ALTERNATING CURRENT

ALTERNATING CURRENT AND VOLTAGE

Voltage or current is said to be alternating if it changes continously in magnitude and periodically in direction. It can be represented by a sine curve or cosine curve

 $I = I_0 \sin \omega t \quad \text{or} \quad I = I_0 \cos \omega t$ where I = Instantaneous value of current at time t, I_0 = Amplitude or peak value $\omega = \text{Angular frequency } \omega = \frac{2\pi}{T} = 2\pi f \qquad T = \text{time period } f = \text{frequency}$ $I_0 = \frac{1}{\frac{T}{2} + \frac{3}{4}T} + \frac{1}{\frac{T}{4} + \frac{1}{\frac{T}{4} + \frac{1}{4}} + \frac{1}{\frac{T}{4} + \frac{1}{\frac{T$

1. Amplitude of AC

The maximum value of current in either direction is called peak value or the amplitude of current. It is represented by I_0 . Peak to peak value = $2I_0$

2. Periodic Time

The time taken by alternating current to complete one cycle of variation is called periodic time or time period of the current.

3. Frequency

The number of cycle completed by an alternating current in one second is called the frequency of the current.

UNIT : (cycle/s) or (Hz)

In India : f = 50 Hz, supply voltage = 220 volt

In USA : f = 60 Hz, supply voltage = 110 volt

4. **Condition required for current/voltage to be Alternating**

- Amplitude is constant. Alternate half cycle is positive and half negative.
- The alternating current continuously varies in magnitude and periodically reverses its direction.



5. Average Value of Mean Value

The mean value of A.C over any half cycle (either positive or negative) is that value of DC which would send same amount of charge through a circuit as is sent by the AC through same circuit in the same time.

average value of current for half cycle $\langle I \rangle = \frac{\int_{0}^{1/2} I dt}{\int_{0}^{T/2} \int dt}$

Average value of $I = I_0 \sin \omega t$ over the positive half cycle :

$$I_{av} = \frac{\int_{0}^{\frac{1}{2}} I_{0} \sin \omega dt}{\int_{0}^{\frac{T}{2}} dt} = \frac{2I_{0}}{\omega T} \left[-\cos \omega t \right]_{0}^{\frac{T}{2}} = \frac{2I_{0}}{\pi}$$

- $<\sin\theta> = <\sin 2\theta> = 0$ $<\cos\theta> = <\cos 2\theta> = 0$ $<\sin\theta\cos\theta> = 0$ $<\sin^{2}\theta> = <\cos^{2}\theta> = \frac{1}{2}$
- For symmetric AC, average value over full cycle = 0, Average value of sinusoidal AC

Full cycle	(+ve) half cycle	(-ve) half cycle	
0	$2I_0$	$-2I_{0}$	
0	π	π	

As the average value of AC over a
complete cycle is zero, it is always
defined over a half cycle which must
be either positive or negative

- 6. Maximum Value
 - $I = a \sin \theta \implies I_{Max.} = a$ $I = a + b \sin \theta \implies I_{Max.} = I_{Max.} = a + b$ (if a and b > 0)
 - $I = a \sin \theta + b \cos \theta \Rightarrow I_{Max.} = I_{Max.} = I_{Max.} = \sqrt{a^2 + b^2}$ $I = a \sin^2 \theta \Rightarrow I_{Max.} = a (a > 0)$

7. Root Mean square (rms) Value

It is value of DC which would produce same heat in given resistance in given time as is done by the alternating current when passed through the same resistance for the same time.

$$I_{rms} = \sqrt{\frac{\int_{0}^{T} I^{2} dt}{\int_{0}^{T} dt}} rms value = virtual value = apparent value$$

rms value of $I = I_0 \sin \omega t$:

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$$I_{\rm rms} = \sqrt{\frac{\int_{0}^{T} (I_0 \sin \omega t)^2 dt}{\int_{0}^{T} dt}} = \sqrt{\frac{I_0^2}{T} \int_{0}^{T} \sin^2 \omega t} dt = I_0 \sqrt{\frac{1}{T} \int_{0}^{T} \left[\frac{1 - \cos 2\omega t}{2}\right]} dt$$
$$= I_0 \sqrt{\frac{1}{T} \left[\frac{t}{2} - \frac{\sin 2\omega t}{2 \times 2\omega}\right]_{0}^{T}} = \frac{I_0}{\sqrt{2}}$$

• If nothing is mentioned then values printed in a.c. circuit on electrical appliances, any given or unknown values, reading of AC meters are assumed to be RMS.

Current	Average	Peak	RMS	Angular frequency
$I_1 = I_0 \sin \omega t$	0	I_0	\mathbf{I}_{0}	ω
			$\sqrt{2}$	
$I_2 = I_0 \sin \omega t \cos \omega t = \frac{I_0}{2} \sin 2\omega t$	0	$\frac{I_0}{2}$	$\frac{I_0}{2\sqrt{2}}$	2ω
$I_3 = I_0 \sin \omega t + I_0 \cos \omega t$	0	$\sqrt{2}$ I ₀	I_0	ω

• For above varieties of current

$$rms = \frac{Peak value}{\sqrt{2}}$$

8. Measurement of A.C.

Alternating current and voltages are measured by a.c. ammeter and a.c. voltmeter respectively. Working of these instruments is based on heating effect of current, hence they are also called hot wire instruments.

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Terms	D.C. meter	A.C. meter
Name	moving coil instrument	hot wire instrument
Based on	magnetic effect of current	heating effect of curent
Reads	average value	r.m.s. value
If used in	A.C. circuit then they reads zero	A.C. or D.C. then meter works
	Θ average value of A.C. = zero	properly as it measures rms value
Deflection	deflection \propto current	deflection ∞ heat
2	$\phi \propto I$ (linear)	$\phi \propto I^2$ (non linear)
Scale	Uniform Separation	$\psi \propto 1 \text{ rms}$ (non inical)
~~~~~	$I - 1 \ 2 \ 3 \ 4 \ 5$	I 1 2 3 4 4
$\phi$ = Number	$\phi - 1 \ 2 \ 3 \ 4 \ 5$	
of division		φ-1 4 9 16 25

#### 9. Phase and phase difference

(a) Phase

 $I = I_0 \sin (\omega t \pm \phi)$ Initial phase =  $\phi$  (it does not change with time) Instantaneous phase =  $\omega t \pm \phi$  (it changes with time)

Phase decides both value and sign.
 UNIT : radian

#### (b) Phase difference

Voltage V = V₀ sin (  $\omega t + \phi_1$ )

Current  $I = I_0 \sin(\omega t + \phi_2)$ 

- Phase difference of I w.r.t. V  $\phi = \phi_2 \phi_1$
- Phase difference of V w.r.t I  $\phi = \phi_1 \phi_2$

#### 10. Lagging and loading Concept

(a) V lead I or I lags V  $\rightarrow$  It means, V reach maximum before I

Let if  $V = V_0 \sin \omega t$  then  $I = I_0 \sin (\omega t - \phi)$ and if  $V = V_0 \sin (\omega t + \phi)$  then  $I = I_0 \sin \omega t$ 

(b) V lags I or I leads  $V \rightarrow It$  means V reach maximum after I Let if  $V = V_0 \sin \omega t$  then  $I = I_0 \sin (\omega t + \phi)$ and if  $V = V_0 \sin (\omega t + \phi)$  then  $I = I_0 \sin \omega t$ 





#### 11. Phasor and Phasore diagram

A diagram representing alternating current and voltage (of same frequency) as vectors (phasor) with the phase angel between them is called phasor diagram.

Let  $V = V_0 \sin \omega t$ and  $I = I_0 \sin(\omega t + \phi)$ In figure (a) two arrows represents phasors. The length of phasors represents the maximum value of quantity. The projection of a phasor on y-axis represents the instantaneous value of quantity. in figure (b) two arrows represents phasor. Their length represents maximum value.



#### 12. Advantages of AC

- A.C. is cheaper than D.C
- It can be easily converted into D.C. (by rectifier)
- It can be controlled easily (choke coil)
- It can be transmitted over long distance at low power loss.
- It can be stepped up or stepped down with the help of transformer.

#### **GOLDEN KEY POINTS**

- AC can't be used in
  - (a) Charging of battery or capacitor (as its average value = 0)
  - (b) Electrolysis and electroplating (Due to large inertia, ions can not follows frequency of A.C)
  - -Minimum, at that instant when they are near their peak values The rate of change of A.C. –
- Maximum, at that instant when they change their direction.
- For alternating current  $I_0 > I_{rms} > I_{av}$ .
- Average value over half cycle is zero if one quarter is positive and the other quarter is negative.

Average value of symmetrical AC for a cycle is zero thats why average potential difference on any element in A.C circuit is zero.

- The instrument based on heating effect of current are works on both A.C and D.C supply and also provides same value of A.C (rms) and D.C that's why a bulb bright equally in D.C. and A.C. of same value.
- If the frequency of AC is f then it becomes zero, 2f times in one second and the direction of current changes 2f times in one second. Also it become maximum 2f times in one second.
- Some Important wave forms and their RMS and Average Value

Nature of wave from	Wave-form	RMS Value	Average or mean Value
Sinusoidal	$0 + \pi - 2\pi$	$\frac{I_0}{\sqrt{2}} = 0.707 I_0$	$\frac{2I_0}{\pi} = 0.637 I_0$ (Half)

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Half wave rectifier	$0 \frac{1}{\pi} \frac{1}{2\pi}$	$\frac{I_0}{2} = 0.5 I_0$	$\frac{I_0}{\pi} = 0.318 I_0$ (Full)
Full wave rectifier	0 π 2π	$\frac{I_0}{\sqrt{2}} = 0.707 I_0$	$\frac{2I_0}{\pi} = 0.637 I_0$
Square or Rectangular	+	I ₀	I ₀ (Half)
Saw Tooth wave	0 π 2π	$\frac{I_0}{\sqrt{3}}$	$\frac{I_0}{2}$ (Half)

- D.C meter in AC circuit reads zero because  $\langle AC \rangle = 0$  (for complete cycle)
- AC meter works in both AC and DC

# **ILLUSTRATIONS**

# **Illustration 1.**

If  $I = 2\sqrt{t}$  ampere then calculate average and rms values over t = 2 to 4 s **Solution** 

$$\langle \mathbf{I} \rangle = \frac{\int_{2}^{2} 2\sqrt{t} dt}{\int_{2}^{4} dt} = \frac{4}{3} \frac{(t^{\frac{3}{2}})_{2}^{4}}{(t)_{2}^{4}} = \frac{2}{3} \left[ 8 - 2\sqrt{2} \right] \text{ and } \mathbf{I}_{\text{rms}} = \sqrt{\frac{\int_{2}^{4} (2\sqrt{t})^{2} dt}{\int_{2}^{4} dt}} = \sqrt{\frac{1}{2} \left[ \frac{t^{2}}{2} \right]_{2}^{4}} = 2\sqrt{3} \mathbf{A}$$

# **Illustration 2.**

If E = 20 sin (100
$$\pi$$
 t) volt then calculate value of E at t =  $\frac{1}{600}$  s

Solution

At 
$$t = \frac{1}{600}s$$
  $E = 20sin\left[100\pi \times \frac{1}{600}\right] = 20sin\left[\frac{\pi}{6}\right] = 20 \times \frac{1}{2} = 10V$ 

#### **Illustration 3.**

A periodic voltage wave form has been shown in figure. Determine.

(a) Frequency of the wave form. (b) Average value.

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#### Solution

(a) After 100 ms wave is repeated so time period is T = 100 ms.  $\Rightarrow$  f =  $\frac{1}{T}$  = 10 Hz

(b) Average value = Area/time period = 
$$\frac{(1/2) \times 100 \times 10}{(100)}$$
 = 5 volt

#### **Illustration 4.**

If a direct current of value of ampere is superimposed on an alternating current  $I = b \sin \omega t$  flowing through a wire, what is the effective value of the resulting current in the circuit?



#### Solution

As current at any instant in the circuit will be  $I = I_{DC} + I_{AC} = a + b \sin \omega t$ 

$$\therefore I_{eff} = \sqrt{\frac{1}{T} \int_{0}^{T} I^{2} dt} = \sqrt{\frac{1}{T} \int_{0}^{T} (a + b \sin \omega t)^{2} dt} = \sqrt{\frac{1}{T} \int_{0}^{T} (a^{2} + 2ab \sin \omega t + b^{2} \sin^{2} \omega t) dt}$$
  
but as  $\frac{1}{T} \int_{0}^{T} \sin \omega t dt = 0$  and  $\frac{1}{T} \int_{0}^{T} \sin^{2} \omega dt = \frac{1}{2}$  
$$\therefore I_{eff} = \sqrt{a^{2} + \frac{1}{2}b^{2}}$$

#### **Illustration 5.**

The equation of current in AC circuit is I =  $4\sin\left[100\pi t + \frac{\pi}{3}\right]A$ . Calculate.

(i) RMS Value (ii) Peak Value (iii) Frequency (iv) Initial phase (v) Current at t = 0 Solution

(i) 
$$I_{\rm rms} = \frac{I_0}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2} \text{ A}$$

(ii) Peak value 
$$I_0 = 4A$$

(iii) 
$$\Theta$$
  $\omega = 100 \pi \text{ rad/s}$   $\therefore$  frequency  $f = \frac{100\pi}{2\pi} = 50 \text{ Hz}$ 

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(iv) Initial phase = 
$$\frac{\pi}{3}$$

(v) At t = 0, I = 
$$4\sin\left[100\pi \times 0 + \frac{\pi}{3}\right] = 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}$$
 A

#### **Illustration 6.**

If I = I₀ sin 
$$\omega t$$
, E = E₀cos  $\left[\omega t + \frac{\pi}{3}\right]$ . Calculate phase difference between E and I

#### Solution

I = I₀ sin 
$$\omega t$$
 and E = E₀ sin  $\left[\frac{\pi}{2} + \omega t + \frac{\pi}{3}\right]$   $\therefore$  phase difference =  $\frac{\pi}{2} + \frac{\pi}{3} = \frac{5\pi}{6}$ 

#### **Illustration 7.**

If  $E = 500 \sin (100 \pi t)$  volt then calculate time taken to reach from zero to maximum. Solution.

$$\Theta \omega = 100 \pi \Rightarrow T = \frac{2\pi}{100\pi} = \frac{1}{50} s$$
, time taken to reach from zero to maximum  $= \frac{T}{4} = \frac{1}{200} s$ 

#### **Illustration 8.**

If Phase Difference between E and I is  $\frac{\pi}{4}$  and f = 50 Hz then calculate time difference.

#### Solution

 $\Theta \qquad 2\pi \equiv T \therefore \frac{\text{Phase difference}}{2\pi} = \frac{\text{time difference}}{T}$  $\Rightarrow \qquad \text{Time difference} = \frac{T}{2\pi} \times \frac{\pi}{4} = \frac{T}{8} = \frac{1}{50 \times 8} = 2.5 \text{ ms}$ 

#### **BEGINNER'S BOX-1**

- **1.** Explain why A.C. is more dangerous than D.C. ?
- 2. Show that average heat produced during a cycle of AC is same as produced by DC with  $I = I_{rms}$ .
- **3.** An ordinary moving coil ammeter us2ed for d.c., cannot be used to measure a.c. even if its frequency is low why?
- 4. Find the time required for a 50Hz alternating current to change its value from zero to rms value.
- 5. The current and voltage in a circuit is given by
  - $i = 3.5 \sin (628t + 30^{\circ}) A$ ,  $V = 28 \sin (628t 30^{\circ})$  volt. Find
  - (a) time period of current
  - (b) phase difference between voltage and current.
- 6. The figure given below shows the variation of an alternating emf with time. What is the average value of the emf for the shaded part of the graph?



#### DIFFERENT TYPES OF AC CIRCUITS

In order to study the behaviour of A.C. circuits we classify them into two categories :

- (a) Simple circuits containing only one basic element i.e. resistor (R) or inductor (L) or capacitor (C) only.
- (b) Complicated circuit containing any two of the three circuit elements R, L and C or all of the three elements.

#### 1. AC circuit containing pure resistance

Let at any instant t, the current in the circuit = I. Potential difference across the resistance = IR with the help of kirchoff's circuital law  $E - I R = 0 \Rightarrow E_0 \sin \omega t = IR$ 

 $\Rightarrow I = \frac{E_0}{R} \sin \omega t = I_0 \sin \omega t \ (I_0 = \text{peak or maximum value of current})$ 

Alternating current developed in a pure resistance is also of the sinusoidal nature. In a.c. circuits containing pure resistance, the voltage and current are in the same phase. The vector or phasor diagram which represents the phase relationship between alternating current and alternating e.m.f. are as shown in figure.





In the a.c. circuit having R only, as current and voltage are in the same phase, hence in fig. both phasors  $E_0$  and  $I_0$  are in the same direction, making an angle  $\omega t$  with OX. Their projection on Y-axis represent the instantaneous values of alternating current and voltage.

i.e. 
$$I = I_0 \sin \omega t$$
 and  $E = E_0 \sin \omega t$   
Since  $I_0 = \frac{E_0}{R}$ , hence  $\frac{I_0}{\sqrt{2}} = \frac{E_0}{R\sqrt{2}} \implies I_{rms} = \frac{E_{rms}}{R}$ 

# $E \xrightarrow{\varphi} E_{0}$ $I \xrightarrow{\varphi} I_{0}$ $Q \xrightarrow{\varphi} W \xrightarrow{\varphi} X$

# $E = E_0 \sin \omega t$

#### 2. AC circuit containing pure inductance

A circuit containing a pure inductance L (having zero ohmic resistance) connected with a source of alternating emf. Let the alternating e.m.f.  $E = E_0 \sin \omega t$ 

When a.c. flows through the circuit, emf induced across inductance =  $-L\frac{dI}{dt}$ 

# Note : Negative sign indicates that induced emf acts in opposite direction to that of applied emf.

Because there is no other circuit element present in the circuit other than inductance so with the

help of Kirchoff's circuital law  $E + \left(-L\frac{dI}{dt}\right) = 0 \implies E = L\frac{dI}{dt}$  so we get  $I = \frac{E_0}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$ 

Maximum current 
$$I_0 = \frac{E_0}{\omega L} \times 1 = \frac{E_0}{\omega L}$$
,

Hence,  $I = I_0 \sin \left( \omega t - \frac{\pi}{2} \right)$ In a pure inductive circuit current always lags behind the emf by  $\frac{\pi}{2}$  or alternating emf leads the a.c. by a

phase angle of  $\frac{\pi}{2}$ .

Expression  $I_0 = \frac{E_0}{\omega L}$  resembles the expression  $\frac{E}{I} = R$ .

This non-resistive opposition to the flow of A.C. in a circuit is called the inductive reactance  $(X_L)$  of the circuit.

 $X_L = \omega L = 2 \pi f L$  where f = frequency of A.C. Unit of  $X_L$ : ohm

$$(\omega L) = \text{Unit of } L \times \text{Unit of } (\omega = 2\pi f) = \text{henry} \times \text{sec} - 1$$
$$= \frac{\text{volt}}{\text{ampere/sec}} \times \text{sec}^{-1} = \frac{\text{volt}}{\text{ampere}} = \text{ohm}$$



Higher the frequency of A.C. higher is the inductive reactance offered by an inductor in an A.C. circuit.

**For d.c. circuit,**  $\mathbf{f} = \mathbf{0}$   $\therefore$   $X_L = \omega L = 2 \pi f L = 0$ 

Hence, inductor offers no opposition to the flow of d.c. where as a resistive path to a.c.

#### **3.** AC circuit containing pure capacitance

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A circuit containing an ideal capacitor of capacitance C connected with a source of alternating emf as shown in fig. The alternating e.m.f. in the circuit  $E = E_0 \sin \omega t$ .

When alternating e.m.f. is applied across the capacitor a similarly varying alternating current flows in the circuit.

The two plates of the capacitor become alternately positively and negatively charged and the magnitude of the charge on the plates of the capacitor varies sinusoidally with time. Also the electric field between the plates of the capacitor varies sinusoidally with time.

Let at any instant t charge on the capacitor = q

Instantaneous potential difference across the capacitor  $\mathbf{E} = \mathbf{q}/\mathbf{C}$ 

 $\Rightarrow q = CE \Rightarrow q = CE_0 \sin \omega t$ 

The instantane3ous value of current  $I = \frac{dq}{dt} = \frac{d}{dt} (CE_0 \sin \omega t)$ 

 $= CE_0 \omega \cos \omega t$ 

$$\Rightarrow I = \frac{E_0}{(1/\omega C)} \sin\left(\omega t \frac{\pi}{2}\right) = I_0 \sin\left(\omega t + \frac{\pi}{2}\right) \text{ where } I0 = \omega CE_0$$

In a pure capacitive circuit, the current always leads the e.m.f. by a phase angle of  $\pi/2$ . The alternating emf lags behinds the alternating current by a phase angle of  $\pi/2$ .

#### **IMPORTANT POINTS**

E/I is the resistance R when both E and I are in phase, in present case they differ in phase by  $\frac{\pi}{2}$ , hence  $\frac{1}{\omega C}$  is not the resistance of the capacitor, the capacitor offer opposition to the flow of A.C. This non-resistive opposition to the flow of A.C. in a pure capacitive circuit is known as capacitive reactance X_C.

$$X_{\rm C} = \frac{1}{\omega \rm C} = \frac{1}{2\pi \rm f \rm C}$$

#### Unit of XC : ohm

Capacitive reactance  $X_C$  is inversely proportional to frequency of A.C.  $X_C$  decreases as the frequency increases.

**For d.c. circuit f = 0**  $\therefore$   $X_{C} = \frac{1}{2\pi fC} = \infty$  but has a very small value for a.c.

This shows that capacitor blocks the flow of d.c. but provides an easy path for a.c.

#### **INDIVIDUAL COMPONENTS (R or L or C)**







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#### 4. Resistance and inductance in series (R–L circuit)

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A circuit containing a series combination of a resistance R and an inductance L, connected with a source of alternating e.m.f. E as shown in figure.



#### • Phasor diagram For L-R circuit

Let in a L–R series circuit, applied alternating emf is  $E = E_0 \sin \omega t$ . As R and L are joined in series, hence current flowing through both will be same at each instant. Let I be the current in the circuit at any instant and V_L and V_R the potential differences across L and R respectively at that instant. Then V_L = IX_L and V_R = IR

Now,  $V_R$  is in phase with the current while  $V_L$  leads the current by  $_{\pi}$ 

$$\overline{2}$$

So  $V_R$  and  $V_L$  are mutually perpendicular (Note :  $E \neq V_R + V_L$ ) The vector OP represents  $V_R$  (which is in phase with I), while OQ represents VL (which leads I by 90°).

The resultant of  $V_R$  and  $V_L$  = the magnitude of vector OR  $E = \sqrt{V_R^2 + V_L^2}$ 

Thus 
$$E^2 = V_R^2 + V_L^2 = I^2(R^2 + X_L^2) \Rightarrow I = \frac{E}{\sqrt{R^2 + X_L^2}}$$

The phasor diagram shown in fig. also shows that in L-R circuit the applied emf E leads the current I or conversely the current I lags behind the e.m.f. E by a phase angle  $\phi$ 

$$\tan \phi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R} = \frac{\omega L}{R} \implies \phi = \tan^{-1} \left(\frac{\omega L}{R}\right)$$



E

V_P

V,

#### **Inductive Impedance ZL :**

In L–R circuit the maximum value of current  $I_0 = \frac{E_0}{\sqrt{R^2 + \omega^2 L^2}}$  Here  $\sqrt{R^2 + \omega^2 L^2}$  represents the effective opposition offered by L–R circuit to the flow of a.c. through it. It is known as impedance of L–R circuit and is represented by Z_L.  $Z_L = \sqrt{R^2 + \omega^2 L^2} = \sqrt{R^2 + (2\pi fL)^2}$ 

The reciprocal of impedance is called admittance  $Y_L = Y_L = \frac{1}{Z_L} = \frac{1}{\sqrt{R^2 + \omega^2 L^2}}$ 

#### 5. Resistance and capacitor in series (R–C circuit)

A circuit containing a series combination of a resistance R and a capacitor C, connected with a source a source of e.m.f. of peak value  $E_0$  as shown in fig.

#### • Phasor diagram for R-C circuit

Current through both the resistance and capacitor will be same at every instant and the instantaneous potential differences across C and R are

$$V_C = I X_C$$
 and  $V_R = IR$ 

where  $X_C$  = capacitive reactance and I = instantaneous current. Now,  $V_R$  is in phase with I, while  $V_C$  lags behind I by 90°.

The phasor diagram is shown in fig.

The vector OP represents  $V_R$  (which is in phase with I) and the

vector OQ represents V_C (which lags behind I by  $\frac{\pi}{2}$ ).

The vector OS represents the resultant of  $V_R$  and  $V_C$  = the applied e.m.f. E.

Hence 
$$V_R^2 + V_C^2 = E^2 \implies E = \sqrt{V_R^2 + V_C^2} \implies E^2 = I2 (R^2 + X_C^2) \implies I = \frac{E}{\sqrt{R^2 + X_C^2}}$$

The term  $\sqrt{(R^2 + X_C^2)}$  represents the effective resistance of the R–C circuit and called the capacitive impedance  $Z_C$  of the circuit.

Hence, in C–R circuit 
$$Z_{C} = \sqrt{R^{2} + X_{C}^{2}} = \sqrt{R^{2} + \left(\frac{1}{\omega C}\right)^{2}}$$







## Capacitive Impedance Z_C :

In R–C circuit the term  $\sqrt{R^2 + X_C^2}$  effective opposition offered by R–C circuit to the flow of a.c. through it. It is known as impedance of R–C circuit and is represented by Z_C. The phasor diagram also shows that in R–C circuit the applied e.m.f. lags behind the current I (or the current I leads the emf E) by a phase angle  $\phi$  given by

$$\tan \phi = \frac{V_{\rm C}}{V_{\rm R}} = \frac{X_{\rm C}}{R} = \frac{1/\omega C}{R} = \frac{1}{\omega CR}, \ \tan \phi = \frac{X_{\rm C}}{R} = \frac{1}{\omega CR} \implies \phi = \tan^{-1} \left(\frac{1}{\omega CR}\right)$$

6. Combination of components (R–L or R–C or L–C)

TERM	R–L	R–C	L–C	
	•		•	

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#### **GOLDEN KEY POINTS**

- Phase difference between capacitive and inductive reactance is  $\pi$ .
- Inductor is called Low pass filter because it allows low frequency signal to pass.
- Capacitor is called high pass filter because it allows high frequency signal to pass.

# ILLUSTRATIONS

#### **Illustration 9.**

what is the inductive reactance of a coil if the current through it is 20 mA and voltage across it is 100 V.

#### Solutions

$$\Theta$$
  $V_L = IX_L$   $\therefore X_L = \frac{V_L}{I} = \frac{100}{20 \times 10^{-3}} = 5 \text{ k}\Omega$ 

#### **Illustration 10.**

The reactance of capacitor is 20 ohm. What does it mean? What will be its reactance if frequency of AC is doubled? What will be its, reactance when connected in DC circuit? What is its consequence?

#### Solution

The reactance of capacitor is 20 ohm. It means that the hinderance offered by it to the flow of AC at a specific frequency is equivalent to a resistance of 20 ohm. The reactance of capacitance

$$X_{\rm C} = \frac{1}{\omega \rm C} = \frac{1}{2\pi \rm f \rm C}$$

Therefore by doubling frequency, the reactance is halved i.e., it becomes 10 ohm. In DC circuit f = 0. Therefore reactance of capacitor  $= \infty$  (infinite). Hence the capacitor can not be used to control DC/

#### **Illustration 11.**

A capacitor of 50 pF is connected to an a.c. source of frequency 1kHz. Calculate its reactance. Solution

$$X_{\rm C} = \frac{1}{\omega {\rm C}} = \frac{1}{2\pi \times 10^3 \times 50 \times 10^{-12}} = \frac{10^7}{\pi} = \Omega$$

#### **Illustration 12.**

In given circuit applied voltage V =  $50\sqrt{2} \sin(100\pi t)$  volt and ammeter reading is 2A then calculate value of L



#### Solution

 $V_{\rm rms} = I_{\rm rms} X_{\rm L} \qquad \Theta \text{ Reading of ammeter} = I_{\rm rms}$  $X_{\rm L} = \frac{V_{\rm rms}}{I_{\rm rms}} = \frac{V_0}{\sqrt{2} I_{\rm rms}} = \frac{50\sqrt{2}}{\sqrt{2} \times 2} = 25 \ \Omega \Rightarrow L = \frac{X_{\rm L}}{\omega} = \frac{25}{100\pi} = \frac{1}{4\pi} \text{ H}$ 

#### **Illustration 13.**

A 50 W, 100 V lamp is to be connected to an AC mains of 200 V, 50 Hz. What capacitance is essential to be put in series with the lamp?

#### Solution

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30Ω

40Ω

Θ resistance of the lamp R = 
$$\frac{V_s^2}{W} = \frac{(100)^2}{50} = 200 \ \Omega$$
 and the maximum current V = 100 = 1

$$I = \frac{V}{R} = \frac{100}{200} = \frac{1}{2}A$$

 $\therefore$  when the lamp is put in series with a capacitance and run at 200 V AC, from V = IZ

$$Z = \frac{V}{I} = \frac{200}{\frac{1}{2}} = 400 \ \Omega$$
 Now as in case of C-R circuit  $Z = \sqrt{R^2 + \frac{1}{(\omega C)^2}}$ ,  

$$\Rightarrow R^2 + \frac{1}{(\omega C)^2} = (400)^2 \Rightarrow \frac{1}{(\omega C)^2} = 16 \times 10^4 - (200)^2 = 12 \times 10^4 \Rightarrow \frac{1}{\omega C} = \sqrt{12} \times 10^2$$

$$\Rightarrow C = \frac{1}{100\pi \times \sqrt{12} \times 10^2} F = \frac{100}{\pi \sqrt{12}} \mu F = 9.2 \mu F$$

#### **Illustration 14.**

Calculate the impedance of the circuit shown in the figure.

#### Solution

$$Z = \sqrt{R^2 + (X_c)^2} = \sqrt{(30)^2 + (40)^2} = \sqrt{2500} = 50 \ \Omega$$

#### **Illustration 15.**

If  $X_L = 50 \Omega$  and  $X_C = 40 \Omega$ . Calculate effective value of current in given circuit.



Solution

$$Z = X_L - X_C = 10 \Omega$$
  
 $I_0 = \frac{V_0}{Z} = \frac{40}{10} = 4 A \implies I_{rms} = \frac{4}{\sqrt{2}} = 2\sqrt{2} A$ 

# **Illustration 16.**

In given circuit calculate, voltage across inductor

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#### Solution

$$\Theta V^{2} = V_{R}^{2} + V_{L}^{2} \qquad \therefore \quad V_{L}^{2} = V^{2} - V_{R}^{2}$$
$$V_{L} = \sqrt{V^{2} - V_{R}^{2}} = \sqrt{(100)^{2} - (60)^{2}} = \sqrt{6400} = 80 \text{ V}$$

#### **Illustration 17.**

In given circuit find out (i) impedance of circuit (ii) current in circuit



#### Solution

(i) 
$$Z = \sqrt{R^2 + X_c^2} = \sqrt{(6)^2 + (8)^2} = 10 \Omega$$
  
(ii)  $V = IZ \Rightarrow I = \frac{V_0}{Z} = \frac{20}{10} = 2A$ , so  $I_{rms} = \frac{2}{\sqrt{2}} = \sqrt{2}A$ 

#### **Illustration 18.**

When 10V, DC is applied across a coil current through it is 2.5 A, if 10V, 50 Hz A.C. is applied current reduce to 2 A. Calculate reactance of the coil.

#### Solution

For 10 V D.C. 
$$\Theta$$
 V = IR  $\therefore$  Resistance of coil R =  $\frac{10}{2.5} = 4\Omega$   
For 10 V A.C. V = IZ  $\Rightarrow$  Z =  $\frac{V}{I} = \frac{20}{10} = 5\Omega$   
 $\Theta$  Z =  $\sqrt{R^2 + X_L^2} = 5 \Rightarrow R^2 + X_L^2 = 25 \Rightarrow X_L^2 = 5^2 - 4^2 \Rightarrow X_L = 3 \Omega$ 

#### **Illustration 19.**

When an alternating voltage of 220V is applied across a device X, a current of 0.5 A flows through the circuit and is in phase with the applied voltage. When the same voltage is applied across another device Y, the same current again flows through the circuit but it leads the applied voltage by  $\pi/2$  radians.

(a) Name the device X and Y.

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(b) Calculate the current flowing in the circuit when same voltage is applied across the series combination of X and Y.

#### Solution

- (a) X is resistor and Y is a capacitor
- (b) Since the current in the two devices is the same (0.5A at 220 volt) When R and C are in series across the same voltage then

$$R = X_{C} = \frac{220}{0.5} = 440 \ \Omega \Rightarrow I_{rms} = \frac{V_{rms}}{\sqrt{R^{2} + X_{C}^{2}}} = \frac{220}{\sqrt{(440)^{2} + (440)^{2}}} = \frac{220}{440\sqrt{2}} = 0.35 \text{ A}$$

#### **BEGINNER'S BOX-2**

- 1. A voltage V = 60 sin  $\pi$ t volt is applied across a 20  $\Omega$  resistor. What will an ac ammeter in series with the resistor read?
- 2. An alternating current source  $E = 100 \sin (1000t)$  volt is connected through a inductor of 10  $\mu$ H then write down the equation of current.
- 3. An alternating voltage  $E = 200\sqrt{2} \sin (100t)$  volt is applied to 2H inductor through an a.c. ammeter. What will be reading of the ammeter?
- 4. A 15.0  $\mu$ F capacitor is connected to a 220 V, 50 Hz source. Find the capacitive reactance and the current (rms and peak) in the circuit. If the frequency is doubled, what happens to the capacitive reactance and the current?
- 5. A 60  $\mu$ F capacitor is connected to a 110 V. 60 Hz a.c. supply. Determine the rms value of the current in the circuit.
- 6. The given graphs (a) and (b) represent the variation of the opposition offered by the circuit element to the flow of alternating current, with frequency of the applied emf. Identify the circuit element corresponding to each graph.



 $314\Omega$ 

- 7. When a series combination of inductance and resistance are connected with a 1 OV, 50 Hz a.c. source, a current of IA flows in the circuit. The voltage leads the current by a phase angle of  $\frac{\pi}{2}$ radian. Calculate the values of resistance and inductive reactance.
- 8. Calculate the impedance of a condenser in order to run a bulb rated at 10 watt 60 volt when connected in series to an A. C. source of 100 volt.
- The current in the shown circuit is found to be  $4\sin\left(314t-\frac{\pi}{4}\right)A$ . 9. Find the value of inductance. V= V_sin314t
- 10. A current of 4A flows in a coil when connected to a 12V de source. If the same coil is connected to a 12V, 50 rad/s a.c. source, a current of 2.4A flows in the circuit. Determine the inductance of the coil.
- 11. An alternating voltage E=200 sin (300t) volt is applied across a series combination of R=100 and inductance of 800mH. Calculate the impedance of the circuit.
- 12. A coil of reactance 100  $\Omega$  and resistance 100  $\Omega$  is connected to a 240 V, 50 Hz a.c. supply.
  - What is the maximum current in the coil? (a)
  - (b) What is the time lag between the voltage maximum and the current maximum ?
- 13. An inductance has a resistance of 100  $\Omega$ . When a.c. signal of frequency 1000 Hz is applied to the coil, the applied voltage leads the current by 45°. Calculate the self inductance of the coil.
- 14. Match the following options –

Circuit component across an ac source	Phase difference between current and		
$(\omega = 220 \text{ rad/sec})$	source voltage		
(A) $-\underbrace{10\Omega}_{500\mu F}$	(p) $\frac{\pi}{2}$		
(B) 5H	$\begin{array}{c c} (q) & \frac{\pi}{6} \end{array}$		
(C) 500µF	(r) $\frac{\pi}{4}$		
(D) $\begin{array}{c} - & \hline \\ & 4H & 3\mu F \end{array}$	(s) $\frac{\pi}{3}$		

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(E)  $lk\Omega$  5H

(t) None of the above

#### INDUCTANCE, CAPACITANCE AND RESISTANCE IN SERIES

#### 1. L–C–R series circuit

A circuit containing a series combination of an resistance R, a toil of inductance Land a capacitor of capacitance C, connected with a source of alternating e.m.f. of peak value of  $E_0$ , as shown in figure.



#### • Phasor Diagram for Series L–C–R circuit

Let in series LCR circuit applied alternating emf is  $E = E_0 \sin wt$ . As L, C and R are joined in series, therefore, current at any instant through the three elements has the same amplitude and phase.

#### However voltage across each element bears a different phase relationship with the current.

Let at any instant of time t the current in the circuit is I.

Let at this time t the potential difference across L, C, and R

 $V_L = I X_L, V_C = I X_C \text{ and } V_R = IR$ 

Now,  $V_R$  is in phase with current I but  $V_L$  leads I by 90°

While  $V_C$  lags behind I by 90°.

 $\frac{1}{p} X$ 

X to

×

X

The vector OP: represents  $V_R$  (which is in phase with I) the vector OQ represent  $V_L$  (which leads I by 90°) and the vector OS represents  $V_C$  (which legs behind I by 90°)  $V_L$  and  $V_C$  are opposite to each other.

If  $V_L > V_C$  (as shown in figure) the their resultant will be  $(V_L - V_C)$  which is represented by OT. Finally, the vector OK represents the resultant of  $V_R$  and  $(V_L - V_C)$ , that is, the resultant of all the three ; applied e.m.f.

Thus 
$$E = \sqrt{V_R^2 + (V_L - V_C)^2} = I\sqrt{R_2 + (X_L - X_C)^2}$$
  
 $\Rightarrow I = \frac{E}{E}$ 

$$\Rightarrow I = \frac{E}{\sqrt{R^2 + (X_L - X_C)^2}}$$



Impedance 
$$Z = \sqrt{R2 + (X_L - X_C)^2} = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$$

The phasor diagram also shown that in LCR circuit the applied e.m.f. leads the current I by a phase angle  $\phi$  tan  $\phi = \frac{X_L - X_C}{R}$ 

2. Series LCR and parallel LCR combination



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#### 3. Resonance

A circuit is said to be resonant when the natural frequency of crcuit Is equal to frequency of the applied voltage.

For resonance both L and C must be present in circuit.

#### 4. Series Resonance

- (a) At Resonance
  - (i)  $X_L = X_C$  (ii)  $V_L = V_C$

(iv)  $Z_{min} = R$  (impedance minimum)

#### (b) Resonance frequency

$$\Theta X_{L} = X_{C} \implies \omega_{r}L = \frac{1}{\omega_{r}C} \implies \omega_{r}^{2}\frac{1}{LC} \implies \omega_{r} = \frac{1}{\sqrt{LC}} \implies f_{r} = \frac{1}{2\pi\sqrt{LC}}$$

#### (c) Variation of with f

(i) If  $f < f_r$  then  $X_L < X_C$ circuit nature capacitive,  $\phi$  (negative)(ii) At  $f = f_r$  then  $X_L = X_C$ circuit nature, Resistive,  $\phi = zero$ (iii) If  $f > f_r$ , then  $X_L > X_C$ circuit nature is inductive,  $\phi$  (positive)



(iii)  $\phi = 0$  (V and I in same phase)

(v)  $I_{max} = \frac{V}{R}$  (current maximum)

## (d) Variation of I with f as f increase, Z first decrease then increase



as f increase, I first increase then decreases

- At resonance, impedance of the series resonant circuit is minimum so it is called 'acceptor circuit' as it most readily accepts that current out of many currents whose frequency is equal to its natural frequency. In radio or TV tuning we receive the desired station by making the frequency of the circuit equal to that of the desired station.
- Half power frequencies

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The frequencies at which, power become half of its maximum value is called half power frequencies

- **Band width** =  $\Delta f = f_2 f_1$
- Quality factor Q : Q-factor of AC circuit basically gives an idea about stored energy & lost energy.

$$Q = 2\pi \frac{\text{maximum energy stored per cycle}}{\text{maximum energy loss per cycle}}$$

- (i) It represents the sharpness of resonance.
- (ii) It is unit less and dimension less quantity

(iii) 
$$Q = \frac{(X_L)_r}{R} = \frac{(X_C)_r}{R} = \frac{2\pi f_r L}{R} = \frac{1}{R} \sqrt{\frac{L}{C}} = \frac{f_r}{\Delta f} = \frac{f_r}{band width}$$

#### Magnification

At resonanceVL or  $V_C = QE$  (where E = supplied voltage)So at resonanceMagnification factor = Q-factor

#### Sharpness

Sharpness  $\propto$  Quality factor  $\propto$  Magnification factor R decrease  $\Rightarrow$  Q increase  $\Rightarrow$  Sharpness increases



#### **GOLDEN KEY POINTS**

- In A.C. circuit voltage for L or C may be greater than source voltage or current but it happens only when circuit contains L and C both and in R it is never greater than source voltage or current.
- In parallel A.C. circuit phase difference between  $I_L$  and  $I_C$  is  $\pi$
- Series resonance circuit gives voltage amplification while parallel resonance circuit gives current amplification.
- At resonance current does not depend on L and C, it depends only on R and V.
- At half power frequencies : net reactance = net resistance.
- As R increases, bandwidth increases
- To obtain resonance in a circuit following parameter can be altered :
  - (i) L (ii) C (iii) frequency of source.
- Two series LCR circuit of same resonance frequency fare joined in series then resonance frequency of series combination is also f
- The series resonance circuit called acceptor whereas parallel resonance circuit called rejector circuit.
- Unit of  $\sqrt{LC}$  is second.

# ILLUSTRATIONS

#### **Illustration 20.**



Find out the impedance of given circuit.

#### Solution.

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{4^2 + (9 - 6)^2} = \sqrt{4^2 + 3^2} = \sqrt{25} = 5W \quad (\Theta | X_L > X_C :: Inductive)$$

#### **Illustration 21.**

Find out reading of AC ammeter and also calculate the potential difference across, resistance and capacitor.

#### Solution

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = 10\sqrt{2} \ \Omega \Rightarrow I_0 = \frac{V_0}{Z} = \frac{100}{10\sqrt{2}} = \frac{10}{\sqrt{2}} A$$
  

$$\Theta \quad \text{anmeter reads RMS value, so its reading} = \frac{10}{\sqrt{2}\sqrt{2}} = 5A$$
  
so  $V_R = 5 \times 10 = 50 \text{ V}$  and  $V_C = 5 \times 10 = 50 \text{ V}$   

$$E = 100 \text{sinot} 100\pi \text{t volt}$$

#### **Illustration 22.**

In LCR circuit with an AC source  $R = 300 \Omega$ ,  $C = 20 \mu$ F, L = 1.0 H,  $E_{rms} = 50V$  and  $f = 50/\pi$  Hz. Find RMS current in the circuit.

#### Solution

$$I_{\rm rms} = \frac{E_{\rm rms}}{Z} = \frac{E_{\rm rms}}{\sqrt{R^2 + \left[\omega L - \frac{1}{\omega C}\right]^2}} = \frac{50}{\sqrt{300^2 + \left[2\pi \times \frac{50}{\pi} \times 1 - \frac{1}{20 \times 10^{-6} \times 2\pi \times \frac{50}{\pi}\right]^2}}}$$
  
$$\Rightarrow I_{\rm rms} = \frac{50}{\sqrt{(300)^2 + \left[100 - \frac{10^3}{2}\right]^2}} = \frac{50}{100\sqrt{9 + 16}} = \frac{1}{10} = 0.1A$$

#### **Illustration 23.**

For what frequency the voltage across the resistance R will be maximum. **Solution** 

It happens at resonance

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$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{\frac{1}{\pi} \times 10^{-6} \times \frac{1}{\pi}}} = 500 \text{ Hz}$$

#### **Illustration 24.**

A capacitor, a resistor and a 40 mH inductor are connected in series to an AC source of frequency 60Hz, calculate the capacitance of the capacitor, if the current is in phase with the voltage. Also find the reading of voltmeter  $V_3$  and Ammeter.



Solution

At resonance 
$$\omega L = \frac{1}{\omega C}$$
,  $C = \frac{1}{\omega^2 L} = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4\pi^2 \times (60)^2 \times 40 \times 10^{-3}} = 176 \mu F$   
 $V_3 = V_R \Rightarrow V_3 = 110 \text{ V}$  and  $I = \frac{V}{R} = \frac{110}{220} = 0.5 \text{ A}$ 

#### **Illustration 25.**

A coil, a capacitor and an A C. source of rms voltage 24 V are connected in series, By varying the frequency of the source, a maximum rms current 6 A is observed, If this coil is connected to a battery of emf 12 V, and internal resistance 40, then calculate the current through the coil.

#### Solution

At resonance current is maximum, 
$$I = \frac{V}{R} \Rightarrow$$
 Resistance of coil  $R = \frac{V}{I} = \frac{24}{6} = 4 \Omega$   
When coil is connected to battery, suppose I current flow through it then  $I = \frac{E}{R+r} = \frac{12}{4+4}$   
= 1.5 A

#### **BEGINNER'S BOX-3**

1. In given circuit find the reading of ammeter and voltmeter.

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2. For the circuit shown in figure, write down the instantaneous current through each element



- 3. A variable frequency 230V alternating voltage source is connected across a series combination of L = 5.0H,  $C = 80\mu$ F and  $R = 40\Omega$ . Calculate.
  - (a) The angular frequency of the source at resonance.
  - (b) Amplitude of current at resonance frequency
- 4. A coil of resistance R and inductance L is connected in series with a capacitor C and complete combination is connected to a.c. voltage, Circuit resonates when angular frequency of supply is  $\omega = \omega_0$ .

Find out relation between  $\omega_0$ , L and C



- 5. Find the phase difference between voltage and current in series LCR circuit at half power frequencies.
- 6. A series LCR circuit with L = 0.12H, C = 480 nF, R =  $23\Omega$  is connected to a 230 V variable frequency supply Find
  - (a) Source frequency for which current is maximum.
  - (b) Q-factor of the given circuit.

#### **POWER IN AC CIRCUIT**

1. The average power dissipation in LCR AC circuit

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Let  $V = V_0 \sin \omega t$  and  $I = I_0 \sin (\omega t - \phi)$ Instantaneous power  $P = (V_0 \sin \omega t) (I_0 \sin(\omega t - \phi) = V_0 I_0 \sin \omega t (\sin \omega t \cos \phi - \sin \phi \cos \omega t))$ Average power  $\langle P \rangle = \frac{1}{T} \int_0^T (V_0 I_0 \sin^2 \omega t \cos \phi - V_0 I_0 \sin \omega t \cos \omega t \sin \phi) dt$   $= V_0 I_0 \left[ \frac{1}{T} \int_0^T \sin^2 \omega t \cos \phi dt - \frac{1}{T} \int_0^T \sin \omega t \cos \omega t \sin \phi dt \right]$   $= V_0 I_0 \left[ \frac{1}{2} \cos \phi - 0 \times \sin \phi \right]$  $\Rightarrow \langle P \rangle \frac{V_0 I_0 \cos \phi}{2} = V_{rms} I_{rms} \cos \phi$ 

Instantaneous	Average power/actual	Virtual	Peak power
power	power/dissipated power/power loss	power/apparent	
		power/rms power	
$\mathbf{P} = \mathbf{VI}$	$P = V_{rms} I_{rms} \cos \phi$	$\mathbf{P} = \mathbf{V}_{\mathrm{rms}} \ \mathbf{I}_{\mathrm{rms}}$	$\mathbf{P} = \mathbf{V}_0 \ \mathbf{I}_0$

- $I_{rms} \cos \phi$  is known as active part of current or wattfull current, workfull current. It is in phase with voltage.
- $I_{rms} \sin \phi$  is known as inactive part of current, wattless current, workless current. It is in quadrature (90°) with voltage.

# 2. Power factor :

Average power  $\overline{\mathbf{P}} = \mathbf{E}_{\text{rms}} \mathbf{I}_{\text{rms}} \cos \phi = \text{rms power} \times \cos \phi$ 

Power factor (cos  $\phi$ ) =  $\frac{\text{Average power}}{\text{rms power}}$  and cos  $\phi \frac{\text{R}}{\text{Z}}$ 

#### 3. Choke Coil

In a direct current circuit, current is reduced with the ~elp of a resistance.

Hence there is a loss of electrical energy F R per sec in the form of heat in the resistance. But in an AC circuit the current can be reduced by choke coil which involves very small amount of loss of energy. Choke coil is a copper coil wound over a soft iron laminated core. This coil is put in series with the circuit in which current is to be reduced. Circuit with a choke coil is a series L-R circuit. If resistance of choke coil = r (very small)



The current in the circuit  $I = \frac{E}{Z}$  with  $Z = \sqrt{(R + r)^2 + (\omega L)^2}$  So due to large inductance L of the coil, the current in the circuit is decreased appreciably. However, due to small resistance of the coil r,

the power loss in the choke  $P_{av} = V_{rms} I_{rms} \cos \phi \rightarrow 0$ 

$$\Theta \qquad \cos\phi \frac{\mathbf{r}}{\mathbf{Z}} = \frac{\mathbf{r}}{\sqrt{\mathbf{r}^2 + \omega^2 \mathbf{L}^2}} \approx \frac{\mathbf{r}}{\omega \mathbf{L}} \to 0$$

#### **GOLDEN KEY POINTS**

- $P_{av} \le P_{rms}$ .
- Power factor varies from 0 to 1
- •

Pure/Ideal	V	Power factor = $\cos\phi$	Average power
R	V, I same Phase	1 (maximum)	V _{rms} . I _{rms}
L	V leads I by $\frac{\pi}{2}$	0, lagging	0
С	V lags I by $\frac{\pi}{2}$	0, leading	0
Choke coil	V leads I by $\frac{\pi}{2}$	0, lagging	0

- At resonance power factor is maximum ( $\phi = 0$  so  $\cos \phi$ ) and  $P_{av} = V_{rms} I_{rms}$
- Choke coil is an inductor having high inductance and negligible resistance.
- Choke coil is used to control current in A.C. circuit at negligible power loss
- Choke coil used only in A. C. and not in D.C. circuit
- Choke coil is based on the principle of wattless current.
- Iron cored choke coil is used generally at low frequency and air cored at high frequency.
- Resistance of ideal choke coil is zero

# **ILLUSTRATIONS**

#### **Illustration 26.**

A voltage of 10V and frequency  $10^3$  Hz is applied to  $\frac{1}{\pi}\mu$ F capacitor in series with a resistor of

500 $\Omega$ . Find the power factor of the circuit and the power dissipated.

#### Solution

Θ

$$X_{\rm C} = \frac{1}{2\pi f {\rm C}} = \frac{1}{2\pi \times 10^3 \times \frac{10^{-6}}{\pi}} = 500\Omega \qquad \therefore \quad {\rm Z} = \sqrt{{\rm R}^2 + {\rm X}_{\rm C}^2} = \sqrt{(500)^2 + (500)^2} = 500\sqrt{2}\Omega$$

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Power factor 
$$\cos f = \frac{R}{Z} = \frac{500}{500\sqrt{2}} = \frac{1}{\sqrt{2}}$$
, Power  
dissipated  $= V_{rms}I_{rms}\cos\phi = \frac{V_{rms}^2}{Z}\cos\phi$   
 $= \frac{(10)^2}{500\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1}{10}W$ 

#### **Illustrating 27.**

If V = 100 sin 100 t volt and I = 100 sin (100 t +  $\frac{\pi}{3}$ ) mA for an A.C. circuit then find out

- (a) phase difference between V and I
- (b) total impedance, reactance, resistance
- (b) power factor and power dissipated
- (d) components contains by circuits

- Solution
  - Phase difference  $\phi = -\frac{\pi}{3}$  (I leads V) (a)

(b) Total impedance 
$$Z = \frac{V_0}{I_0} = \frac{100}{100 \times 10^{-3}} = 1k\Omega$$
 Now resistance  $R = Z \cos 60^\circ =$ 

$$1000 \times \frac{1}{2} = 500\Omega$$
 reactance X = Zsin60° =  $1000 \times \frac{\sqrt{3}}{2} = 500\sqrt{3}\Omega$ 



 $\phi = -60^{\circ} \Rightarrow$  Power factor =  $\cos\phi = \cos(-60^{\circ}) = 0.5$  (leading) (c) Power dissipated P = V_{rms} I_{rms} cos  $\phi = \frac{100}{\sqrt{2}} \times \frac{0.1}{\sqrt{2}} \times \frac{1}{2} = 2.5$  W

Circuit must contains R as  $\phi \neq \frac{\pi}{2}$  and as  $\phi$  is negative so C must be there, (L may exist (d) but  $X_C > X_L$ )

#### **Illustrating 28.**

If power factor of a R–L series circuit is  $\frac{1}{2}$  when applied voltage is V = 100 sin 100 $\pi$ t volt and

resistance of circuit is  $200\Omega$  then calculate the inductance of the circuit.

#### Solution

$$\cos \phi = \frac{R}{Z} \Rightarrow \frac{1}{2} = \frac{R}{2} \Rightarrow Z = 2R \Rightarrow \sqrt{R^2 + X_L^2} = 2R$$
$$\Rightarrow X_L = \sqrt{3} R$$
$$\omega L = \sqrt{3} R \Rightarrow L = \frac{\sqrt{3}R}{\omega} = \frac{\sqrt{3} \times 200}{100\pi} = \frac{2\sqrt{3}}{\pi} H$$

#### **Illustrating 29.**

A circuit consisting of an inductance and a resistance joined to a 200 volt supply (A.C.). It draws a current of 10 ampere. If the power used in the circuit is 1500 watt. Calculate the wattless current.

#### Solution

Apparent power =  $200 \times 10 = 2000$  W  $\therefore$  Power factor  $\cos \phi = \frac{\text{True power}}{\text{Apparent power}} = \frac{1500}{2000} = \frac{3}{4}$ Watless current =  $I_{\text{rms}} \sin \phi = 10\sqrt{1 - \left(\frac{3}{4}\right)^2} = \frac{10\sqrt{7}}{4}$  A

#### **Illustration 30.**

A choke coil and a resistance are connected in series in an a.c circuit and a potential of 130 volt is applied to the circuit. If the potential across the resistance, is 50 V. What would be the potential difference across the choke coil.

#### Solution

$$V = \sqrt{V_R^2 + V_L^2} \implies V_L = \sqrt{V^2 - V_R^2} = \sqrt{(130)^2 - (50)^2} = 120 \text{ v}$$

#### **Illustrating 31.**

An electric lamp which runs at 80V DC consumes 10 A current. The lamp is connected to 100 V- 50 Hz ac source compute the inductance of the choke required.

#### Solution

Resistance of lamp R  $\frac{V}{I} = \frac{80}{10} = 8\Omega$ 

Let Z be the impedance which would maintain- a current of 10 A through the Lamp when it is run on 100 Volt a.c. then.  $Z = \frac{V}{I} = \frac{100}{10} = 10 \Omega$  but  $Z = \sqrt{R^2 + (\omega L)^2}$  $\Rightarrow (\omega L)^2 = Z^2 - R^2 = (10)^2 - (8)^2 = 36 \Rightarrow \omega L = 6 \Rightarrow L = \frac{6}{\omega} = \frac{6}{2\pi \times 50} = 0.02H$ 

#### **Illustrating 32.**

Calculate the resistance or inductance required to operate a lamp (60V, 1 OW) from a source of (100 V, 50 Hz)

#### Solution



100V, 50Hz

But 
$$V_R = IR \implies 40 = \frac{1}{6}(R) \implies R = 240 \Omega$$

(b) Now in this case 
$$(V_{Lamp})^2 + (V_L)^2 = (V)^2$$
  
 $(60)^2 + (V_L)^2 = (100)^2 \Rightarrow V_L = 80 V$   
Also  $V_L = IX_L = \frac{1}{6}X_L$  so  $X_L = 80 \times 6 = 480 \Omega = L (2\pi f) \Rightarrow L = 1.5 H$ 

A capacitor of suitable capacitance replace a choke coil in an AC circuit, the average power consumed in a capacitor is also zero. Hence, like a choke coil, a capacitor can reduce current in AC circuit without power dissipation.

Cost of capacitor is much more than the cost of inductance of same reactance that's why choke coil is used.

#### **Illustration 33.**

A choke coil of resistance R and inductance L is connected in series with a capacitor C and complete combination is connected to a.c. voltage, Circuit resonates when angular frequency of supply is

$$\omega = \omega_0.$$

- (a) Find out relation between  $\mu_0$ , L and C
- What is phase difference between V and I at resonance, is it (b) changes when resistance of choke coil is zero.

#### Solution

(a) At resonance conduction 
$$X_L = X_C \Rightarrow \omega_0 L = \frac{1}{\omega_0 C} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

(b)

 $\Theta v \cos \phi = \frac{R}{Z} = \frac{R}{R} = 1$ 

No, it is always zero.

#### **BEGINNER'S BOX -4**

 $\therefore \phi = 0^{\circ}$ 

- What is the power factor of a circuit that draws 5A at 160 V and whose power consumption is 1. 600W?
- 2. In a series LCR circuit as shown in fig.
  - Find heat developed in 80 seconds (a)
  - (b) Find wattless current



then

3. For a series LCR circuit

> $I = 100 \sin (100\pi t - \pi/3) mA$  and  $V = 100 \sin(100\pi t) \text{volt},$

(a) Calculate resistance and reactance of circuit.

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- (b) Find average power loss.
- 4. The source voltage and current in the circuit are represented by the folfowir19' equations –

$$E = 110 \sin (\omega t + \frac{\pi}{6}) \text{ volt}, \quad I = 5 \sin (\omega t - \frac{\pi}{6}) \text{ ampere}$$

Find :-

- (a) Impedance of circuit.
- (b) Power factor with nature
- 5. In given circuit  $R = 100\Omega$ . If voltage leads current by  $60^{\circ}$  then find
  - (a) Current supply by source
  - (b) Average power
- 6. An inductor of reactance  $4\Omega$  and a resistor of resistance  $3\Omega$  are connected in series with 100V ac supply, calculate wattless current in circuit.
- 7. A 100  $\Omega$  resistor is connected to a 220 V, 50 Hz a.c. Supply.
  - (a) What is the rms value of current in the circuit?
  - (b) What is the net power consumed over a full cycle?
- 8. A choke coil and a resistance are connected in series in an a.c. circuit and a potential of 136 volt is applied to the circuit. If the potential across the resistance is 50V. What would be the potential difference across the choke coil.

# LC OSCILLATION

The oscillation of energy between capacitor (electric field energy) and inductor (magnetic field energy) is called LC Oscillation.

# 1. Undamped oscillation

When the circuit has no resistance, the energy taken once from the source and given to capacitor keeps on oscillating between C and L then the oscillation produced will be of constant amplitude. These are called undamped oscillation.



After switch is closed

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R	-mir		
V=200	-O- sin100	πt volt	

$$\frac{Q}{C} + L\frac{di}{dt} = 0 \implies \frac{Q}{C} + L\frac{d^2Q}{dt^2} = 0 \implies \frac{d^2Q}{dt^2} + \frac{1}{LC}Q = 0$$

By comparing with standard equation of free oscillation  $\left[\frac{d^2x}{dt^2} + \omega^2 x = 0\right]$ 

$$\omega^2 = \frac{1}{LC}$$
 Frequency of oscillation  $f = \frac{1}{2\pi\sqrt{LC}}$ 

Charge varies sinusoidally with time  $q = q_m \cos \omega t$ 

current also varies periodically with t  $I = \frac{dq}{dt} = q_m \omega \cos(\omega t + \frac{\pi}{2})$ 

If initial charge on capacitor is  $q_m$  then electrical energy strored in capacitor is  $U_E = \frac{1}{2} \frac{q_m^2}{C}$ 

At t = 0 switch is closed, capacitor starts to discharge.

As the capacitor is fully discharged, the total electrical energy is stored in the inductor in the form of magnetic energy.

$$U_{\rm B} = \frac{1}{2} L I_{\rm m}^2$$
 where  $I_{\rm m} = {\rm max. \ current}$ 

$$(U_{\text{max}})_{\text{EPE}} = (U_{\text{max}})_{\text{MPE}} \Rightarrow \frac{1}{2} \frac{q_{\text{m}}^2}{C} = \frac{1}{2} L I_{\text{m}}^2$$

#### **GOLDEN KEY POINTS**

• In damped oscillation amplitude of oscillation decreases exponentially With time.

• At 
$$t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}, \dots$$
 energy stored is completely magnetic.

• At 
$$t = \frac{T}{8}, \frac{3T}{8}, \frac{5T}{8}, \dots$$
 energy is shared equally between L and C

• Phase difference between charge and current is  $\frac{\pi}{2}$  (when charge is maximum, current, minimum) when charge is minimum, current maximum

#### **Illustrating 34.**

An LC circuit contains a 20mH inductor and a  $50\mu$ F capacitor with an initial charge of 10mC. The resistance of the circuit is negligible. Let the instant the circuit is closed to be t = 0

- (a) What is the total energy stored initially.
- (b) What is the natural frequency of the circuit.
- (c) At what minimum time is the energy stored is completely magnetic.
- (d) At what minimum time is the total energy shared equally between inductor and the capacitor.

#### Solution

(a) 
$$U_{\rm E} = \frac{1}{2} \frac{q^2}{C} = \frac{1}{2} \times \frac{(10 \times 10^{-3})^2}{50 \times 10^{-6}} = 1.0 \text{ J}$$

(b) 
$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{20 \times 10^{-3} \times 50 \times 10^{-6}}} = 10^3 \text{ rad/sec} \implies \text{f} = 159 \text{ Hz}$$

(c) 
$$\Theta$$
  $q = q_0 \cos \omega t$   
Energy stored is completely magnetic (i.e. electrical energy is zero,  $q = 0$ )

at 
$$t = \frac{T}{4}$$
, where  $T = \frac{1}{f} = 6.3$  ms

(d) Energy is shared equally between L and C when charge on capacitor become  $\frac{q_0}{\sqrt{2}}$ 

so, at  $t = \frac{T}{8}$ , energy is shared equally between L and C

#### **BEGINNER'S BOX-5**

1. Initially key was placed on (1) till the capacitor got fully charged. Key is placed on (2) at t=O. The minimum time when the energy in both capacitor and inductor will be same-



- 2. An inductor of inductance 2.0 mH is connected across a charged capacitor of capacitance 5.0 μF and the resulting L-C circuit is set oscillating at its natural frequency. Let Q denote the instantaneous charge on the capacitor and I the current in the circuit. It is found that the maximum value of Q is 200 μC.
  [IIT–JEE 2006]
  - (i) Find the maximum value of I.
  - (ii) When  $Q = 200 \ \mu C$ , what is the value of I?
  - (iii) When  $Q = 100 \mu C$ , what is the value | dI/dt | ?
  - (iv) When I is equal to one-half its maximum value, what is the value of |Q|?

# ANSWERS BEGINNER'S BOX–1

**1.** There are two reasons for it :



2. For AC,  $I = I_0 \sin \omega t$ , the instantaneous value of heat produced (per second) in a resistance R is,  $H = I^2 R = I_0^2 \sin^2 \omega t \times R$  the average value of heat produced during a cycle is :-

$$H_{av} = \frac{\int_{0}^{T} Hdt}{\int_{0}^{T} dt} = \frac{\int_{0}^{T} (I_{0}^{2} \sin^{2} \omega t \times R) dt}{\int_{0}^{T} dt} = \frac{1}{2} I_{0}^{2} R \qquad \left[ Q \quad \int_{0}^{T} I_{0}^{2} \sin^{2} \omega t dt = \frac{1}{2} I_{0}^{2} T \right]$$
$$H_{av} = \left(\frac{I_{0}}{\int_{0}^{T}}\right) R = I_{rms}^{2} R \qquad \dots (i)$$

$$\Rightarrow \qquad H_{av} = \left(\frac{I_0}{\sqrt{2}}\right) R = I_{rms}^2 R \qquad \dots (i$$

However, in case of DC,  $H_{DC} = I^2 R$  ...(ii)  $\Theta I = I_{rms}$  so from equation (i) and (ii) HDC =  $H_{av}$ 

AC produces same heating effects as DC of value  $I = I_{rms}$ . This is also why AC instruments which are based on heating effect of current give rms value.

- **3.** The average value of a.c. for a cycle is zero. So a d.c. ammeter will always read zero in a.c. circuit.
- **4.** 2.5 ms **5.** (a) 0.01 s, (b) 60° **6.** 200 V

#### **BEGINNER'S BOX-2**

- **1.** 2.1 A **2.**  $10000 \sin\left(1000t \frac{\pi}{2}\right) A$  **3.** 1A
- 4. The capacitive reactance  $X_C = 212 \Omega$

 $I_{rms} = 1.03 \text{ A}$ 

The peak current  $I_0 = 1.46 \text{ A}$ 

If the frequency is doubled the capacitive reactances is halved, the consequently, the current is doubled.

5.	2.49 A	<b>6.</b> (a)	resistor (b) in	ductor	7. $R = 5\Omega$ and	$d X_{\rm L} = 5\sqrt{3} \Omega$	8.	480Ω
9.	1H	10.	0.08 henry	11.	240.2 Ω	<b>12.</b> (a) 2.4	(b) 2.5	5 ms
13.	15.9 mH	14.	(A) - r, (B) -	p, (C) -	-p, (D) - p, (E)	) — r		

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#### **Beginner's Box-3**

- 1. Reading of ammeter =  $.2 \cdot 5A$ Reading of voltmeter = 25V
- 2. The three current equations are,

V = i_RR, V = L
$$\frac{di_L}{dt}$$
 and  $\frac{dV}{dt} = \frac{1}{C}i_C$   
so  $i_R = \frac{V_0}{R}\sin\omega t$ ,  
 $i_L = -\frac{V_0}{\omega L}\cos\omega t$  and  $i_C = V_0\omega C\cos\omega t$ 

3. (a) angular frequency at resonance  $\omega_r = 50 \text{ rad/s}$ (b) amplitude of current at resonance  $I_m = 8.13 \text{ A}$ 

4. (a) 
$$\omega_0 = \frac{1}{\sqrt{LC}}$$
, (b)  $\phi = 0^\circ$  No, It is alw2ays zero.

- 5.  $\phi = \frac{\pi}{4}$
- 6. (a)  $6.63 \times 10^2$  Hz (b) Quality factor Q = 21.7

#### **BEGINNER'S BOX-4**

- **1.** 0.75 **2.**(a) 4000 joule (b) 2.12 A
- 3. (a) R = 500 ohm,  $X = 500\sqrt{3}$  ohm (b) 2.5 watts
- 4. (a) Impedance Z = 22 $\Omega$  (b) Power factor =  $\frac{1}{2}$  (lagging)
- 5. (a)  $\frac{1}{\sqrt{2}}$  A, (b) 50W
- **6.** Wattless current = 16A
- **7.** (a) 2.2 A (b) 484 watt
- **8.** 120V

#### **BEGINNER'S BOX-5**

- 1.  $t = \frac{\pi\sqrt{LC}}{4}$
- **2.** (i) 2.0 A, (ii) Zero, (iii)  $10^4$  A/s, (iv)  $1.732 \times 10^{-4}$  C