# PERMUTATIONS AND COMBINATIONS

## FACTORIAL NOTATION

The continuous product of first n natural numbers is called **factorial** and it can be represented by notation $\underline{|n}$ or n! .

So $\quad n! = 1.2.3 \ldots\ldots\ldots (n-1).n$

or $\quad n! = n(n-1)(n-2) \ldots\ldots\ldots 3.2.1$

$\quad = n\{(n-1)(n-2) \ldots\ldots\ldots 3.2.1.\}$

$\therefore \quad n! = n(n-1)! = n(n-1)(n-2)!$

$\quad = n(n-1)(n-2)(n-3)!$

$$n(n-1) \ldots\ldots\ldots (n-r+1) = \frac{n!}{(n-r)!}$$

**Some useful results :**

| | | |
|---|---|---|
| 0! = 1 | 4! = 24 | 8! = 40320 |
| 1! = 1 | 5! = 120 | 9! = 362880 |
| 2! = 2 | 6! = 720 | 10! = 3628800 |
| 3! = 6 | 7! = 5040 | –n! = Meaningless |

## FUNDAMENTAL PRINCIPLES OF COUNTING

When one or more operations can be accomplished by number of ways then there are two principles to find the total number of ways to accomplish one, two, or all of the operations without counting them as follows :

**1. Fundamental Principle of Multiplication :**

Let there are two parts A and B of an operation and if these two parts can be performed in m and n different number of ways respectively, then that operation can be completed in m × n ways.

## *Solved Examples*

**Ex.1** A Hall has 3 gates. In how many ways can a man enter the hall through one gate and come out through a different gate?

**Sol.** Suppose the gates are A, B and C. Now there are 3 ways (A, B or C) of entering into the hall. After entering into the hall, the man come out through a different gate in 2 ways. Hence, by the multiplication principle, total number of ways is

A < B, C
B < A, C
C < A, B

$\Rightarrow 3 \times 2 = 6$ ways.

**Ex.2** The number of three digit numbers can be formed without using the digits 0,2,3,4, 5 and 6 is (if repetition of digit is allowed)–

(A) 54 (B) 64

(C) 44 (D) None of these

**Sol.** We have to determine the total number of three digit numbers formed by using the digits 1,7,8,9. clearly the repetition of digits is allowed.

A three digit number has three places viz. unit's, ten's and hundred's. Unit's place can be filled by any of digit 1,7,8,9, so units place can be filled in 4 ways. Similarly each one of the ten's and hundred's places can be filled in 4 ways.

$\therefore$ Total number of required numbers

$= 4 \times 4 \times 4 = 64$ **Ans.[B]**

**Ex.3** The number of numbers are there between 100 and 1000 in which all the digits are distinct is –

(A) 648 (B) 548

(C) 448 (D) None of these

**Sol.** 4 number between 100 and 1000 has three digits, so we have to form all possible 3- digit numbers with distinct digits. We cannot have at the hundred's place so the hundred's place can be filled with any of the digits 1,2,3, ....9. So there are 9 ways of filling the hundred's place.

Now 9 digits are left including 0. so ten's place can be filled with any of the remaining 9 digits in 9 ways. Now, the unit's place can be filled within any of the numbering 8 digits, so there are 8 ways of filling the unit's place.

Hence the total number of required numbers

$= 9 \times 9 \times 8 = 648$ **Ans.[A]**

**Ex.4** The number of three digit numbers greater than 600 can be formed by using the digits 2,3,4, 6,7 if repetition of digits is allowed-

(A) 50 (B) 20

(C) 30 (D) None of these

**Sol.** Since three digit number greaterthan 600 will have 6 or 7 at hundred's place. So hundred's place can be filled in 2 ways. Each of ten's and one's place can be filled in 5 ways. Hence total no. of required numbers

$= 2 \times 5 \times 5 = 50$ **Ans.[A]**

**2. Fundamental Principle of addition :**

If there are two operations such that they can be done independently in m and n ways respectively, then any one of these two operations can be done by (m + n) number of ways.

## *Solved Examples*

**Ex.5** Find the number of two digit numbers (having different digits) which are divisible by 5.

**Sol.** Any number of required type either ends in 5 or in 0. Number of two digit numbers (with different digits) ends in 5 is 8 and that of ending in 0 is 9.

Hence, by addition principle the required number of numbers is 8 + 9 = 17.

**Ex.6** A book seller has 5 books of mathematics, 6 books of physics, and 4 books of chemistry. A student can purchase any one book of either mathematics or physics or chemistry by 5 + 6 + 4 = 15 number of ways but the same student can purchase one book of mathematics, physics and chemistry each by $5 \times 6 \times 4 = 120$ number of ways.

## PERMUTATIONS

An arrangement of some given things taking some or all of them, is called a **permutation** of these things.

For Example, three different things a, b and c are given, then different arrangements which can be made by taking two things from the three given things are ab, ac, bc, ba, ca, cb

Therefore, the number of permutations will be 6.

**Important Results**

**1.** The number of permutations of n different things, taking r at a time is denoted by ${}^nP_r$ or P (n, r)

$${}^nP_r = \frac{n!}{(n-r)!} \qquad (0 \le r \le n)$$

$$= n\,(n-1)\,(n-2)\,.\,.\,.\,(n-r+1).$$

**Note:**

**(i)** The number of permutations of n different things taken all at a time = ${}^nP_n = n!$

**(ii)** ${}^nP_0 = 1$, ${}^nP_1 = n$ and ${}^nP_{n-1} = {}^nP_n = n!$.

**(iii)** ${}^nP_r = n\,({}^{n-1}P_{r-1}) = n\,(n-1)\,(n-2)({}^{n-2}P_{r-2})$

$= n\,(n-1)\,(n-2)\,({}^{n-3}P_{r-3}) = \ldots$

**Explanation :** Arrangement of n things at r places may be done as follows :

First we make arrangement for first place which may be done in n ways. After this, we make arrangement for second place which may be done in (n–1) ways. Continuing this process, we shall find the arrangements for third, fourth....$r^{th}$ places may be done in (n–2), (n–3) ....(n–r +1) ways respectively.

Hence total number of permutations of n things taking r at a time

$= n\,(n-1)\,(n-2)\,.......(n-r+1)$

$.{}^nP_1 = n$

**Note :** $.{}^nP_r = n.\,{}^{n-1}P_{r-1}$

$.{}^nP_r = (n-r+1)\,.{}^nP_{r-1}$

$.{}^nP_n = {}^nP_{n-1}$

## *Solved Examples*

**Ex.7** If ${}^{56}P_{r+6} : {}^{54}P_{r+3} = 30800 : 1$, find ${}^rP_2$.

**Sol.** We have

$$\frac{{}^{56}P_{r+6}}{{}^{54}P_{r+3}} = \frac{30800}{1}$$

$$\Rightarrow \frac{56!}{(50-r)!} \times \frac{(51-r)!}{54!} = \frac{30800}{1}$$

$\Rightarrow$ 56.55 (51 – r) = 30800

$\Rightarrow$ r = 41

$\therefore$ ${}^rP_2 = {}^{41}P_2 = 41.40 = 1640$.

**Ex.8** Prove that ${}^nP_r = {}^{n-1}P_r + r.\,{}^{n-1}P_{r-1}$

**Sol.** R.H.S $= {}^{n-1}P_r + r\,{}^{n-1}P_{r-1}$

$$= \frac{(n-1)!}{(n-1-r)!} + r\frac{(n-1)!}{(n-1-r+1)!}$$

$$= \frac{(n-1)!}{(n-r-1)!} + \frac{r(n-1)!}{(n-r)!}$$

$$= \frac{(n-r)(n-1)!}{(n-r)!(n-r-1)!} + \frac{r(n-1)!}{(n-r)!}$$

$$= \frac{(n-r)(n-1)!}{(n-r)!} + \frac{r(n-1)!}{(n-r)!}$$

$$= \frac{(n-1)!}{(n-r)!}[n-r+r] = \frac{n(n-1)!}{(n-r)!} = \frac{n!}{(n-r)!}$$

$= {}^nP_r$ = L.H.S.

**Ex.9** The value of ${}^8P_3$ is -

(A) 336 (B) 56

(C) 386 (D) None of these

**Sol.** ${}^8P_3 = 8.7.6 = 336$

With the help of formula

$${}^8P_3 = \frac{8!}{(8-3)!} = \frac{8.7.6.5.4.3.2.1}{5.4.3.2.1} = 8.7.6 = 336$$

**Ans.[A]**

**Ex.10** The number of numbers which can be formed with the digits 2, 3, 4, 5, 6 by taking 4 digits at a time are-

(A) 135 (B) 120

(C) 150 (D) None of these

**Sol.** The required numbers

$$= {}^5P_4 = \frac{5!}{(5-4)!} = 5.4.3.2.1. = 120$$

**Ans.[B]**

**Ex.11** In how many ways can three persons sit on 6 chairs?

(A) 150 (B) 140

(C)120 (D) 110

**Sol.** The Required number of ways are

${}^6P_3 = 6.5.4 = 120$ **Ans.[C]**

**Ex.12** How many different signals can be made by 5 flags from 8 flags of different colours?

(A) 6720 (B) 5720

(C) 4720 (D) None of these

**Sol.** The total number of signals is the number of arrangements of 8 flags by taking 5 flags at a time.

Hence required number of signals

$$= {}^8P_5 = \frac{8!}{(8-5)!} = \frac{8!}{3!} = \frac{8\times7\times6\times5\times4\times3!}{3!}$$

$= 6720$ **Ans.[A]**

**Ex.13** How many numbers lying between 100 and 1000 can be formed with the digits 1,2,3,4,5,6 if the repetition of digits is not allowed?

(A) 30 (B) 120

(C) 50 (D) None of these

**Sol.** Every number lying between 100 and 1000 is a three digits number. Therefore we have to find the number of permutations of six digits 1,2,3,4,5,6 taken three at a time.

Hence, the required number of numbers

$$= {}^6P_3 = \frac{6\times5\times4\times3!}{3!} = 120$$ **Ans.[B]**

**Ex.14** How many four digit numbers are there with distinct digits?

(A) 4536 (B) 4526

(C) 4516 (D) None of these

**Sol.** The total number of arrangements of ten digits 0,1,2,3,4,5,6,7,8,9 taking 4 at a time is ${}^{10}P_4$. But these arrangements also include those number which have 0 at thousand's place. Such numbers are not four digit numbers. When 0 is fixed at thousand's place, we have to arrange remaining 9 digits by taking 3 at a time. The number of such arrangements is ${}^9P_3$.

So, the total number of numbers having 0 at thousand's place = ${}^8P_3$.

Hence, the total numbers four digit numbers

$$= {}^{10}P_4 - {}^9P_3 = 5040 - 504 = 4536$$

**Ans.[A]**

**2. Permutations in which all things are not different:**

The number of permutations of n things taken **<u>all at a time</u>** when p of them are alike and of one kind, q of them are alike and of second kind, r of them are alike and of third kind and all remaining being different is

$$\frac{n!}{p!\,q!\,r!}$$

**Explanation :** If all n things were different then total number of permutations will be n!, but if p things are similar, then these p things can be mutually arranged only in one way, Whereas in the case when they were different, their mutual arrangements could be p!. Similarly when q things and r things were different, their mutual arrangements could be q! and r! respectively. Thus the total number of permutations is obtained by dividing n! by p! . q! .r!.

**Note :** Above formula is applicable only when all n things are taken at a time.

## *Solved Examples*

**Ex.15** How many 7 letter words can be formed using the letters of the word 'ARIHANT'?

**Sol.** Here total letters is 7, in which 2A's, but the rest are different. Hence the number of words formed

$$= \frac{7!}{2!} = 2520$$

**Ex.16** How many different words can be formed with the letters of the word "ALLAHABAD" ?

(A) 10080 (B) 8640

(C) 15120 (D) 7560

**Sol.** There are in all 9 letters in the given word. Out of them there are 4 'A's, 2 'L's and the remaining 3 are different . So the total number of permutations

$$\frac{9!}{4!\,2!} = 7560$$ **Ans.[D]**

**Ex.17** How many numbers can be formed with the digits 2,3,3,4,2,3 taken all at a time.

(A) 460 (B) 60

(C) 260 (D) None of these

**Sol.** We have to arrange these six digits, out of which 2 occurs twice, 3 occurs thrice and rest are distinct.

The number of such arrangement = $\frac{6!}{2! \times 3!} = 60$

**Ans.[B]**

**3. Permutations in which things may be repeated:**

The number of permutations of n **different things** taken r at a time when each thing can be used once, twice, .....upto r times in any permutation is $\mathbf{n^r}$.

In particular, in above case when n things are taken at a time then total number of permutation is $\mathbf{n^n}$.

## *Solved Examples*

**Ex.18** A child has four pockets and three marbles. In how many ways can the child put the marbles in his pockets?

**Sol.** The first marble can be put into the pocket in 4 ways, so the second can also be put in the pocket in 4 ways so can the third . Thus, the number of ways in which the child can put the marbles = 4 × 4 × 4 = 64 ways.

**Ex.19** There are 6 pockets in the coat of a person. In how many ways can he put 4 pens in these pockets?

(A) 360 (B) 1296

(C) 4096 (D) None of these

**Sol.** First pen can be put in 6 ways.

Similarly each of second, third and fourth pen can be put in 6 ways.

Hence total number of ways

= 6 × 6 × 6 × 6 = 1296 **Ans.[B]**

**Ex.20** In how many ways 3 prizes can be distributed among 5 students, when-

**(a)** no student receives more than a prize.

**(b)** a student can receive any number of prizes.

**(c)** a student does not get all prizes.

(A) 60,125,120 (B) 125,60,120

(C) 125,120,60 (D) None of these

**Sol. (a)** Since the first prize can be given to any one of the student so it can be given in 5 ways. The second prize can be given in 4 ways and the third prize can be given in 3 ways as no student can get more than one prize. Therefore, number of ways of distributing 3 prizes among 5 boys.

= 5 × 4 × 3 = 60

**(b)** Since a student can receive any number of prizes, therefore each prize can be distributed in 5 ways. Hence the number of ways of distribution of 3 prizes among 5 students is = 5 × 5 × 5 = $5^3$ = 125.

**(c)** Here the number of ways so that a student can receive all prizes will be 5, therefore if a student doesn't receive all prizes, then number of ways of distributing 3 prizes among 5 students.

= $5^3 - 5 = 120$ **Ans.[A]**

**4. Restricted Permutations :** If in a permutation, some particular things are to be placed at some particular places or some particular things are always to be included or excluded, then it is called a **restricted permutation.** The following are some of the restricted permutations.

**(a)** The number of permutations of n dissimilar things taken r at a time, when m particular things always occupy definite places = $\mathbf{{}^{n-m}P_{r-m}}$

**(b)** The number of permutations of n different things taken altogether when r particular things are to be placed at some **r** given places.

= $\mathbf{{}^{n-r}P_{n-r} = (n-r)!}$

**(c)** The number of permutations of n different things taken r at a time, when m particular things are always to be excluded = $\mathbf{{}^{n-m}P_r}$

**(d)** The number of permutations of n different things taken r at a time when m particular things are always to be included = $\mathbf{{}^{n-m}C_{r-m} \times r!}$

## Solved Examples

**Ex.21** How many numbers lying between 1000 and 2000 can be formed with the digits 1, 2, 3, 4, 5 which are divisible by 5.

(A) 3 (B) 6

(C) 12 (D) 18

**Sol.** The required numbers will contain 4 digits in which first and last digits will be 1 and 5 respectively. The remaining two places are to be filled up from the remaining three digits and it can be done in $^3P_2 = 6$ ways. Hence the required number = 1 × 6 = 6

**Ans.[B]**

**Ex.22** How many different words beginning with S and ending with K can be made by using the letters of the word 'SIKAR' ?

(A) 6 (B) 12

(C) 48 (D) 60

**Sol.** After putting S and K at their respective places the remaining 3 letters can be arranged in 3! ways. Therefore required number of words = 1 × 3! = 6

**Ans.[A]**

**Ex.23** How many different 3 letter words can be formed with the letters of the word 'JAIPUR' when A and I are always to be excluded ?

(A) 12 (B) 24

(C) 48 (D) 60

**Sol.** After leaving A and I, we are remained with 4 different letters which are to be used for forming 3- letter words. Hence the required number = $^4P_3$ = 4.3.2 = 24. **Ans.[B]**

**5. Permutation of numbers when given digits include zero :**

If the given digits include 0, then two or more digit numbers formed with these digits cannot have 0 on the extreme left. In such cases we find the number of permutations in the following two ways.

**(a)** (The number of digits which may be used at the extreme left) x (The number of ways in which the remaining places may be filled up)

**(b)** If given digits be n (including 0) then total number of m- digit numbers formed with them

$= {}^nP_m - {}^{n-1}P_{m-1}$.

because $^{n-1}P_{m-1}$ is the number of such numbers which contain 0 at extreme left.

## Solved Examples

**Ex.24** How many six digit numbers can be formed by using the digits 0,1,2,3,4,5 and 6?

(A) 5040 (B) 4320

(C) 720 (4 ) 5760

**Sol. First Method :** The total numbers by taking any six digits from the given seven digits $= {}^7P_6 = 5040$

The six digits numbers containing zero at first place $= {}^6P_5 = 720$

$\therefore$ Required Numbers = 5040 - 720 = 4320

**Second Method :** Here first we make six squares in a row in the following way

6 6 5 4 3 2

Obviously first square on extreme left can be filled in 6 ways, since any non zero six digits 1,2,3,4,5,6 can occupy this square .

The remaining squares can be filled in

6 × 5 × 4 × 3 × 2 = 720 no. of ways

$\therefore$ Required Numbers = 6 × 720 = 4320

**Ans.[B]**

## COMBINATIONS

The different groups or selections of a given number of things by taking some or all at a time without paying any regard to their order, are called their **combinations.**

For example, if three things a, b and c are given then ab, bc and ac are three different groups, because ab and ba will give only one group, similarly bc and cb will give one group and ac and ca will give another group. Thus taking two things out of three different things a, b and c, the following three groups can be formed :

ab, bc, ca

The number of combinations of n **different** things taken r at a time is denoted by

$^nC_r$ or C (n, r )

$$^nC_r = \frac{n!}{r!(n-r)!}$$

So $$^nC_r = \frac{n(n-1)(n-2).....(n-r+1)}{r!}$$

**Particular cases :** $^nC_r = \frac{^np_r}{r!}$

$^nC_n = 1$

$^nC_0 = 1$

**Some Important Results :**

* $^nC_r = {^nC_{n-r}}$
* $^nC_x = {^nC_y} \Rightarrow x + y = n$
* $^nC_r + {^nC_{r-1}} = {^{n+1}C_r}$
* $^nC_r = \frac{n}{r}.{^{n-1}C_{r-1}}$
* $^nC_r = \frac{1}{r}(n-r+1)\,{^nC_{r-1}}$
* $^nC_1 = {^nC_{n-1}} = n$
* Greatest value of $^nC_r$

  $= {^nC_{n/2}}$, when n is even

  $= {^nC_{(n-1)/2}}$ or $^nC_{(n+1)/2}$ , when n is odd

**Difference between Permutation and Combination :**

**(a)** In combinations the order of things has no role to play, while in permutations the order of things plays an important role.

**(b)** In combinations the different groups are formed by taking some or all given things, while in case of permutations all possible arrangements of things are made in each group. Thus the number of permutations is always greater than the number of combinations.

**Note:**

In general, we use the methods of finding combinations in case of forming groups, teams or committees, while the methods of finding permutations are used in forming numbers with the help of digits, words with the help of letters, distribution of prizes etc.

## *Solved Examples*

**Ex.25** If $^nC_{r-1} = 36$, $^nC_r = 84$ and $^nC_{r+1} = 126$, then find r.

**Sol.** $$\frac{^nC_r}{^nC_{r-1}} = \frac{84}{36}$$

$$\Rightarrow \frac{n-r+1}{r} = \frac{7}{3}$$

$$\left(\because \frac{^nC_r}{^nC_{r-1}} = \frac{n-r+1}{r}\right)$$

$\Rightarrow$ 3n – 3r + 3 = 7r

$\Rightarrow$ 10r – 3n = 3 . . . (i)

and $$\frac{^nC_{r+1}}{^nC_r} = \frac{n-(r+1)+1}{(r+1)} = \frac{126}{84}$$

$$\left(\because \frac{^nC_r}{^nC_{r-1}} = \frac{n-r+1}{r}\right)$$

$$\Rightarrow \frac{n-r}{r+1} = \frac{3}{2}$$

$\Rightarrow$ 2n – 2r = 3r + 3

$\Rightarrow$ 5r – 2n = – 3

$\Rightarrow$ 10 r – 4n = – 6

Substracting (ii) from (i) n = 9

10r – 27 = 3 $\Rightarrow$ 10 r = 30

$\Rightarrow$ r = 3

**Ex.26** In how many ways can a committee of 6 persons be made out of 10 persons ?

(A) 210 (B) 300

(C) 151200 (D) None of these

**Sol.** We have to select 6 persons from 10 given persons. This can be done in $^{10}C_6$ ways therefore number of committees.

$$= {^{10}C_6} = {^{10}C_4} = \frac{10!}{4!\,6!} = \frac{10.9.8.7}{4.3.2.1} = 210$$

**Ans.[A]**

**Ex.27** In how many ways a committee of 5 members can be selected from 6 men and 5 women, consisting of 3 men and 2 women?

(A) 200 (B) 100

(C) 300 (D) None of these

**Sol.** Three men out of 6 men can be selected in $^6C_3$ ways. Two women out of 5 women can be selected in $^5C_2$ ways. Therefore total number of ways = $^6C_3 \times {}^5C_2 = 200$ ways. **Ans.[A]**

**Ex.28** Out of 5 men and 2 women, a committee of 3 is to be formed. In how many ways can it be formed if atleast one woman is to be included?

(A) 20 (B) 30

(C) 25 (D) None of these

**Sol.** The committee can be formed in the following ways:

(i) By selecting 2 men and 1 woman

(ii) By selecting 1 man and 2 women

2 men out of 5 men and 1 woman out of 2 women can be chosen in $^5C_2 \times {}^2C_1$ ways.

And 1 man out of 5 men and 2 women out of 2 women can be chosen in $^5C_1 \times {}^2C_2$ ways.

Total number of ways of forming the committee

$= {}^5C_2 \times {}^2C_1 + {}^5C_1 \times {}^2C_2 = 20 + 5 = 25$

**Ans.[C]**

**1. Restricted Combinations :**

The number of combinations of n different things taking r at a time

**(a)** When p particular things are always to be included $= {}^{n-p}C_{r-p}$

**(b)** When p particular things are always to be excluded $= {}^{n-p}C_r$

**(c)** When p particular things are always included and q particular things are always excluded

$= {}^{n-p-q}C_{r-p}$

## *Solved Examples*

**Ex.29** In how many ways 11 players can be selected out of 15 players when

(a) one particular player is always to be selected.

(b) one particular player is never to be selected.

(A) 364,1365 (B) 1001,364

(C) 3003, 364 (D) 3003,1001

**Sol.** (a) In this case 10 players are to be selected out of 14 players and it can be done in

$$^{14}C_{10} = {}^{14}C_4 = \frac{14.13.12.11}{1.2.3.4} = 1001 \text{ ways}$$

(b) In this case 11 players are to be selected out of 14 players and it can be done in

$$^{14}C_{11} = {}^{14}C_3 = \frac{14.13.12}{1.2.3} = 364 \text{ ways}$$

**Ans.[B]**

**2. Total number of combinations in different cases:**

**(a)** The number of combinations of n **different things** taking some or all (or atleast one) at a time

$$= {}^nC_1 + {}^nC_2 + ..... + {}^nC_n = \mathbf{2^n - 1}$$

**(b)** The number of ways to select some or all out of (p + q + r) things where p are alike of first kind, q are alike of second kind and r are alike of third kind is = **(p +1) (q +1) (r +1) – 1**

**Explanation :** Since p things are similar, therefore these p things may or may not be selected in (p +1) ways. Similarly q things of second type may or may not be selected in (q +1) ways and r things of third type may or may not be selected in (r +1) ways. Thus the number of ways in which the given objects may or may not be chosen

= (p + 1) (q +1) (r + 1)

Since this includes one way in which nothing is chosen. Hence the number of ways of selecting the things is = (p + 1) (q + 1) (r + 1) – 1

**(c)** The number of ways to select some or all out of (p + q + t) things where p are alike of first kind, q are alike of second kind and remaining t are different is = **(p +1) (q +1) $2^t$ – 1**

## Solved Examples

**Ex.30** In how many ways can I purchase one or more shirts if 6 different shirts are available ?

(A) 64 (B) 62
(C) 63 (D) 126

**Sol.** Total number of ways

$= {}^6C_1 + {}^6C_2 + {}^6C_3 + {}^6C_4 + {}^6C_5 + {}^6C_6$

$= 2^6 - 1 = 63$ **Ans. [C]**

**Ex.31** A bag contains 3 one rupee coins, 4 fifty paise coins and 5 ten paise coins. How many selections of money can be formed by taking atleast one coin from the bag ?

(A) 120 (B) 60
(C) 119 (D) 59

**Sol.** Here are 3 things of first kind, 4 things of second kind and 5 things of third kind so the total number of selections

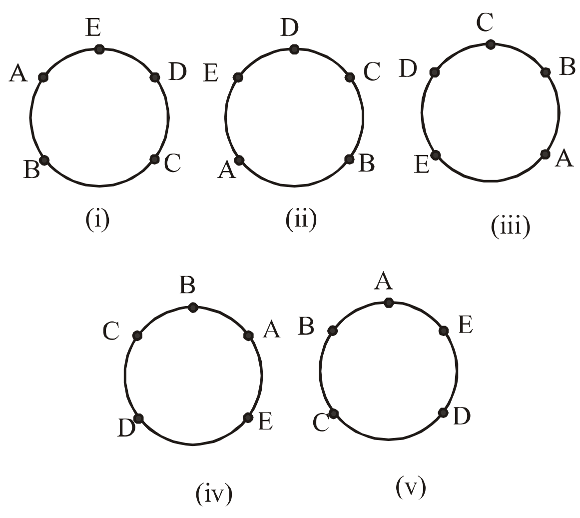
$= (3+1)(4+1)(5+1) - 1 = 119$

**Ans.[C]**

## CIRCULAR PERMUTATIONS

**(i) Arrangements round a circular table :**

Consider five persons A, B, C, D and E to be seated on the circumference of a circular table in order (which has no head). Now, shifting A, B, C, D and E one position in anticlockwise direction we will get arrangements as follows:



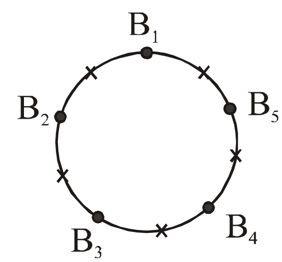

we see that arrangements in all figures are same.

$\therefore$ The number of circular permutations of n different things taken all at a time is $\frac{{}^nP_n}{n} = (n-1)!$, if clockwise and anticlockwise orders are taken as different.

## Solved Examples

**Ex.32** In how many different ways can five boys and five girls form a circle such that the boys and girls are alternate?

**Sol.** After fixing up one boy on the table the remaining can be arranged
in 4! ways. There will be 5 places, one place each between two
boys which can be filled by 5 girls in 5! ways.

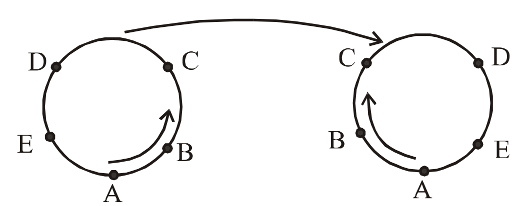



Hence by the principle of multiplication, the required number of ways

$= 4! \times 5! = 2880.$

**(ii) Arrangements of beads or flowers (all different) around a circular necklace or garland:**

Consider five beads A, B, C, D and E in a necklace or five flowers A, B, C and D, E in a garland etc. If the necklace or garland on the left is turned over we obtain the arrangement on the right, i.e., anticlockwise and clockwise order of arrangements are not different.




Thus the number of circular permutations of 'n' different things taken all at a time is $\frac{1}{2}(n-1)!$, if clockwise and anticlockwise orders are taken as not different.

## Solved Examples

**Ex.33** Find the number of ways in which 12 different beads can be arranged to form a necklace.

**Sol.** 12 different beads can be arranged among themselves in a circular order in (12 – 1)! = 11! ways. Now in the case of necklace there is no distinction between clockwise and anticlockwise arrangements. So the required number of arrangements $= \frac{1}{2}(11!)$.

(iii) number of circular permutations of 'n' different things taken 'r' at a time.

**Case I:**

If clockwise and anticlockwise orders are taken as different, then the required number of circular permutation = $({}^nP_r) / r$

## Solved Examples

**Ex.34** In how many ways can 13 persons out of 24 persons be seated around a table.

**Sol.** In case of circular table the clockwise and anticlockwise orders are different, thus the required number of circular permutations $= \frac{{}^{24}P_{13}}{13} = \frac{24!}{13 \times 11!}$.

**Case II:**

If clockwise and anticlockwise orders are taken as same, then the required number of circular permutations = $({}^nP_r) / (2r)$

## Solved Examples

**Ex.35** How many necklace of 12 beads each can be made from 18 beads of various colours?

Sol. In the case of necklace there is no distinction between the clockwise and anticlockwise arrangements, thus the required number of circular permutations

$$= \frac{{}^{18}P_{12}}{2 \times 12} = \frac{18!}{6! \times 24}$$

$$= \frac{18 \times 17 \times 16 \times 15 \times 14 \times 13!}{6 \times 5 \times 4 \times 3 \times 2 \times 1 \times 24} = \frac{119 \times 13!}{2}.$$

**Note** :

**(i)** In a circular permutation the relative position among the things is important whereas the place of a thing has no significance. Thus in a circular permutation first thing can be placed anywhere. This operation can be done only in one way, then relative order begins. Thus the ways for performing remaining parts of the operation can be calculated just like the calculation of linear permutation for example to place 8 different things round a circle, first we place any one thing at any one place, there will be only one way to perform this operation. Then remaining 7 things will

occupy remaining 7 places, for which the numbers of ways = 7! . Thus required number of circular permutations is 7!

**(ii)** In a garland of flowers or a necklace of beads, it is difficult to distinguish clockwise and anticlockwise orders of things, so a circular permutation under both these orders is considered to be the same.

## Solved Examples

**Ex.36** The number of ways in which 7 persons be seated at 5 places round a table are-

(A) 252 (B) 504

(C) 2520 (D) None of these

**Sol.** The required number

$$= \frac{{}^7P_5}{5} = \frac{7.6.5.4.3}{5} = 504$$ **Ans.[B]**

**Ex.37** In how many ways can 5 beads out 7 different beads be strung into a ring ?

(A) 504 (B) 2520
(C) 252 (D) None of these

**Sol.** In this case a clockwise and corresponding anti clockwise order will give the same circular permutation. So the required number

$$= \frac{^7P_5}{2.5} = \frac{7.6.5.4.3}{2.5} = 252$$ **Ans.[C]**

**2 Restricted Circular Permutations :** When there is a restriction in a Circular permutation then first of all we shall perform the restricted part of the operation and then perform the remaining part treating it similar to a linear permutation.

## *Solved Examples*

**Ex.38** In how many ways can 6 persons be seated round a circular table when two particular persons sit together ?

(A) 120 (B) 240
(C) 48 (D) 24

**Sol.** First of all let two particular persons be seated together. They can sit together in 2! = 2 ways. Then the remaining four persons may sit on remaining four places in 4 ! = 24 ways, so the total number of ways

$= 2 \times 24 = 48$ **Ans.[C]**

## DIVISION INTO GROUPS

**(a)** The number of ways in which (p + q) things can be divided into two groups of p and q things is

$$^{p+q}C_p = {}^{p+q}C_q = \frac{(p+q)!}{p!\,q!}$$

**Particular case :** when p = q, then total number of combinations are

(i) $\frac{2p!}{(p!)^2}$ when groups are differentiable.

(ii) $\frac{2p!}{2!\,(p!)^2}$ when groups are not differentiable.

**(b)** The number of ways in which (p + q + r) things can be divided into three groups containing p, q and r things is

$$\frac{(p+q+r)!}{p!\,q!\,r!}$$

**Particular case :**

when p = q = r, then total number of combinations are

(i) $\frac{3p!}{(p!)^3}$ when groups are differentiable.

(ii) $\frac{3p!}{3!\,(p!)^3}$ when groups are not differentiable.

For example if 36 distinct objects are to be divided among 9 groups such that four groups have 2 objects each, three groups have 5 objects each and remaining two groups having 6 and 7 objects, then the required

number of ways = $\frac{36!}{(2!)^4 .4!(5!)^3 .3!.6!.7!}$.

## DISTRIBUTION AMONG PERSONS

The number of ways in which n distinct objects can be distributed among r persons in a required way = number of ways of dividing n distinct objects in r-groups in the required way $\times$ r!.

For example if 36 distinct objects are to be divided among 9 persons such that four of the persons are getting 2 objects each, three of the persons are getting 5 objects each and the remaining two persons are getting 6 and 7 objects, then the number of ways of doing so is

$$\frac{36!}{(2!)^4 .4!(5!)^3 .3!.6!.7!} \times 9!.$$

## *Solved Examples*

**Ex.39** In how many ways can 15 students

(i) be divided into 3 groups of 5 each

(ii) be sent to three different colleges in groups of 5 each.

(A) $\frac{15!}{3!\,(5!)^3}, \frac{15!}{(5!)^3}$  (B) $\frac{15!}{(5!)^3}, \frac{15!}{(5!)^3}$

(C) $\frac{15!}{3!\,(5!)^3}, \frac{15!}{3!\,(5!)^3}$  (D) $\frac{15!}{(5!)^3}, \frac{15!}{3!\,(5!)^3}$

**Sol.** (i) Total number of combinations of 15 students into 3 groups of 5 each $= \frac{15!}{3!\,(5!)^3}$

(ii) In this case the groups are associated with different colleges, so the required number

$= \frac{15!}{(5!)^3}$  **Ans.[A]**

**Ex.40** In how many way can 52 playing cards be distributed into 3 groups of 17 cards each and one group of one card.

(A) $\frac{52!}{(17!)^3\ 3!}$  (B) $\frac{52!}{(17!)^3}$

(C) $\frac{52!}{(17!)^3\, 3!\, 2!}$  (D) None of these

**Sol.** The total number of ways

$$\frac{52!}{17!\,17!\,17!\ 3!} = \frac{52!}{(17!)^3\ 3!}$$

**Ans.[A]**

**Ex.41** 3 copies each of 4 different books are available. The number of ways in which these can be arranged on the shelf is-

(A) 12!  (B) $\frac{12!}{3!\,4!}$

(C) 369,600  (D) 369,000

**Sol.** The total no. of ways to arrange 3 copies each of 4 different books-

$= \frac{12\,!}{(3\,!)^4} = 369{,}600$  **Ans.[C]**

**Ex.42** The number of ways of dividing 20 persons into 10 couples is-

(A) $\frac{20!}{2^{10}}$  (B) ${}^{20}C_{10}$

(C) $\frac{20!}{(2!)^9}$  (D) None of these

**Sol.** Here the order of the couples is not important. So, required number of ways is $\frac{20!}{2^{10}10!}$

**Ans. [D]**

## PERMUTATIONS IN WHICH THE OPERATION OF SELECTION IS NECESSARY

There are questions of permutation in which we have to start with the operation of selection for the given number of things. After this we calculate the number of different arrangements for each of such selected group.

## *Solved Examples*

**Ex.43** How many words can be formed containing 4 consonants and 3 vowels out of 6 consonants and 5 vowels ?

(A) ${}^6C_4 \times {}^5C_3$  (B) ${}^6C_4 \times {}^5C_3 \times 7!$

(C) ${}^6P_4 \times {}^5P_3$  (D) ${}^6P_4 \times {}^5P_3 \times 7!$

**Sol.** First we select 4 consonant out of 6 and 3 vowels out of 5. This can be done in ${}^6C_4 \times {}^5C_3$ ways. After such a selection of 7 letters then can be arranged in 7! ways. So the total number of words

$= {}^6C_4 \times {}^5C_3 \times 7!$  **Ans. [B]**

**Ex.44** In how many ways can 7 persons be seated round two circular tables when 4 persons can sit on the first table and 3 can sit on the other ?

(A) 420  (B) 35

(C) 210  (D) 2520

**Sol.** First we divide 7 persons into two groups of 4 and 3 persons. The total number of such division

$$= \frac{7!}{4!\ 3!} = 35$$

Now for such a division of 4 and 3 persons there are 3! x 2! ways of sitting round the given two tables. Hence total number of required arrangements

$= 35 \times 12 = 420$  **Ans.[A]**

**Ex.45** The number of words by taking 4 letters out of the letters of the word 'COURTESY', when T and S are always included are-

(A) 120 (B) 720
(C) 360 (D) None of these

**Sol.** Since T and S are to be included in every word, therefore first we choose 2 letters out of the remaining letters which can be done in ${}^6C_2$ ways. Now each group of letters will give 4! words. Therefore number of words

$= {}^6C_2 \times 4! = 15 \times 24 = 360$ **Ans.[C]**

## DEARRANGEMENT THEOREM

Any change in the given order of the things is called a **Dearrangement**.

**(a)** If n items are arranged in a row, then the number of ways in which they can be rearranged so that no one of them occupies the place assigned to it is

$$n!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}+.....+(-1)^n\frac{1}{n!}\right]$$

**(b)** If n things are arranged at n places then the number of ways to rearrange exactly r things

at right places is

$$\frac{n!}{r!}\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}+....+(-1)^{n-r}\frac{1}{(n-r)!}\right]$$

## *Solved Examples*

**Ex.46** There are 3 letters and 3 envelopes. Find the number of ways in which all letters are put in the wrong envelopes.

(A) 6 (B) 4
(C) 2 (D) None of these

**Sol.** The required number of ways

$$= 3!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}\right]$$

$$= 3! - 3! + \frac{3!}{2!} - 1$$

$\Rightarrow 3 - 1 = 2$ **Ans.[C]**

**Ex.47** There are 4 balls of different colour and 4 boxes of colours same as those of the balls. Then find the number of ways to place two balls in the boxes with respect to their colour.

(A) 6 (B) 4
(C) 2 (D) None of these

**Sol.** The required number of ways

$$\frac{4!}{2!}\left[1-\frac{1}{1!}+\frac{1}{2!}\right]$$

$$= 4.3\left[1-1+\frac{1}{2}\right] = 6$$ **Ans.[A]**

## MULTINOMIAL THEOREM

Consider the equation $x_1 + x_2 + \ldots + x_r = n$, where $a_i \le x_i \le b_i;\ a_i, b_i, x_i \in I; i = 1, 2, \ldots, r$. In order to find the number of solutions of the given equation satisfying the given conditions we observe that the number of solutions is the same as the coefficient of $x^n$ in the product

$$\left(x^{a_1} + x^{a_1+1} + x^{a_1+2} + \ldots x^{b_1}\right)$$

$$\left(x^{a_2} + x^{a_2+1} + x^{a_2+2} + \ldots x^{b_2}\right)$$

$$\left(x^{a_3} + x^{a_3+1} + x^{a_3+2} + \ldots x^{b_3}\right) \;.\;.\;.\;.\;.\;.\;.\;.$$

$$\left(x^{a_r} + x^{a_r+1} + x^{a_r+2} + \ldots x^{b_r}\right).$$

For example, if we have to find the number of nonnegative integral solutions of $x_1 + x_2 + \ldots + x_r = n$, then as above the required number is the coefficient of $x^n$ in

$$\left(x^0 + x^1 + \ldots + x^n\right)\left(x^0 + x^1 + \ldots + x^n\right)\ldots$$

$$\left(x^0 + x^1 + \ldots + x^n\right)(r - \text{brackets})$$

= Coefficient of $x^n$ in $(1 + x + x^2 + \ldots + x^n)^r$

= Coefficient of $x^n$ in $(1 + x + x^2 + \ldots)^r$

= Coefficient of $x^n$ in $(1 - x)^{-r}$

= Coefficient of $x^n$ in

$$\left[1+(-r)(-x)+\frac{(-r)(-r-1)}{2!}(-x)^2+\frac{(-r)(-r-1)(-r-2)}{3!}(-x)^3..\right]$$

= Coefficient of $x^n$ in

$$\left[1+{}^{r}C_1x+{}^{r+1}C_2x^2+{}^{r+2}C_3x^3+...\right]$$

$\Rightarrow {}^{n+r-1}C_n = {}^{n+r-1}C_{r-1}$.

**Note :** If there are $\ell$ objects of one kind, m objects of second kind, n objects of third kind and so on; then the number of ways of choosing r objects out of these objects is the coefficient of $x^r$ in the expansion of $(1 + x + x^2 + x^3 + ... + x^{\ell})$ $(1 + x + x^2 + ... + x^m)$ $(1 + x + x^2 + ... + x^n)$.

Further if one object of each kind is to be included, then the number of ways of choosing r objects out of these objects is the coefficient of $x^r$ in the expansion of

$(x + x^2 + x^3 + ... + x^{\ell})$

$(x + x^2 + x^3 + ... + x^m)$

$(x + x^2 + x^3 + ... + x^n) ...$

## Solved Examples

**Ex.48** Find the number of non negative integral solutions of

$x_1 + x_2 + x_3 + 4x_4 = 20$.

Sol. Number of non negative integral solutions of the given equation

= coefficient of $x^{20}$ in

$(1-x)^{-1}(1-x)^{-1}(1-x)^{-1}(1-x^4)^{-1}$

= coefficient of $x^{20}$ in $(1-x)^{-3}(1-x^4)^{-1}$

= coefficient of $x^{20}$ in

$(1 + {}^{3}C_1x + {}^{4}C_2x^2 + {}^{5}C_3x^3 + {}^{6}C_4x^4 + ...)$

$(1 + x^4 + x^8 + ...)$

$= 1 + {}^{6}C_4 + {}^{10}C_8 + {}^{14}C_{12} + {}^{18}C_{16} + {}^{22}C_{20} = 536$.

**Application of multinomial theorem**

If we want to distribute n identical objects in r different groups under the condition that empty groups are not allowed. $a_1 + a_2 + a_3 + ..... + a_r = n$

Boundary conditions are $1 \le a_1, a_2 ...... a_r \le n$

(As each box contains at least one object)

Number of ways

= coefficient of $x^n$ in $(x^1 + x^2 + ...... + x^n)^r$

= coefficient of $x^{n-r}$ in $(1 + x + x^2 + ..... + x^{n-1})^r$

$= {}^{(n-r)+r-1}C_{r-1} = {}^{n-1}C_{r-1}$

## Solved Examples

**Ex.49** Find the number of ways in which 16 identical toys are to be distributed among 3 children such that each child does not receive less than 3 toys.

**Sol.** Let $x_1, x_2, x_3$ be the number of toys received by the three children

Then, $x_1, x_2, x_3, \ge 3$ and $x_1, x_2, x_3 = 16$

Let $u_1 = x_1 - 3$, $u_2 = x_2 - 3$ and $u_3 = x_3 - 3$

Then, $u_1, u_2, u_3 \ge 0$ and $u_1 + u_2 + u_3 = 7$

Here, n = 7 and r = 3

$\therefore$ Number of ways $= {}^{n+r-1}C_{r-1} = {}^{9}C_2 = 36$

**Ex.50** Find the number of non-negative integral solutions of $x_1 + x_2 + x_3 + 4x_4 = 20$.

**Sol.** Here, clearly $0 \le x_4 \le 5$, $x_1, x_2, x_3, \ge 0$

and $x_1 + x_2 + x_3 = 20 - 4x_4$

$\Rightarrow$ r = 3 and $n = 20 - 4x_1$

If $x_4 = 0$ number of ways ${}^{20+3-1}C_{3-1} = {}^{22}C_2$

If $x_4 = 1$ number of ways ${}^{16+3-1}C_{3-1} = {}^{18}C_2$

Similarly, if $x_4 = 2, 3, 4, 5$, number of ways $= {}^{14}C_2$, ${}^{10}C_2$, ${}^{6}C_2$, ${}^{2}C_2$ respectively

$\therefore$ Total number of ways

$= {}^{22}C_2 + {}^{18}C_2 + {}^{14}C_2 + {}^{10}C_2 + {}^{6}C_2 + {}^{2}C_2 = 536$

**Ex.51** In how many ways can 10 identical toys be distributed among 3 children such that the first receives a maximum of 6 toys, the second receives a maximum of 7 toys and the third receives a maximum of 8 toys.

**Sol.** Let $x_1, x_2$ and $x_3$ be the number of toys received by the three children.

Then,

$x_1+x_2+x_3=10,\ 0 \le x_1 \le 6,$

$0 \le x_2 \le 7, \quad 0 \le x_3 \le 8$

$\Rightarrow$ Required number of ways = coeff of $x^n$ in

$(x^0 + x^1 + x^2 + ...... x^6)(x^0 + x^1 + x^2 + ......x^7)(x^0 + x^1 + x^2 + .....x^8)$

= coeff of $x^{10}$ in

$$\left[\frac{1(1-x^7)}{1-x}\right]\left[\frac{1(1-x^8)}{1-x}\right]\left[\frac{1(1-x^9)}{1-x}\right]$$

= coeff of $x^{10}$ in

$[(1-x^7)(1-x^8)(1-x^9)]\,[1-x]^{-3}$

= coeff of $x^{10}$ in

$(1 - x^7 - x^8 - x^9......)(1 + {}^3C_1x + {}^4C_2x^2 + {}^5C_3x^3 + ......{}^{12}C_{10}x^{10} + .....)$

{Note that powers >10 are unimportant and hence ignored}

$= {}^{12}C_{10} - {}^5C_3 - {}^4C_2 - {}^3C_1 = 66 - 10 - 6 - 3 - = 47$

**Ex.52** In how many ways 5 identical balls can be distributed into 3 different boxes so that no box remains empty?

**Sol.** The required number of ways $= {}^{5-1}C_{3-1}$

$$= {}^4C_2 = \frac{4.3}{1.2} = 6$$

**Ex.53** Find the number of permutation of 4 letters taken from the word EXAMINATION.

**Sol.** Number of permutations are :

(AA), (I I), (NN), E, X, M, T, O

$$= \text{Coefficient of } x^4 \text{ in } 4!\left(1+\frac{x}{1!}+\frac{x^2}{2!}\right)^3\left(1+\frac{x}{1!}\right)^5$$

$$= \text{Coefficient of } x^4 \text{ in } 4!\left(1+x+\frac{x^2}{2!}\right)^3(1+x)^5$$

= Coefficient of $x^4$ in 4!

$$\left\{(1+x)^3+\frac{x^6}{8}+\frac{3}{2}(1+x)^2x^2+\frac{3}{4}x^4(1+x)\right\}.(1+x)^5$$

= Coefficient of $x^4$ in 4!

$$\left\{(1+x)^8+\frac{x^6}{8}(1+x)^5+\frac{3}{2}x^2(1+x^7)+\frac{3}{4}x^4(1+x)^6\right\}$$

$$= 4!\left\{{}^8C_4+0+\frac{3}{2}\,{}^7C_2+\frac{3}{4}\right\}$$

$$= 24\left\{\frac{8.7.6.5}{1.2.3.4}+\frac{3}{2}+\frac{7.6}{1.2}+\frac{34}{3}\right\}$$

$= 8.7.6.5 + 6\,(3.7.6) + 6.3$

$= 1680 + 756 + 18 = 2454$

## NUMBER OF DIVISORS

(i) Every natural number N can always be put in the form $N = p_1^{\alpha_1}\, p_2^{\alpha_2}\, . . .\, p_k^{\alpha_k}$ where $p_1, p_2 . . . p_k$ are distinct primes and $\alpha_1, \alpha_2, . . . \alpha_k$ are non negative integers.

(ii) If $N = p_1^{\alpha_1}\, p_2^{\alpha_2} . . . p_k^{\alpha_k}$ then number of divisor of N $= (\alpha_1 + 1)(\alpha_2 + 1) . . . (\alpha_k + 1)$ which includes 1 and N also. **Note :** All the divisors excluding 1 and N are called proper divisors.

(iii) The sum of the divisors of N

$= (1 + p_1 + p_1^2 + . . . + p_1^{\alpha_1})(1 + p_2 + p_2^2 + . . . + p_2^{\alpha_2}) . . . (1 + p_k + p_k^2 + . . . + p_k^{\alpha_k})$

$$= \frac{p_1^{\alpha_1+1}-1}{p_1-1}.\frac{p_2^{\alpha_2+1}-1}{p_2-1}...\frac{p_k^{\alpha_k+1}-1}{p_k-1}$$

(iv) The number of ways of putting N as a product of two natural numbers is

$\frac{1}{2}(\alpha_1 + 1)(\alpha_2 + 1) . . . (\alpha_k + 1)$ if N is not a perfect square. If N is a perfect square then this is

$\frac{1}{2}[(\alpha_1 + 1)(\alpha_2 + 1) . . . (\alpha_k + 1) + 1]$

## EXPONENT OF PRIME IN n!

Let p be a given prime and n any positive integer, then maximum power of p present in n! is

$\left[\frac{n}{p}\right]+\left[\frac{n}{p^2}\right]+\left[\frac{n}{p^3}\right]+\cdots,$ where $[\cdot]$ denotes the greatest integer function. The proof of the above formula can be obtained using the fact that $\left[\frac{n}{m}\right]$ gives the number of integral multiples of m in 1, 2, . . . , n; for any positive integers n and m. The above formula does not work for composite numbers. For example if we have to find the maximum power of 6 present in 32!, then the answer is not $\left[\frac{32}{6}\right]+\left[\frac{32}{6^2}\right]+\ldots = 5$, as 5 is the number of integral multiples of 6 in 1, 2, . . . , 32; and 6 can be obtained on multiplying 2 by 3 also. Hence for the required number, we find the maximum powers of 2 and 3 (say r and s) present in 32!. Using the above formula r = 31 and s = 14. Hence 2 and 3 will be combined (to form 6) 14 times. Thus maximum power of 6 present in 32! is 14.

## SUM OF NUMBERS

**(a)** For given n different digits $a_1, a_2, a_3 ......, a_n$ the sum of the digits in the unit place of all numbers formed (if numbers are not repeated) is

$$(a_1 + a_2 + a_3 + ...... + a_n)\ (n-1)!$$

i.e. (sum of the digits) (n-1)!

**(b)** Sum of the total numbers which can be formed with given n different digits $a_1, a_2, a_3 .....a_n$ is $\mathbf{(a_1 + a_2 + a_3 + .... + a_n)(n-1)!.(111 ...n\ ties)}$

## *Solved Examples*

**Ex.54** The sum of all numbers which can be formed with digits 1,2 and 3 is-

(A) 716 (B) 1432

(C) 2148 (D) None of these

**Sol.** The sum of the digits = 1 + 2 + 3 = 6 and n = 3, so the sum of all numbers formed

= 6 . 2! (111)

= 12 × 111 = 1432 **Ans.[B]**

## SOME IMPORTANT RESULTS ABOUT POINTS

If there are n points in a plane of which m (< n) are collinear, then

**(a)** Total number of different straight lines obtained by joining these n points is

$$^nC_2 - {}^mC_2 + 1$$

**(b)** Total number of different triangles formed by joining these n points is

$$^nC_3 - {}^mC_3$$

**(c)** Number of diagonals in polygon of n sides is

$$^nC_2 - n \quad \text{i.e.} \quad \frac{n(n-3)}{2}$$

**(d)** If m parallel lines in a plane are intersected by a family of other n parallel lines. Then total number of parallelograms so formed is

$$^mC_2 \times {}^nC_2 \quad \text{i.e.} \quad \frac{mn(m-1)(n-1)}{4}$$