charged particles 1, 2 and 3 in a uniform electric field. Give the signs of the three charges which particle has highest charge to mass ratio ?



Ans. Unlike charges attract each other, therefore particle 1 and 2 are negatively charged whereas particle 3 has positive charge. Particle 3 gets maximum deflection so it has highest charge (e) to mass (m) ratio because deflection, $y \propto e/m$.

57. A sphere S_1 of radius r_1 encloses a total charge Q. If there is another concentric shpere S_2 of radius r_2 ($r_2 > r_1$) enclosing charge 2Q, find the ratio of the electric flux through S_1 and S_2. How will the electric flux through sphere S_1 change, if a medium of dielectric constant 5 is introduced in the space inside S_2 inplace of air ?



Ans. As per Gauss' theorem, electric flux

$$\phi = \frac{q}{\in_0}$$

$$\therefore \frac{\phi_1}{\phi_2} = \frac{q_1/\in_0}{q_2/\in_0} = \frac{q_1}{q_2}$$

Here $q_1 = Q$ and $q_2 = Q + 2Q = 3Q$

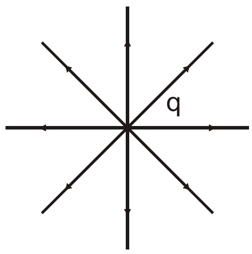
$$\therefore \frac{\phi_1}{\phi_2} = \frac{Q}{3Q} \text{ i.e. } \frac{\phi_1}{\phi_2} = \frac{1}{3}$$

With dielectric, flux $\phi_1' = \frac{\phi_1}{K} = \frac{\phi_1}{5}$.

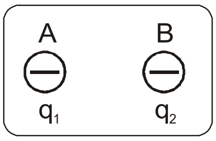
58. Give two properties of electric lines of force. Sketch them for an isolated positive point charge.

Ans. (i) The lines of force start from a positive charge and end at negative charge.

(ii) Thel lines of force do not intersect.

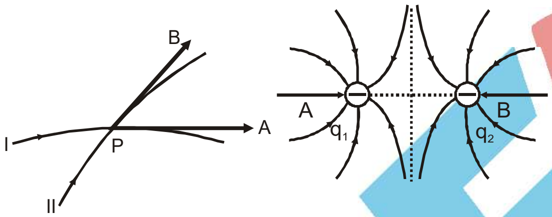


59. Two electric lines of force cannot intersect each other. Why ?



Sketch the lines of force for two equation negatie charges q_1 and q_2 placed at points A and B, separated by a small distance in air.

Ans. Let two eletric lines of force I and II intersect at point P. Clearly there wil be two tangents at this point say PA and PB. Two different tangents at a point will mean two different directions of electric field at the same point which is not possible. So, two electric lines of force cannot intersect.



The sketch of lines of force for two equal negative charges q_1 and q_2 placed at A and B separated by a small distance in air is shown in the figure.

60. (a) Why does the electric field inside a dielectric decreases when it is placed in an external eletric field ?

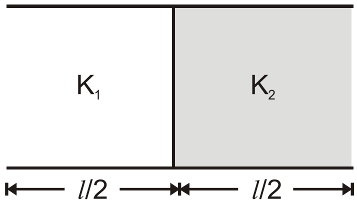
(b) A parallel plate capacitor with air between the plates has a capacitance of 8 pF. What wil be the capacitance if the distance between plates be reduced by half and space between then is filled with a substance of dieletric constant K = 6.

Ans. (a) Due to polarisation of the dielectric an opposite field is set up in it which reduce the net field.

(b) Here $\dfrac{C}{C_0} = \dfrac{K \in_0 A/(d/2)}{\in_0 A/d} = 2K$

i.e. $C = 2KC_0 = 2 \times 6 \times 8 = 96$ pF.

61. Two dielectric slabs of dielectric constants K_1 and K_2 are filled in between two plates, each of area A, of a parlallel plate capacitor. Find net capacitance.



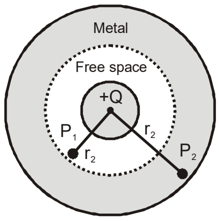
Ans. Here total capacitance is equivalent to resultant capacitance of parallel combination.

$$C = C_1 + C_2 = \frac{K_1 \in_0 A}{l/2} + \frac{K_2 \in_0 A}{l/2}$$

$$= \frac{2 \in_0 A}{l}(K_1 + K_2)$$

62. A small metal sphere carrying charge +Q is located at the centre of a spherical cavity ina large uncharged metal sphere as shown in figure. use Gauss' theorem to find electric field at point P_1 and P_2.

Ans. As per Gauss's thorem, flux $\phi = \dfrac{Q}{\in_0}$



Assuming a Gaussian spherical shell of radius r_1 at P_1.

we get $\oint \vec{E}_1 . d\vec{s}_1 = \dfrac{Q}{\in_0}$

i.e. $E_1 \times 4\pi r_1^2 = \dfrac{Q}{\in_0}$

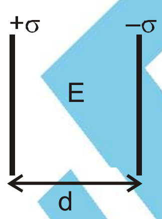
i.e. $E_1 = \dfrac{Q}{4\pi \in_0 r_1^2}$

Similarly assuming a Gaussian spherical shell at P_2, we get $E_2 = 0$ because charge inside a condcutor is zero (whole charge resides at the outer surface)

63. Explain te underlying principle of a parallel plate capacitor. If two similar plates, each of area A having surface charge densities + σ and – σ are separated by a distnce d in air, write expression for
(i) the electric field at points between the two plates
(ii) the potential differeneces between the plates
(iii) the capacitance of the capacitor so formed.

Ans. To increase the capacitance of a condcuting plate, an uncharged grounded conducting plate is brought near it. An opposite charge is induced on the grounded plate which reduces the potential of the first plate without bringing any chagne in its potential. This potential difference allows the conducting plates to have more and more charge.

(i) $E = \dfrac{\sigma}{\epsilon_0}$



(ii) $| E | = \dfrac{dV}{dr} = \dfrac{V}{d}$

i.e. $V = Ed = \dfrac{\sigma d}{\epsilon_0}$

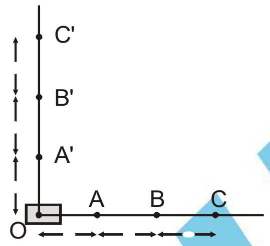
(iii) $C = \dfrac{\epsilon_0 A}{d}$

64. A parallel plate capacitor, each with plate area A and separation d is charged to a potential diference V. The battery used to charg it is then disconnected. A dielectric slab of thickness d and dielectric constant K is now placed between the plates. What change, if any, will take place in (i) charge on plates (ii) electric field intensity between plates
(iii) capacitance.

Ans. (i) Since battery is disconnected, the charge remains same.
(ii) Electrric field will polarise the dielecric i.e. an opposite field develops in it so net electric field decreases.
(iii) Capacitance increases K times as

$C_{new} = \dfrac{K \epsilon_0 A}{d}$

65. The following data was obtained for the depedence of the magnitdue of electric field, with distance, from a reference point O, within the charge distribution in the shaded region.



Field Point	A	B	C	A'	B'	C'
Magnitude of Electric Field	E	E / 8	E / 27	E / 2	E / 16	E / 54

(i) Identify the charge distribution and justify your answer.
(ii) If the potential due to this charge distribution, has a value V at the point A, what is its value at the point A ?

Ans. (i) Given charge distribution is identified as a small electric dipole having centre a O. Points A, B and C are lying on the axial line of this dipole.
Points A′, B′ and C′ lie on the equatorial line of this dipole.
i.e. If E is the electric field at point A distnce l from O then electric field at B distance $2l$ from O has to be $\dfrac{E}{2^3}$ i.e. $\dfrac{E}{8}$ and so on (because electric field $\propto \dfrac{1}{(\text{distnce})^3}$).
Moreover if E is the field at A then at A′, the field will be $\dfrac{E}{2}$ (and so on because electric field, $E_{equatorial} = \dfrac{E_{axial}}{2}$).
(ii) Potential at any point on the equatorial line of a dipole is zero so at A′, value of potential is zero.

66. A charge Q located at a point $\vec{r}$ is in equilibrium under the combined electric field of three charges q_1, q_2, q_3. If the charges q_1, q_2 are located at point $\vec{r}_1$ and $\vec{r}_2$ respectively, find the direction of the force on Q, due to q_3 in terms of q_1 q_2, $\vec{r}_2$, $\vec{r}_2$ and $\vec{r}$.

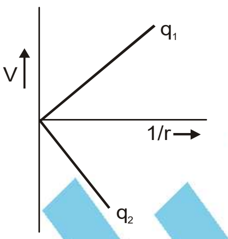
Ans. For equilibrium, $\vec{F}_{q_1Q} + \vec{F}_{q_2Q} + \vec{F}_{q_3Q} = 0$

i.e. $\cdot\vec{F}_{q_3Q} = -\left[\vec{F}_{q_1Q} + \vec{F}_{q_2Q}\right]$

$$= -\frac{Q}{4\pi \in_0}\left[\frac{q_1(\vec{r}-\vec{r}_1)}{|(\vec{r}-\vec{r}_1)|^3} + \frac{q_2(\vec{r}-\vec{r}_2)}{|(\vec{r}-\vec{r}_2)|^3}\right]$$

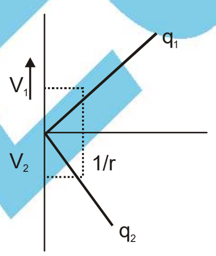
which gives the magnitude and direction of the force.

67. The two graphs drawn below, show the variation of electrostatic potential (V) with $\frac{1}{r}$ (r being distance of the field point from the point charge) for two point charges q_1 and q_2.



(i) What are the signs of the two charges ?
(ii) Which of the two charges has a larger magnitude and why ?

Ans. $V = \frac{kq}{r}$.

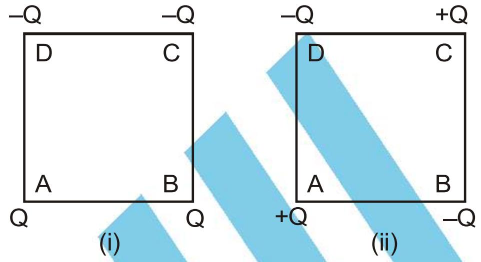


(i) q_1 is positive and q_2 is negative.
(ii) q_2 is of larger magnitude

$\because$ for same $\frac{1}{r}$ we get $V \propto q$.

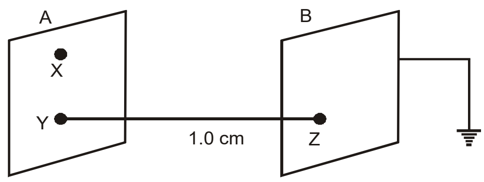
Then V_2 due to q_2 is larger than V_1 due to q_1.

68. Four point charges are placed at four corners of a square in the two ways (i) and (ii) as shown below. Will the (a) electric field (b) electric potential, at the centre of the square, be the same or different in the two configuration and why ?



Ans. (a) Electric field at centre of figure (ii) will be zero because same charges on the diagonally opposite corners of a sqaure give zero electric field at the centre whereas it will be nonzero in figure (i). (b) Electric potential will be same i.e. zero in the case of figure (i) and (ii) becuase there are two plus and two minus charges of same magnitude equidistant from centres in both figrues.

69. Two identical plane metallic surfaces A and B are ketp parallel to each other in air separated by a distnce of 1.0 cm as show nin the figure.



Surface A is given a positive potential of 10 V an outer surface of B is earthed. (i) Wat is the magnitude and direction of the uniform electric field between points Y and Z ? (ii) What is the work done in moving a charge of 20 mC from point X and point Y ?

Ans. (i) $E = \frac{V}{r} = \frac{10}{1\times10^{-2}} = 10^3$ V m^{-1}

Direction of E is from Y to Z
(ii) No work is done because plate A is an euipotential surface.
$\because$ W = q ΔV = q × 0 = 0.

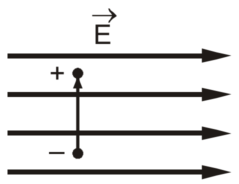
70. An electric dipole of dipole moment $\vec{p}$ is placed in a uniform electric field $\vec{E}$. Write the expression for the torque $\vec{\tau}$ experienced by the dipole. Identify two pairs of perpendicular vectors in the expression. Show diagrammatically the orientation of te dipole in the field for which the torque is (i) Maximum (ii) Half the maximum value (iii) Zero.

Ans. Torque $\vec{\tau} = \vec{p} \times \vec{E}$ i.e. $\tau = pE \sin\theta$

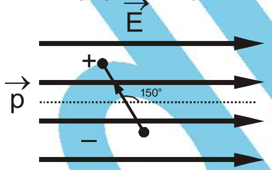
Vectors $\vec{\tau}$ is normal to vector $\vec{p}$ as well as $\vec{E}$ i.e. $\vec{\tau} \perp \vec{p}$ as well as $\vec{\tau} \perp \vec{E}$

(i) Maximum torque occurs when $\vec{p}$ is perpendicular to $\vec{E}$.

($\because \tau = pE \sin 90° = pE$)

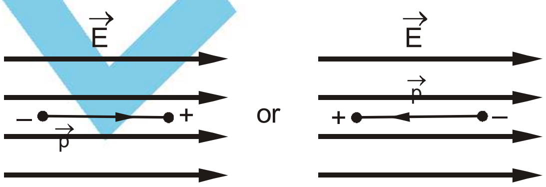


(ii) Half the maximum value of torque occurs when angle θ between $\vec{p}$ and $\vec{E}$ is 30° (or 150°) ($\because \tau = pE \sin 30° = \frac{pE}{2}$)



(iii) Zero torque ocurs when θ is 0° or 180°.

($\because \tau = pE \sin 0° = 0 = pE \sin 180°$)



71. If a capcitor is disconnected from the battery, what will be the energy stored in the capacitor when (i) separation between plates is doubled and (ii) an uncharged and identical capacitor is connected across it.

Ans. (i) $U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$ and $C = \frac{\epsilon_0 A}{d}$,

Here d is made twice, therefore, C will become half but Q will remain same.

$\therefore U = \frac{1}{2}\frac{Q^2}{C/2} = \frac{Q^2}{C}$ i.e. total energy tends to increase.

(ii) When an uncharged capacitor is connected across the capacitor, capacitance of the combination becomes C + C = 2C and the charge is equally distributed i.e. $\frac{q}{2}$ on each capacitor.

$\therefore$ Energy stored in the capacitor = $\frac{1}{2} \times \left(\frac{q}{2}\right)^2 \times \frac{1}{C} = \frac{1}{2}\frac{1}{4}\cdot\frac{q^2}{C}$ i.e. $\frac{1}{4}$th of original.

72. Show mathematically that electri field intenstiy due to a short dipole at a distance of along its axis is twice the intensity at the same distance along the equatorial axis.

(i) An electric dipole is held in a uniform electric field. What happens to dipole, when released in uniform electric field

(ii) The dipole is aligned parallel to the field. Calculate the work done in rotating it through 180°.

Ans. Electric field at a point on the axial line is

$$E_a = \frac{1}{4\pi\epsilon_0}\frac{2pr}{(r^2 - a^2)^2}$$

Electric field at a point on the equatorial line is

$$E_e = \frac{1}{4\pi\epsilon_0}\frac{2pr}{(r^2 + a^2)^{3/2}}$$

For a short dipole a is negligible as compared to r.

$\therefore$ Eq. (i) and (ii) reduce to

$$E_a = \frac{1}{4\pi\epsilon_0}\frac{2p}{r^3} \quad \text{....(iii)}$$

and $$E_e = \frac{1}{4\pi\epsilon_0}\frac{p}{r^3} \quad \text{...(iv)}$$

Comparing (iii) and (iv)

$E_a = 2E_e$ **(proved)**

(i) When a dipole is released in uniform electric field, it shall align in the direction of the electric field.

(ii) Work done in rotating a dipole is given by

$W = p E (\cos \theta_1 - \cos \theta_2)$

Hence $\theta_1 = 0°$ and $\theta_2 = 180°$

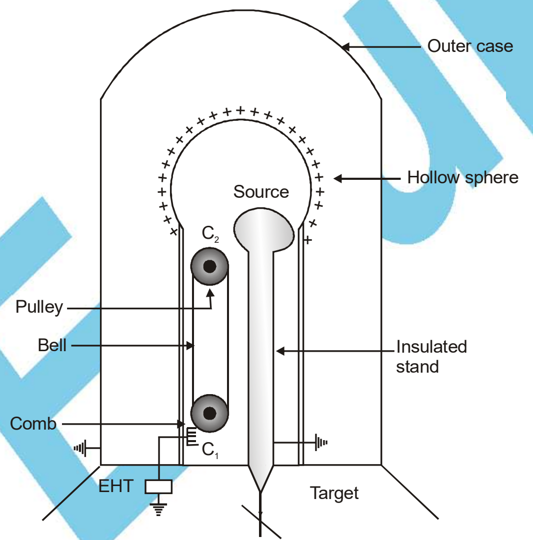
$W = p E (\cos 0° - \cos 180°)$

$= p E [1 - (-1)] = 2 pE$

73. Give the principle and explain the working of a van de Graff generator with the help of labelled diagram. Mention its use.

Ans. ***Van de Graff*** generator is a device used to build up very high potential to accelerate particls which are required to conduct various experiments related with atomic physics and to start nuclear reactions etc.

Principle: Charge resides at the surface of a sphere. If a hollow conductor is in contact w ith another conductor, the charge given to conductor is immediately transferred to the hollow conductor enabling the h ollowing coductor to accept large charge. Thus builing up of a high potential is made possible.



Construction and Working:

It consists of hollow metallic sphere mounted on insulated stand. A belt is run on two pulleys. A metal comb C_1 sprays charge on the belt with the help of an EHT source.

Comb C_2 picks up the charge and hands it over to sphere. This charge and hands it over to sphere. This charge is immediately handed over to the outer surface of the sphere. Ths process contrinues til very high potential difference is built up. Outer enclosure prevents the leakage of the charge to surrounding.s The source provides the necessary ionisation and thus beams of high energy ions are produced which are used to hit the target.

Uses: Van de Graff generators are used to produced accelerated beams of high energy ions to

(a) Start the nueclear reaction

(b) treat disease like cancer

(c) break atoms for various experiments in physics.

74. Assuming that a capacitor is disconnected from the charging battery, explains how the (i) capacitance (ii) p.d. across the plates, and (iii) energy stored in parallel plate capacitor change, when a medium of dielectric constant 'K' is introduced between the plates.

Ans. Let U be the energy stored with air as medium, V the potential difference with air as medium and C the capacitance with air as medium. on insertion of the dielectric medium,

(i) Capacitance C′ = KC i.e. it increases K times

(ii) As $C = \dfrac{q}{V}$ $\therefore$ $C' = \dfrac{q}{V'}$

[Charge q remains same because capacitor is disconnected from supply]

or $V' = \dfrac{q}{C'} = \dfrac{q}{KC} = \dfrac{V}{K}$ $\therefore$ New P.D. = $\dfrac{1}{K}V$ i.e. it decreases $\dfrac{1}{K}$ times

(iii) Eenrgy stored in capacitor,

$$U' = \frac{1}{2}C'V'^2 = \frac{1}{2}(KC)\left(\frac{V}{K}\right)^2$$

$$= \frac{1}{K}\left(\frac{1}{2}CV^2\right) = \frac{U}{K}$$

i.e. it decreases $\dfrac{1}{K}$ times.

75. Derive an expression for energy stored in a parallel plate capacitor with air as medium between the plates.

Ans. Energy stored in a capacitor = $\frac{1}{2}CV^2$

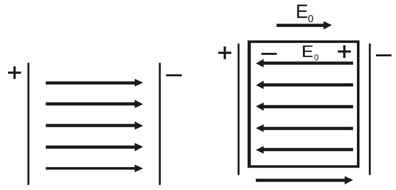
$\therefore$ Energy stored in a capacitor with air as dielectric = $\frac{1}{2}\frac{\epsilon_0 A}{d}V^2$

and Energy stored in a capacitor with dielectric medium = $\frac{1}{2}\frac{\epsilon_0 KA}{d}V^2$

Comparing (iii) and (iv), it is clear that energy stored will increase K times if air is replaced with a dielectric medium of dielectric constant K if the source i.e. battery remains conneted.

76. Draw electric field lines between the plates of a paralle plate capacitor with (i) air (ii) dielectric as the medium.
A parallel plate capacitor with air as dielecric is connected to a power supply and charged to a potential difference V_0. After disconnecting from power supply a sheet of insulating material is inserted between the plates compltely filling the space between them. How will its (i) capacity (ii) electric field (iii) energy change ? Given capacitance with air as C_0 and permittivity for air as ϵ_0 and for medium ϵ.

Ans. (i) Caacity with air, $C_0 = \frac{\epsilon_0 A}{d}$



$E = (E_0 - Ep)$

Capacity with medium, $C_m = \frac{\epsilon_0 A}{d}$

$\therefore \frac{C_m}{C_0} = \frac{\epsilon}{\epsilon_0}$ i.e. $C_m = \frac{\epsilon}{\epsilon_0}C_0$

Since $\epsilon > \epsilon_0 \therefore C_m > C_0$

(ii) Electric field, $E = \frac{V}{d}$

Here $V_m = \frac{V_0}{\epsilon / \epsilon_0} = \frac{\epsilon_0 V_0}{\epsilon}$

$\therefore E_m = \frac{\epsilon_0}{\epsilon}E_0$

Since $\epsilon_0 < \epsilon \therefore E_m < E_0$

(iii) Using $U = \frac{1}{2}CV^2$ we get

$$U_m = \frac{1}{2}\left(\frac{\epsilon_0}{\epsilon}C_0\right)\frac{V_0^2 \epsilon_0^2}{\epsilon^2} = \left(\frac{\epsilon_0}{\epsilon}U_0\right)$$

$$\left(\because U_0 = \frac{1}{2}C_0V_0^2\right)$$

Since $\epsilon_0 < \epsilon \therefore U_m < U_0$

EXERCISE # I — UNSOLVED QUESTION

1. State Coulomb's law of force in electrostatics.

2. What is the work done in moving a 100nC charge between two point 5 cm apart on an equipotential surface ?

3. Which physical quantity has unit NC^{-1} ? is it a vector or a scalar quantity ?

4. Two charges q_1 and q_2, separated by a smal distance satisfy the equation $q_1 + q_2 = 0$. What does it tell us about the charges.

5. Name the physical quantity which has joule coulomb^{-1} as its unit. Is it a scalar or a vector quantity ?

6. Write the S.I. unit of (i) electric field intensity and (ii) electric dipole moment.

7. Define dipole moment. Is it a scalar or a vector quantity ?

8. A 500 μC charge is at the centre ofa square of side 10 cm. Find the work done in moving a charge of 10 μC between two diagonally opposite points on the square.

9. Draw an equipotential surface for a system, consiting of two charges Q, – Q separated a distance 'r' in air.

10. Draw an equipotential surface for uniform electric field.

11. Derie an expression for the electric potential at a distance r from a poit charge Q.

12. Deriv an expression for the electric field intensity at a distance r from the point charge q.

13. Derive the expression for the capacitance of a parallel plate capacitor, having two identical plates each of area A and separated by a distance d, when the space between the plates is filled by a dielectric medium.

14. Derive an expression for the total work done in rotating an electric dipole through an angle θ in a uniform electric field.

15. State Gauss' theorem in electrostatics and express it mathematically. Using it derive as expression for electric field intenstiy at a point near a thin infinite plane sheet of electric charge.

16. Show mathematically that the potential at a point on the equatorial line of an electric dipole is zero.

17. A conducting slab of thikness 't' is introduced, without touching, between the plates of a parallel plate capacitor, separated by distance 'd' (t < d). Derive an expression for the capacitance of the capacitor.

18. State Gauss' theorem in elecrostatics. Prove that no electric field exists inside a hollow charged sphere.

19. Drive an expression for energy stored in a charged parallel plate capacitor with air as the medium between its plates.

20. Derive an expression for the electric potential at a point along the axial line of an electric dipole.

21. Define the rerm 'electric dipole moment'. Give its unit.
    Derive an expression for the maximum torque acting on an acting dipole, whern held in a uniform electric field.

22. Draw a labelled diagram of Van de Graaff generator. State its principle of working.

23. Two capacitors with capacity C_1 and C_2 are charged to potential V_1 and V_2 respectiely and then connected in parallel. Calculate the common potential across the combination, the charge on each capacitor, the elecrostatic energy stored in the system and the change in the electrostatic energy from its initial value.

24. Derive the expression for the electric potential at any poit along the axial line of an electric dipole ?

25. Derive an expresion for the potential energy of an electric dipole of dipole moment $\vec{p}$ in an electric field $\vec{E}$.

26. Three condensers C_1, C_2 and C_3 are connected in sries. Derive an expression for the equivalent capacitance.

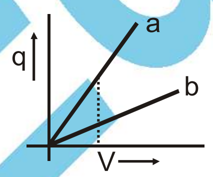
27. Derive expression for the electric field due to a uniformly charges spherical shell at point
(i) inside and (ii) outside the shell.

28. State Gauss' theorem in electrostatics, Using this thoerem derive an expression for the electric field intenstiy due to an infinitely long, straight wire of linear charge density λ C m^{-1}.

30. Define electric line of force and given its two important properties.

31. Define electric field intensity. Write its S.I. unit. Write the magnitdue and direction of electric field intenstiy due to an electric dipole of legnth 2a at the mid point of line joining the charges.

32. The give graph shows variation of charge q versus p.d. V for two capacitors C_1 and C_2 both have same reparation but plate area of C_2 is double than that of C_1. Which line corresponds to C_1 and why



33. Draw equipotential surfaces and corresponding elecric field lines for (i) Single point charge q < 0 (ii) Uniform electric field.

34. State Gauss theorem. Expressit mathematically. Apply this thoerem to obtain an expression for electric field due to an infinite plane sheet of charge, of charge density σ C m^{-2}.

35. State Gauss's theorem in electrostatics. Apply this theorem to derive an expression for electric field intensity at a point near an infinity long straight charge wire.

36. What is a dielectric ? Why does capacitance of a parallel plate capacitor on introduction of a dielectric in between the two plates. Derive an expression for the capacitance of such a capacitor having two identical plates each of area A and separated by distance d. The space between the plates has a medium of dielectric constant K.

37. Derive an expression for energy stored in a parallel plate capacitor with air as medium between its plates.
Air is now replaced by a dielecric medium of constant K. How does it change the total energy of capacitor if.
(i) the capacitor remains connected to some battery.
(ii) the capacitor is disconnected from battery.

38. Using Gauss' thoerem, derive an expression for elecric field at a point near a thin infinite plane sheet of elctric charge. How does this electric field change for a uniformly thick sheet of charge ?

39. State Gauss' theorem in electrostaitcs. Apply this theorem to derive an expression for electric field intenstiy at a point outside a uniformly charged thin spherical shell.

40. The electric field E due to a point charge at any point near it is defined as $E = \lim_{q \to 0} \frac{F}{q}$

where q is test charge and F is the force acting on it. What is the physical significance of $\lim_{q \to 0}$ in this expression ? Dar electric field lines of a point charge Q when (i) Q > 0 (ii) Q < 0.

41. Deduce an expression for electric potential due to an electric dipole at any point on its axis. Mention one contrasting feature of electrric potential of a dipole at a point as compared to that due to a single charge.

42. Derive the relation $C = \frac{\epsilon_0 A}{d}$ for the capacitance of a parallel plate capacitor, where symbols have their usual meaning. A parallel plate capacitor is charged toa potential difference of V volt and disconnected from supply. if distance between plates is doubled explain how does (i) electric field and (ii) enery stored in the capacitor change ?

43. Define 'electric dipole moment'. give its units. Derive an expression for torque acting on an electric dipole held in uniform electric field. An electric dipole is placed in uniform electric field and is free to move. Explain what happens hen it is placed (i) parallel (ii) perpendicular to the field.

44. Derive an expression for the energy stored in a parallel plate capacitor
A parallel plate capacitor with air as dielectric is charged by a d.c. source to a potential V. Without disconnecting the capacitor from the source, air is replaced by another dielectric medium of dielectric constant 10. State with reason, how does (i) electric field between plates and (ii) energy stored in the capacitor change.

45. Give principle of working of Van de Graff genertor. With the help of a labelled diagram, describe its construction and working. How is the leakage of charge minimised from the generator.

46. Obtain the expression for the capacitance of a parallel plate capacitor. Three capacitors of capacitances C_1, C_2 and C_3 are connected (i) in seris (ii) in parallel. Show that the energy stored in the series combination is the same as that in the parallel combination.

47. Derive an expression for the energy density of a capacitor.

48. A parallel plate capacitor has two identical plates each of area A, separated by distance d. The space between the plates is partially filled with a dielectric medium. Derive an expression for the capacitance of the capacitor.

EXERCISE # II BOARD QUESTION

Q.1 Two charges of magnitudes - 3Q and +2Q are located at points (a,0) and (4a,0) respectively. What is the electric flux due to these charges through a sphere of radius '5a' with its centre at the origin ?

Q.2 A slab of material of dielectric constant K has the same area as that of the plats of a parallel plate capacitor but has thickness d/3, where d is the separation between the plates. Find out the expression for its capacitance when the slab is inserted between the plates of the capacitor.

Q.3 A capacitor, made of two parallel plates each of plate area A and separation d, is being charged by a external ac source. Show that the displacement current inside the capacitor is the same as the current charging the capacitor.

Q.4 (a) Define electric dipole moment. Is it a scalar or a vector ? Derive the expression for the electric field of a dipole at a point on the equatorial plane of the dipole.

(b) Draw the equipotential surfaces due to an electric dipole. Locate the points where the potential due to the dipole is zero.

OR

Using Gauss' law deduce the expression for the electric field due to a uniformly charged spherical conducting shell of radius R at a point (i) outside and (ii) inside the shell.

Plot a graph showing variation of electric field as a function of r > R and r < R.

(r being the distance from the centre of the shell)