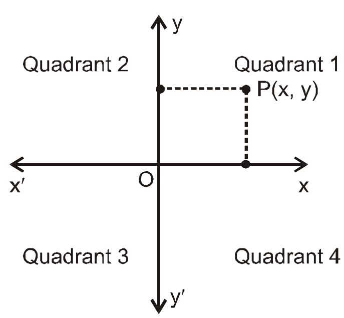
Point

RECTANGULAR CARTESIAN CO-ORDINATE SYSTEMS

We shall right now focus on two-dimensional co-ordinate geometry in which two perpendicular lines called co-ordinate axes (x-axis and y-axis) are used to locate a point in the plane.



O is called origin. Any point P in this plane can be represented by a unique ordered pair (x, y), which are called co-ordinates of that point. x is called x co-ordinate or abscissa and y is called y co-ordinate or ordinate. The two perpendicular lines xox′ and yoy′ divide the plane in four regions which are called quadrants, numbered as shown in the figure.

Let us look at some of the formulae linked with points now.

DISTANCE FORMULA

The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by $PQ = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Note :

(i) In particular the Distance of a point $P(x, y)$ from the origin $= \sqrt{x^2 + y^2}$

(ii) Distance between two polar co-ordinates $A(r_1, \theta_1)$ and $B(r_2, \theta_2)$ is given by $AB = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_1 - \theta_2)}$

Solved Examples

Ex.1 The distance between $P(3, -2)$ and $Q(-7, -5)$ is

[1] $\sqrt{115}$ [2] $\sqrt{109}$ [3] $\sqrt{91}$ [4] 11

Sol. $PQ = \sqrt{(3+7)^2 + (-2+5)^2} = \sqrt{100+9} = \sqrt{109}$

Ans. [2]

Ex.2 The distance between $P\left(2, -\frac{\pi}{6}\right)$ and $Q\left(3, \frac{\pi}{6}\right)$ is

[1] 3 [2] 1/2 [3] $\sqrt{7}$ [4] $\sqrt{5}$

Sol. $PQ = \sqrt{2^2 + 3^2 - 2.2.3\cos\left(-\frac{\pi}{6} - \frac{\pi}{6}\right)}$

$= \sqrt{13 - 12 \times \cos\left(-\frac{\pi}{3}\right)} = \sqrt{13 - 12 \times \frac{1}{2}} = \sqrt{7}$ **Ans. [3]**

Ex.3 Find the value of x, if the distance between the points (x, –1) and (3, 2) is 5

Sol. Let P(x ,–1) and Q(3, 2) be the given points. Then PQ = 5 (given)

$\sqrt{(x-3)^2+(-1-2)^2} = 5$

$\Rightarrow (x-3)^2 + 9 = 25 \qquad \Rightarrow x = 7 \text{ or } x = -1$

APPLICATION OF DISTANCE FORMULAE

(i) For given three points A, B, C to decide whether they are collinear or vertices of a particular triangle. After finding AB, BC and CA we shall find that the point are:

* Collinear: If the sum of any two distances is equal to the third.
* Vertices of an equilateral triangle if AB = BC = CA
* Vertices of an isosceles triangle if AB = BC or BC = CA or CA = AB
* Vertices of a right angled triangle $AB^2 + BC^2 = CA^2$ etc.,

(ii) For given four points:

* $AB = BC = CD = DA;\ AC = BD \Rightarrow ABCD$ square
* $AB = BC = CD = DA;\ AC \neq BD \Rightarrow ABCD$ rhombus
* $AB = CD,\ BC = DA,\ AC = BD \Rightarrow ABCD$ is a rectangle
* $AB = CD,\ BC = DA,\ AC \neq BD$

  $\Rightarrow$ ABCD is a parallelogram

Solved Examples

Ex.4 If $A(2,-2); B(-2,1)$ and $C(5,2)$ are three points then A, B, C are

[1] collinear

[2] vertices of an equilateral triangle

[3] vertices of right angled triangle

[4] none of these

Sol.

$AB = \sqrt{(-2-2)^2+(1+2)^2} = 5$

$BC = \sqrt{(5+2)^2+(2-1)^2} = 5\sqrt{2}$

$CA = \sqrt{(2-5)^2+(-2-2)^2} = 5$

Since the sum of any two distances is not equal to the third so A, B, C are not collinear. They are vertices of a triangle. Also $AB^2 + CA^2 = BC^2$

$\Rightarrow A, B, C$ are vertices of a right angled triangle.

Ans. [3]

QUADRILATERAL DIAGONALS ANGLE BETWEEN DIAGONALS

(i)	Parallelogram	Not equal	$\theta \neq \frac{\pi}{2}$
(ii)	Rectangle	Equal	$\theta \neq \frac{\pi}{2}$
(iii)	Rhombus	Not equal	$\theta = \frac{\pi}{2}$
(iv)	Square	Equal	$\theta = \frac{\pi}{2}$

Note :

(i) Diagonal of square, rhombus, rectangle and parallelogram always bisect each other.

(ii) Diagonal of rhombus and square bisect each other at right angle.

(iii) Four given points are collinear, if area of quadrilateral is zero.

Solved Examples

Ex.5 Points $A(1,1), B(-2,7)$ and $C(3,-3)$ are

[1] collinear

[2] vertices of equilateral triangle

[3] vertices of isoscele triangle

[4] none of these

Sol. $AB = \sqrt{(1+2)^2+(1-7)^2} = \sqrt{9+36} = 3\sqrt{5}$

$BC = \sqrt{(-2-3)^2+(7+3)^2} = \sqrt{25+100} = 5\sqrt{5}$

$CA = \sqrt{(3-1)^2+(-3-1)^2} = \sqrt{4+16} = 2\sqrt{5}$

Clearly $BC = AB + AC$. Hence A, B, C are collinear.

Ans. [1]

SECTION FORMULA

Co-ordinates of a point which divides the line segment joining two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ in the ratio $m_1 : m_2$ are

(1) For internal division

$$=\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

(2) For external division $=\left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}\right)$

(3) Co-ordinates of mid point of PQ $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Put $m_1 = m_2$

(4) Co-ordinates of any point on the line segment joining two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ are

$\left(\frac{x_1 + \lambda x_2}{\lambda + 1}, \frac{y_1 + \lambda y_2}{1 + \lambda}\right), \lambda \neq -1$

1. DIVISION BY AXES: PQ is divided by

(i) x-axis in the ratio $= \frac{-y_1}{y_2}$

(ii) y-axis in the ratio $= -\frac{x_1}{x_2}$

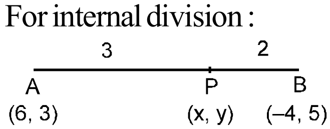
2. DIVISION BY A LINE: A line $ax + by + c = 0$ divides PQ in the ratio $-\frac{ax_1 + by_1 + c}{ax_2 + by_2 + c}$

Solved Examples

Ex.6 Find the co-ordinates of the point which divides the line segment joining the points (6, 3) and (– 4, 5) in the ratio 3 : 2 (i) internally and (ii) externally.

Sol. Let P (x, y) be the required point.

(i) For internal division :

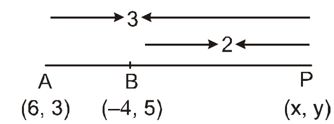


$x = \frac{3 \times -4 + 2 \times 6}{3 + 2}$ and $y = \frac{3 \times 5 + 2 \times 3}{3 + 2}$ or

$x = 0$ and $y = \frac{21}{5}$

So the co-ordinates of P are $\left(0, \frac{21}{5}\right)$

(ii) For external division



$x = \frac{3 \times -4 - 2 \times 6}{3 - 2}$ and $y = \frac{3 \times 5 - 2 \times 3}{3 - 2}$

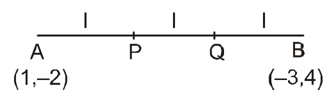
or $x = -24$ and $y = 9$

So the co-ordinates of P are (–24, 9)

Ex.7 Find the co-ordinates of points which trisect the line segment joining (1, – 2) and (– 3, 4).

Sol. Let A (1, –2) and B(–3, 4) be the given points. Let the points of trisection be P and Q.

Then AP = PQ = QB = λ (say)



$\therefore$ PB = PQ + QB = 2λ and AQ = AP + PQ = 2λ

$\Rightarrow$ AP : PB = $\lambda : 2\lambda = 1 : 2$ and AQ : QB = $2\lambda : \lambda = 2 : 1$

So P divides AB internally in the ratio 1 : 2 while Q divides internally in the ratio 2 : 1

$\therefore$ the co-ordinates of P are

$\left(\frac{1 \times -3 + 2 \times 1}{1 + 2}, \frac{1 \times 4 + 2 \times -2}{1 + 2}\right)$ or $\left(-\frac{1}{3}, 0\right)$

and the co-ordinates of Q are

$\left(\frac{2 \times -3 + 1 \times 1}{2 + 1}, \frac{2 \times 4 + 1 \times (-2)}{2 + 1}\right)$ or $\left(-\frac{5}{3}, 2\right)$

Hence, the points of trisection are $\left(-\frac{1}{3}, 0\right)$ and $\left(-\frac{5}{3}, 2\right)$.

Ex.8 The co-ordinates of point of internal and external division of the line segment joining two points $(3, -1)$ and $(3, 4)$ in the ratio 2 : 3 are respectively

[1] (2, 3) (11, 3) [2] (3, 1) (3, –11)

[3] (1, 3) (–11, 3) [4] (1, –3) (11, –3)

Sol. Internal division

$x = \frac{2(3)+3(3)}{2+3} = 3$ $\quad y = \frac{2(4)+3(-1)}{2+3} = 1$

Hence point $(3,1)$

External division

$x = \frac{2(3)-3(3)}{2-3} = 3$ $\quad y = \frac{2(4)-3(-1)}{2-3} = -11$

Hence point $(3,-11)$ **Ans.[2]**

Ex.9 The ratio in which the line $3x+4y=7$ divides the line segment joining the points $(1,2)$ and $(-2,1)$ is

[1] $\frac{4}{9}$ [2] $\frac{9}{4}$ [3] $\frac{1}{3}$ [4] $\frac{3}{4}$

Sol. Required ratio $= -\frac{ax_1+by_1+c}{ax_2+by_2+c}$

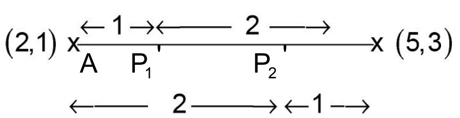
$= -\frac{3(1)+4(2)-7}{3(-2)+4(1)-7} = -\frac{4}{-9} = \frac{4}{9}$ **Ans. [1]**

Ex.10 The points of trisection of line joining the points $A(2,1)$ and $B(5,3)$ are

[1] $\left(3,\frac{5}{3}\right),\left(4,\frac{7}{3}\right)$ [2] $\left(3,\frac{3}{5}\right)\left(4,\frac{3}{7}\right)$

[3] $\left(-3,\frac{5}{3}\right)\left(4,-\frac{7}{3}\right)$ [4] $\left(3,-\frac{5}{3}\right)\left(4,\frac{3}{7}\right)$

Sol.



$P_1(x,y) = \left(\frac{1\times5+2\times2}{1+2}, \frac{1\times3+2\times1}{1+2}\right) = \left(3,\frac{5}{3}\right)$

$P_2(x,y) = \left(\frac{2\times5+1\times2}{2+1}, \frac{2\times3+1\times1}{2+1}\right) = \left(4,\frac{7}{3}\right)$

Ans.[1]

Ex.11 The ratio in which the lines joining the $(3,-4)$ and $(-5,6)$ divided by x-axis is

[1] 3 : 2 [2] 2 : 3 [3] 4 : 3 [3] 3 : 4

Sol. Required ratio $= -\frac{y_1}{y_2} = -\left(\frac{-4}{6}\right) = 2:3$ **Ans.[2]**

Centroid, Incentre & Excentre :

If A (x_1, y_1), B(x_2, y_2), C(x_3, y_3) are the vertices of triangle ABC, whose sides BC, CA, AB are of lengths a, b, c respectively, then the co-ordinates of the special points of triangle ABC are as follows :

Centroid G $\equiv \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

Incentre I $\equiv \left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right)$,and

Excentre (to A) I_1

$\equiv \left(\frac{-ax_1+bx_2+cx_3}{-a+b+c}, \frac{-ay_1+by_2+cy_3}{-a+b+c}\right)$ and so on.

Notes :

(i) Incentre divides the angle bisectors in the ratio, $(b+c) : a; (c+a) : b \; \& \; (a+b) : c$.

(ii) Incentre and excentre are harmonic conjugate of each other w.r.t. the angle bisector on which they lie.

(iii) Orthocentre, Centroid & Circumcentre are always collinear & centroid divides the line joining orthocentre & circumcentre in the ratio 2 : 1.

(iv) In an isosceles triangle G, O, I & C lie on the same line and in an equilateral triangle, all these four points coincide.

(v) In a right angled triangle orthocentre is at right angled vertex and circumcentre is mid point of hypotenuse

(vi) In case of an obtuse angled triangle circumcentre and orthocentre both are out side the triangle.

Solved Examples

Ex.12 Find the co-ordinates of (i) centroid (ii) in-centre of the triangle whose vertices are (0, 6), (8, 12) and (8, 0).

Sol. (i) We know that the co-ordinates of the centroid of a triangle whose angular points are $(x_1, y_1), (x_2, y_2)\,(x_3, y_3)$ are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

So the co-ordinates of the centroid of a triangle whose vertices are (0, 6), (8, 12) and (8, 0) are $\left(\frac{0+8+8}{3}, \frac{6+12+0}{3}\right)$ or $\left(\frac{16}{3}, 6\right)$.

(ii) Let A (0, 6), B (8, 12) and C(8, 0) be the vertices of triangle ABC.

Then $c = AB = \sqrt{(0-8)^2+(6-12)^2} = 10$,

$b = CA = \sqrt{(0-8)^2+(6-0)^2} = 10$ and

$a = BC = \sqrt{(8-8)^2+(12-0)^2} = 12$.

The co-ordinates of the in-centre are

$\left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right)$ or

$\left(\frac{12\times0+10\times8+10\times8}{12+10+10}, \frac{12\times6+10\times12+10\times0}{12+10+10}\right)$ or

$\left(\frac{160}{32}, \frac{192}{32}\right)$ or (5, 6)

Ex.13 Centroid of the triangle whose vertices are $(0,0),(2,5)$ and $(7,4)$ is

[1] (4, 3) [2] (3, 4) [3] (3, 3) [4] (3, 5)

Sol. $\left(\frac{0+2+7}{3}, \frac{0+5+4}{3}\right) = (3,3)$ **Ans.[3]**

Ex.14 Incentre of triangle whose vertices are $A(-36,7), B(20,7), C(0,-8)$ is

[1] (1, 1) [2] (0, –1) [3] (–1, 0) [4] (1, 0)

Sol. Using distance formula

$a = BC = \sqrt{20^2+(7+8)^2} = 25$,

$b = CA = \sqrt{36^2+(7+8)^2} = 39$

$c = AB = \sqrt{(36+20)^2+(7-7)^2} = 56$,

$I = \left(\frac{25(-36)+39(20)+56(0)}{25+39+56}, \frac{25(7)+39(7)+56(-8)}{25+39+56}\right)$

$I = (-1,0)$ **Ans.[3]**

Ex.15 If $(1,4)$ is the centroid of a triangle and its two vertices are $(4,-3)$ and $(-9,7)$ then third vertices is

[1] (7, 8) [2] (8, 8) [3] (8, 7) [4] (6, 8)

Sol. Let the third vertices of triangle be (x,y) then

$1 = \frac{x+4-9}{3} \Rightarrow x = 8$

$4 = \frac{y-3+7}{3} \Rightarrow y = 8$ So third vertex is (8, 8).

Ans.[2]

Ex.16 If $(0,1),(1,1)$ and $(1,0)$ are middle points of the sides of a triangle, then find its incentre is

[1] $(2-\sqrt{2},\ 2-\sqrt{2})$ [2] $(2-\sqrt{2},\ -2+\sqrt{2})$

[3] $(2+\sqrt{2},\ 2+\sqrt{2})$ [4] $(2+\sqrt{2},\ -2-\sqrt{2})$

Sol. Let $A(x_1,y_1), B(x_2,y_2)$ and $C(x_3,y_3)$ are vertices of a triangle, then

$x_1+x_2=0,\ x_2+x_3=2,\ x_3+x_1=2$

$y_1+y_2=2,\ y_2+y_3=2,\ y_3+y_1=0$

Solving these equations, we get $A(0,0), B(0,2)$ and $C(2,0)$ Now $a = BC = 2\sqrt{2}, b = CA = 2, c = AB = 2$

Thus incentre of ΔABC is $(2-\sqrt{2}, 2-\sqrt{2})$ **Ans.[1]**

Ex. 17 Two vertices of a triagle are (5, –1) and (–2, 3). If origin is the orthocentre, then the third vertex of the triangle is

(1) (4, –7) (2) (–4, 7)

(3) (–4, –7) (4) (4 , 7)

Sol. Let C (α , β) be the third vertex

$\overline{AO} \perp \overline{BC} = \left(\frac{\beta-3}{\alpha+2}\right)\left(\frac{-1}{5}\right) = -1$

$\Rightarrow 5\alpha - \beta = -13$ .... (1)

$\overline{BO} \perp \overline{AC} \Rightarrow \left(\frac{\beta+1}{\alpha-5}\right)\left(\frac{3}{-2}\right) = -1$

$\Rightarrow 2\alpha - 3\beta = 13$ ..... (2)

Solving (1) and (2) , $(\alpha , \beta) = (-4 , -7)$ **Ans.[3]**

Ex.18 If $O(0,0)$; $A(3,0)$ and $B(0,4)$ are vertices of a triangle, then its circumcentre is

[1] (1, 1) [2] $\left(\frac{3}{2},2\right)$ [3] $\left(1,\frac{4}{3}\right)$ [4] $\left(2,\frac{3}{2}\right)$

Sol. Let it be $P(x,y)$. Then $PO^2 = PA^2 = PB^2$

$$\Rightarrow x^2 + y^2 = (x-3)^2 + y^2 = x^2 + (y-4)^2$$

$$\Rightarrow 0 = -6x + 9 = -8y + 16 \qquad \Rightarrow x = \frac{3}{2},\ y = 2$$

$\therefore$ circumcentre $\equiv \left(\frac{3}{2},2\right)$ **Ans.[2]**

AREA OF TRIANGLE AND QUADRILATERAL

1. AREA OF TRIANGLE (Cartesian Coordinates)

Let $A(x_1,y_1), B(x_2,y_2)$ and $C(x_3,y_3)$ are vertices of a triangle, then

Area of triangle $ABC = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$$= \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$$

Condition of collinearity:

Three points $A(x_1,y_1), B(x_2,y_2), C(x_3,y_3)$ are collinear if the area of $\Delta ABC = 0$ i.e.,

if $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

Particular cases:

(i) When one vertex is origin i.e., if the vertices are $(0,0)$; (x_1,y_1) and (x_2,y_2) then its area $\Delta = \frac{1}{2}|x_1y_2 - x_2y_1|$

(ii) When abscissae or ordinates of all vertices are equal then its area is zero.

(iii) When two vertices be on x-axis say $(a,0)$, $(b,0)$ and third vertex be (h,k), then its area $= \frac{1}{2}|a-b|k$

(iv) When two vertices be on y-axis say $(0,c),(0,d)$ and third vertex be (h,k), then its area $= \frac{1}{2}|c-d|h$

(v) Area of the triangle formed by coordinate axes and the line $ax+by+c=0$ is $\frac{c^2}{2ab}$

(vi) When ABC is right angled triangle and $\angle B = 90^0$, then $\Delta = \frac{1}{2}(AB \times BC)$

(vii) When ABC is equilateral triangle, then $\Delta = \frac{\sqrt{3}}{4}(\text{side})^2 = \frac{1}{\sqrt{3}}(\text{height})^2$

(viii) When D, E, F are the mid points of the sides AB, BC, CA of the triangle ABC, then its area $\Delta = 4(\Delta DEF)$

Note:

Area of a triangle is always taken to be non-negative. So always use mod sign while using area formula.

Ex.19 If the vertices of a triangle are $(1,2)(4,-6)$ and $(3,5)$ then its area is

[1] $\frac{25}{2}$ sq. unit [2] 12 sq. unit

[3] 5 sq. unit [4] 25 sq. unit

Sol. $\Delta = \frac{1}{2}\left[1(-6-5) + 4(5-2) + 3(2+6)\right]$

$= \frac{1}{2}[-11+12+24] = \frac{25}{2}$ square unit **Ans.[1]**

2. AREA OF QUADRILATERAL

If $(x_1,y_1),(x_2,y_2),(x_3,y_3)$ and (x_4,y_4) are vertices of a quadrilateral then its area

$$= \frac{1}{2}\left[(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_4 - x_4y_3) + (x_4y_1 - x_1y_4)\right]$$

Note:

(i) If the area of quadrilateral joining four points is zero then those four points are collinear.

(ii) If two opposite vertex of rectangle are (x_1, y_1) and (x_2, y_2) then its area may be $= |(y_2 - y_1)(x_2 - x_1)|$

(iii) If two opposite vertex of a square are $A(x_1, y_1)$ and $C(x_2, y_2)$ then its area is

$$= \frac{1}{2}AC^2 = \frac{1}{2}\left[(x_2 - x_1)^2 + (y_2 - y_1)^2\right]$$

Solved Examples

Ex.20 If $(1,1)(3,4)(5,-2)$ and $(4,-7)$ are vertices of a quadrilateral then its area is

[1] $\frac{41}{2}$ sq. units [2] 41 sq. units

[3] 20 sq. units [4] 22 sq. units

Sol. $= \frac{1}{2}[1(4) - 3(1) + 3(-2) - 5(4) + 5(-7) - 4(-2) + 4(1) - 1(-7)]$

$$= \frac{1}{2}[4 - 3 - 6 - 20 - 35 + 8 + 4 + 7] = \frac{41}{2} \text{sq. units}$$

Ans.[1]

Ex.21 If the coordinates of two opposite vertex of a square are (a,b) and (b,a) then area of square is

[1] $(a + b)^2$ [2] $2(a + b)^2$

[3] $(a - b)^2$ [4] $2(a - b)^2$

Sol. We know that area of square $= \frac{1}{2}d^2$

$$= \frac{1}{2}\left[(a-b)^2 + (b-a)^2\right] = (a-b)^2$$ **Ans.[3]**

Ex.22 If the co-ordinates of two points A and B are (3, 4) and (5, –2) respectively. Find the co-ordinates of any point P if PA = PB and Area of ΔPAB = 10.

Sol. Let the co-ordinates of P be (x, y). Then

PA = PB $\Rightarrow$ $PA^2 = PB^2$

$\Rightarrow (x - 3)^2 + (y - 4)^2 = (x - 5)^2 + (y + 2)^2$

$\Rightarrow x - 3y - 1 = 0$

Now, Area of ΔPAB = 10

$$\Rightarrow \frac{1}{2}\begin{vmatrix} x & y & 1 \\ 3 & 4 & 1 \\ 5 & -2 & 1 \end{vmatrix} = \pm 10$$

$\Rightarrow 6x + 2y - 26 = \pm 20$

$\Rightarrow 6x + 2y - 46 = 0$ or $6x + 2y - 6 = 0$

$\Rightarrow 3x + y - 23 = 0$ or $3x + y - 3 = 0$

Solving $x - 3y - 1 = 0$ and $3x + y - 23 = 0$ we get x = 7, y = 2. Solving $x - 3y - 1 = 0$ and $3x + y - 3 = 0$, we get x = 1, y = 0. Thus, the co-ordinates of P are (7, 2) or (1, 0)

3. AREA OF A TRIANGLE (Polar Coordinates)

If (r_1, θ_1), (r_2, θ_2) and (r_3, θ_3) are vertices of a triangle then its area Δ.

$$\Delta = \frac{1}{2}\left[r_1 r_2 \sin(\theta_2 - \theta_1) + r_2 r_3 \sin(\theta_3 - \theta_2) + r_3 r_1 (\theta_1 - \theta_3)\right]$$

Solved Examples

Ex.23 The area of a triangle with vertices $(a, \theta); \left(2a, \theta + \frac{\pi}{3}\right); \left(3a, \theta + \frac{2\pi}{3}\right)$

[1] $\frac{5\sqrt{2}}{4}a^2$ [2] $\frac{2\sqrt{5}}{4}a^2$

[3] $\frac{5\sqrt{3}}{4}a^2$ [4] $\frac{2\sqrt{3}}{4}a^2$

Sol. $= \frac{1}{2}\left[2a^2 \sin\frac{\pi}{3} + 6a^2 \sin\frac{\pi}{3} + 3a^2 \sin\left(\frac{-2\pi}{3}\right)\right] = \frac{5\sqrt{3}}{4}a^2$

Ans.[3]

4. AREA OF A TRIANGLE WHEN EQUATIONS OF ITS SIDES ARE GIVEN:

If $a_r x + b_r y + c_r = 0 \, (r = 1, 2, 3)$ are sides of a triangle then its area is given by

$$\Delta = \frac{1}{2C_1C_2C_3}\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}^2$$

when C_1, C_2, C_3 are cofactors of c_1, c_2, c_3 in the determinant.

Ex.24 If $x - y = 1$, $x + 2y = 0$ and $2x + y = 3$ are sides of a triangle, then its area is

[1] $\frac{2}{3}$ [2] $\frac{3}{2}$ [3] 2 [4] $\frac{1}{2}$

Sol. Area = $\frac{1}{2(-3)(-3)3}\begin{vmatrix} 1 & -1 & -1 \\ 1 & 2 & 0 \\ 2 & 1 & -3 \end{vmatrix}^2 = \frac{1}{54}\times 36 = \frac{2}{3}$.

Ans.[1]

LOCUS OF A POINT

The locus of a moving point is the path traced out by that point under one or more given conditions.

How to find the locus of a point : Let (x_1, y_1) be the co-ordinate of the moving points say P. Now apply the geometrical conditions on x_1, y_1. This gives a relation between x_1, and y_1. Now replace x_1 by x and y_1 by y in the eleminant and resulting equation would be the equation of the locus.

Note :

(i) Locus of of a point P which is equidistant from the two point A and B is straight line and is a perpendicular bisector of line AB.

(ii) In above case if

$PA = KPB$ where $K \neq 1$ then the locus of P is a circle.

(iii) Locus of P if A and B are fixed.

(a) Circle if $\angle APB = \text{constant}$

(b) Circle with diameter AB if $\angle APB = \frac{\pi}{2}$

(c) Ellipse if $PA + PB = \text{constant}$

(d) Hyperbola if $PA - PB = \text{constant}$

Solved Examples

Ex.25 The locus of a point such that the sum of its distances from the points $(0,2)$ and $(0,-2)$ is 6 is

[1] $9x^2 + 5y^2 = 45$

[2] $5x^2 + 9y^2 = 45$

[3] $4x^2 + 7y^2 = 35$

[4] $9x^2 + 5y^2 = 50$

Sol. Let $P(h,k)$ be any point on the locus and let $A(0,2)$ and $B(0,-2)$ be the given points.

By the given condition $PA + PB = 6$

$$\Rightarrow \sqrt{(h-0)^2 + (k-2)^2} + \sqrt{(h-0)^2 + (k+2)^2} = 6$$

$$\Rightarrow \sqrt{h^2 + (k-2)^2} = 6 - \sqrt{h^2 + (k+2)^2}$$

$$\Rightarrow h^2 + (k-2)^2 = 36 - 12\sqrt{h^2 + (k+2)^2} + h^2 + (k+2)^2$$

$$\Rightarrow -8k - 36 = -12\sqrt{h^2 + (k+2)^2}$$

$$\Rightarrow (2k+9)^2 = 9\left(h^2 + (k+2)^2\right)$$

$$\Rightarrow 4k^2 + 36k + 81 = 9h^2 + 9k^2 + 36k + 36$$

$$\Rightarrow 9h^2 + 5k^2 = 45$$

Hence, locus of (h,k) is $9x^2 + 5y^2 = 45$

Ans.[1]