SETS

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A set is a collection of well defined objects which are distinct from each other. Set are generally denoted by capital letters A, B, C, ... etc. and the elements of the set by small letters a, b, c ..... etc.

If a is an element of a set A, then we write $a \in A$ and say a belongs to A.

If a does not belong to A then we write $a \notin A$, e.g. the collection of first five prime natural numbers is a set containing the elements 2, 3, 5, 7, 11.

METHODS TO WRITE A SET :

(i) Roster Method or Tabular Method :

In this method a set is described by listing elements, separated by commas and enclose then by curly brackets. Note that while writing the set in roster form, an element is not generally repeated e.g. the set of letters of word SCHOOL may be written as {S, C, H, O, L}.

(ii) Set builder form (Property Method) :

In this we write down a property or rule which gives us all the element of the set.

$A = \{x : P(x)\}$ or $\{x / P(x)\}$ where P(x) is the property by which $x \in A$ and colon (:) or Slash (/) stands for 'such that'

Solved Examples

Ex.1 Express set $A = \{x : x \in N \text{ and } x = 2n \text{ for } n \in N\}$ in roster form

Sol. $A = \{2, 4, 6, ...\}$

Ex.2 Express set $B = \{x^2 : x < 4, x \in W\}$ in roster form

Sol. $B = \{0, 1, 4, 9\}$

Ex.3 Express set $A = \{2, 5, 10, 17, 26\}$ in set builder form

Sol. $A = \{x : x = n^2 + 1, n \in N, 1 \le n \le 5\}$

TYPES OF SETS

Null set or empty set : A set having no element in it is called an empty set or a null set or void set, it is denoted by ϕ or { }. A set consisting of at least one element is called a non-empty set or a non-void set.

Singleton set : A set consisting of a single element is called a singleton set.

Finite set : A set which has only finite number of elements is called a finite set.

Order of a finite set : The number of elements in a finite set A is called the order of this set and denoted by O(A) or n(A). It is also called cardinal number of the set. e.g. $A = \{a, b, c, d\}$

$\Rightarrow n(A) = 4$

Infinite set : A set which has an infinite number of elements is called an infinite set.

Equal sets : Two sets A and B are said to be equal if every element of A is member of B, and every element of B is a member of A. If sets A and B are equal, we write A = B and if A and B are not equal then $A \neq B$

Equivalent sets : Two finite sets A and B are equivalent if their number of elements are same i.e. n(A) = n(B)

e.g. A = {1, 3, 5, 7}, B = {a, b, c, d}

$\Rightarrow$ n(A) = 4 and n(B) = 4

$\Rightarrow$ A and B are equivalent sets

Note - Equal sets are always equivalent but equivalent sets may not be equal

Solved Examples

Ex.4 Identify the type of set :

(i) $A = \{x \in N : 5 < x < 6\}$

(ii) A = {a, b, c}

(iii) A = {1, 2, 3, 4, ...}

(iv) A = {1, 2, 6, 7} and B = {6, 1, 2, 7, 7}

(v) A = {0}

Sol. (i) Null set (ii) finite set

(iii) infinite set (iv) equal sets

(v) singleton set

SUBSET AND SUPERSET :

Let A and B be two sets. If every element of A is an element B then A is called a subset of B and B is called superset of A. We write it as $A \subseteq B$. e.g. A = {1, 2, 3, 4} and B = {1, 2, 3, 4, 5, 6, 7} $\Rightarrow A \subseteq B$

If A is not a subset of B then we write $A \not\subseteq B$

PROPER SUBSET :

If A is a subset of B but $A \neq B$ then A is a proper subset of B and we write $A \subset B$. Set A is not proper subset of A so this is improper subset of A

Note :

(i) Every set is a subset of itself

(ii) Empty set ϕ is a subset of every set

(iii) $A \subseteq B$ and $B \subseteq A \Leftrightarrow A = B$

(iv) The total number of subsets of a finite set containing n elements is 2^n.

(v) Number of proper subsets of a set having n elements is $2^n - 1$.

(vi) Empty set ϕ is proper subset of every set except itself.

POWER SET :

Let A be any set. The set of all subsets of A is called power set of A and is denoted by P(A)

$X = \{x_1, x_2, x_3, \ldots\ldots\ldots. x_n\}$ then $n(P(X)) = 2^n$; $n(P(P(x))) = 2^{2^n}$

Solved Examples

Ex.5 Examine whether the following statements are true or false :

(i) $\{a, b\} \not\subseteq \{b, c, a\}$

(ii) $\{a, e\} \not\subseteq \{x : x$ is a vowel in the English alphabet$\}$

(iii) $\{1, 2, 3\} \subseteq \{1, 3, 5\}$

(iv) $\{a\} \in \{a, b, c\}$

Sol. (i) False as {a, b} is subset of {b, c, a}

(ii) True as a, e are vowels

(iii) False as element 2 is not in the set {1, 3, 5}

(iv) False as $a \in \{a, b, c\}$ and $\{a\} \subseteq \{a, b, c\}$

Ex.6 If a set A = {a, b, c} then find the number of subsets of the set A and also mention the set of all the subsets of A.

Sol. Since n(A) = 3

$\therefore$ number of subsets of A is $2^3 = 8$

and set of all those subsets is P(A) named as power set

P(A): {ϕ, {a}, {b}, {c}, {a,b}, {b,c}, {a,c}, {a,b,c}}

Ex.7 Find power set of set A = {1, 2}

Sol. P(A) = {ϕ, {1}, {2}, {1, 2}}

Ex.8 If ϕ denotes null set then find P(P(P(ϕ)))

Sol. Let P(ϕ) = {ϕ}

P(P(ϕ)) = {ϕ,{ϕ}}

P(P(P(ϕ))) = {ϕ, {ϕ}, {{ϕ}}, {ϕ, {ϕ}}}

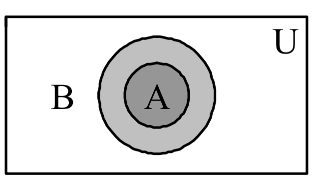
UNIVERSAL SET :

A set consisting of all possible elements which occur in the discussion is called a universal set i.e. it is a super set of each of the given set. Thus, a set that contains all sets in a given context is called the universal set. It is denoted by U. e.g. if A = {1, 2, 3}, B = {2, 4, 5, 6}, C = {1, 3, 5, 7} then U = {1, 2, 3, 4, 5, 6, 7} can be taken as the universal set.

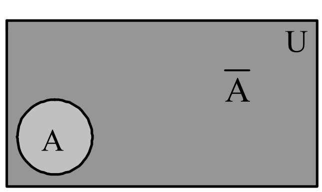
VENN (EULER) DIAGRAMS

The diagrams drawn to represent sets are called Venn diagram or Euler-Venn diagrams. Here we represents the universal U as set of all points within rectangle and the subset A of the set U is represented by the interior of a circle. If a set A is a subset of a set B, then the circle representing A is drawn inside the circle representing B. If A and B are not equal but they have some common elements, then to represent A and B by two intersecting circles.

e.g. If A is subset of B then it is represented diagrammatically in fig.



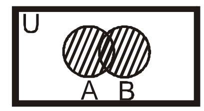
e.g. If A is a set then the complement of A is represented in fig.



SOME OPERATION ON SETS :

(i) Union of two sets : If A and B are two sets then union ($\cup$) of A and B is the set of all those elements which belong either to A or to B or to both A and B.
A $\cup$ B = {x : x $\in$ A or x $\in$ B}

e.g. A = {1, 2, 3}, B = {2, 3, 4} then A $\cup$ B = {1, 2, 3, 4}

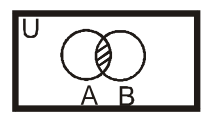


A $\cup$ B

(ii) Intersection of two sets : If A and B are two sets then intersection ($\cap$) of A and B is the set of all those elements which belong to both A and B.

A $\cap$ B = {x : x $\in$ A and x $\in$ B}

e.g. A = {1, 2, 3}, B = {2, 3, 4} then A $\cap$ B = {2, 3}

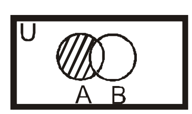


A $\cap$ B

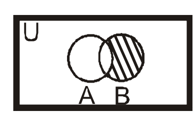
(iii) Difference of two sets : If A and B are two sets then the difference of A and B, is the set of all those elements of A which do not belong to B.

A – B = {x : x $\in$ A and x $\notin$ B}. It is also written as A$\cap$B'.

Similarly B – A = B $\cap$ A' e.g. A = {1, 2, 3}, B = {2, 3, 4} ; A – B = {1}

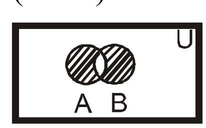


A – B



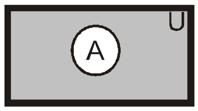
B – A

(iv) Symmetric difference of sets : Set of those elements which are obtained by taking the union of the difference of A & B is (A – B) & the difference of B & A is (B – A), is known as the symmetric difference of two sets A & B. It is denoted by A Δ B and A Δ B = (A – B) $\cup$ (B – A)



(A Δ B) = (A – B) $\cup$ (B – A)

(v) Complement of a set : Complement of a set A is a set containing all those elements of universal set which are not in A. It is denoted by $\overline{A}$, A^C or A'. So $A^C = \{x : x \in U \text{ but } x \notin A\}$. e.g. If set $A = \{1, 2, 3, 4, 5\}$ and universal set $U = \{1, 2, 3, 4, ...50\}$ then $\overline{A} = \{6, 7, ...50\}$

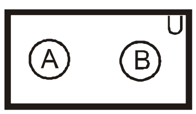


A'

NOTE :

All disjoint sets are not complementary sets but all complementary sets are disjoint.

(vi) Disjoint sets : If $A \cap B = \phi$, then A, B are disjoint e.g. If $A = \{1, 2, 3\}$, $B = \{7, 8, 9\}$ then $A \cap B = \phi$



Disjoint

LAWS OF ALGEBRA OF SETS (PROPERTIES OF SETS):

(i) Idempotent Law :

For any set A, we have (i) $A \cup A = A$ and (ii) $A \cap A = A$

Proof :

(i) $A \cup A = \{x : x \in A \text{ or } x \in A\} = \{x : x \in A\} = A$

(ii) $A \cap A = \{x : x \in A \;\&\; x \in A\} = \{x : x \in A\} = A$

(ii) Identity Law :

For any set A, we have

(i) $A \cup \phi = A$ and

(ii) $A \cap U = A$ i.e. ϕ and U are identity elements for union and intersection respectively

Proof :

(i) $A \cup \phi = \{x : x \in A \text{ or } x \in \phi\}$

$= \{x : x \in A\} = A$

(ii) $A \cap U = \{x : x \in A \text{ and } x \in U\}$

$= \{x : x \in A\} = A$

(iii) Commutative Law :

For any set A and B, we have

(i) $A \cup B = B \cup A$ and (ii) $A \cap B = B \cap A$

i.e. union and intersection are commutative.

(iv) Associative Law :

If A, B and C are any three sets then

(i) $(A \cup B) \cup C = A \cup (B \cup C)$

(ii) $(A \cap B) \cap C = A \cap (B \cap C)$

i.e. union and intersection are associative.

(v) Distributive Law :

If A, B and C are any three sets then

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

i.e. union and intersection are distributive over intersection and union respectively.

(vi) De-Morgan's Principle :

If A and B are any two sets, then

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

Proof : (i) Let x be an arbitrary element of $(A \cup B)'$. Then $x \in (A \cup B)' \Rightarrow x \notin (A \cup B)$

$\Rightarrow x \notin A$ and $x \notin B$ $\qquad \Rightarrow x \in A' \cap B'$

Again let y be an arbitrary element of $A' \cap B'$. Then $y \in A' \cap B'$

$\Rightarrow y \in A'$ and $y \in B'$ $\qquad \Rightarrow y \notin A$ and $y \notin B$

$\Rightarrow y \notin (A \cup B)$ $\qquad \Rightarrow y \in (A \cup B)'$

$\therefore A' \cap B' \subseteq (A \cup B)'$.

Hence $(A \cup B)' = A' \cap B'$

Similarly (ii) can be proved.

NOTE :

(i) $A - (B \cup C) = (A - B) \cap (A - C)$; $A - (B \cap C) = (A - B) \cup (A - C)$

(ii) $A \cap \phi = \phi$, $A \cup U = U$

Solved Examples

Ex.9 Let $A = \{2, 4, 6, 8\}$ and $B = \{6, 8, 10, 12\}$ then find $A \cup B$

Sol. $A \cup B = \{2, 4, 6, 8, 10, 12\}$

Ex.10 Let $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{2, 4, 6, 8\}$. Find $A - B$ and $B - A$.

Sol. $A - B = \{x : x \in A \text{ and } x \notin B\} = \{1, 3, 5\}$

similarly $B - A = \{8\}$

Ex.11 State true or false :

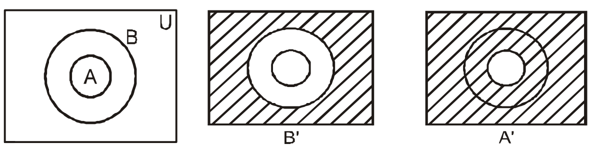
(i) $A \cup A' = \phi$ (ii) $\phi' \cap A = A$

Sol. (i) false because $A \cup A' = U$

(ii) true as $\phi' \cap A = U \cap A = A$

Ex.12 Use Venn diagram to prove that $A \subseteq B \Rightarrow B' \subseteq A'$.

Sol.



From venn diagram we can conclude that $B' \subseteq A'$.

Ex.13 Prove that if $A \cup B = C$ and $A \cap B = \phi$ then $A = C - B$.

Sol. Let $x \in A \Rightarrow x \in A \cup B \Rightarrow x \in C$ $(\because A \cup B = C)$

Now $A \cap B = \phi \Rightarrow x \notin B$ $(\because x \in A)$

$\Rightarrow x \in C - B$ $(\because x \in C \text{ and } x \notin B) \Rightarrow A \subseteq C - B$

Let $x \in C - B \Rightarrow x \in C$ and $x \notin B$

$\Rightarrow x \in A \cup B$ and $x \notin B \Rightarrow x \in A$

$\Rightarrow C - B \subseteq A$ $\therefore A = C - B$

Ex.14 If $A = \{x : x = 2n + 1, n \in Z$ and $B = \{x : x = 2n, n \in Z\}$, then find $A \cup B$.

Sol. $A \cup B = \{x : x$ is an odd integer$\} \cup \{x : x$ is an even integer$\} = \{x : x$ is an integer$\} = Z$

Ex.15 If $A = \{x : x = 3n, n \in Z\}$ and $B = \{x : x = 4n, n \in Z\}$ then find $A \cap B$.

Sol. We have,

$x \in A \cap B \Leftrightarrow x = 3n, n \in Z$ and $x = 4n, n \in Z$

$\Leftrightarrow x$ is a multiple of 3 and x is a multiple of 4

$\Leftrightarrow x$ is a multiple of 3 and 4 both

$\Leftrightarrow x$ is a multiple of 12 $\Leftrightarrow x = 12n, n \in Z$

Hence $A \cap B = \{x : x = 12n, n \in Z\}$

Ex.16 If $A = \{2, 3, 4, 5, 6, 7\}$ and $B = \{3, 5, 7, 9, 11, 13\}$ then find $A - B$ and $B - A$..

Sol. $A - B = \{2, 4, 6\}$ & $B - A = \{9, 11, 13)$

SOME IMPORTANT RESULTS ON NUMBER OF ELEMENTS IN SETS :

If A, B & C are finite sets and U be the finite universal set, then

(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(ii) $n(A \cup B) = n(A) + n(B)$ (if A & B are disjoint sets)

(iii) $n(A - B) = n(A) - n(A \cap B)$

(iv) $n(A \Delta B) = n[(A - B) \cup (B - A)]$

$= n(A) + n(B) - 2n(A \cap B)$

(v) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B)$

$- n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

(vi) $n(A' \cup B') = n(A \cap B)' = n(U) - n(A \cap B)$

(vii) $n(A' \cap B') = n(A \cup B)' = n(U) - n(A \cup B)$

Solved Examples

Ex.17 In a group of 40 students, 26 take tea, 18 take coffee and 8 take neither of the two. How many take both tea and coffee ?

Sol. $n(U) = 40, n(T) = 26,$ $n(C) = 18$

$n(T' \cap C') = 8 \Rightarrow n(T \cup C)' = 8$

$\Rightarrow n(U) - n(T \cup C) = 8$

$\Rightarrow n(T \cup C) = 32$

$\Rightarrow n(T) + n(C) - n(T \cap C) = 32$

$\Rightarrow n(T \cap C) = 12$

Ex.18 In a group of 50 persons, 14 drink tea but not coffee and 30 drink tea. Find

(i) How many drink tea and coffee both ?

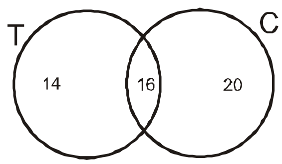
(ii) How many drink coffee but not tea ?

Sol. T : people drinking tea

C : people drinking coffee

(i) $n(T) = n(T - C) + n(T \cap C)$

$\Rightarrow 30 = 14 + n(T \cap C) \quad \Rightarrow n(T \cap C) = 16$



(ii) $n(C - T) = n(T \cup C) - n(T) = 50 - 30 = 20$

Ex.19 If A and B be two sets containing 3 and 6 elements respectively, what can be the minimum number of elements in $A \cup B$? Find also, the maximum number of elements in $A \cup B$.

Sol. We have, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

This shows that $n(A \cup B)$ is minimum or maximum according as $n(A \cap B)$ is maximum or minimum respectively.

Case-I

When $n(A \cap B)$ is minimum, i.e., $n(A \cap B) = 0$

This is possible only when $A \cap B = \phi$. In this case,

$n(A \cup B) = n(A) + n(B) - 0 = n(A) + n(B) = 3 + 6 = 9$.

So, maximum number of elements in $A \cup B$ is 9.

Case-II

When $n(A \cap B)$ is maximum.

This is possible only when $A \subseteq B$. In this case, $n(A \cap B) = 3$

$\therefore \quad n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= (3 + 6 - 3) = 6$

So, minimum number of elements in $A \cup B$ is 6.