BOOLEAN LOGIC

Introduction:

George Boole, a nineteenth-century English Mathematician, developed a system of logical algebra by which reasoning can be expressed mathematically. In 1854, Boole published a classic book, “An Investigation of the Laws of thought” on which he founded the Mathematical theories of Logic and Probabilities,

Boole’s system of logical algebra, now called Boolean algebra, was investigated as a tool for analyzing and designing relay switching circuits by Claude E. Shannon at the Massachusetts institute of Technology in 1938. Shannon, a research assistant in the Electrical Engineering Department, wrote a thesis entitled “A” symbolic Analysis of Relay and Switching Circuits. As a result of his work, Boolean algebra is now, used extensively in the analysis and design of logical circuits. Today Boolean algebra is the backbone of computer circuit analysis.

Two Valued Logical Symbol:

Aristotle made use of a two valued logical system in devising a method for getting to the truth, given a set of true assumptions. The symbols that are used to represent the two levels of a two valued logical system are 1 and 0. The symbol 1 may represent a closed switch, a true statement, an “on” lamp, a correct action, a high voltage, or many other things. The symbol “O” may represent on open switch, a false statement, an “off” lamp, an incorrect action, a low voltage, or many other things.

For the electronics circuits and signals a logic 1 will represent closed switch, a high voltage, or an “on” lamp, and a logic 0 will represent an open switch, low voltage, or an “off” lamp. These describe the only two states that exist in digital logic systems and will be used to represent the in and out conditions of logic gates.

Fundamental Concepts of Boolean Algebra:

Boolean algebra is a logical algebra in which symbols are used to represent logic levels. Any symbol can be used, however, letters of the alphabet are generally used. Since the logic levels are generally associated with the symbols 1 and 0, whatever letters are used as variables that can take the values of 1 or 0.

Boolean algebra has only two mathematical operations, addition and multiplication. These operations are associated with the OR gate and the AND gate, respectively.

Logical Addition:

When the + (the logical addition) symbol is placed between two variables, say X and Y, since both X and Y can take only the role 0 and 1, we can define the + Symbol by listing, all possible combinations for X and Y and the resulting value of X + Y.

The possible input and out put combinations may arranged as follows:

$$0 + 0 = 0$$

$$0 + 1 = 1$$

$$1 + 0 = 1$$

$$1 + 1 = 1$$

This table represents a standard binary addition, except for the last entry. When both' X and Y represents 1’s, the value of X + Y is 1. The symbol + therefore does not has the “Normal” meaning, but is a Logical addition symbol. The plus symbol (+) read as "OR", therefore X +Y is read as X or Y.

This concept may be extended to any number of variables for example A + B + C +D = E Even if A, B, C and D all had the values 1, the sum of the values i.e. is 1.

Logical Multiplication:

We can define the "." (logical multiplication) symbol or AND operator by listing all possible combinations for (input) variables X and Y and the resulting (output) value of X. Y as,

$$0 . 0 = 0$$

$$0 . 1 = 0$$

$$1 . 0 = 0$$

$$1 . 1 = 1$$

Note : Three of the basic laws of Boolean algebra are the same as in ordinary algebra; the commutative law, the associative law and the distributive law.

The commutative law for addition and multiplication of two variables is written as,

$$A + B = B + A$$

And $$A . B = B . A$$

The associative law for addition and multiplication of three variables is written as,

$$(A + B) + C = A + (B + C)$$

And $$(A . B) . C = A . (B . C)$$

The distributive law for three variables involves' both addition and multiplication and is written as,

$$A (B + C) = A B + AC$$

Note that while either '+' and '.' s can be used freely. The two cannot be mixed without ambiguity in the absence of further rules.

For example does A . B + C means (A . B) + C or A . (B+ C)? These two form different values for A = O, B = 1 and C = 1, because we have

$$(A . B) + C = (0.1) + 1 = 1$$

and $$A . (B + C) = 0 . (1 + 1) = 0$$

which are different. The rule which is used is that '.' is always performed before '+'. Thus X . Y + Z is (X.Y) + Z.

Logic Gates:

A logic gate is defined as a electronics circuit with two or more input signals and one output signal. The most basic logic Circuits are OR gates, AND gates, and invertors or NOT gates. Strictly speaking, invertors are not logic gates since they have only one input signal; however They are best introduced at the same time as basic gates and will therefore be dealt in this section.

OR Gate:

An OR gate is a logic circuit with two or more input signals and one output signal. The output signal will be high (logic 1) if any one input signal is high (logic 1). OR gate performs logical addition.

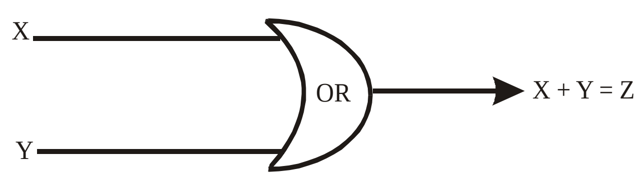
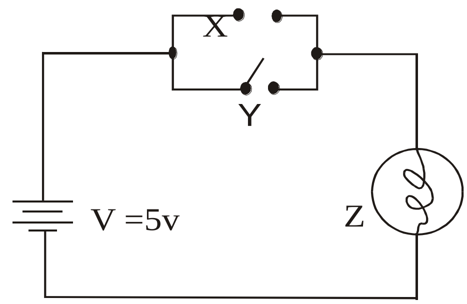


Fig. 1

A circuit that will functions as an OR gate can be implemented in several ways. A mechanical OR gate can be fabricated by connecting two switches in parallel as shown in figure 2.

Fig. 2



Truth Table for a switch circuit operation as an OR gate.

Table – 1

Switch X	**Switch Y**	**Output Z**
Open	Open	0
Open	Closed	5V
Closed	Open	5V
Closed	Closed	5V

Note that for the switch circuit were use diodes and resistors, Transistors and resistors and other techniques to control the voltage and resistance.

Note: If the switch is "on", it is represented by 1, and if, it is "off", it is represented by 0.

Truth Table for a Two-input **OR** gate.

Table - 2

In Put		**Out Put**
X	Y	Z
0	0	0
0	1	1
1	0	1
1	1	1

Truth table for a three in put **OR** gate.

Table – 3

A	**B**	**C**	**X**
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

No. of combinations = 2^n, where n is number of variables.

AND Gate:

An **AND** gate is a logic circuit with two or more input signals and one output signal. The output signal of an **AND** gate is high (logic 1) only if all inputs signals are high (Logic 1).

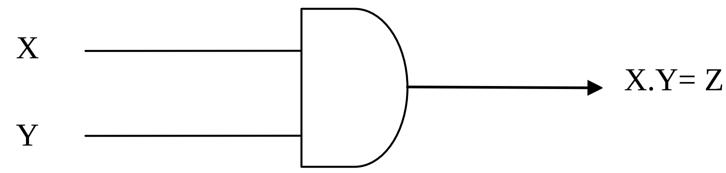


Fig.3

An **AND** gate performs logical multiplication on inputs. The symbol for **AND** gate is

A circuit that will functions as an **AND** gate can be implemented in several ways. A mechanical **AND** gate can be fabricated by connecting two switches in series as show in fig. 4

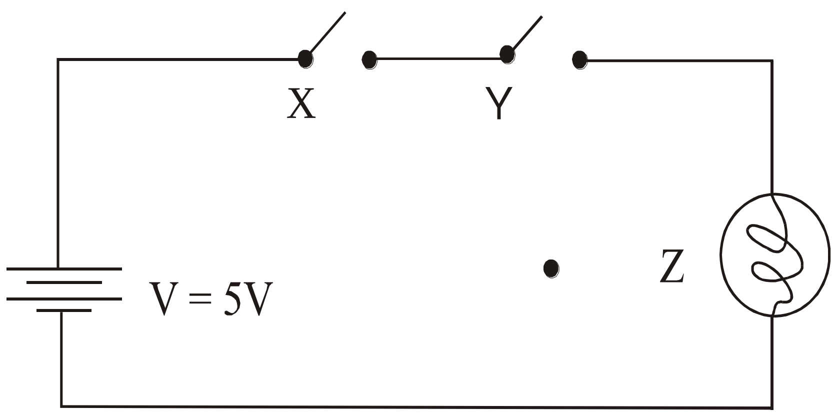


Fig. 4

Truth Table for a switch circuit operation as an AND gate.

Table – 4

Switch X	Switch Y	Output Z
Open	Open	0
Open	Closed	0
Closed	Open	0
Closed	Closed	5V

Truth Table for a Two-input **AND** gate

Table - 5

In Put		Out Put
X	**Y**	**Z**
0	0	0
0	1	0
1	0	0
1	1	1

Truth Table for a three input **AND** gate.

Table 6

Inputs			Output
A	**B**	**C**	**X**
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

Complementation:

The logical operation of complementary or inverting a variable is performed in the Boolean Algebra. The purpose of complementation is to

invert the, input signal, since there are only two values that variables can assume in two-value logic system, therefore if the input is 1, the output is 0 and if the input is 0 the output is 1. The symbol used to represent complementation of a variable is a bar (-) above the variable, for example the complementation of A is written as $\overline{A}$ and is read as "complement of A" or "A not".

Since variables can only be equal to 0 or 1, we can say that

$\overline{O} = 1$ Or $\overline{1} = O$

Also $\overline{\overline{O}} = O$ Or $\overline{\overline{1}} = 1$

Invertors Or NOT gate:

An inventor is a gate with only one input signal and one output signal; the output signal is always the opposite or complement of the input signal.

An invertor is also called a **NOT** gate because the output not the same as the input.

Symbol of Invertor or NOT gate is

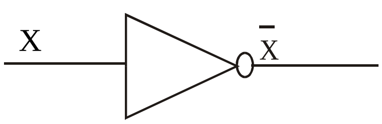


Fig.5 (i)

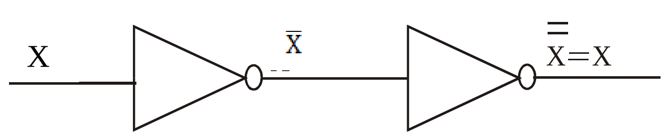


Fig.5 (ii)

Fig. 5

The circle at the output or input indicates inversion. It also distinguish between the symbol for the **NOT** gate or the symbol for an operational amplifier or certain types of buffers, because the symbol -▶- can also be used for diode.

Truth Table for a **NOT** circuit

Table – 7

In put	Out put
0	1
1	0

NOTE :A word is a group (or string) of binary bits that represents a closed instruction or data,

Example 1: How many input words in the Truth Table of an 6 - input OR gate? Which input word produce a high output?

Solution:

The total number of input word's = $2^n = 2^6$ =32, where n is number of inputs. In an OR gate 1 or more-high inputs produce a high output. Therefore the word of 000000 results in low outputs all other input words produce a high output.

Basic Duality in Boolean Algebra:

We state the duality theorem without proof. Starting with a Boolean relation, *we* can derive another Boolean relation by

1. Changing each **OR (+)** sign to an **AND (.)** sign
2. Changing each **AND (.)** sign to an **OR (+)** sign.
3. Complementary each 0 and 1 For instance

$$A + 0 = A$$

The dual relation is $A \cdot 1 = A$

Also since A (B + C) = AB + AC by distributive law. Its dual relation is

$$A + B C = (A + B) (A + C)$$

Fundamental Laws and Theorems of Boolean Algebra:

1. $X + 0 = X$
2. $X + 1 = 1$
3. $X + X = X$
4. $X + \overline{X} = 1$

OR operations

5. $X \cdot 0 = 0$
6. $X \cdot 1 = X$
7. $X \cdot X = X$
8. $X \cdot \overline{X} = 0$

AND operations

9. $\overline{\overline{X}} = X$ Double complement

10. $X + Y = Y + X$
11. $XY = YX$

Commutative laws

12. $(X + Y) + Z = X + (Y + Z)$ Associative laws

13. $(X . Y). Z = X. (Y. Z)$

14. $X (Y + Z) = XY + XZ$ Distribution Law

15. $X + Y .Z = (X + Y) . (X + Z)$ Dual of Distributive Law

16. $X + XZ = X$ Laws of absorption

17. $X (X + Z) = X$

18. $X + \overline{X} Y = X + Y$ Identity Theorems

19. $X (\overline{X} + Y) = X.Y$

20. $\overline{X+Y} = \overline{X} . \overline{Y}$ De Morgan's Theorems

21. $\overline{X.Y} = \overline{X} + \overline{Y}$

Proof of Boolean Algebra Rules:

Every rule can be proved by the application of rules and by perfect Induction.

Rule 15:

(i) This rule does not apply to normal algebra We follow:

$(X + Y) (X + Z) = XX + XZ + YX + YZ$

$= X + XZ + YX + YZ,$ $X.X=X$

$= X (1 + Z) + YX + YZ$

$= X + YX + YZ,$ $1 + Z = 1$

$= X (1 + Y) + Y \cancel{Z}$

$= X + YZ$ $1 + Y = 1$

(ii) Proof by Perfect induction Method:

Truth Table-8 for the R.H.S. $(X + Y) (X + Z)$

and for L.H.S. $X + YZ$

X	Y	Z	X+Y	X+Z	YZ	(X+Y)(X+Z)	X+YZ
0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	1	0	1	1
1	0	0	1	1	0	1	1
1	0	1	1	1	0	1	1
1	1	0	1	1	1	1	1
1	1	1	1	1	1	1	1

R.H.S. = L.H.S.

Rule .16 $X + XZ = X$

L.H.S. $= X + XZ = X(1 + Z) = X.\ 1 = X,$ $I + Z = 1$

$= \text{R.H.S.}$

Rule 17: $X(X + Z) = X$

L.H.S. $= X\ (X + Z)$

$= X\ X + XZ$ By distributive law

$= X + XZ,$ as $X.X = X$

$= X\ (1 + Z),$ As $1 + Z = 1$

$= X\ .\ 1$

$= X$

L.H.S. = R.H.S.

Rule 18: **(i)** $X + \overline{X}\ Y$ $= X + Y$

L.H.S. $= X + \overline{X}\ Y = (X + \overline{X}\ )\ .\ (X + Y)$ By rule 15 dual

Of distributive law.

$= 1\ .\ (X + Y)$ as $X + \overline{X} = 1$

$= X + Y$

L.H.S. = R.H.S.

(ii) Proof by perfect Induction Method:

Truth Table 9 for L.H.S. $X + \overline{X}\ Y$ and for R.H.S. $X + Y$

X	Y	$\overline{X}$	$\overline{X}\ Y$	$X + \overline{X}\ Y$	$X + Y$
0	0	1	0	0	0
0	1	1	1	1	1
1	0	0	0	1	1
1	1	0	0	1	1

L.H.S. =R.H.S.

Rule 19:

(i) $X.(\ \overline{X} + Y) = X\ .\ Y$

L.H.S. $= X\ (\ \overline{X} + Y) = X\ \overline{X} + X\ Y$ By distributive law

$= 0 + XY$ as $X\ .\ \overline{X} = 0$

$= X\ Y$

L.H.S. = R.H.S.

(ii) Proof by Perfect Induction Method:

Truth Table 10 for L.H.S. $X.\ (\ \overline{X} + Y)$ and for R.H.S. $X.Y.$

X	Y	$\overline{X}$	$\overline{X} + Y$	$X\ (\ \overline{X} + Y)$	$X\ .\ Y$
0	0	1	1	0	0
0	1	1	1	0	0
1	0	0	0	0	0
1	1	0	1	1	1

L.H.S. = R.H.S.

De Morgan’s Theorems:

(i) $\overline{X + Y} = \overline{X} \,.\, \overline{Y}$

(ii) $\overline{X \,.\, Y} = \overline{X} + \overline{Y}$

Proof: (i) By Perfect induction

(i)Truth Table 11 for L.H.S. $\overline{X + Y}$ and for R.H.S. $\overline{X} \,.\, \overline{Y}$

X	Y	$\overline{X}$	$\overline{Y}$	X +Y	$\overline{X + Y}$	$\overline{X} \,.\, \overline{Y}$
0	0	1	1	0	1	1
0	1	1	0	1	0	0
1	0	0	1	1	0	0
1	1	0	0	1	0	0

L.H.S. =R.H.S.

(ii)Truth Table 12 **for L.H.S.** $\overline{X \,.\, Y}$ and for R.H.S. $\overline{X} + \overline{Y}$

X	Y	$\overline{X}$	$\overline{Y}$	X .Y	$\overline{X \,.\, Y}$	$\overline{X} + \overline{Y}$
0	0	1	1	0	1	1
0	1	1	0	0	1	1
1	0	0	1	0	1	1
1	1	0	0	1	0	0

L.H.S. = R.H.S.

Rules:

3rd and 7th called idempotent. These shows that Boolean algebra is idempotent.

i.e. A + A = A and A. A = A

Proof:

The variable A can have only the value 0 or 1.

(3) If A =0, then 0 + 0 = 0

If A = 1, then 1 + 1 = 1

(7) If A = 0, then 0.0. = 0

If A =1, then 1 . 1 =1

Rule 2: X +1 =1

If X = 0 then 0 + 1 = 1

If X = 1, then 1 + 1 =1

Rule 5: X . 0 = 0

If X = 0, Then 0.0 = 1

If X= 1, Then 1 .0 = 1

Rule 9: $\overline{\overline{X}} = X$, i.e., the Boolean algebra is involuted.

If X =0, Then $\overline{O} = 1$ and $\overline{1} = 0$

So $\overline{\overline{O}} = \overline{1} = 0$

If X=1, Then $\overline{1} = 0$ and $\overline{O} = 1$

So $\bar{\bar{1}} = \bar{0} = 1$

Similarly we can prove the remaining rules by setting the values of variables as 0 and 1 or by perfect induction

Example:2: **Express the Boolean function**

$$XY + YZ + \bar{Y}Z = XY + Z$$

Solution:

$$
\begin{aligned}
\text{L.H.S.} &= XY + YZ + \bar{Y}Z \\
&= XY + Z(Y + \bar{Y}) \\
&= XY + Z.1 \\
&= XY + Z \\
\text{L.H.S} &= \text{R.H.S.}
\end{aligned}
$$

Example 3: **Find the complement of the expression: X + YZ and verified the result by perfect induction.**

Solution:

$$\overline{X + YZ} = \bar{X} . \overline{YZ}$$

$$= \bar{X} .(\bar{Y} + \bar{Z}) \text{ by DeMorgan's Law}$$

This relation can be verified by perfect induction.

Truth Table 13 for L.H.S. $\overline{X+YZ}$ and for R.H.S. $\bar{X} . (\bar{Y} + \bar{Z})$

X	Y	Z	$\bar{X}$	$\bar{Y}$	$\bar{Z}$	YZ	X+YZ	$\bar{Y} + \bar{Z}$	$\overline{X+YZ}$	$\bar{X}(\bar{Y}+\bar{Z})$
0	0	0	1	1	1	0	0	1	1	1
0	0	1	1	1	0	0	0	1	1	1
0	1	0	1	0	1	0	0	1	1	1
0	1	1	1	0	0	1	1	0	0	0
1	0	0	0	1	1	0	1	1	0	0
1	0	1	0	1	0	0	1	1	0	0
1	1	0	0	0	1	0	1	1	0	0
1	1	1	0	0	0	1	1	0	0	0

L.H.S. = R.H.S.

Example 4: **Find the complement of $\bar{A}B + C\bar{D}$, (b) AB +CD = 0**

Solution:

(a) $\overline{\bar{A}B + C\bar{D}} = (\overline{\bar{A}B}) . (\overline{C\bar{D}})$

$= (\bar{\bar{A}} + \bar{B}) . (\bar{C} + \bar{\bar{D}})$

$(A + \bar{B}) . (\bar{C} + D)$

(b) $AB + CD = 0$

Taking complement on both sides.

$$= \overline{AB + CD} = \overline{O}$$

$$= \overline{AB} \,.\, \overline{CD} = 1$$

$$(\overline{A} + \overline{B}) \,.\, (\overline{C} + \overline{D}) = 1$$

Example 5: Simplify the Boolean expressions:

(i) $(X + Y)(X + \overline{Y})(\overline{X} + Z)$

(ii) $XYZ + X\overline{Y}Z + XY\overline{Z}$

Solution:

(i) First simplify $(X + Y)(X + \overline{Y})$

$(X + Y)(X + \overline{Y}) = XX + X\overline{Y} + YX + Y\overline{Y}$

$= X + X\overline{Y} + YX + O$, as $XX = X$

as $Y\overline{Y} = 0$

$= X + X(\overline{Y} + Y)$, as $\overline{Y} + Y = 1$

$= X + X \,.\, 1$, as $X \,.1 = X$

$= X + X$

$= X$

Now $(X + Y)(X + \overline{Y})(\overline{X} + Z)$

$= X(\overline{X} + Z)$

$= X\overline{X} + XZ$, by distributive law

$= 0 + XZ$

$= XZ$

(ii) $XYZ + X\overline{Y}Z + XY\overline{Z}$

$= XZ(Y + \overline{Y}) + XY\overline{Z}$

$= XZ + XY\overline{Z}$, as $Y + \overline{Y} = 1$

$= X(Z + Y\overline{Z})$

$= X[(Z + Y).(Z + \overline{Z})]$, (By Rule 15 dual of distributive

$= X [(Z + Y). 1] = X (Z + Y)$

$=X (Y + Z)$, by commutative law.

Example 6: Minimize the following expression by use of Boolean rules.

(a) $X = A B C + \overline{A} B + A B \overline{C}$

(b) $X = \overline{A} B \overline{C} + A \overline{B} \overline{C} + \overline{A} \overline{B} \overline{C} + \overline{A} \overline{B} \overline{C}$

(c) $AB + \overline{A} C + B C = AB + \overline{A} C$

(d) $(A + B) (\overline{A} + C) (B + C) =(A + B) (\overline{A} + C)$

Solution:

(a) $X \quad =ABC + \overline{A} B + AB\overline{C}$

$= ABC + AB \overline{C} + \overline{A} B$

$= AB (C + \overline{C}) + \overline{A} B$

$=AB + \overline{A} B$ as $C + \overline{C} = 1$

$= (A + \overline{A}) B.$

$= 1. B$

$=B$

(b) $X \quad = \overline{A} B\overline{C} + A \overline{B} \overline{C} + \overline{A} \overline{B} \overline{C} + \overline{A} \overline{B} \overline{C}$

$=\overline{A} B\overline{C} + A\overline{B} \overline{C} + \overline{A} \overline{B} \overline{C}$ as $\overline{A} + \overline{A} = \overline{A}$

$=\overline{A} B\overline{C} +(A + \overline{A}) \overline{B} \overline{C}$

$=\overline{A} B\overline{C} + 1 . B\overline{C}$

$= (\overline{A} B + \overline{B}) \overline{C}$

$= [(\overline{A} + \overline{B}) . (\overline{B} + \overline{B})] \overline{C}$ by the dual of distribution, rules 15

$= (\overline{A} + \overline{B}) . 1] \overline{C}$

$= (\overline{A} + \overline{B}) \overline{C}$

(c) L.H.S. $= AB + \overline{A}C + BC$

$= AB + \overline{A}C + BC$

$= AB + \overline{A}C + 1.BC$ as $1 = A + \overline{A}$

$= AB + \overline{A}C + (A + \overline{A})BC$

$= AB + \overline{A}C + ABC + \overline{A}BC$, by distributive law

$= AB + ABC + \overline{A}C + \overline{A}BC$, by commutative law

$= AB(1 + C) + \overline{A}C(1 + B)$, AS $1 + X = 1$

$= AB + \overline{A}C$

L.H.S. = R.H.S.

(d) L.H.S. $= (A + B)(\overline{A} + C)(B + C)$

$= (A\overline{A} + AC + B\overline{A} + BC)(B + C)$

$= (0 + AC + B\overline{A} + BC)(B + C)$

$= (AC + B\overline{A} + BC)(B + C)$

$= [AC + B(\overline{A} + C)](B + C)$

$= ABC + ACC + BB(\overline{A} + C) + BC(\overline{A} + C)$

$= ABC + AC + B(\overline{A} + C) + BC(\overline{A} + C)$

$= AC(B + 1) + B(\overline{A} + C)(1 + C)$

$= AC + B(\overline{A} + C)$

$= A\overline{A} + AC + B(\overline{A} + C)$ as $A\overline{A} = 0$

$= A(\overline{A} + C) + B(\overline{A} + C)$ or by rule 19.

$= (A + B)(\overline{A} + C)$

L.H.S. = R.H.S.

Sum of Product (Minterm):

The **Sum of Product** means that the products of the variables that are seperated by a plus sign. The variables can be complemented or uncomplemented, **for example,**

$$AB + A\overline{B} + \overline{A}B + \overline{A}\,\overline{B} + AB\overline{C} + A\overline{B}C + A\overline{B}C$$

Product of sum (Maxterm):

The **Product of Sum** means that the sum of variables that are seperated by a multiplication sign. For example,

$$(A + B)(\overline{A} + B)(A + \overline{B})(\overline{A} + \overline{B}),$$

$$(A + \overline{B} + C)(A + \overline{B} + \overline{C})(\overline{A} + \overline{B} + C)$$

Fundamental Products:

The products that produce a high (1) output are called Fundamental products. For example. for the two input variables A and B.

We have four possible combination's, which are shown in the table below and the fundamental product's corresponding to each:

Truth Table 14 Two Variables

Table 14

A	B	Fundamental Product	Output Z
0	0	$\overline{A}\,\overline{B}$	1
0	1	$\overline{A}B$	1
1	0	$A\overline{B}$	1
1	1	AB	1

For three input variables or signals a similar idea is applied. Whenever the input variable is 0, the same variable is complemented in the fundamental product.

Truth Table 15. Three variables

A	B	C	Output Z	Fundamental Product	Output for product	Sum terms	Output for Sum
0	0	0	0	$\overline{A}\,\overline{B}\,\overline{C}$	1	$A+B+C$	0
0	0	1	0	$\overline{A}\,\overline{B}C$	1	$A+B+\overline{C}$	0
0	1	0	1	$\overline{A}B\overline{C}$	1	$A+\overline{B}+C$	0
0	1	1	1	$\overline{A}BC$	1	$A+\overline{B}+\overline{C}$	0
1	0	0	0	$A\overline{B}\,\overline{C}$	1	$\overline{A}+B+C$	0
1	0	1	0	$A\overline{B}C$	1	$\overline{A}+B+\overline{C}$	0
1	1	0	1	$AB\overline{C}$	1	$\overline{A}+\overline{B}+C$	0
1	1	1	0	ABC	1	$\overline{A}+\overline{B}+\overline{C}$	0

Sum of product(SOP) $= \overline{A}B\overline{C} + \overline{A}BC + AB\overline{C}$

Product of sum(POS) $= (A + B + C)(A + B + \overline{C})(\overline{A} + B + C)$

$$(\overline{A} + B + \overline{C})(\overline{A} + \overline{B} + \overline{C})$$

Note: See remarks for sum of product and product of sums.

Remarks:

(1) A sum of product (minterm) is obtained as follows: For each row of the truth table for which the out put is 1, the Boolean term is the product of variables that are equal to 1 and the complement of variable that are equal to 0. The sum of these products is the desired Boolean equation.

(2) A product of sum expression is obtained as follows: each row of the truth table for which the output is 0, the Boolean term is the sum of the variables that are equal 0 plus the complement of the variables that are equal to 1. The product of these sum is the desired Boolean equation.

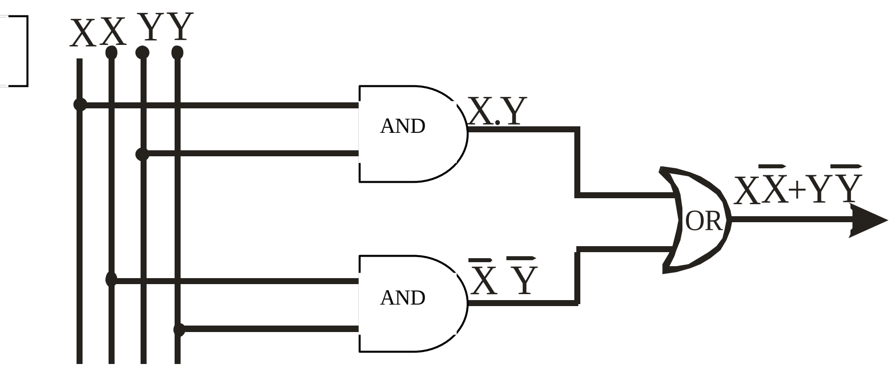
Combination of Gates:

The OR gate and AND gates and invertors can be interconnected to form gatting or logic networks, in the switching theory, these are also called combinational networks. The Boolean algebra expression corresponding to a given Network can be driven by systematically progressing from input to output on the gates.

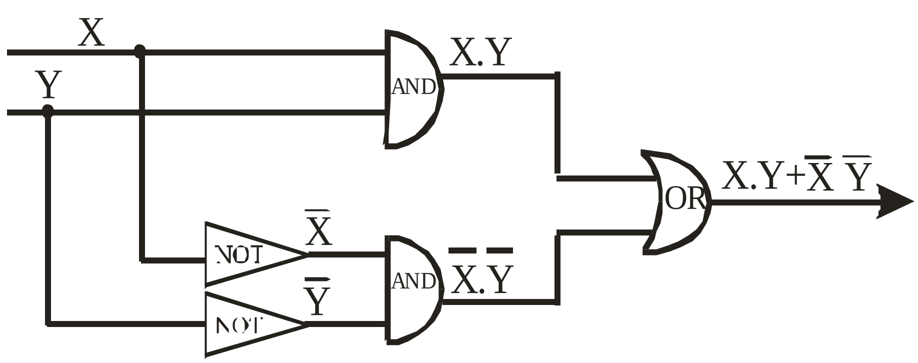
A net work that forms (i) $(X.Y) + (\bar{X} . \bar{Y})$

and another net work that forms (ii) $(X + Y). (\bar{X} + \bar{Y})$ are shown as

(i)



Or,



(ii)

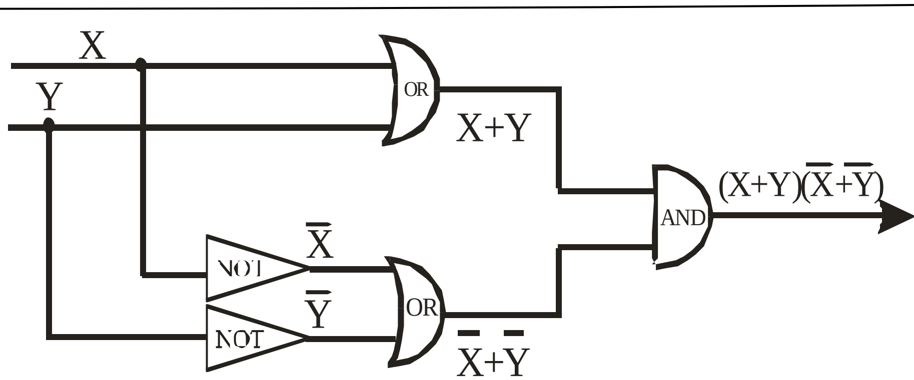


Fig. 8

Boolean Expression and Logic Diagrams:

Boolean expressions are frequently written to describe mathematically the behavior of a logic circuit. Using a truth table and the Boolean expression, one can determine which combinations of input signals cause the output signal.

Example 7: Write the Boolean expression that describes mathematically the behaviour of logic circuit shown in fig. 10. Use a truth table to determine what input conditions produce a logic 1 output.

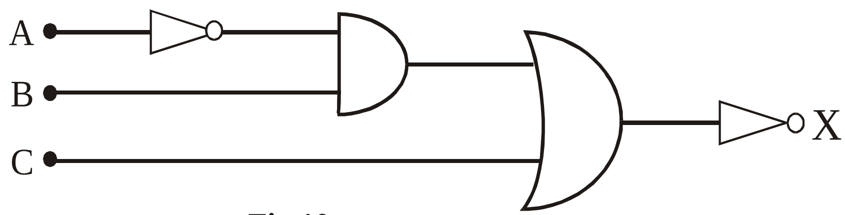


Fig.10

Solution:

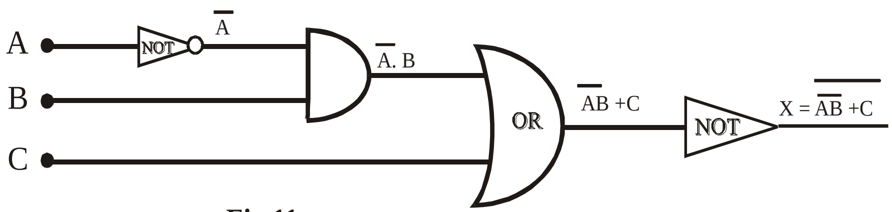


Fig.11

Fig.11 Circuit showing solution for example 8

Solution:

Truth Table 18 for the Circuit in Fig.11

A	B	C	$\overline{A}$	$\overline{A}\,B$	$\overline{A}\,B+C$	$\overline{\overline{A}\,B+C}$
0	0	0	1	0	0	1
0	0	1	1	1	1	0
0	1	0	1	1	1	0
0	1	1	1	1	1	0
1	0	0	0	0	0	1
1	0	1	0	1	1	0
1	1	0	0	0	0	1
1	1	1	0	1	1	0

Thus the input conditions those produce a logic 1 output are : 0 0 0 , 100, 110

Example 8: Given the Boolean expression

$X = AB + ABC + A\,\overline{B}\,\overline{C} + A\,\overline{C}$

(a) Draw the logic diagram for the expression.

(b) Minimize the expression.

(c) Draw the logic diagram for the reduced expression.

Solution: (a)The logic diagram is shown in the Fig. 12.

(b) $X = AB + ABC + A\overline{B}\,\overline{C} + A\overline{C}$

$= AB(1 + C) + A\overline{C}(\overline{B} + 1)$

$= AB.1 + A\overline{C}.1 = AB + A\overline{C}$

$= A(B + \overline{C})$

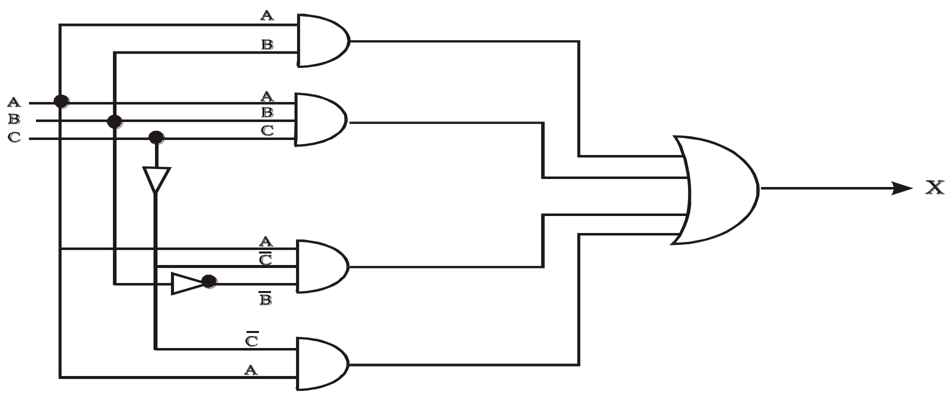


FIG. 12

(c)

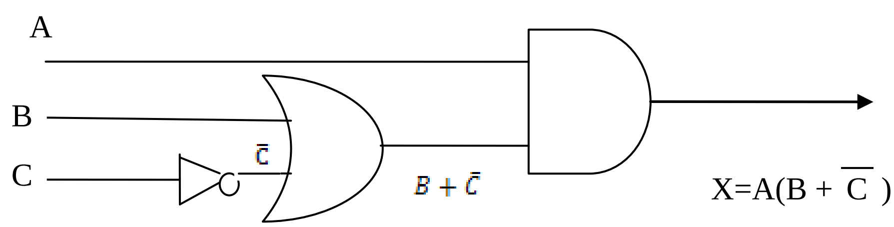


Fig.13 Minimize diagram for example 9.