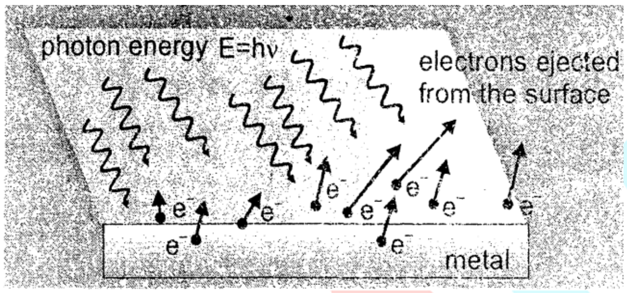
Modern Physics
(Photo Electric Effect & Matter Wave)

PHOTO ELECTRIC EFFECT

It is a phenomenon of ejecting electrons by falling light of suitable frequency or suitable wavelength on a metal. Ejected electron is called photoelectron and current flowing due to the photoelectorns is called photoelectric current.

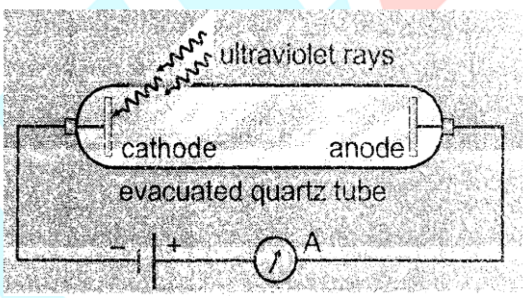


This effect was discovered by Hertz.
Law of photo electric effect were given by Lenard.
It was explained by Einstein using Quantum theory of light.

DIFFERENT EXPERIMENTS

Hertz Experiment

Hertz observed that when ultraviolet rays are incident on a negative plate of electric discharge tube then conduction takes place easily in the tube.



Hallwach Experiment

Hallwach observed that if negatively charged Zn plate of electroscope is illuminated by ultra violet light, its negative charge decreases and becomes neutral and after some time, it gains positive charge. It indicates that under the action of ultra violet light, some negative charged particles are emitted from the metal.

Lenard Experiment

He told that when ultraviolet rays are incident on cathode, electrons are ejected, these electrons are attracted by anode and due to complete path of photo electrons, photo current flows. When ultra violet rays are incident on anode, electrons are ejected but current does not flow.

GOLDEN KEY POINTS

- **Work function :** Minimum energy required by an electron to escape from the metal surface.
- For the photo electric effect the light of short wavelength (or high frequency) is more effective than the light of long wavelength (or low frequency)
- **Type of electron emission**
  - **(i) Therminoic emission :** By suitably heating, sufficient thermal energy can be imparted to the free electrons to enable them to come out of the metal.
  - **(ii) Field emission :** By applying a very strong electric field (of the order of 10^8 Vm^{-1}) to a metal, electrons can be pulled out of the metal, as in a spark plug.
  - **(iii) Photo-electric emission :** When light of suitable frequency illuminates a metal surface, electrons are emitted from the metal surface.

QUANTUM THEORY

1. Energy of photon

Energy radiated from a source propagated (microscopically) in the form of small packets and these are known as photons. According to Planck the energy of a photon is directly proportional to the frequency of the radiation.

$E \propto v$ $\qquad$ $E = hv$

$$E = \frac{hc}{\lambda} \quad (\because c = v\lambda) \quad \Rightarrow \quad E = \frac{hc}{\lambda} = \frac{12400}{\lambda} eV - A \qquad [\because \ hc = 12400 \ (Å - eV)]$$

Here $\quad$ E = energy of photon, $\quad$ c = speed of light,

h = Planck's constant ($h = 6.62 \times 10^{-34}$ J-s),

E= charge of electron,

v = frequency of photon, $\quad$ λ = wavelength of photon

2. Linear momentum of photon

Linear momentum of photon $p = \frac{E}{c} = \frac{hv}{c} = \frac{h}{\lambda}$

3. Effective mass of photon

Effective mass of photon $m = \frac{E}{c^2} = \frac{hc}{c^2\lambda} = \frac{h}{c\lambda}$ $\qquad$ i.e. effective mass $m \propto \frac{1}{\lambda}$

So mass of violet light photon is greater than the mass of red light photon. ($\because \ \lambda_R > \lambda_V$)

Rest mass of a photon is always zero.

4. Intensity of light

$$I = \frac{E}{At} = \frac{P}{A} \qquad \ldots..(i)$$

SI Unit : $\frac{Joule}{m^2 - s}$ or $\frac{watt}{m^2}$

Here, $\quad$ P = power of source, $\quad$ A = Area, $\quad$ t = time taken,

E = energy incident in t time = Nhv $\quad$ N = number of photon incident in t time

$$\text{Intensity } I = \frac{N(hv)}{At} = \frac{n(hv)}{A} \qquad \ldots.(ii) \qquad \left[\because \ n = \frac{N}{t} = \text{no. of photon per sec.}\right]$$

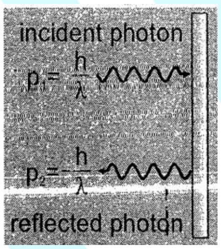
From equation (i) and (ii)

$$\frac{P}{A} - \frac{h(hv)}{A}$$

$$n = \frac{P}{hv} - \frac{P\lambda}{hc} \quad \Rightarrow \quad n = (5 \times 10^{24}\ J^{-1}\ m^{-1})\ P \times \lambda$$

5. Radiation force and Radiation pressure

(i) When radiation are falling normally on a perfectly reflecting surface
Let 'N' photons are falling in time t,



Momentum before striking the surface (p_1) = $\frac{Nh}{\lambda}$

Momentum after striking the surface (p_2) = $-\frac{Nh}{\lambda}$

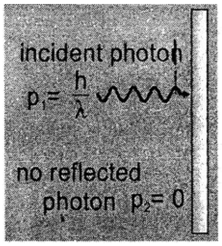
Change in momentum of photos $= p_2 - p_1 = \frac{-2Nh}{\lambda}$

Momentum transferred to the surface $= \Delta p = \frac{2Nh}{\lambda}$

Radiation force on the surface $F = \frac{\Delta p}{\Delta t} = \frac{2Nh}{t\lambda} = n\left[\frac{2h}{\lambda}\right]$ but $n = \frac{P\lambda}{hc}$

$\therefore \quad F = \frac{2h}{\lambda} \times \frac{P\lambda}{hc} = \frac{2P}{c}$ and Radiation pressure $\frac{F}{A} = \frac{2P}{cA} = \frac{2I}{c}\ \left[\because\ I = \frac{P}{A}\right]$

(ii) When radiations are falling normally on a perfectly absorbing surface :



Radiation force $F = \frac{p_1 - p_2}{t} = \frac{\frac{Nh}{\lambda} - 0}{t} = \frac{Nh}{t\lambda}$

$\Rightarrow \quad F = n\frac{h}{\lambda}\ \left[\because\ n = \frac{P\lambda}{hc}\right]$

$F = \frac{P}{c}$ and Pressure $= \frac{F}{A} = \frac{P}{Ac} = \frac{I}{c}$

ILLUSTRATIONS

Illustrations 1

Calculate the energy of photo having λ = 4000Å in eV and in J.

Solution

$$E = \frac{12400}{4000} eV = 3.1\ eV$$

$E = 3.1 \times 1.6 \times 10^{-19} = 4.96 \times 10^{-19}$ J

Illustration 2

The power of a bulb is 60 milliwatt and the wavelength of light is 6000 Å. Calculate the number of photon second emitted by the bulb ?

Solution

Energy released per second

$$nh\nu = \text{power or } \frac{nhc}{\lambda} = P \text{ or } n = \frac{P\lambda}{hc} = \frac{60\times10^{-3}\times6000\times10^{-10}}{6.62\times10^{-34}\times3\times10^{8}} \text{ photon/sec} = 1.8 \times 10^{17} \text{ photon/sec}$$

Illustration 3

The energy flux of sunlight reaching on the Earth's surface is 1.388×10^3 W/m^2. How many photos (nearly) per square metre are incident on the Earth per second? Assume that the photons in the sunlight have an average length of 550 nm.

Solution

$$\text{Intensity } I = \frac{\text{Power emitted}}{\text{Area}} = \frac{P}{A} = 1.388 \times 10^3 \text{ W/m}^2$$

$$\text{Area Energy of a photo } E = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^{8}}{550\times10^{-9}} = 3.616 \times 10^{-19} \text{ J}$$

$$\text{Let n be the total number of photon/area then } n = \frac{\frac{P}{A}}{E} = \frac{I}{E} = \frac{1.388\times10^{3}}{3.616\times10^{-19}} = 3.83 \times 10^{21} \text{ photon/m}^2\text{s}$$

Illustration 4

How many photons of wavelength λ = 6600 nm must strike a totally reflecting screen per second at normal incidence so as to exert a force of 1N? **[AIPMT Mains 2007]**

Solution

$$\text{Momentum of the incident photon } p = \frac{h}{\lambda},$$

$$\text{Momentum after reflection } = -\frac{h}{\lambda}$$

$$\text{Change in momentum } \Delta p = \frac{2h}{\lambda}$$

If n is the number of photons falling per second on the screen then force

$$F = \frac{\Delta p}{\Delta t} = \frac{2nh}{\lambda} \quad \Rightarrow \quad n = \frac{F\lambda}{2h} = \frac{1\times6600\times10^{-9}}{2\times6.6\times10^{-34}} = 5 \times 10^{27} \text{ photons s}^{-1}$$

BEGINNER'S BOX-1

1. Who discovered photo electric effect?
(1) Hertz (2) Lenard (3) Hallwach (4) Einstein

2. The energy of a photon is equal to 3 kilo eV. Calculate its liner momentum?

3. A parallel beam of monochromatic light of wavelength 500 nm is incident normally on a perfectly absorbing surface. The power through any cross-section of the beam is 10. Find
(a) Number of photon absorbed by the surface per second.
(b) The force exerted by light beam on the surface

4. Which color of photon has greater energy either red or violet?

5. A TV station is operated at 100 MW with a signal frequency of 10 Mhz. Calculate the number of photons radiated per second by its antenna.

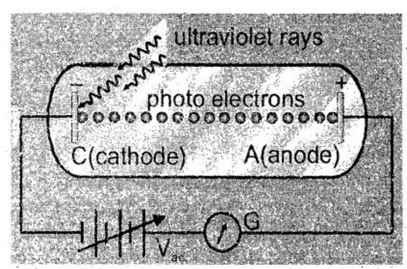
6. Calculate number of photos passing through a ring of unit area in unit time if light of intensity 100 W^{-2} and of wavelength 400 nm is falling normally on the ring.

7. A special kind of light bulb emits monochromatic light of wavelength 700 nm. Electrical energy supply to it at the rate of 60 W and the bulb is 50% efficient at converting that energy to light energy. How many photons are emitted by the bulb during its life time of 1 day.

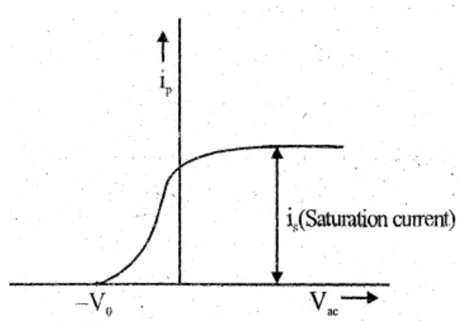
EXPERIMENTAL STUDY OF P.E.E. BY LENARD

1. The effect of potential difference between A and C

Light of fixed frequency (υ) and intensity (I) is incident on photo emissive plate C, keeping A positive w.r.t. C. If positive potential of A gradually increased, it is found that first photo current increases then becomes maximum called **saturation current**. Now we apply negative potential to plate A w.r.t. C and on increasing it gradually then current decreases and becomes zero at certain negative potential.



Magnitude of the minimum negative potential at which i_p becomes zero, called stopping potential (V_0).



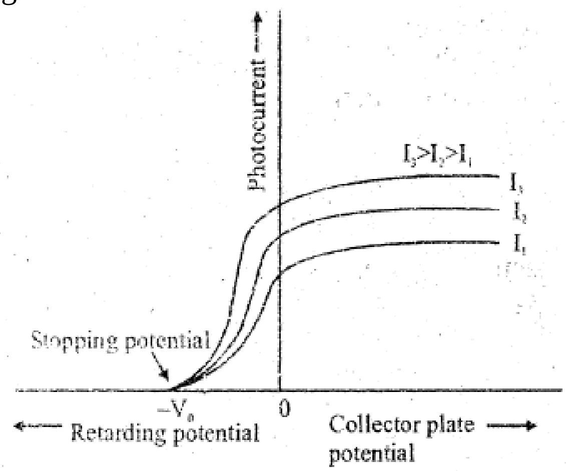
At stopping potential most energetic electron are stopped.

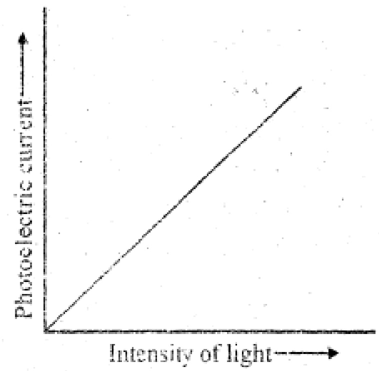
It means stopping potential measures maximum kinetic energy of photo electrons.

i.e. $K_{max} = eV_0$

2. **Effect of intensity of light**

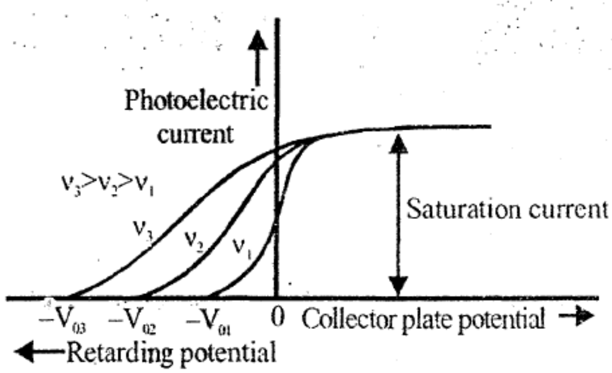
When intensity of given source able to produce PEE, is increased I increases but V_0 remains unchanged.

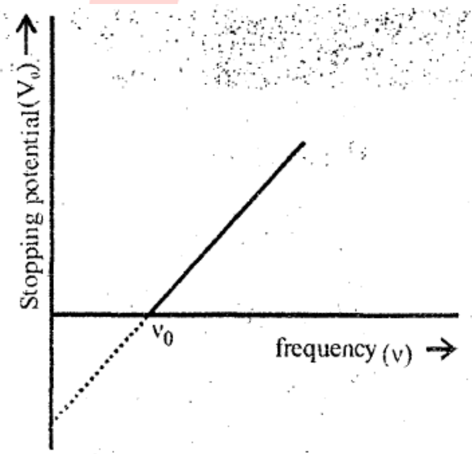




3. Effect of frequency

When frequency of radiations in increased, V_0 increase but i_s almost unchanged. It is also observed that below a certain frequency photoelectrons don't come out.





The minimum frequency which can eject the photo electrons is called cut off or threshold frequency.

Similarly corresponding maximum wavelength is called threshold wavelength.

4. **Time lag**

There is no time lag between the incidence of radiations and emission of electron. It is a spontaneous process.

GOLDEN KEY POINTS

- **Intensity :** Energy light passing through per unit area per unti time is known as intensity of light.

Intensity $\propto$ photon per second $\propto$ current $\propto \frac{1}{d^2}$ (for a given point source)

$\frac{i_2}{i_1} = \frac{d_1^2}{d_2^2}$ i = current flows in the circuit

d = distance between the source of light and electron emitter

- Stopping potential does not depend on the distance between emitter and collector.
- The photoelectric emission is an instantaneous process without any apparent time lag ($\sim 10^{-9}$ s or less), even when the incident radiation is made exceedingly dim.
- Ultra violet light causes photo electric emission from any metal surface, while visible light causes photo emission from the alkali metals.
- The work function represented the energy needed to remove the least tightly bound electrons from the surface. It depends only on nature of the metal and independent of any other factors.

FAILURE OR WAVE THEORY OF LIGHT

(i) According to wage theory when light incident on a surface, energy is distributed continuously over the surface. So the electron must take a time interval to accumulate sufficient energy to come out. But in experiment there is no time lag.

(ii) When intensity is increased, more energetic electrons should be emitted. So that stopping potential should be intensity dependent. But it is not observed.

(iii) According to wave theory, if intensity is sufficient then, at each frequency, electron emission is possible. It means there should not be existence of threshold frequency.

EXPLANATION BY EINSTEIN

1. Radiations absorbed by the surface are in the form of quanta (photon). Energy of each photon depends on frequency. One photon can interact with one electron at a time. In the interaction between photon and electron incident photon transfers its whole energy to the electron.
If energy is sufficient then electron comes out without any time delay. It means photo electric effect is an instantaneous process.

2. If intensity of the given source is increased then number of photon increases. So that, more number of electrons are emitted and greater saturation current is obtained. It means saturation current depends upon intensity of the given source $i_s \propto I$

3. At a time, only one photon can interact with one electron.
Energy of photon used by the electron is
hv = Kinetic energy of electron + Energy required to make electron free from the metal surface (ϕ_0)
+ Energy lost in collision before emission (Q)
If Q = 0, means there is no heat loss, then kinetic energy of electron is maximum.
Now, $hv = (K.E._{max}) + \phi_0$ It is known as Einstein's equation of P.E.E.
$(K.E._{max}) = hv - \phi_0$ or $eV_0 = hv - \phi_0$ or $eV_0 = hv - hv_0$

Here v_0 is threshold frequency for that $V_0 = 0$
It means maximum K.E. and stopping potential (V_0) depends on frequency. It is independent of intensity of the given source.

4. Kinetic energy cannot be negative so that, $h\nu > \phi_0$,

$h\nu = h\nu_0$ $\quad \left[\text{Here} \quad \phi_0 = h\nu_0 = \frac{hc}{\lambda_0},\ \phi_0 = \frac{12400}{\lambda_0} eV - Å\right]$

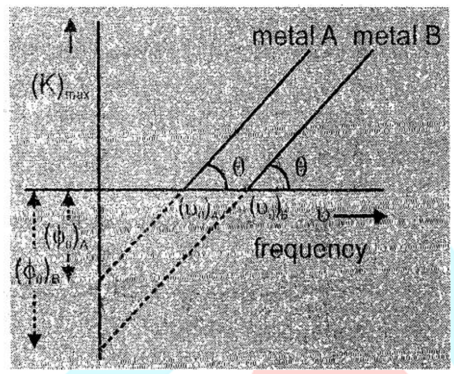
$\nu > \nu_0$ It means if frequency is less than 'ν_0' electron does not come out.

Graph between $(K.E.)_{max}$ and frequency

$(K)_{max} = h\nu - \phi_0$ $\quad$ [$\because$ $Y = mx - c$]

slope = m = tan θ = h(same for all metals)

$(\phi_0)_B > (\phi_0)_A$

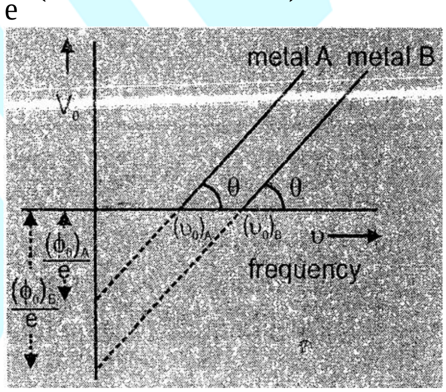


Graph between $(K.E.)_{max}$ and frequency

$\because$ $\quad eV_0 = h\nu - \theta_0$

$$V_0 = \left[\frac{h}{e}\right]\nu - \left[\frac{\phi_0}{e}\right]$$

Slope = m = tan θ = $\frac{h}{e}$ (same for all metals)



QUANTUM EFFICIENCY

$$\text{Quantum efficiency} = \frac{\text{number of electron emitted per second}}{\text{total number of photons incident per second}} = \frac{n_c}{n_{ph}}$$

$$x = \frac{n_e}{n_{ph}} \qquad \text{.....(i)}$$

If quantum efficiency is x% then $n_e = \frac{x}{100} n_{ph}$ [from equation (i)] [Here $n_{ph} = (5 \times 10^{24}\ J^{-1}\ m^{-1})P\lambda$]

PHOTOELECTRIC CURRENT

Photoelectric current $I_e = \dfrac{\text{charge}}{\text{time}} = \dfrac{Q}{t} = n_e e = 1.6 \times 10^{-19} n_e$

GOLDEN KEY POINTS

- Einstein's Photo Electric equation is based on conservation of energy.
- Einstein explained P.E.E. on the basis of quantum theory, from which he was awarded Nobel prize.
- According to Einstein one photon can eject one e^- only. But here the energy of incident photon should be greater than work function (threshold energy) to bring out the electron.
- Particle nature of light is also supported by Compton effect and Raman effect.

  Compton effect: The reduction in the energy (hence increase in wavelength) of high-energy (X-ray or gamma ray) photons when they are scattered by the free electrons, or loosely bound electrons which thereby gain energy.

  Raman effect: It is the inelastic scattering of monochromatic light as it passes through a transparent medium due to interaction of photons with the molecules of medium. This result in wavelength being increased or decreased.

ILLUSTRATIONS

Illustrations 5

The threshold wavelength of a metal is 400 nm. Photo electrons have kinetic energy maximum 1.5 eV. Find the wavelength of incident photon.

Solution

$\lambda_0 = 400 \text{ nm} = 4000 \text{ Å}$

$$KE_{max.} = \frac{12400 \text{ eVÅ}}{\lambda} - \frac{12400 \text{ eVÅ}}{\lambda_0}$$

$$\Rightarrow \quad 1.5 \text{ eV} = \frac{12400 \text{ eVÅ}}{\lambda} - \frac{12400 \text{ eV}}{4000} \quad \Rightarrow \quad 1.5 \text{ eV} = \frac{12400 \text{ eVÅ}}{\lambda} - 3.1 \text{ eV}$$

$$\Rightarrow \quad (1.5 + 3.1) \text{ eV} = \frac{12400 \text{ eVÅ}}{\lambda} \quad \Rightarrow \quad \lambda = \frac{12400 \text{Å}}{4.6} \quad \Rightarrow \quad \lambda = 2696 \text{ Å}$$

Illustration 6

The work function of a metal is 2.3 eV and the wavelength of incident photon is 4.8×10^{-7} m. Find maximum kinetic energy of photo electrons.

Solution

$\phi = 2.3 \text{ eV}$ and $\lambda = 4.8 \times 10^{-7} \text{ m} = 4800 \text{ Å}$

$$K.E._{max} = \frac{hc}{\lambda} - \phi = \frac{(6.62 \times 10^{-34} \text{ J s})(3 \times 10^{8} \text{ m s}^{-1})}{\lambda} - \phi = \frac{12400 \text{eVÅ}}{\lambda} - \phi$$

$$K.E._{max} = \left[\frac{12400}{4800} - 2.3\right] eV$$

$\Rightarrow$ 0.28 eV

Illustration 7

Light quanta with an energy 4.9 eV eject photoelectrons from metal with work function 4.5 eV. Find the maximum impulse transmitted to the surface of the metal when each electron flies out.

Solution

According to Einstein's photoelectric equation

$$K_{max} = \frac{1}{2}mv_{max}^2 = hv - \phi_0 = 4.9 - 4.5 = 0.4\ eV$$

Q Change of momentum = impulse

$\therefore$ Impulse = $mv - \left(-\frac{E}{c}\right) = mv + \frac{E}{c}$

Maximum impulse $= \sqrt{2mK_{max}} + \frac{E}{c}$

$$= \sqrt{2 \times 0.4 \times 1.6 \times 10^{-19} \times 9.1 \times 10^{-31}} + \frac{4.9 \times 1.6 \times 10^{-19}}{3 \times 10^8}$$

$= 3.43 \times 10^{-25}$ kg m/s

Illustration 8

The stopping potential for the photoelectrons emitted from a metal surface of work function 1.7 eV is 10.4 V. .Find the wavelength of the radiation used. Also identify the energy levels in hydrogen atom which will emit this wavelength.

Solution

Energy of radiation hv = $KE_{max} + \phi_0 = eV_0 + \lambda_0$ = 10.4 eV + 1.7 eV = 12.1 eV

Wavelength corresponding to this energy $\lambda = \frac{hc}{E} = \frac{12400}{12.1\,eV} eV - Å = 1024Å$

As $\Delta E = 13.6\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = 12.1\,eV$

So this radiation will be emitted by transition

$n = 3 \rightarrow n = 1$

Illustrations 9

A metallic surface is illuminated alternatively with light of wavelength 3000 Å and 6000 Å respectively. It is observed that the maximum speeds of the photoelectrons under these illuminations are in the ratio 3 : 1. Calculate the work function of the surface in eV. ($h = 6.62 \times 10^{-34}$ J-s, $c = 3 \times 10^8$ m/s)

Solution

Maximum kinetic energy of photo electron $K_{max.} = \frac{1}{2}mv_{max}^2 = \frac{hc}{\lambda} - \phi_0$

Now let 3000 Å = λ then 6000 Å = 2λ

$$\therefore \qquad \frac{\frac{hc}{\lambda}-\phi_0}{\frac{hc}{2\lambda}-\phi_0}=\frac{(v_{max})_1^2}{(v_{max})_2^2}=\frac{9}{1}$$

$$\Rightarrow \qquad \phi_0=\frac{7hc}{16\lambda}=\frac{7\times6.62\times10^{-34}\times3\times10^{8}}{16\times3000\times10^{-10}\times1.6\times10^{-19}}=1.81\,\text{eV}$$

Illustration 10

A light of wavelength 1240 Å incident on a metal having threshold frequency 4.8×10^{14} Hz. What is maximum kinetic energy of photo electron ? If light has intensity 100 W/cm^2 then calculate number of incident photons per m^2 per second.

Solution

Maximum kinetic energy of photo electrons

$$K_{max}=\frac{hc}{\lambda}-\phi=\frac{hc}{\lambda}-hv_0$$

$$K_{max}=\frac{6.62\times10^{-34}\times3\times10^{8}}{1240\times10^{-10}\times1.6\times10^{-19}}\text{eV}-\frac{6.62\times10^{-34}\times4.8\times10^{14}}{1.6\times10^{-19}}\text{eV}$$

$K_{max} = (10 – 2)\text{eV} = 8 \text{ eV} = 12.8 \times 10^{-19}$ J

Number of photons per unit area per unit time $n = \dfrac{P\lambda}{hcA}=\dfrac{I\lambda}{hc}$

$$n=\frac{100\times1240\times10^{-10}}{10^{-4}\times6.62\times10^{-34}\times3\times10^{8}}\,m^{-2}s^{-1}=62.43\times10^{22}\,m^{-2}s^{-1}$$

Illustration 11

When a metal is irradiated by monochromatic light, the maximum kinetic energy of the photo-electrons is 1.2eV. If frequency of the light is increased 50% then maximum kinetic energy of photo-electron is 3.6 eV. Evaluate the work function of the metal.

Solution

Einstein's equation of photo electric effect is $(KE)_{max} = hv – \phi_0 \quad \Rightarrow \quad 1.2 \text{ eV} = hv – \phi_0$

When frequency of light is increased 50% then $3.6 \text{ eV} = 1.5\, hv – \phi_0$

From above equation $3.6 \text{ eV} = 1.5(1.2 \text{ eV} + \phi_0) – \phi_0$

$\Rightarrow \qquad 3.6 \text{ eV} = 1.8 \text{ eV} + 0.5\, \phi_0$

$\Rightarrow \qquad \phi_0 = 3.6$ eV

Illustration 12

A light beam of 2mW power and 6000 Å wavelength incident on a photo-cell. If threshold wavelength of emitter is 7000A and 2% incident photons eject the photo-electron then find out value of saturation current in the photo cell.

Solution

Let n photons per second incident on the emitter then power $P = \frac{nhc}{\lambda}$

$$\Rightarrow \quad n = \frac{P\lambda}{hc} = \frac{2\times10^{-3}\times6\times10^{-7}}{6.62\times10^{-34}\times3\times10^{8}} = 6\times10^{15}$$

If per second number of emitted electrons is n_e then

$$n_e = \frac{2}{100}\times n = \frac{2}{100}\times6\times10^{15} = 12\times10^{13}$$

Saturated current $i_s = (n_e)e = 12 \times 10^{13} \times 1.6 \times 10^{-19} = 19.2 \times 10^{-6}$ A

Illustrations 13

The strength of magnetic field required to bend photoelectrons of maximum energy in a circle of radius 50 cm when light of wavelength 3300 Å is incident on a barium emitter is 6.7×10^{-6} T. What value of charge on the photoelectrons is obtained from this data ?

(Given : Work function of barium = 2.5 eV; Mass of the electron = 9×10^{-31} kg)

[AIPMT Mains 2007]

Solution

Maximum KE of photoelectron $\frac{1}{2}mv_{max}^2 = \frac{hc}{\lambda} - \phi$

$$\Rightarrow \quad v_{max} = \sqrt{\frac{2}{m}\left(\frac{hc}{\lambda} - \phi\right)}$$

$$\Rightarrow \quad v_{max} = \sqrt{\frac{2}{9\times10^{-31}}\left(\frac{6.6\times10^{-34}\times3\times10^{8}}{3300\times10^{-10}} - 2.5\times1.6\times10^{-19}\right)}$$

$$\Rightarrow \quad v_{max} = \sqrt{\frac{4}{9}\times10^{12}} = \frac{2}{3}\times10^{6}\,ms^{-1}$$

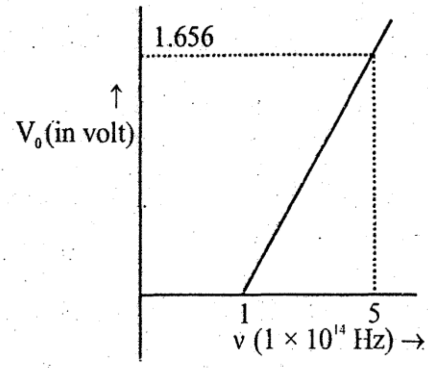
Now, $\quad Bev_{max} = \frac{Mv_{max}^2}{R_{max}}$

$$\Rightarrow \quad e = \frac{Mv_{max}}{BR_{max}} = \frac{9\times10^{-31}\times\frac{2}{3}\times10^{6}}{6.7\times10^{-6}\times0.5} = 1.8\times10^{-19}\,C$$

Illustration 14

The graph between the stopping potential and frequency of the incident radiation is shown in figure. Calculate. **[AIPMT Mains 2008]**

(i) Planck's constant (ii) Work function



Solution

(i) According to Einstein's equation of photo electric effect $eV_0 = h\nu - h\nu_0$

$$\Rightarrow \quad h = \frac{eV_0}{(\nu - \nu_0)} = \frac{1.6\times10^{-19}\times1.656}{(5-1)\times10^{14}} = \frac{1.6\times1.656}{4}\times10^{-33} = 6.624 \times 10^{-34} \text{ J-s}$$

(ii) Work function $\phi_0 = h\nu_0 = 6.624 \times 10^{-34} \times 1 \times 10^{14} = 6.624 \times 10^{-20}$ J

$$= \frac{6.624\times10^{-20}}{1.6\times10^{-19}}\text{eV} - 0.414\text{ eV}$$

BEGINNER'S BOX-2

1. The work function of a metal is 4 eV if 5000 Å wavelength of light is incident on the metal. Is there any photo electric effect?
2. In a photo cell 4 unit photo electric current is flowing the distance between source and cathode is 4 unit. Now distance between source and cathode becomes 1 unit. What will be photo electric current now?
3. The wavelength of photons in two cases are 4000 Å and 3600 Å respectively what is difference in stopping potential for these two ?
4. Threshold frequency of a surface is v_0. It is illuminated by $3v_0$ frequency, then the maximum speed of photo electrons is V m/sec. What will be maximum speed if incident frequency is 9 v_0?
5. When incident wavelength is λ, stopping potential is $3V_0$. If incident wavelength is 2λ then stopping potential is V_0. Find out threshold wavelength in terms of λ.
6. A light beam of power 1.5 mW and 400 nm wavelength incident on a cathode. If quantum efficiency is 0.1% then find out obtained photo current and number of photoelectron per second.
7. A metallic surface of work function hv is illuminated by a radiation beam of frequency 5v. Stopping potential observed is X. What will be stopping potential if the surface is illuminated by radiation of 7v frequency?
8. The kinetic energy of the fastest moving photo electron from a metal of work function 2.8 eV is 2eV. If the frequency of light is doubled, then find the maximum kinetic energy of photo electron.
9. A monochromatic light incident on metal 'A' having threshold frequency v_0. It emits photo electrons of maximum kinetic energy K Now incident frequency is made three times and fall on a metal 'B' havirjg threshold frequency $2v_0$. What will maximum kinetic energy of photo electrons emitted by metal 'B' ?
10. If light of wavelength 4000 Å falls on a metal which has a stopping potential 1.4 volt against photoelectric emission then what is the work function of the metal. [Take $h = 6.6 \times 10^{-34}$ Js and $c = 3 \times 10^{8}$ ms^{-1}]

PHOTO CELL

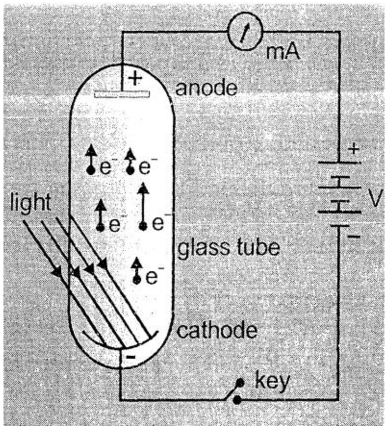
A photo cell is a practical application of the phenomenon of photo electric effect. It converts light energy into electrical energy.

Construction

A photo cell consists of an evacuated sealed glass tube containing anode and a concave cathode of suitable emitting material such as Cesium (Cs).

Working

When light of frequency greater than the threshold frequency of cathode material falls on the cathode, emitted photoelectrons are collected by the anode and an electric current starts flowing in the external circuit. The current increases with the increase in the intensity of light. The current would stop, if the light does not fall on the cathode.



Application

(i) In television camera.

(ii) In automatic door

(iii) Burglar's alarm

(iv) Automatic switching of street light and traffic signals.

MATTER WAVES THEORY

DUAL NATURE OF LIGHT

Experimental phenomena of light reflection, refraction, interference, diffraction are explained only on the basis of wave theory of light. These phenomena verify the wave nature of light.

Experimental phenomena of light photoelectric effect and Compton effect, pair production and pair annihilation can be explained only on the basis of the particle nature of light. These phenomena verify the particle nature of light. It is inferred that light does not have any definite nature, rather its nature depends on its experimental phenomenon. This is known as the dual nature of light. The wave nature and particle nature both cannot be possible simultaneously.

de-Broglie HYPOTHESIS

De-Broglie imagined that as light possess both wave and particle nature, similarly matter must also posses both nature, particle as well as wave.

De Broglie imagined that despite particle nature of matter, waves must also be associated with material particles. Wave associated with material particles, are defined as matter waves.

1. de-Broglie wavelength associated with moving particles

If a particle of mass m moving with velocity v

Kinetic energy of the particle E = $\frac{1}{2}mv^2 = \frac{p^2}{2m}$

Velocity of the particle $v = \sqrt{\frac{2E}{m}}$

Momentum of the particle p = mv = $\sqrt{2mE}$

The wave length associated with the particles is $\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$

The order of magnitude of wave lengths associated with macroscopic particles is 10^{-24} Å

The smallest wavelength whose measurement is possible is that of γ-rays (λ ; 10^{-5} Å). This is the reason why the wave nature of macroscopic particles is not observable.

The wavelength of matter waves associated with the microscopic particles like electron, proton, neutron, α-particle, atom, molecule etc. is of the order of 10^{-10} m, it is equal to the wavelength of X-rays, which is within the limit of measurement Hence the wave nature of these particles is observable.

2. de-Broglie wavelength associated with the charged particles

Let a charged particle having charge q is accelerated by potential difference V

Kinetic energy of this particle E = $\frac{1}{2}mv^2 = qV$

Momentum of particle p = mv = $\sqrt{2mE} = \sqrt{2mqV}$

The De Broglie wavelength associated with charged particle $\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mqV}}$

e.g. de-Broglie wavelength for an electron

$m_e = 9.1 \times 10^{-31}$ kg , $q = 1.6 \times 10^{-19}$ C, $h = 6.62 \times 10^{-34}$ J-s

De-Broglie wavelength associated with electron $\lambda = \frac{6.62\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1.6\times10^{-19}\,V}}$ m

$$\lambda = \frac{12.27\times10^{-10}}{\sqrt{V}}\,\text{metre}\sqrt{\text{volt}} \Rightarrow \quad \lambda = \frac{12.27}{\sqrt{V}}\,\text{Å}\sqrt{\text{volt}}$$

So, $\lambda \propto \frac{1}{\sqrt{V}}$

Potential difference required to stop an electron o wavelength λ is $V = \frac{150.6}{\lambda^2}$ volt (Å)^2

3. de-Broglie Wavelength Associated with Uncharged Particles

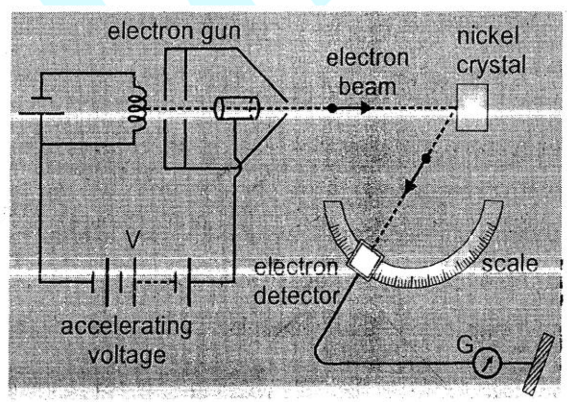
If an uncharged particle has thermal kinetic energy $E = \frac{3}{2}kT$, then its de-Broglie wavelength

$$\lambda = \frac{h}{\sqrt{2m\left(\frac{3}{2}kT\right)}}; \quad \lambda = \frac{h}{\sqrt{3m\,kT}}$$

where k = Bolzmann's constants and T = temperature in K scale.

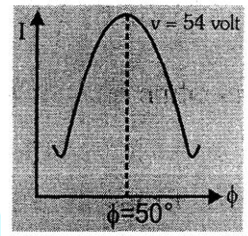
DAVISSON GERMER EXPERIMENT

The experimental arrangement is shown in figure. There are three main parts of this experiment.



(i) Electron gun : Electrons of desired energy are produced in it by the process of thermionic emission.

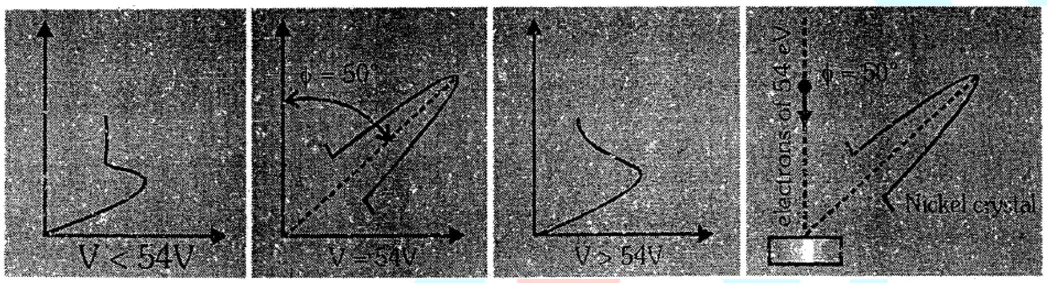
(ii) Nickle crystal : Diffracts the electros beam obtained from electron gun. Nickle crystal behaves like a three dimensional diffraction grating.



(iii) Detector (Ionisation chamber) : It detects the electron beam diffracted by the nickel crystal.

(iv) Conclusion and results : Curve between the intensity (I) of diffracted electrons and diffracting angle (ϕ).

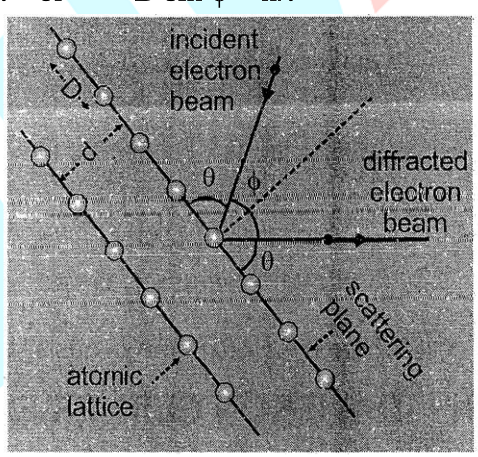
Graph I versus ϕ for different accelerating potential V



In this experiment, on drawing different I–ϕ curve intensity maxima is obtained at an angle of diffraction of 50° and accelerating potential 54 volt.

From the diffraction measurements wavelength associated with electron is obtained as 1.65 Å whereas according to de-Broglie theory this wavelength comes out to be 1.66 Å. Because both the results are same, therefore we can say wave nature is associated with the moving electrons. diffraction from crystal is studied using two equations called Bragg's equations. This are -

$$2d \sin\theta = n\lambda \quad \text{or} \quad D \sin \phi = n\lambda$$



Where d = distance between two consequtive crystal plane or interplanar distance.

D = Distance between two atoms in the same lattice plane

n = order of diffraction

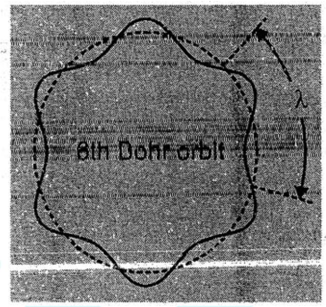
λ = De Broglie wavelength associated with electron.

θ = glancing angle

ϕ = angle of diffraction

Relation between θ and ϕ : $\phi = 180° - 2\theta$ or $\theta = 90° - \dfrac{\phi}{2}$

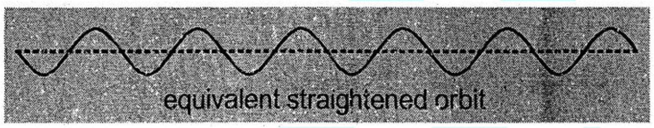
EXPLANATION OF BOHR QUANTISATION CONDITION



According to De Broglie electron revolves round the nucleus in the form of stationary waves (i.e.; wave packet) in the similar fashion as stationary waves in a vibrating string. Electron revolves those circular orbits whose circumference is an integral multiple of de-Broglie wavelength associated with the electron. $2\pi r = n\lambda$

$\because \quad \lambda = \frac{h}{mv}$ and $2\pi r = n\lambda$

$\therefore \quad mvr = \frac{nh}{2\pi}$

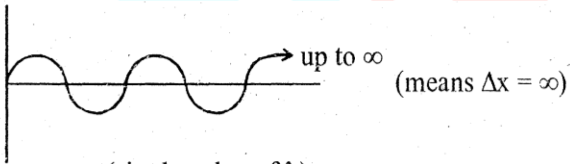


This is the Bohr quantisation condition.

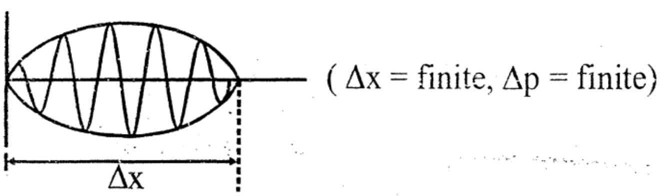
GOLDEN KEY POINTS

Quantum view of probable waves

- If a particle has definite momentum p, ($\Delta p = 0$) then it has definite wavelength $\left(\lambda = \frac{h}{p}\right)$, with extends in entire space. By max born it means particle can be find in whole universe.



But for a good description, particle should in a finite region (means Δx = finite). So, it is rather good idea to consider a particle have group of wave (multiple value of λ) with central wavelength 'h/p'. In this case superposition of constituent wave takes place and results out as wavepacket.



Above concepts are consistent with uncertainty principle

- Wave function of matter wave is an imaginary function ψ.
- If amplitude of matter wave is ψ then $\psi^2(\Delta V)$ = probability of finding in ΔV volume.
- Truely satisfactory physics of dual nature of matter has not developed so far. Quantum wave theory is still subject of research.
- Observations on photoelectric effect imply that in the event of matter-light interaction, absorption of energy takes place in discrete units of hv. This is not quite the same as saying that light consists of particles, each of energy $h\upsilon$.

- Observations on the stopping potential (its independence of intensity and dependence on frequency) are the crucial discriminator between the wave-picture and photon-picture of photoelectric effect.
- The wavelength of a matter wave given by $\lambda = \frac{h}{p}$ has physical significance; its phase velocity v_p has no physical significance. However, the group velocity of the matter wave is physically meaningful and equals the velocity of the particle.

ILLUSTRATIONS

Illustrations 1

Find the initial momentum of electron if the momentum of electron is changed by P_m and the De-Broglie wavelength associated with it changes by 0.50%

Solution

$$\frac{d\lambda}{\lambda} \times 100 = 0.5\,b \qquad \Rightarrow \qquad \frac{d\lambda}{\lambda} \times \frac{0.5}{100} = \frac{1}{200} \qquad \text{and} \qquad \Delta P = P_m$$

$$Q \qquad p\frac{h}{\lambda}$$

$$\text{Differentiating } \frac{dp}{d\lambda} = -\frac{h}{\lambda^2} = -\frac{h}{\lambda} \times \frac{1}{\lambda} = -\frac{p}{\lambda}$$

$$\Rightarrow \qquad \frac{|dp|}{p} = \frac{d\lambda}{\lambda}$$

$$\therefore \qquad \frac{P_m}{p} = \frac{1}{200} \qquad \Rightarrow \qquad p = 200\, P_m$$

Illustration 2

A deutron is accelerated through a potential of 500 volts. Find the potential through which a singly ionized helium ion is to be accelerated for the same De Broglie wavelength.

Solution

$$\lambda = \frac{h}{\sqrt{2mqV}} \qquad \text{or} \qquad mV = \text{constant} \qquad \Rightarrow \qquad V = Pd,\ q \text{ is same}$$

$$m_{He} \times V_{He} = m_d V_d \qquad \text{or} \qquad 4\,V_{He} = 2 \times 500 \qquad \Rightarrow \qquad V_{He} = 250\text{ V}$$

Illustration 3

An α-particle moves in circular path of radius 0.83 cm. in the presence of a magnetjc field of 0.25 Wb/m^2. Find the De-Broglie wavelength associated with the particle.

Solution

$$\lambda = \frac{h}{p} = \frac{h}{qBr} = \frac{6.62 \times 10^{-34}}{2 \times 1.6 \times 10^{-19} \times 0.25 \times 53 \times 10^{-4}} \text{ metre}$$

$$\Rightarrow \qquad \lambda = 0.01\ \text{Å} \qquad \left[Q \quad \frac{mv^2}{r} = qvB \right]$$

Illustration 4

For what kinetic energy of a neutron will the associated de-Broglie wavelength be 1.40×10^{-10} m? Mass of neutron is 1.675×10^{-27} kg ($h = 6.6 \times 10^{-34}$ J-s)

Solution

De-Broglie wavelength $\lambda = \dfrac{h}{mv}$

$\therefore \quad v = \dfrac{h}{m\lambda} = \dfrac{6.6\times10^{-34}}{1.675\times10^{-27}\times1.40\times10^{-10}}\,ms^{-1}$

$\therefore \quad$ K.E. of neutron is $\dfrac{1}{2}mv^2 = \dfrac{1}{2}\times(1.675\times10^{-27}\times1.4\times1.4\times10^{-20}) = 6.69\times10^{-21}\,J$

Illustration 5

An electron and a photon have got the same de-Broglie wavelength. Prove that total energy of electron is greater than energy of photon.

Solution

Total energy of electron $E_e = mc^2$ and therefore $\dfrac{E_e}{E_p} = \dfrac{hc^2}{hv}\times\dfrac{\lambda}{hc} = \dfrac{c}{v}$

Q $\quad c > v \quad$ So, $E_e > E_p$

Illustration 6

If λ_e and λ_P denote the de-Broglie wavelength of electron and proton after they are accelerated from rest through potential difference V_0 then find the relation between λ_e and λ_p.

Solution

Q $\quad$ De-Broglie wavelength $\lambda = \dfrac{h}{\sqrt{2mE}} = \dfrac{h}{\sqrt{2mqV_0}}$

Here v_0 is constant so $\lambda \propto \dfrac{1}{\sqrt{m}}$

Q $\quad m_p > m_e$

$\therefore \quad \lambda_e > \lambda_p$

Or

Q $\quad \lambda = \dfrac{h}{\sqrt{2mE}} = \dfrac{h}{\sqrt{2mqV_0}}$

$\therefore \quad \dfrac{\lambda_e}{\lambda_p} = \sqrt{\dfrac{m_p}{m_e}}$

Illustration 7

If λ_e and λ_P denote the de-Broglie wavelength of electron and proton after they are accelerated from rest through potential difference V_0 then find the relation between λ_e and λ_p.

Solution

$\because \quad \lambda = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mqV}}$

$\therefore \quad \lambda \propto \frac{h}{\sqrt{mq}}$

$\Rightarrow \quad \lambda_p : \lambda_d : \lambda_\alpha = \frac{1}{m_p q_p} : \frac{1}{m_d q_d} : \frac{1}{m_\alpha q_\alpha} = \frac{1}{\sqrt{m_p q_p}} : \frac{1}{\sqrt{2m_p q_p}} : \frac{1}{\sqrt{4m_p 2q_p}} = 1 : \frac{1}{\sqrt{2}} : \frac{1}{2\sqrt{2}}$

BEGINNER'S BOX-3

1. de-Broglie wavelength of an electron, accelerated by potential V is λ. What will be de-Broglie wavelength of the electron which is accelerated by 4V potential?

2. Find the ratio of de Broglie wavelength of molecules of hydrogen and helium which are at temperatures 27°C and 127°C respectively.

3. A particle of mass M at rest decays into two particles of masses m_1 and m_2 having non zero velocities. Find out the ratio of the de-Broglie wavelengths of the two particles.

4. Find out velocity of an electron so that its momentum is equal to that of photon with a wavelength of λ= 5200 Å.

5. Find out voltage applied to an electron microscope to produce electron of wavelength 0.6Å.

6. What is the effective mass of a photon having wavelength A? **[AIPMT 2006]**

7. A particle moving with velocity that is three times that of velocity of electron. If ratio of de-Broglie wave length of particle to that of the electron is 1.8×10^{-4}. Find mass of particle ($m_e = 9.1 \times 10^{-31}$ kg) **[AIPMT 2008]**

ANSWERS

BEGINNER'S BOX-1

1.	Hertz	**2.**	1.6×10^{-24} kg-m/s	**3.**	(a) 2.5×10^{19}	(b) 3.33×10^{-8} N	
4.	Violet	**5.**	1.5×10^{34}	**6.**	2×10^{20}	**7.**	9×10^{24}

BEGINNER'S BOX-2

1.	No	**2.**	64 unit	**3.**	0.34 V	**4.**	2V
5.	4λ	**6.**	0.48 μA, 3×10^{12}	**7.**	1.5 X	**8.**	6.8 eV
9.	$3K + h\nu_0$	**10.**	1.7 eV				

BEGINNER'S BOX-3

1.	$\frac{\lambda}{2}$	**2.**	$\lambda_{H_2} : \lambda_{He} = \sqrt{8} : \sqrt{3}$	**3.**	1 : 1	**4.**	1400 m/s
5.	416.6 volt	**6.**	$m = \frac{h}{\lambda c}$	**7.**	1.68×10^{-27} kg		