Trigonometric Equation

1. Definition

An equation containing trigonometric function of unknown angles are known as trigonometric equations.

Ex. $\cos\theta = \dfrac{1}{2}$, $\tan\theta = \dfrac{1}{\sqrt{3}}$ and $\sin\theta = \dfrac{1}{2}$ etc. are trigonometric equations.

2. Periodic Function

A function $f(x)$ is said to be periodic if there exists $T > 0$ such that $f(x + T) = f(x)$ for all x in the domain of definitions of $f(x)$. If T is the smallest positive real numbers such that $f(x + T) = f(x)$, then it is called the period of $f(x)$.

The period of sin x, cos x, sec x, cosec x is 2π and period of tan x and cot x is π.

3. General solution of standard trigonometrical equation

Since Trigonometrical functions are periodic functions, therefore, solutions of Trigonometrical equations can be generalised with the help of periodicity of Trigonometrical functions. The solution consisting of all possible solutions of a Trigonometrical equation is called its general solution.

3.1 General Solution of the equation $\sin\theta = 0$:

By Graphical approach,
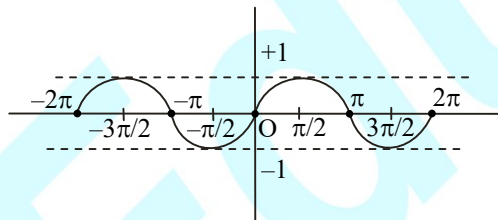
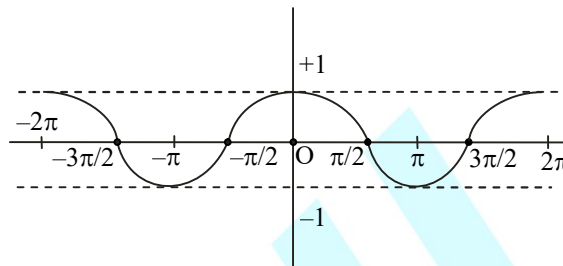The above graph of $\sin\theta$ clearly shows that $\sin\theta = 0$ at

$\theta = 0, \pm\pi, \pm 2\pi, \pm 3\pi$ ....

$\sin\theta = 0$ is

$\theta = n\pi$: $n \in I$ i.e. $n = 0, \pm 1, \pm 2$ ....

3.2 General solution of $\cos\theta = 0$:

By graphical approach,

The above graph of $\cos\theta$ clearly shows that $\cos\theta = 0$ at

$\theta = \pm\pi/2, \pm 3\pi/2, \pm 5\pi/2,$ ....

$\cos\theta = 0$

$\theta = (2n + 1)\pi/2$, $n \in I$.

i.e. $n = 0, \pm 1, \pm 2$ ....

3.3 General solution of $\tan\theta = 0$:

Proof: If $\tan\theta = 0$

or $\dfrac{\sin\theta}{\cos\theta} = 0$

$\sin\theta = 0$,

it follows that general solution of $\tan\theta = 0$ it same as of $\sin\theta = 0$

general solution of $\tan\theta = 0$ is

$\theta = n\pi$; $n \in I$

Note : General solution of $\sec\theta = 0$ and $\mathrm{cosec}\,\theta = 0$ does not exist because $\sec\theta$ and $\mathrm{cosec}\,\theta$ can never be equal to 0.

3.4 General solution of the equation $\sin\theta = \sin\alpha$:

is $\theta = n\pi + (-1)^n\alpha$; $n \in I$

3.5 General solution of the equation $\cos\theta = \cos\alpha$:

is $\theta = 2n\pi \pm \alpha$, $n \in I$

3.6 General solution of the equation $\tan\theta = \tan\alpha$:

is $\theta = n\pi + \alpha$; $n \in I$

4. General solution of square of the trigonometrical equations

4.1 General solution of $\sin^2\theta = \sin^2\alpha$

is $\theta = n\pi \pm \alpha$; $n \in I$

4.2 General solution of $\cos^2\theta = \cos^2\alpha$

is $\theta = n\pi \pm \alpha$; $n \in I$

4.3 General solution of $\tan^2\theta = \tan^2\alpha$:

If $\tan^2\theta = \tan^2\alpha$ $\Rightarrow \theta = n\pi \pm \alpha$; $n \in I$