Fractions And Mixed Numbers

Revision Notes on Fractions

Fraction

A Fraction is a part of whole. The 'whole' here could be an object or the group of objects. But all the parts of the whole must be equal.



- 1. The **first** one is the whole i.e. a complete circle.
- 2. In the **second** circle, if we divide the circle into two equal parts then the shaded portion is the half i.e. ½ of the circle.
- 3. In the **third** circle, if we divide the circle into four equal parts and shade only one part then the shaded part is the one fourth i.e. ¼ of the whole circle.
- 4. In the **fourth** circle, if we divide the circle into four equal parts and shade three parts then the shaded part is the three fourth i.e. ³/₄ of the whole circle.

Numerator and Denominator



The upper part of the fraction is called **Numerator**. It tells the number of parts we have.

The lower part of the fraction is called **Denominator**. It tells the total parts in a whole.

It reads as "three-fifths".

Representation of fraction on Number line

Example

Mark $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$ and $\frac{1}{8}$ on the different number lines.

Solution

- Draw a number line.
- We know that ½ is less than 1 and greater than 0, so we have to divide the gap between two equal parts and then mark the middle point as ½.

As the denominator is the whole and the numerator is the part, so we have to divide the gap between 0 and 1 in the number of parts as the denominator is given.

For 1/3, divide into 3 equal parts.

For ¼, divide into 4 equal parts and so on.



Proper Fractions

If the numerator is less than the denominator then it is called **proper fraction**. If we represent a proper fraction on the number line than it will always lie between 0 and 1.



Improper fractions and Mixed fractions

When the numerator is greater than the denominator then it is called **Improper** fraction.



The above fraction is made by adding one whole part and one-fourth part.

 $1 + \frac{1}{4} = 1\frac{1}{4} = \frac{5}{4}$

The fraction made by the combination of whole and a part is called **Mixed fraction**.

Convert Mixed fraction into Improper fraction

A mixed fraction is in the form of

 $Quotient \frac{Remainder}{Divisor}$

We can convert it in the form of an improper fraction by

 $\frac{\text{Quotient} \times \text{Divisor} + \text{Remainder}}{\text{Divisor}} \text{ or } \frac{\text{whole} \times \text{Denominator} + \text{Numerator}}{\text{Denominator}}$

Example

Convert $2\frac{3}{4}$ into improper fraction.

Solution

$$2\frac{3}{4} = \frac{(4\times2)+3}{4} = \frac{11}{4}$$

Equivalent Fractions

Equivalent fractions are those fractions which represent the same part of a whole.



All the above images are different but equivalent fractions as they represent the same i.e. half part of a whole circle.

Finding equivalent fractions

1. Multiplying the same number

If we multiply the numerator and denominator of any fraction with the same number then we will get the equivalent fraction. There could be more than one equivalent fractions of one fraction.

Example

Find three equivalent fraction of ½.

Solution

1	×	2		2
2	×	2	=	4
1	×	3		3
2	×	3	=	6
1	×	4		4
2	×	4	=	8

2. Dividing the same number

If we divide the numerator and denominator of any fraction with the same number then we will get the equivalent fraction.

Example

Find the equivalent fraction of 18/27 with denominator 9.

Solution

To get the denominator 9 we need to divide it by 3.

So, to find the equivalent fraction we need to divide the fraction by 3.

$$\frac{18 \div 3}{27 \div 3} = \frac{6}{9}$$

Hence the equivalent fraction with denominator 9 is 6/9.

The simplest form of a Fraction

If the numerator and denominator do not have any other common factor than 1 then it is said to be the simplest or lowest form of that fraction.

Example-

$$\frac{2}{3}, \frac{4}{5}$$
 etc.

To find the equivalent fraction which is the simplest form we have to find the **HCF** of numerator and denominator and then divide them both by that HCF.

Example

Reduce the fraction 18/27 in the simplest form.

Solution

HCF of 18 and 27 is 9.

Hence,

$$\frac{18 \div 9}{27 \div 9} = \frac{2}{3}$$

2/3 is the lowest form of 18/27.

Like Fractions and Unlike Fractions

Fractions which have same denominators are known as Like fractions.

Example

$$\frac{5}{13}, \frac{3}{13}, \frac{1}{13}$$
 etc.

Fractions which have different denominators are known as **unlike fractions**.

Example

$$\frac{2}{3}, \frac{5}{9}, \frac{13}{17}$$
 etc

Comparing fractions

 $2\frac{1}{6}$ $3\frac{3}{8}$

If we have to compare the above two fractions then it is easy as the first one is less than 3 and the second one is greater than 3. So we can clearly say that

$$3\frac{3}{8} > 2\frac{1}{6}$$

But sometimes it is not easy to compare it so easily. So we need some accurate procedure.

1. Comparing like fractions

Like fractions are the fractions with the same denominator so we have to compare them with the numerator only. The fraction with greater numerator is greater.



In the above example, both are divided into 8 equal parts, so the fraction with seven shaded part is greater than the 5 shaded parts.

2. Comparing unlike fractions

The fractions with different denominators are unlike fractions.

1. Unlike fraction with the same numerator

If we have to compare the fractions with different denominator but same numerator, we have to compare with the denominator only.

In that case, the fraction with the small denominator is greater than the other.

Example



Here the numerator is same i.e.3 so we will compare with the denominator.

The fraction with small denominator i.e. $\frac{3}{4}$ is greater than the fraction with the large denominator i.e. $\frac{3}{8}$.

2. Unlike fraction with different numerators

If the numerator and denominator both are different then we have to make the denominator same by finding the equivalent fraction of both the fractions then compare the fractions as like fractions.

To find the equivalent fraction of both the fractions with the same denominator, we have to take the LCM of the denominator.

Example

Compare 6/7 and 3/5.

Solution

The product of 7 and 5 is 35.

So we will find the equivalent fraction of both the fractions with the denominator 35.

6	×	5		30
7	×	5	_	35
3	×	7	_	21
5	×	7	_	35

Now we can compare them as like fractions.

 $\frac{30}{35} > \frac{21}{35}.$

Addition and Subtraction of Fractions

1. Adding like fractions

In case of like fractions, the denominator is same so we can add them easily.

Steps to add like fractions-

- a. Add the numerators.
- b. Leave the common denominator same. (Don't add the denominator).
- c. Write the answer as

Numerator 1 + Numerator 2 Common Denominator

Example

 $\operatorname{Add} \frac{1}{4} + \frac{2}{4}.$



Solution

$$\frac{1}{4} + \frac{2}{4} = \frac{1+2}{4} = \frac{3}{4}$$

2. Subtracting like fractions

Steps to subtract the like fractions-

- a. Subtract the small numerator from the bigger one.
- b. Leave the common denominator same.
- c. Write the answer as

Big numerator – small numerator common denominator

Subtract $\frac{2}{4}$ from $\frac{3}{4}$.

Solution

 $\frac{3}{4} - \frac{2}{4} = \frac{(3-2)}{4} = \frac{1}{4}$



3. Adding unlike fraction

If we have to add the unlike fractions, first we have to find the equivalent fraction of the given fractions with the same denominator then add them.

Steps to add unlike fractions-

- a. Take the LCM of the denominator of the given fractions.
- b. Find the equivalent fractions of both fractions with LCM as the denominator.
- c. Add them as the like fractions.

Example

Find $\frac{4}{5} + \frac{3}{8}$.

Solution

Take the LCM of 5 and 8, which is 40.

$$\frac{4}{5} \times \frac{8}{8} = \frac{32}{40}$$
$$\frac{3}{8} \times \frac{5}{5} = \frac{15}{40}$$
$$\frac{32}{40} + \frac{15}{40} = \frac{47}{40} = 1\frac{7}{40}$$

4. Subtracting unlike fractions

Steps to subtract unlike fractions-

- a. Take the LCM of the denominator of the given fractions.
- b. Find the equivalent fractions of both fractions with LCM as the denominator.
- c. Subtract them as the like fractions.

Example

Find
$$\frac{3}{4} - \frac{1}{5}$$
.

Solution

LCM of 4 and 5 is 20.

 $\frac{3}{4} \times \frac{5}{5} = \frac{15}{20}$ $\frac{1}{5} \times \frac{4}{4} = \frac{4}{20}$ $\frac{15}{20} - \frac{4}{20} = \frac{11}{20}$

Mixed operations

Revision Notes on Mixed operations

Fractions tell about "a part of a whole".



Here the pizza is divided into 4 equal parts and there are 3 parts left with us.

We will write it in a fraction as 3/4, in which 3 is numerator which tells the number of parts we have and 4 is denominator which tells the total parts in a whole.

The General form of a Fraction

 $Fraction = \frac{Numerator}{Denominator}$

Where, denominator $\neq 0$

If numerator = denominator then the fraction becomes a whole i.e. 1. This is called unity of fraction.

Types of Fraction

Type of Fraction	Meaning	Example
Proper fraction	When numerator is less than the denominator. It shows the part of a whole.	3 4
Improper fraction	When numerator is more than the denominator. It represents the mixture of whole and a proper fraction.	$\frac{7}{4}$
Mixed Fraction	The improper fraction can be written in the mixed form as it is the mixture of whole number and a fraction.	$\frac{1\frac{3}{4}}{1\frac{3}{4}}$
Like Fraction	The fractions with the same denominator are like fractions.	
Unlike Fraction	The fractions with different denominators are unlike fractions.	$\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$
Equivalent Fraction	The fractions proportional to each other are called equivalent fractions. It represents the same amount with different fractions.	$\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$

Converting a Mixed Fraction into an Improper Fraction



Converting an Improper Fraction into a Mixed Fraction

Divide the Numerator by the denominators that the quotient will be the whole number and remainder will be the numerator, while denominator will remain the same.



How to find the equivalent fractions?

To find the equivalent fraction of proper and improper fraction, we have the multiply both the numerator and denominator with the same number.



Reciprocal of a Fraction

If we have two non-zero numbers whose product is one then these numbers must be the reciprocals of each other.



To find the reciprocal of any fraction, we just need to flip the numerator with the denominator.

Multiplication of Fractions

1. How to multiply a fraction with a whole number?

a. If we have to multiply the **proper or improper fraction with the whole number** then we simply multiply the numerator with that whole number and the denominator will remain the same.

Example

$$2\times\frac{3}{4}=\frac{6}{4}$$

b. If we have to multiply the **mixed fraction with the whole number** then first convert it in the form of improper fraction then multiply as above.

Example

 $2\frac{3}{5} \times 2 = \frac{13}{5} \times 2 = \frac{26}{5}$

c. Fraction as an operator "of".

If it is written that find the 1/2 of 24 then what does 'of' means here?



Here 'of' represents the multiplication.

$$\frac{1}{2}$$
 of 24 = $\frac{1}{2} \times 24 = 12$

2. How to multiply a fraction with another fraction?

If we have to multiply the **proper or improper fraction with another fraction** then we simply multiply the numerator of both the fractions and the denominator of both the fractions separately and write them as the new fraction.

 $Fraction \times Fraction = \frac{Product of Numerators}{Product Of Denominators}$

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

Example

a.
$$\frac{2}{5} \times \frac{3}{4} = \frac{2 \times 3}{5 \times 4} = \frac{6}{20}$$

b. $\frac{1}{2}$ of $\frac{9}{4} = \frac{1}{2} \times \frac{9}{4} = \frac{9}{8}$

Value of the products of the fractions

Generally when we multiply two numbers then we got the result which is greater than the numbers.

5 × 6 = 30, where, 30 > 5 and 30 > 6

But in case of a fraction, it is not always like that.

a. The product of two proper fractions

If we multiply two proper fractions then their product will be less than the given fractions.

Example

 $\frac{2}{5} \times \frac{4}{9} = \frac{8}{45}$ where, $\frac{8}{45} < \frac{2}{5}$ and $\frac{8}{45} < \frac{4}{9}$

b. The product of two improper fractions

If we multiply two improper fractions then their product will be greater than the given fractions.

Example

 $\frac{9}{2} \times \frac{7}{4} = \frac{63}{8}$, where $\frac{63}{8} > \frac{9}{2}$ and $\frac{63}{8} > \frac{7}{4}$

c. The product of one proper and one improper fraction

If we multiply proper fraction with the improper fraction then the product will be less than the improper fraction and greater than the proper fraction.

Example

2	7	14	. 14	2	14	7
5	$\times \frac{-}{3}$	$=\frac{1}{15}, w$	$here \frac{15}{15} >$	$\frac{-}{5}$ and	$\frac{1}{15} <$	3

Division of Fractions

1. How to divide a whole number by a Fraction?

a. If we have to divide the **whole number with the proper or improper fraction** then we will multiply that whole number with the reciprocal of the given fraction.

Example

$$2 \div \frac{9}{5} = 2 \times \frac{5}{9} = \frac{10}{9}$$

b. If we have to divide the **whole number with the mixed fraction** then we will convert it into improper fraction then multiply it's reciprocal with the whole number.

$$2 \div 1\frac{4}{5} = 2 \div \frac{9}{5} = 2 \times \frac{5}{9} = \frac{10}{9}$$

2. How to divide a Fraction with a whole number?

To divide the fraction with a whole number, we have to take the reciprocal of the whole number then divide it with the whole number as usual

Example

 $\frac{3}{10} \div 2 = \frac{3}{10} \div \frac{2}{1} = \frac{3}{10} \times \frac{1}{2} = \frac{3}{20}$

3. How to divide a fraction with another Fraction?

To divide a fraction with another fraction, we have to multiply the first fraction with the reciprocal of the second fraction.

 $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$

Invert the fraction that you are dividing by	$\frac{4}{5} \div \frac{2}{3} = \frac{4}{5} \times \frac{3}{2}$
Multiply the numerators and denominators	$\frac{4}{5} \times \frac{3}{2} = \frac{12}{10}$
Simplify the fraction if necessary	$\frac{12}{10} = 1\frac{1}{5}$