# Mathematical Reasoning

## MATHEMATICAL REASONING

### STATEMENTS

In reasoning we communicate our ideas or thoughts with the help of sentences in a particular language.

"A sentence is called a mathematically acceptable statement if it is either true or false but not both".

A statement is assumed to be either true or false. A true statement is known as a valid statement and a false statement is known as an invalid statement.

**Note :**

A statement can not be both true and false at the same time.

**Illustration 1 :**

consider the following sentences :

(i) Three plus two equals five.

(ii) The sum of two negative number is negative.

(iii) Every square is a rectangle.

Each of these sentences is a true sentence, therefore they are statements.

**Illustration 2 :**

Consider the following sentences :

(i) Three plus four equals six.

(ii) All prime numbers are odd.

(iii) Every relation is a function.

Each of these sentences is a false sentence, therefore they are statements.

**Illustration 3 :**

Consider the following sentences :

(i) The sum of x and y is greater than 0 (ii)

The square of a number is even.

Here, we are not in a position to determine whether it is true of false unless we know what the numbers are. Therefore these sentences are not a statement.

### *Solved Examples*

**Ex.10** Which of the following sentences are statements :

(i) Three plus two equals five.

(ii) The sum of two negative number is negative

(iii)Every square is a rectangle.

**Sol.** Each of these sentences is a true sentence therefore they all are statements.

**Ex.11** Which of the following sentences are statements :

(i) Three plus four equals six.

(ii) All prime numbers are odd.

(iii) Every relation is a function.

**Sol.** Each of these sentences is a false sentence therefore all of these are statements.

**Ex.12** Which of the following sentences are statements :

(i) The sum of x and y is greater than 0.

(ii) The square of a number is even.

**Sol.** Here, we are not in a position to determine whether it is true of false unless we know what the numbers are. Therefore these sentences are not a statement.

**Ex.13** Which of the following sentances are statements :

(i) Give me a glass of water.

(ii) Is every set finite ?

(iii) How beautiful ?

(iv) Tomorrow is Monday.

(v) May God bless you !

**Sol.** None of these sentences is a statement.

**Note :**

(i) Imperative (expresses a request or command), exclaimatory sentences (expresses some strong feeling), Interrogative sentences (asks some question) are not considered as a statement in mathematical language.

(ii) Sentences involving variable time such as "today", "tomorrow" or "yesterday" are not statements.

(iii) Scientifically established facts are considered true.

(iv) Optative (blessing & wishes) sentences are not a statement.

**Note :**

(i) Imperative sentences (expresses a repuest or command), exclamatory sentences (expresses some strong feeling), Interrogative sentences (asks some questions) do not considered as a statement in mathematical language.

(ii) Sentences involving variable time such as "today", "tomorrow" or "yesterday" are not statements.

**Illustration 4 :** Consider the sentences :

(i) Give me a glass of water.

(ii) Is every set finite ?

(iii) How beautiful ?

(iv) Tomorrow is Monday.

These all the sentences are not a statement.

## TRUTH TABLE

Truth table is that which gives truth values of compound statements.

It has a number of rows and columns. The number of rows depend upon the number of simple statements.

Note that for n statements, there are $2^n$ rows.

**(i)** Truth table for single statement p :

Number of rows = $2^1 = 2$

| p |
|---|
| T |
| F |

**(ii)** Truth table for two statements p and q :

Number of rows = $2^2 = 4$

| p | q |
|---|---|
| T | T |
| T | F |
| F | T |
| F | F |

**(iii)** Truth table for three statements p, q and r.

Number of rows = $2^3 = 8$

| p | q | r |
|---|---|---|
| T | T | T |
| T | T | F |
| T | F | T |
| T | F | F |
| F | T | T |
| F | T | F |
| F | F | T |
| F | F | F |

## NEGATION OF A STATEMENT

The denial of a statement p is called its negation and is written as

| p | ~p |
|---|---|
| T | F |
| F | T |

**Truth table**

~ p and read as 'not p'. Negation of any statement p is formed by writing "It is not the case that .........."

or "It is false that ............"

or inserting the word "not" in p.

## *Solved Examples*

**Ex.14** Write negation of following statements :

(i) "All cats scratch"

(ii) "$\sqrt{5}$ is a rational number".

**Sol.** (i) Some cats do not scratch

OR

There exists a cat which does not scratch

OR

At least one cat does not scratch

(ii) $\sqrt{5}$ is an irrational number

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**Ex.(1)** p : The number 2 is greater than 7.

The negation of this statement is -

~ p : It is not the case that the number 2 is greater than 7.

or

~ p : It is false that the number 2 is greater than 7.

or

~ p : The number 2 is not greater than 7.

**Ex.(2)** p : 7 > 9 The negation of this statement is

$\sim p : 7 \ngtr 9$ or $\sim p : 7 \le 9$

**Truth table for negation of a statement :**

| p | ~p |
|---|---|
| T | F |
| F | T |

### COMPOUND STATEMENTS

If a statement is combination of two or more statements, then it is said to be a compound statement.

And each statement which form a compound statement are known as its sub-statements or component statements.

**Ex.(1)** Number 3 is prime or it is odd

The component statements are :

p : Number 3 is prime

q : Number 3 is odd.

The connecting word is "or"

**Ex.(2)** 25 is a multiple of 5 and 8

The component statements are

p : 25 is a multiple of 5

q : 25 is a multiple of 8

The connecting word is "and"

### BASIC CONNECTIVES

In the compound statement, two or more statements are connected by words like 'and', 'or', 'if . . . . . then', 'only if', 'if and only if', 'there exists', 'for all' etc. These are called connectives. When we use these compound statements, it is necessary to understand the role of these words.

### THE WORD "AND" (CONJUCTION)

Any two statements can be connected by the word "and" to form a compound statement. The compound statement with word "and" is true if all its component statements are true.

| p | q | $p \wedge q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

**Truth table**

The compound statement with word "and" is false if any or all of its component statements are false. The compound statement "p and q" is denoted by "$p \wedge q$".

**Ex.1** 42 is divisible by 5, 6 and 7.

The component statements of this statement are-

p : 42 is divisible by 5.

q : 42 is divisible by 6.

r : 42 is divisible by 7.

Clearly statement p is false, while the q and r are true statements.

i.e. The compound statement is false.

**Ex.2** $9 > 4$ and $2 < 7$

The component statements of this statement are

$p : 9 > 4$

$q : 2 > 7$

Clearly p, q are true statement

i.e. the given compound statement is true.

**Ex.3** $5 < 12$ and $15 < 7$

p : $5 < 12$

q : $15 < 7$

Clearly p, q are false statements.

i.e. the given compound statement is false.

The above discussion suggests us the following rules :

**Rule -**

**(1)** The compound statement with word "and" is true if all its component statements are true.

**(2)** The compound statement with word "and" is false if any or all of its component statements are false.

**Truth table for compound statement with word "And"**

The compound statement "p and q" is denoted by "p ^ q".

| p | q | $p \wedge q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

## THE WORD "OR" (DISJUNCTION)

Any two statements can be connected by the word "OR" to form a compound statement. The compound statement with word "or" is true if any or all of its component statements are true.

| p | q | p v q |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

**Truth table**

The compound statement with word "or" is false if all its component statement are false. The compound statement "p or q" is denoted by "p v q."

**Ex.1** 125 is a multiple of 7 or 8.

Its component statements are.

p : 125 is a multiple of 7

q : 125 is a multiple of 8

clearly, both p and q are false

i.e. the compound statement is false.

**Ex.2** The earth is round or the sun is hot

Its component statements are :

p : the earth is round.

q : the sun is hot.

clearly, both p and q are true.

i.e. The compound statement is true.

**Ex.3** $\sqrt{2}$ is a rational number or an irrational number

its component statements are :

p : $\sqrt{2}$ is a rational number.

q : $\sqrt{2}$ is an irrational number.

clearly p is false and q is true statement i.e. The compound statement is true.

The above discussion suggest us the following rules :

**Rule**

**(1)** The compound statement with word "or" is true if any or all of its component statements are true.

**(2)** The compound statement with word "or" is false if all its component statement are false.

**Truth table for compound statement with word "OR"** :

The compound statement "p or q" is denoted by "$p \vee q$".

| p | q | $p \vee q$ |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

## TYPES OF "OR"

**(i)** **Exclusive OR :** If in statement $p \vee q$ i.e. p or q, happening of any one of p, q excludes the happening of the other then it is exclusive or. Here both p and q cannot occur together. For example in statement "I will go to delhi either by bus or by train", the use of 'or' is exclusive.

**(ii) Inclusive OR :** If in statement p or q, both p and q can also occur together then it is inclusive or. The statement 'In senior secondary exam, you can take optional subject as physical education or computers' is an example of use of inclusive OR.

## Solved Examples

**Ex.15** Find the truth value of the statement "2 divides 4 and 3 + 7 = 8"

**Sol.** 2 divides 4 is true and 3 + 7 = 8 is false. so given statement is false.

**Ex.16** Write component statements of the statement "All living things have two legs and two eyes".

**Sol.** Component statements are :

All living things have two legs

All living things have two eyes

## IMPLICATION

There are three types of implications which are "if . . . . then", "only if" and "if and only if".

**Conditional Connective :**

**'IF . . . . THEN' :**

If p and q are any two statements then the compound statement in the form "If p then q" is called a conditional statement. The statement "If p then q" is denoted by $p \rightarrow q$ or $p \Rightarrow q$ (to be read as p implies q). In the implication $p \rightarrow q$, p is called the antecedent (or the hypothesis) and q the consequent (or the conclusion)

If p then q reveals the following facts :

(i) p is a sufficient condition for q

(ii) q is a necessary condition for p

(iii) 'If p then q' has same meaning as that of 'p only if q'

(iv) $p \rightarrow q$ has same meaning as that of $\sim q \rightarrow \sim p$

| p | q | $p \rightarrow q$ | $q \rightarrow p$ |
|---|---|---|---|
| T | T | T | T |
| T | F | F | T |
| F | T | T | F |
| F | F | T | T |

**Truth table**

**Examples :**

(i) If x = 4, then $x^2 = 16$

(ii) If ABCD is a parallelogram, then AB = CD

(iii) If Mumbai is in England, then 2 + 2 = 5

(iv) If Shikha works hard, then it will rain today.

## CONTRAPOSITIVE, CONTRADICTION AND CONVERSE OF A CONDITIONAL STATEMENT :

If p and q are two statements then

Let $p \Rightarrow q$ Then
(i) (Contrapostive of $p \Rightarrow q$)is($\sim q \Rightarrow \sim p$)
(ii) (Contradicton of $p \Rightarrow q$)is($q \Rightarrow \sim p$)
(iii) (Converseof $p \Rightarrow q$)is($q \Rightarrow p$)

**Note :** A statement and its contrapositive convey the same meaning.

## Solved Examples

**Ex.17** Write the contrapositive of the following statement : "If Mohan is poet, then he is poor"

**Sol.** Consider the following statements :

p : Mohan is a poet

q : Mohan is poor

Clearly, the given statement in symbolic form is $p \rightarrow q$.Therefore, its contrapositive is given by $\sim q \rightarrow \sim p$.

Now, $\sim p$ : Mohan is not a poet.

$\sim q$ : Mohan is not poor.

$\therefore$ $\sim q \rightarrow \sim p$ : If Mohan is not poor, then he is not a poet.

Hence the contrapositive of the given statement is "If Mohan is not poor, then he is not a poet".

**Ex.18** Write the converse and the contrapositive of the statement "If x is a prime number, then x is odd".

**Sol.** Given statement is :

"If x is a prime number then x is odd".

Let p : x is a prime number and q : x is odd

$\therefore$ Given statement is $p \rightarrow q$

The converse of $p \rightarrow q$ is $q \rightarrow p$

i.e. "If x is odd then x is a prime number"

The contrapositive of $p \rightarrow q$ is $\sim q \rightarrow \sim p$

i.e. "If x is not odd then x is not a prime number".

**Ex.19** Write the contradiction of "If it rains, then I stay" at home.

**Sol.** If I stay at home then It does not rain.

## BICONDITIONAL STATEMENTS

If p and q are any two statements then the compound statement in the form of "p if and only if q" is called a biconditional statements and is written in symbolic form $p \leftrightarrow q$ or $p \Leftrightarrow q$

**Ex.** The following statements are biconditional statements

(i) A number is divisible by 3 if and only if the sum of the digits forming the number is divisible by 3.

(ii) One is less than seven if and only if two is less than eight.

(iii) A triangle is equilateral if and only if it is equiangular.

**Truth table for a biconditional statement :**

**Rule :**

$p \leftrightarrow q$ is true only when both p and q have the same value.

| p | q | $p \to q$ | $q \leftrightarrow p$ |
|---|---|---|---|
| T | T | T | T |
| T | F | F | F |
| F | T | F | F |
| F | F | T | T |

**Negation of biconditional statement :**

$\sim (p \leftrightarrow q) = (p \wedge \sim q) \vee (\sim p \wedge q)$

## *Solved Examples*

**Ex.20** Write the negation of each of the following statements :

(i) ABC is an equilateral triangle if and only if it is equiangular.

(ii) Two lines are parallel if and only if they have the same slope.

**Sol.** (i) Let p : ABC is an equilateral triangle

q : It is equiangular

$\therefore \sim (p \leftrightarrow q) = (p \wedge \sim q) \vee (\sim p \wedge q)$

$\sim (p \leftrightarrow q)$ : Either ABC is an equilateral triangle and it is not equiangular or ABC is not an equilateral triangle and it is equiangular.

(ii) Let p : Two lines are parallel

q : They have the same slope

$\therefore \sim (p \leftrightarrow q) = (p \wedge \sim q) \vee (\sim p \wedge q)$

$\sim (p \leftrightarrow q)$ = Either two lines are parallel and they not have the same slopes or two lines are not parallel and they have the same slope.

## TAUTOLOGY AND FALLACY (CONTRADICTIONS)

**(a) Tautology :** This is a statement which always true for all truth values of its components.

**Ex.** Consider $p \vee \sim p$

Truth table

| p | ~p | $p \vee \sim p$ |
|---|---|---|
| T | F | T |
| F | T | T |

we observe that last columns is always true. Hence $p \vee \sim p$ is a tautology.

**(b) Fallacy (contradiction) :** This is statement which is always false for all truth values of its components.

**Ex.** Consider $p \wedge \sim p$

| p | ~p | $p \wedge \sim p$ |
|---|---|---|
| T | F | F |
| F | T | F |

we observe that last columns is always false. Hence $p \wedge \sim p$ is a fallacy (contradiction).

## *Solved Examples*

**Ex.21** Prove by construction of truth table that $p \vee \sim(p \wedge q)$ is a tautology.

**Sol.** Truth table

| p | q | $p \wedge q$ | $\sim (p \wedge p)$ | $p \vee \sim (p \wedge q)$ |
|---|---|---|---|---|
| T | T | T | F | T |
| T | F | F | T | T |
| F | T | F | T | T |
| F | F | F | T | T |

Since the last column shows T's only, therefore $p \vee \sim (p \wedge q)$ is a tautology.

**Ex.22** Prove by constructing truth table that $(p \wedge q) \wedge \sim (p \vee q)$ is a fallacy (contradiction)

**Sol.** Truth table

| p | q | $p \wedge q$ | $p \vee q$ | $\sim (p \vee q)$ | $(p \wedge q) \wedge \sim (p \vee q)$ |
|---|---|---|---|---|---|
| T | T | T | T | F | F |
| T | F | F | T | F | F |

| | | | | | |
|---|---|---|---|---|---|
| F | T | F | T | F | F |
| F | F | F | F | T | F |

Since the last column shows F's only, therefore $(p \wedge q) \wedge \sim (p \vee q)$ is a fallacy.

## ARGUMENTS AND THEIR VALIDITY

An argument is an assertion that a given set of statements $s_1, s_2, ......s_n$ implies other statement. In other word, an argument is an assertion that the statement s follows from statements $s_1, s_2, .....s_n$ . Statements $s_1, s_2, .....s_n$ are called hypotheses (or premises) and the statement s is called the conclusion.

We denote the argument containing hypotheses $s_1, s_2, .....s_n$ and conclusion s by

$s_1, s_2, ......s_n$ ; s

or

$s_1, s_2, ......s_n$ /- s

or

$(s_1 \wedge s_2 \wedge ..... \wedge s_n) \rightarrow s$

or

$s_1$
$s_2$
$s_3$
...
...
$s_n$
_____
so s

The symbol "/-" is read as turnstile.

## LOGICALLY EQUIVALENT STATEMENTS

If truth values of statements p and q are same then they are logically equivalent and written as $p \equiv q$.

## ALGEBRA OF STATEMENTS

If p, q, r are any three statements and t is a tautology, c is a contradiction then

(1) Commutative Law :

(i) $p \vee q = q \vee p$ (ii) $p \wedge q = q \wedge p$

| p | q | $(p \wedge q)$ | $(q \wedge p)$ | $(p \vee q)$ | $(q \vee p)$ |
|---|---|---|---|---|---|
| T | T | T | T | T | T |
| T | F | F | F | T | T |
| F | T | F | F | T | T |
| F | F | F | F | F | F |

(2) Associative Law :

(i) $p \vee (q \vee r) = (p \vee q) \vee r$
(ii) $p \wedge (q \wedge r) = (p \wedge q) \wedge r$

| p | q | r | $(p \wedge q)$ | $(q \wedge r)$ | $(p \wedge q) \wedge r$ | $p \wedge (q \wedge r)$ |
|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T |
| T | T | F | T | F | F | F |
| T | F | T | F | F | F | F |
| T | F | F | F | F | F | F |
| F | T | T | F | T | F | F |
| F | T | F | F | F | F | F |
| F | F | T | F | F | F | F |
| F | F | F | F | F | F | F |

(3) Distributive Law :

(i) $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$
(ii) $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$
(iii) $p \wedge (q \wedge r) \equiv (p \wedge q) \wedge (p \wedge r)$
(iv) $p \vee (q \vee r) \equiv (p \vee q) \vee (p \vee r)$

| p | q | r | $(q \vee r)$ | $(p \wedge q)$ | $(p \wedge r)$ | $p \wedge (q \vee r)$ | $(p \wedge q) \vee (p \wedge r)$ |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| T | T | F | T | T | F | T | T |
| T | F | T | T | F | T | T | T |
| T | F | F | F | F | F | F | F |
| F | T | T | T | F | F | F | F |
| F | T | F | T | F | F | F | F |
| F | F | T | T | F | F | F | F |
| F | F | F | F | F | F | F | F |

(4) Identity Law :

(i) $p \vee t = t$ (ii) $p \wedge t = p$
(iii) $p \vee c = p$ (iv) $p \wedge c = c$

| p | t | c | $(p \wedge t)$ | $(p \vee t)$ | $(p \wedge c)$ | $(p \vee c)$ |
|---|---|---|---|---|---|---|
| T | T | F | T | T | F | T |
| F | T | F | F | T | F | F |

(5) Complement Law :

(i) $p \vee (\sim p) = t$ (ii) $p \wedge (\sim p) = c$
(iii) $\sim t = c$ (iv) $\sim c = t$
(v) $\sim(\sim p) = p$

| p | $\sim p$ | $(p \wedge \sim p)$ | $(p \vee \sim p)$ |
|---|---|---|---|
| T | F | F | T |
| F | T | F | T |

(6) Idempotent Law :

(i) $p \vee p = p$ (ii) $p \wedge p = p$

| p | $(p \wedge p)$ | $(p \vee p)$ |
|---|---|---|
| T | T | T |
| F | F | F |

(7) De Morgan's law :

(i) $\sim(p \vee q) = (\sim p) \wedge (\sim q)$

(ii) $\sim(p \wedge q) = (\sim p) \vee (\sim q)$

| p | q | $\sim p$ | $\sim q$ | $(p \wedge q)$ | $\sim(p \wedge q)$ | $(\sim p \vee \sim q)$ |
|---|---|---|---|---|---|---|
| T | T | F | F | T | F | F |
| T | F | F | T | F | T | T |
| F | T | T | F | F | T | T |
| F | F | T | T | F | T | T |

(8) Involution laws (or Double negation laws) :

$\sim(\sim p) \equiv p$

| p | $\sim q$ | $\sim(\sim p)$ |
|---|---|---|
| T | F | T |
| F | T | F |

(9) Contrapositive Laws : $p \to q \equiv \sim q \to \sim p$

| p | q | $\sim p$ | $\sim q$ | $p \to q$ | $\sim q \to \sim p$ |
|---|---|---|---|---|---|
| T | T | F | F | T | T |
| T | F | F | T | F | F |
| F | T | T | F | T | T |
| F | F | T | T | T | T |

(10) Truth table of biconditional statement :

| p | q | $p \leftrightarrow q$ | $q \leftrightarrow p$ | $p \to q$ | $q \to p$ | $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$ |
|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T |
| T | F | F | F | F | T | F |
| F | T | F | F | T | F | F |
| F | F | T | T | T | T | T |

## NEGATION OF COMPOUND STATEMENTS

If p and q are two statements then

(i) Negation of conjunction : $\sim(p \wedge q) \equiv \sim p \vee \sim q$

| p | q | $\sim p$ | $\sim q$ | $(p \wedge q)$ | $\sim(p \wedge q)$ | $(\sim p \vee \sim q)$ |
|---|---|---|---|---|---|---|
| T | T | F | F | T | F | F |
| T | F | F | T | F | T | T |
| F | T | T | F | F | T | T |
| F | F | T | T | F | T | T |

(ii) Negation of disjunction : $\sim(p \vee q) \equiv \sim p \wedge \sim q$

| p | q | $\sim p$ | $\sim q$ | $(p \vee q)$ | $\sim(p \vee q)$ | $(\sim p \wedge \sim q)$ |
|---|---|---|---|---|---|---|
| T | T | F | F | T | F | F |
| T | F | F | T | T | F | F |
| F | T | T | F | T | F | F |
| F | F | T | T | F | T | T |

(iii) Negation of conditional : $\sim(p \to q) \equiv p \wedge \sim q$

| p | q | $\sim q$ | $(p \to q)$ | $\sim(p \to q)$ | $(p \wedge \sim q)$ |
|---|---|---|---|---|---|
| T | T | F | T | F | F |
| T | F | T | F | T | T |
| F | T | F | T | F | F |
| F | F | T | T | F | F |

(iv) Negation of biconditional : $\sim(p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$ or $p \leftrightarrow \sim q$

| p | q | $\sim p$ | $\sim q$ | $(p \to q)$ | $\sim(p \to q)$ | $(p \wedge \sim q)$ | $(p \leftrightarrow q)$ |
|---|---|---|---|---|---|---|---|
| T | T | F | F | T | F | F | T |
| T | F | F | T | F | T | T | F |
| F | T | T | F | T | F | F | F |
| F | F | T | T | T | F | F | T |

| $\sim(p \leftrightarrow q)$ | $p \leftrightarrow \sim q$ | $q \wedge \sim p$ | $(p \wedge \sim q) \vee (q \wedge \sim p)$ |
|---|---|---|---|
| F | F | F | F |
| T | T | F | T |
| T | T | T | T |
| F | F | F | F |

we know that $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

$\therefore \sim(p \leftrightarrow q) \equiv \sim[(p \to q) \wedge (q \to p)]$

$\equiv \sim(p \to q) \vee \sim(q \to p) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$

**SUMMARY**

(i) $\sim(p \wedge q) \equiv (\sim p) \vee (\sim q)$

(ii) $\sim(p \vee q) \equiv (\sim p) \wedge (\sim q)$

(iii) $\sim(p \Rightarrow q) \equiv \sim(\sim p \vee q) = p \wedge (\sim q)$

(iv) $\sim(p \Leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$ or $p \Leftrightarrow \sim q$

## *Solved Examples*

**Ex.23** Write the negation of the following compound statements :

(i) All the students completed their homework and the teacher is present.

(ii) Square of an integer is positive or negative.

(iii) If my car is not in workshop, then I can go college.

(iv) ABC is an equilateral triangle if and only if it is equiangular

**Sol.** (i) The component statements of the given statement are :

p : all the students completed their homework.

q : The teacher is present.

The given statement is p and q. so its negation is ~p or ~q = Some of the students did not complete their home work or the teacher is not present.

(ii) The component statement of the given statements are :

p : Square of an integer is positive.

q : Square of an integer is negative.

The given statement is p or q. so its negation is ~p and ~q = Their exists an integer whose square is neither positive nor negative.

(iii) Consider the following statements :

p : My car is not in workshop

q : I can go to college

The given statement in symbolic form is $p \rightarrow q$

Now, $\sim(p \rightarrow q) \cong p \wedge (\sim q)$

$\Rightarrow$ $\sim(p \rightarrow q)$ : My car is not in workshop and I cannot go to college.

Hence the negation of the given statements is "My car is not in workshop and i can not go to college".

(iv) Consider the following statements :

p : ABC is an equilateral triangle.

q : It is equiangular

Clearly, the given statement is symbolic form is $p \leftrightarrow q$.

Now, $\sim(p \leftrightarrow q) \cong (p \wedge \sim q) \vee (\sim p \wedge q)$

$\Rightarrow$ $\sim(p \rightarrow q)$ : Either ABC is an equilateral triangle and it is not equiangular or ABC is not an equilateral triangle and it is equiangular.

**Ex.24** Show that $p \rightarrow (p \vee q)$ is a tautology

**Sol.**

| p | q | $p \vee q$ | $p \rightarrow (p \vee q)$ |
|---|---|---|---|
| T | T | T | T |
| T | F | T | T |
| F | T | T | T |
| F | F | F | T |

**Ex.25** By using laws of algebra of statements show that $(p \vee q) \wedge \sim p \equiv \sim p \wedge q$.

**Sol.** $(p \vee q) \wedge \sim p \equiv (\sim p) \wedge (p \wedge q)$

$\equiv (\sim p \wedge p) \vee (\sim p \wedge q)$

$\equiv f \vee (\sim p \wedge q)$

$\equiv \sim p \wedge q$

**Ex.26** Find the negation of statement $p \wedge \sim q$

**Sol.** Negation of $(p \wedge \sim q) \equiv \sim(p \wedge \sim q)$

$\equiv \sim p \vee \sim\sim q \equiv \sim p \vee q$

## VALID ARGUMENT

An argument is said to be a valid argument if the conclusion s is true whenever all the hypotheses $s_1, s_2, .... s_n$ are true or equivalently argument is valid when it is a tautology, otherwise the argument is called an invalid argument.

**Method of testing the validity of arguments :**

**Step I -**

Construct the truth table for conditional statements $s_1 \wedge s_2 \wedge s_3 \wedge ..... \wedge s_n \rightarrow s$.

**Step II -**

Check the last column of truth table. If the last column contains T only, then the given argument is valid. Otherwise, it is an invalid argument.

## *Solved Examples*

**Ex.27** Test the validity of argument $(s_1, s_2 ; s)$ where $s_1 : p \vee q$, $s_2 \sim p$ and $s : q$

**Sol.** We first construct the truth table for conditional statement

$s_1 \wedge s_2 \rightarrow s$ i.e. $(p \vee q) \wedge \sim p \rightarrow q$

Truth table

| p | q | $p \vee q$ | $\sim p$ | $(p \vee q) \wedge \sim p$ | $(p \vee q) \wedge \sim p \rightarrow q$ |
|---|---|---|---|---|---|
| T | T | T | F | F | T |
| T | F | T | F | F | T |
| F | T | T | T | T | T |
| F | F | F | T | F | T |

we observe that the last column of the truth table for $s_1 \wedge s_2 \rightarrow s$ contains T only. Thus it is a tautology. Hence the given argument is valid.

**Ex.28** Examine the validity of argument :

$[(p \to q) \wedge \sim q] \to (\sim p)$.

**Sol.**

| p | q | ~ p | ~ q | p → q | (p → q) ∧ ~ q | [(p → q) ∧ ~ q] → (~p) |
|---|---|---|---|---|---|---|
| T | T | F | F | T | F | T |
| T | F | F | T | F | F | T |
| F | T | T | F | T | F | T |
| F | F | T | T | T | T | T |

Since the last column shows T only therefore

$[(p \to q) \wedge \sim q] \to (\sim p)$ is a valid argument.

## DUALITY

The compound statements $s_1$ and $s_2$ are said to be duals of each other if one can be obtained from the other by replacing $\wedge$ by $\vee$ and $\vee$ by $\wedge$. The connectives $\wedge$ and $\vee$ are also called duals of each other.

**Duality symbol :**

Let $s(p, q) = p \wedge q$ be a compound statement Then $s^*(p, q) = p \vee q$ where $s^*(p, q)$ is the dual statement of s(p,q).

## *Solved Examples*

**Ex.29** Write the duals of the following statements

(i) $(p \vee q) \vee r$

(ii) $(p \vee q) \wedge (r \vee s)$

(iii) $[\sim(p \wedge q)] \vee [p \wedge \{q \vee s\}]$

**Sol.** The required duals are given by

(i) $(p \wedge q) \wedge r$

(ii) $(p \wedge q) \vee (r \wedge s)$

(iii) $[\sim(p \vee q)] \wedge [p \vee \{q \wedge s\}]$

**Note :** If the compound statements s contains the special variables t(tautology) or c (contradiction), then dual of s is obtained by replacing t by c and c by t and $\wedge$ by $\vee$ and $\vee$ and $\wedge$ as usual.

**Ex.30** Write the dual of the following statements :

(i) $(p \vee q) \wedge c$  (ii) $(p \vee t) \wedge r$

(iii) $(p \wedge q) \vee t$  (iv) $(p \vee q) \vee c$

**Sol.** The required duals are given by :

(i) $(p \wedge q) \vee t$  (ii) $(p \wedge c) \vee r$

(iii) $(p \vee q) \wedge c$  (iv) $(p \wedge q) \wedge t$