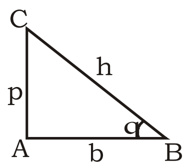
Trigonometry

1. **Trigonometric Ratio:**



To study different trigonometric ratio functions we will consider a right angled triangle. Suppose ABC is a right angled triangle with Ð A = 90°.

We can obtain six different trigonometric ratio from the sides of these triangle. They are respectively $\frac{AC}{BC}, \frac{AB}{BC}, \frac{AC}{AB}, \frac{AB}{AC}, \frac{BC}{AB}$ and $\frac{BC}{AB}$. If Ð B = q then these ratio are respectively called sin q, cos q, tan q, cot q sec q and cosec q. Clearly for the given angle q, AC (p) is perpendicular, AB (b) is base and BC (h) is hypotenuse. Hence six different trigonometric ratios are follows (see the given figure)

Trigonometric Ratios:-

$$\sin\theta = \frac{AC}{BC} = \frac{p}{h} = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\cos\theta = \frac{AB}{BC} = \frac{b}{h} = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$\tan\theta = \frac{AC}{AB} = \frac{p}{b} = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\text{cosec}\theta = \frac{BC}{AC} = \frac{h}{p} = \frac{\text{Hypotenuse}}{\text{Perpendicular}}$$

$$\sec\theta = \frac{BC}{AB} = \frac{h}{b} = \frac{\text{Hypotenuse}}{\text{Base}}$$

$$\cot\theta = \frac{AB}{AC} = \frac{b}{p} = \frac{\text{Base}}{\text{Perpendicular}}$$

Clearly sin q and cosec q are reciprocals to each other. Similarly cos q and sec q are reciprocals to each other while tan q and cot q are reciprocals to each other.

Relations between Trigonometric Ratios :-

(i) $\text{cosec}\,\theta = \frac{1}{\sin\theta}$

or $\text{cosec}\,\theta \times \sin\theta = 1$

(ii) $\sec\theta = \frac{1}{\cos\theta}$

or $\sec\theta \times \cos\theta = 1$

(iii) $\cot\theta = \frac{1}{\tan\theta}$

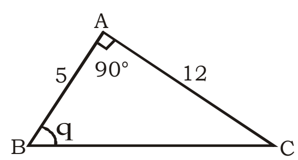
or $\cot\theta \times \tan\theta = 1$

(iv) $\tan\theta = \frac{\sin\theta}{\cos\theta}$

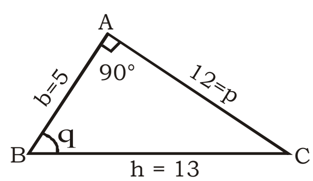
(v) $\cot\theta = \frac{\cos\theta}{\sin\theta}$

TYPE - 1

Ex.1 Write all the six t-ratios value in the given figure:



Sol. In DABC is, a right angle triangle with Ð A = 90°,



Let AC = 12 = p and AB = 5 = b

Then from Pythagoras theorem,

$BC = \sqrt{AB^2 + AC^2} = \sqrt{5^2 + 12^2}$

$= \sqrt{25 + 144} = \sqrt{169} = 13$

Here side opposite to q is AC which is p.

Side adjacent to q is AB, which is b.

Side opposite to right angle is BC, which is hypotenuse h.

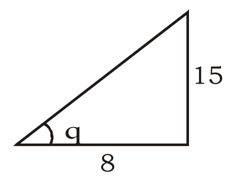
\ $\sin q = \frac{p}{h} = \frac{12}{13}$, $\text{cosec}\, q = \frac{h}{p} = \frac{13}{12}$

$\cos q = \frac{b}{h} = \frac{5}{13}$, $\sec q = \frac{h}{b} = \frac{13}{5}$

$\tan q = \frac{p}{b} = \frac{12}{5}$, $\cot q = \frac{b}{p} = \frac{5}{12}$

Ex.2 If 15 cot q = 8 then calculate the remaining trigonometric ratio.

Sol.



$\cot q = \frac{8}{15} = \frac{b}{p}$

Let b = 8k p = 15 k

From pythagoras theorem, $h^2 = p^2 + b^2 = (15k)^2 + (8k)^2$

or, $h^2 = 225\,k^2 + 64k^2 = 289\,k^2$

or, $h = \sqrt{289k^2} = 17k$

Hence, $\sin q = \frac{p}{h} = \frac{15k}{17k} = \frac{15}{17}$

$\cos q = \frac{b}{h} = \frac{8k}{17k} = \frac{8}{17}$

$\tan q = \frac{p}{b} = \frac{15k}{8k} = \frac{15}{8}$

$\sec q = \frac{h}{b} = \frac{17k}{8k} = \frac{17}{8}$

$\text{cosec}\, q = \frac{h}{p} = \frac{17k}{15k} = \frac{17}{15}$

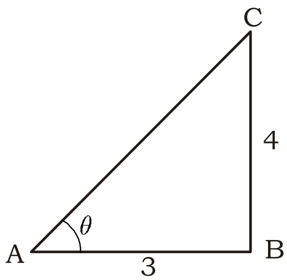
Ex.3 If $\tan\theta = \frac{4}{3}$, then $\cos\theta = ?$

(a) $4/5$ (b) $3/5$

(c) $3/4$ (d) $1/5$

Sol.(b) $\tan\theta = \frac{BC}{AB} = \frac{4}{3}$

$\therefore AC = \sqrt{(4)^2 + (3)^2} = 5$



$\therefore \cos\theta = \frac{AB}{AC} = \frac{3}{5}$

Ex.4 If $\tan\theta = \frac{4}{3}$, the value of $\frac{1-\sin\theta}{1+\sin\theta}$ is:-

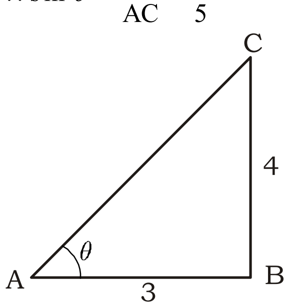
(a) $1/2$ (b) $1/3$

(c) $1/9$ (d) $1/13$

Sol.(c) $\tan\theta = \frac{4}{3} = \frac{BC}{AB}$

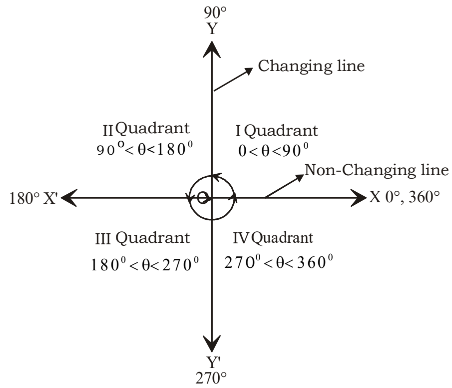
and $AC = \sqrt{(3)^2 + (4)^2} = 5$

$\therefore \sin\theta = \frac{BC}{AC} = \frac{4}{5}$



$$\therefore \frac{1-\sin\theta}{1+\sin\theta} = \frac{1-\frac{4}{5}}{1+\frac{4}{5}} = \frac{1}{9}$$

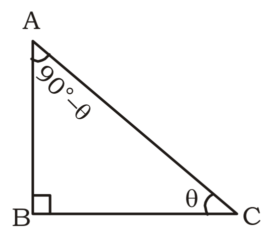
Quadrants:-



Let XOX' and YOY' be two mutually perpendicular lines. These lines divide the plane into four parts and each one of them is called a quadrant.

Complementary Angle.

For a given angle q its complementary angle is (90°– q).



From definition,

$\sin q = \frac{\text{side opposite angle } \theta}{\text{hypotenuse}} = \frac{AB}{AC}$

and cos (90° – q)

$= \frac{\text{side along with angle } (90° - \theta)}{\text{hypotenuse}} = \frac{AB}{AC}$

\ $\boxed{\sin q = \cos(90° - q)}$

Similarly, we can prove that

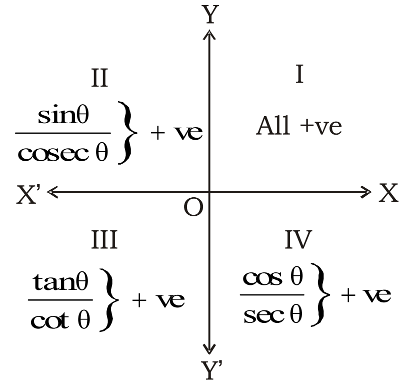
\ $\boxed{\cos q = \sin(90° - q)}$

⇒ 90°,270°....(odd multiple of 90°) will be changed

⇒ 0°,180°,360.°.........(multiple of 180°) will not be changed

Change will be in following manner:
sinθ → cosθ & cos θ → sin θ
tanθ → cotθ & cotθ → tanθ
secθ→ cosecθ & cosec θ → sec θ

Signs of Trignometric Ratios:-



Trigonometric Ratios of Allied Angles

(A) T-ratios of $(-\theta)$ in terms of those of θ :-

1. $\sin(-\theta) = -\sin\theta$
2. $\cos(-\theta) = \cos\theta$
3. $\tan(-\theta) = -\tan\theta$
4. $\cot(-\theta) = -\cot\theta$
5. $\sec(-\theta) = \sec\theta$
6. $\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$

(B) T-ratios of $(90^0 - \theta)$ in terms of those of θ :-

1. $\sin(90^0 - \theta) = \cos\theta$
2. $\cos(90^0 - \theta) = \sin\theta$
3. $\tan(90^0 - \theta) = \cot\theta$
4. $\cot(90^0 - \theta) = \tan\theta$
5. $\sec(90^0 - \theta) = \operatorname{cosec}\theta$
6. $\operatorname{cosec}(90^0 - \theta) = \sec\theta$

(C) T-ratios of $(90^0 + \theta)$ in terms of those of θ :-

1. $\sin(90^0 + \theta) = \cos\theta$
2. $\cos(90^0 + \theta) = -\sin\theta$
3. $\tan(90^0 + \theta) = -\cot\theta$
4. $\cot(90^0 + \theta) = -\tan\theta$
5. $\sec(90^0 + \theta) = -\operatorname{cosec}\theta$
6. $\operatorname{cosec}(90^0 + \theta) = \sec\theta$

(D) T-ratios of $(180^0 - \theta)$ in terms of those of θ :-

1. $\sin(180^0 - \theta) = \sin\theta$
2. $\cos(180^0 - \theta) = -\cos\theta$
3. $\tan(180^0 - \theta) = -\tan\theta$

4. $\cot(180^0 - \theta) = -\cot\theta$

5. $\sec(180^0 - \theta) = -\sec\theta$

6. $\text{cosec}\,(180^0 - \theta) = \text{cosec}\,\theta$

(E) T-ratios of $(180° + \theta)$ in terms of those of θ :-

1. $\sin(180^0 + \theta) = -\sin\theta$

2. $\cos(180^0 + \theta) = -\cos\theta$

3. $\tan(180^0 + \theta) = \tan\theta$

4. $\cot(180^0 + \theta) = \cot\theta$

5. $\sec(180^0 + \theta) = -\sec\theta$

6. $\text{cosec}\,(180^0 + \theta) = -\text{cosec}\,\theta$

(F) T-ratios of $(270° - \theta)$ in terms of those of θ :-

1. $\sin(270^0 - \theta) = -\cos\theta$

2. $\cos(270^0 - \theta) = -\sin\theta$

3. $\tan(270^0 - \theta) = \cot\theta$

4. $\cot(270^0 - \theta) = \tan\theta$

5. $\sec(270^0 - \theta) = -\text{cos ec}\,\theta$

6. $\text{cos ec}(270^0 - \theta) = -\sec\theta$

(G) T-ratios of $(270° + \theta)$ in terms of those of θ :-

1. $\sin(270^0 + \theta) = -\cos\theta$

2. $\cos(270^0 + \theta) = \sin\theta$

3. $\tan(270^0 + \theta) = -\cot\theta$

4. $\cot(270^0 + \theta) = -\tan\theta$

5. $\sec(270^0 + \theta) = \text{cos ec}\,\theta$

6. $\text{cosec}\,(270^0 + \theta) = -\sec\theta$

(H) T-ratios of $(360° - \theta)$ in terms of those of θ :-

1. $\sin(360^0 - \theta) = -\sin\theta$

2. $\cos(360^0 - \theta) = \cos\theta$

3. $\tan(360^0 - \theta) = -\tan\theta$

4. $\cot(360^0 - \theta) = -\cot\theta$

5. $\sec(360^0 - \theta) = \sec\theta$

6. $\text{cosec}\,(360^0 - \theta) = -\text{cos ec}\,\theta$

(I) T-ratios of $(360° + \theta)$ in terms of those of θ :-

1. $\sin(360^0 + \theta) = \sin\theta$

2. $\cos(360^0 + \theta) = \cos\theta$

3. $\tan(360^0 + \theta) = \tan\theta$

4. $\cot(360^0 + \theta) = \cot\theta$

5. $\sec(360^0 + \theta) = \sec\theta$

6. $\text{cosec}\,(360^0 + \theta) = \text{cosec}\,\theta$

(J) T-ratios of $(n \times 360° + \theta)$ in terms of those of θ :-

1. $\sin(n \times 360^0 + \theta) = \sin\theta$

2. $\cos(n \times 360^0 + \theta) = \cos\theta$

3. $\tan(n \times 360^0 + \theta) = \tan\theta$

4. $\cot(n \times 360^0 + \theta) = \cot\theta$

5. $\sec(n \times 360^0 + \theta) = \sec\theta$

6. $\text{cosec}\,(n \times 360^0 + \theta) = \text{cosec}\,\theta$

Value of some specific angle of trigonometrical (t)-ratio function.
We must learn the following table to solve the question based on trigonometrical (t)-ratio angle 0°, 30°, 45°, 60°, 90°

θ	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
cot	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
cosec	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

TYPE - II

Ex.5 find the value of following
(i) sin120° (ii) cos 210°
(iii)Tan570° (iv) cot 780°
(v) sin960° (vi) cos1020°
(vii) sec1500°

Sol. (i) sin 120°
= sin (90 + 30)°
$\left(\sin(90 + q) = \cos q\right)$
= cos 30° = $\frac{\sqrt{3}}{2}$

Sol.(ii) cos210°
= cos (180 + 30)°
$\left(\cos(180 + q) = -\cos q\right)$
= – cos30° = $\frac{-\sqrt{3}}{2}$

Sol.(iii) Tan 570°
= Tan (540 + 30)°
(540° multiple of 180°, Then no change Tan (540 + q) = Tan q)
= Tan 30° = $\frac{1}{\sqrt{3}}$

Sol.(iv) cot 780°
= cot (720 + 60)°
\ cot (n × 360 + q) = cot q
= cot (2 × 360 + 60)°
= cot 60° = $\frac{1}{\sqrt{3}}$

Sol.(v) sin 960°
= sin(900 + 60)°
\ 900° multiple of 180°, so no change of Trignometry function.
= sin(2 × 360 + 180 + 60) = sin (180 + 60) = –sin60°
= $\frac{-\sqrt{3}}{2}$ $\left(\sin(180 + q) = -\sin q\right)$

Sol.(vi) cos(1020)°
= cos (1080– 60)°
1080 multiple of 180°, so no change In Trignometry function.
= cos (3 × 360 – 60)°
= cos 60° = $\frac{1}{2}$

Sol.(vii) sec (1500°)
= sec (1440 + 60)°
= sec (4 × 360 + 60)°
$\left(\sec(n \times 360 + q) = \sec q\right)$
= sec 60° = 2

Ex.6
$\frac{\cos(90^0 + A).\sec(360^0 - A).\tan(180^0 - A)}{\sec(A - 720^0).\sin(A + 540^0).\cot(A - 90^0)} = ?$

(a) 0 (b) 1 (c) -1
(d) None of these

Sol. (b)
$\frac{\cos\left(90^0 + A\right).\sec\left(360^0 - A\right).\tan\left(180^0 - A\right)}{\sec\left(A - 720^0\right).\sin\left(A + 540^0\right).\cot\left(A - 90^0\right)}$

$= \frac{(-\sin A).(\sec A)(-\tan A)}{\sec(2\times 360 - A).\sin\left[3\times 180^0 + A\right].\left[-\cot\left(90^0 - A\right)\right]}$

$\left(\because \sec(-\theta) = \sec\theta\right)$

and $\cot(-\theta) = -\cot\theta]$

$$= \frac{\sin A.\sec A.\tan A}{\sec A(-\sin A)(-\tan A)}$$

$$= \frac{\sin A.\sec A.\tan A}{\sin A.\sec A.\tan A} = 1$$

Ex.7 sin720° – cot 270° – sin 150° cos120° is equal to:–

(a) $1/2$ (b) $1/3$

(c) $1/5$ (d) $1/4$

Sol. (d).sin720°–cot270° – sin150°. cos120°

= sin (2×360°+0°) – cot(360°–90°) –

$\sin(90^0+60^0).\cos(90^0+30^0)$

$= \sin 0^0 + \cot 90^0 + \cos 60^0.\sin 30^0$

$= 0 + 0 + \frac{1}{2}\times\frac{1}{2} = \frac{1}{4}$

Ex.8 Find the value of :- $\frac{\sin 37^0}{\cos 53^0}$

(a) 1 (b) -1 (c) 0 (d) 0

Sol. (a)

$$\frac{\sin 37^0}{\cos 53^0} = \frac{\sin 37^0}{\cos(90^0-37^0)} = \frac{\sin 37^0}{\sin 37^0} = 1$$

Ex.9 Evaluate :- sin² 60°+cos² 30° + cot²45° + sec² 60 ° - cosec²30° + cos² 0°:-

(a) $3/2$ (b) $5/2$

(c) $7/2$ (d) 2

Sol. (c) We know that $\sin 60^0 = \frac{\sqrt{3}}{2}$

$\cos 30^0 = \frac{\sqrt{3}}{2}, \cot 45^0 = 1$

sec 60° = 2

$\therefore \sin^2 60^0 + \cos^2 30^0 + \cot^2 45^0 + \sec^2 60^0 - \operatorname{cosec}^2 30^0 + \cos^2 0^0$

$$= \left[\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + 1^2 + 2^2 - 2^2 + 1^2\right] = \frac{7}{2}$$

Ex.10 If $\frac{x\operatorname{cosec}^2 30^0.\sec^2 45^0}{8\cos^2 45^0.\sin^2 60^0}$ = tan² 60° - tan² 30°, then the value of x is :-

(a) -1 (b) 0 (c) 1 (d) 2

Sol. (c)

$$\frac{x\times(2)^2\times(\sqrt{2})^2}{8\times\left(\frac{1}{\sqrt{2}}\right)^2\times\left(\frac{\sqrt{3}}{2}\right)^2} = (\sqrt{3})^2 - \left(\frac{1}{\sqrt{3}}\right)^2$$

$$\Rightarrow 8x = \left(\frac{8}{3}\times 8\times\frac{1}{2}\times\frac{3}{4}\right) \Rightarrow x = 1$$

Some Useful formula

(i) $\sin^2\theta + \cos^2\theta = 1$

or $\sin^2\theta = 1 - \cos^2\theta$

or $\cos^2\theta = 1 - \sin^2\theta$

(ii) $1 + \tan^2\theta = \sec^2\theta$

or $\sec^2\theta - 1 = \tan^2\theta$

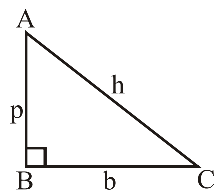
or $\sec^2\theta - \tan^2\theta = 1$

(iii) $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

or $\operatorname{cosec}^2\theta - 1 = \cot^2\theta$

or $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

Proof we know,



$\sin\theta = \frac{p}{h}$ $\cos\theta = \frac{b}{h}$

Now,

$\sin^2\theta + \cos^2\theta$

$$= \left(\frac{p}{h}\right)^2 + \left(\frac{b}{h}\right)^2 = \frac{p^2}{h^2} + \frac{b^2}{h^2} = \frac{p^2+b^2}{h^2}$$

$\because$ In right angle ΔABC

$p^2 + b^2 = h^2$

then $\sin^2\theta + \cos^2\theta$

$= \frac{h^2}{h^2} = 1$

★ Same as we can proof all remaining results same this process

TYPE - III

If A + B = 90°,

Results

- **(i) sin A. secB = 1**
  or sinA = cosB
- **(ii) cos A. cosecB = 1**
  or secA = cosecB
- **(iii) tanA. tanB = 1**
  or tanA = cotB
- **(iv) cotA. cotB = 1**
- **(v) $\sin^2 A + \sin^2 B = 1$**
- **(vi) $\cos^2 A + \cos^2 B = 1$**

Proof

(i) **sin A. secB = 1**

A + B = 90° (given)

Then, B = 90 – A

Now, sinA.sec.(90 – A)

$\Rightarrow$ sinA.cosec.A

$\Rightarrow$ $\sin A \times \frac{1}{\sin A} = 1$

★ Same as we can proof all remaining results same this process

★ **And their vice-versa are also true.**

when sinA. secB = 1,

then we can say A + B = 90°

Ex. 11 The value of (sin25°. sec65°) is equal to:-

Sol. 25° + 65° = 90°

$\left(\begin{matrix}\text{If } A+B=90^\circ \\ \sin A.\sec B = 1\end{matrix}\right)$

So, sin25°. sec65° = 1

Ex.12 The value of $(\tan 23^\circ \times \tan 67^\circ)$ is equal to :-

Sol. 23° + 67° = 90°

$\left(\begin{matrix}\text{If } A+B=90^\circ \\ \tan A.\tan B = 1\end{matrix}\right)$

So, $\tan 23^0.\tan 67^0 = 1$

Ex.13 The value of tan10°. tan25°. tan 65°. tan 80° is

Sol.

tan10° tan25° tan65° tan80°

= 1

Ex.14 If sin(3x– 6) = cos (6x – 3) find the value of x.

Sol. 3x– 6 + 6x – 3 = 90°

9x = 99°

x = 11

$\left(\begin{matrix}\text{If } A+B=90, \\ \text{then } \sin A = \cos B\end{matrix}\right)$

Ex.15 The value of cos40°.cosec50°

Sol. 40° + 50° = 90°

$\left(\text{If } A+B=90° \atop \text{cosA. cosecB} = 1\right)$

So, cos40°. cosec50° = 1

TYPE–IV

Sum and Difference Formula

(i) sin (A+B) = sin A. cos B+cos A sin B

(ii) sin (A – B) = sin A. cos B – cos A sin B

(iii) cos (A+B) = cos A. cos B – sin A sin B

(iv) cos (A – B) = cos A. cos B+sin A sin B

(v) 2 sin A. cos B = sin (A+B)+sin (A-B)

(vi) 2 cos A. sin B = sin (A+B)-sin (A-B)

(vii) 2 sin A. sin B = cos (A-B)-cos (A+B)

(viii) 2 cos A. cos B = cos (A+B)+cos (A-B)

(ix) $\sin^2 A - \sin^2 B$ = sin (A+B) . sin (A-B)

(x) $\cos^2 A - \cos^2 B$ = cos (A+B) . cos (A-B)

Tangent Formulae

(i) $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A.\tan B}$

(ii) $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A.\tan B}$

(iii) $\cot(A+B) = \dfrac{\cot A \cdot \cot B - 1}{\cot A + \cot B}$

(iv) $\cot(A-B) = \dfrac{\cot B \cdot \cot A + 1}{\cot B - \cot A}$

(v) $\tan(45+\theta) = \dfrac{1+\tan\theta}{1-\tan\theta}$

$= \dfrac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$

(vi) $\tan(45-\theta) = \dfrac{1-\tan\theta}{1+\tan\theta}$

$= \dfrac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}$

Ex.26 Find the value of the following

(i) sin75° (ii) cos75°

(iii) tan15° (iv) tan75°

Sol.(i) sin75°

sin(45° + 30°)

= sin45° cos30° + cos45°sin30°

$= \dfrac{1}{\sqrt{2}} \times \dfrac{\sqrt{3}}{2} + \dfrac{1}{\sqrt{2}} \times \dfrac{1}{2} = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$

★ $\sin 75° = \dfrac{\sqrt{3}+1}{2\sqrt{2}} = \cos 15°$

(ii) cos75°

cos(45° + 30°)

= cos45°cos30° – sin45°sin30°

$= \dfrac{1}{\sqrt{2}} \times \dfrac{\sqrt{3}}{2} - \dfrac{1}{\sqrt{2}} \times \dfrac{1}{2} = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$

★ $\cos 75° = \dfrac{\sqrt{3}-1}{2\sqrt{2}} = \sin 15°$

(iii) tan15°

tan(45 – 30)

$= \dfrac{\tan 45° - \tan 30°}{1 + \tan 45°.\tan 30°}$

$= \dfrac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \times \frac{1}{\sqrt{3}}} = \dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

★ $\tan 15° = \dfrac{\sqrt{3}-1}{\sqrt{3}+1} = \cot 75°$

(iv) tan75°

tan(45 + 30)

$= \dfrac{\tan 45° + \tan 30°}{1 - \tan 45° \tan 30°}$

$= \dfrac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \times \frac{1}{\sqrt{3}}} = \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$

★ $\tan 75° = \dfrac{\sqrt{3}+1}{\sqrt{3}-1} = \cot 15°$

Trignometric Ratios of Specific Angles

(i) $\sin 18^0 = \left(\dfrac{\sqrt{5}-1}{4}\right)$

(ii) $\cos 18^0 = \dfrac{\sqrt{10+2\sqrt{5}}}{4}$

(iii) $\cos 36^0 = \left(\dfrac{\sqrt{5}+1}{4}\right)$

(iv) $\sin 36^0 = \dfrac{\sqrt{10-2\sqrt{5}}}{4}$

(vii) $\sin 22\dfrac{1^0}{2} = \dfrac{\sqrt{2-\sqrt{2}}}{2}$

(viii) $\cos 22\dfrac{1^0}{2} = \dfrac{\sqrt{2+\sqrt{2}}}{2}$

Ex.16 The value of $\dfrac{\cos 15° - \sin 15°}{\cos 15° + \sin 15°}$ is

Sol. $\dfrac{\cos 15° - \sin 15°}{\cos 15° + \sin 15°}$

= tan (45 – 15)

$= \tan 30° = \dfrac{1}{\sqrt{3}}$

Ex.17 The value of tan40° + 2tan10° is equal to

(a) tan40° (b) cot40°

(c) sin40° (d) cos40°

Sol. We know,

40° + 10° = 50°

both sides take tan

tan(40° + 10°) = tan50°

⇒ $\dfrac{\tan 40° + \tan 10°}{1 - \tan 40°.\tan 10°} = \tan 50°$

⇒ tan40° + tan10°

= tan50° –

tan50°.tan40°.tan10°

1 (∵ tanA.tanB = 1 if A + B = 90)

⇒ tan40° + tan10°

= tan50° – tan10°

⇒ tan40° + 2tan10° = tan50°

Now, tan50° = tan.(90° – 40°)

= cot40°

Ex.18 The value of $\dfrac{\tan 57° + \cot 37°}{\tan 33° + \cot 53°}$ is equal to

(a) tan33°.cot53°

(b) tan53°.cot37°

(c) tan33°.cot57°

(d) tan57°.cot37°

Sol. $\dfrac{\tan 57° + \cot 37°}{\tan 33° + \cot 53°}$

⇒ $\dfrac{\tan 57° + \cot 37°}{\tan(90-57) + \cot 53°}$

⇒ $\dfrac{\tan 57° + \frac{1}{\tan 37°}}{\cot 57° + \cot(90-37)°}$

⇒ $\dfrac{\tan 57° + \frac{1}{\tan 37°}}{\frac{1}{\tan 57°} + \tan 37°}$

⇒ $\dfrac{\frac{(\tan 57° \tan 37° + 1)}{\tan 37°}}{\frac{(\tan 57° \tan 37° + 1)}{\tan 57°}}$

$\Rightarrow \quad \dfrac{1}{\tan 37°} \times \tan 57°$

$\Rightarrow \quad \tan 57°.\ \cot 37°$

TYPE-V

Use of componendo and dividendo-

If $\dfrac{x}{y} = \dfrac{a}{b}$, **Then** $\dfrac{x}{y} = \dfrac{a}{b}$

$\dfrac{x+y}{x-y} = \dfrac{a+b}{a-b}$

Proof $\dfrac{x}{y} = \dfrac{a}{b}$

Add 1 in both side.

$\dfrac{x}{y} + 1 = \dfrac{a}{b} + 1$

$\dfrac{x+y}{y} = \dfrac{a+b}{b}$ ....(i)

subtract 1 in both side.

$\dfrac{x}{y} - 1 = \dfrac{a}{b} - 1$

$\dfrac{x-y}{y} = \dfrac{a-b}{b}$ ....(ii)

(i) / (ii)

$\dfrac{x+y}{x-y} = \dfrac{a+b}{a-b}$

Ex.19 If $\dfrac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = 9$ find the value $\tan\theta$ and $\cos\theta$

Sol. $\dfrac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \dfrac{9}{1}$

Apply C & D

$\Rightarrow \dfrac{(\sin\theta + \cos\theta) + (\sin\theta - \cos\theta)}{(\sin\theta + \cos\theta) - (\sin\theta - \cos\theta)}$

$= \dfrac{9+1}{9-1}$

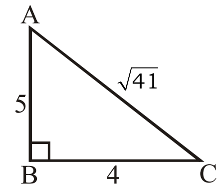
$\Rightarrow \dfrac{2\sin\theta}{2\cos\theta} = \dfrac{10}{8}$

$\tan = \dfrac{5}{4}$

Now, $\tan\theta = \dfrac{\text{Perpendicular}}{\text{Base}}$

Hypotenuse $= \sqrt{(5)^2 + (4)^2}$

$= \sqrt{41}$



Then, $\cos\theta = \dfrac{b}{h} = \dfrac{4}{\sqrt{41}}$

Ex.20 If $\dfrac{\sec\theta + \tan\theta}{\sec\theta - \tan\theta} = \dfrac{5}{3}$, then find The value of $\sin\theta$

Sol. $\dfrac{\sec\theta + \tan\theta}{\sec\theta - \tan\theta} = \dfrac{5}{3}$

Apply C & D

$\Rightarrow \dfrac{(\sec\theta + \tan\theta) + (\sec\theta - \tan\theta)}{(\sec\theta + \tan\theta) - (\sec\theta - \tan\theta)}$

$= \dfrac{5+3}{5-3}$

$\Rightarrow \dfrac{2\sec\theta}{2\tan\theta} = \dfrac{8}{2}$

$\Rightarrow \dfrac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}} = 4 \quad \Rightarrow \dfrac{1}{\sin\theta} = 4$

So, $\sin\theta = \dfrac{1}{4}$

TYPE - VI

Some pythagorean natural number will help in solving the problem on trigonometric ratio angle.

pythagorean theorem

$(\text{Base})^2 + (\text{Perpendicular})^2 = (\text{Hypotenuse})^2$

$3^2 + 4^2 = 5^2$, $6^2 + 8^2 = 10^2$,
$5^2 + 12^2 = 13^2$, $10^2 + 24^2 = 26^2$,
$8^2 + 15^2 = 17^2$, $7^2 + 24^2 = 25^2$,
$20^2 + 21^2 = 29^2$, $9^2 + 40^2 = 41^2$, etc.

Ex.21 If $\sin\theta + \cos\theta = \dfrac{17}{13}$ find the value of $\sin\theta.\cos\theta$

Sol. $\sin\theta + \cos\theta = \dfrac{17}{13}$

squaring of both side

$\Rightarrow (\sin\theta + \cos\theta)^2 = \left(\dfrac{17}{13}\right)^2$

$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = \dfrac{289}{169}$

$\Rightarrow 1 + 2\sin\theta\cos\theta = \dfrac{289}{169}$

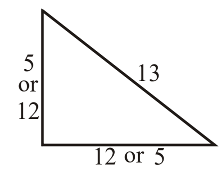
$\Rightarrow 2\sin\theta\cos\theta = \dfrac{289}{169} - 1$

$\Rightarrow 2\sin\theta\cos\theta = \dfrac{289-169}{169}$

$\Rightarrow 2\sin\theta\cos\theta = \dfrac{120}{169}$

$\Rightarrow \sin\theta\cos\theta = \dfrac{60}{169}$

Alternate:-



$\sin\theta + \cos\theta = \dfrac{17}{13} \rightarrow \dfrac{p+b}{h}$

$\sin\theta \downarrow \dfrac{p}{h}$, $\cos\theta \downarrow \dfrac{b}{h}$

Apply pythagorean here hypotenuse is 13, Then other sides of right angle triangle will be 5 and 12.

Now, Check

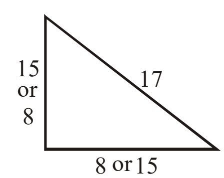
$\dfrac{5}{13} + \dfrac{12}{13} = \dfrac{17}{13}$

But we cannot find exact value of base and perpendicular, here no affect of value of $\sin\theta$ and $\cos\theta$. This question because both are product.

Hence, $\sin\theta\ \cos\theta = \dfrac{5}{13}, \dfrac{12}{13} = \dfrac{60}{169}$

Ex.22 If $\sin\theta + \cos\theta = \dfrac{23}{17}$, find the value of $\sin\theta.\cos\theta$

Sol.



$\sin\theta + \cos\theta = \dfrac{23}{17}$

– –

$\dfrac{p}{h} + \dfrac{b}{h} = \dfrac{p+b}{h}$

so, $h \rightarrow 17$

Apply pythagorean sides here hypotenuse is 17, then other sides 8 and 15

Now, Check $\dfrac{8}{17} + \dfrac{15}{17} = \dfrac{23}{17}$

Hence, sin q .cos q $= \frac{8}{17} \times \frac{15}{17} = \frac{120}{289}$

TYPE -VII

Function and Inverse function ®

(a) **If sinθ + cosecθ = 2**

then sinθ = cosecθ =1

Q **$\sin^n\theta$ + $\text{cosec}^n\theta$ =2**

n ® natural no.

Ex.23 If sinθ + cosecθ = 2 find the value of $\sin^{100}\theta + \text{cosec}^{100}\theta$

Sol. sinθ + cosecθ =2

Then, sinθ = cosecθ = 1

so, $\sin^{100}\theta + \text{cosec}^{100}\theta = 1$

$= (1)^{100} + (1)^{100} = 2$

(b) **If cosθ + secθ = 2**

then cosθ = secθ = 1

Q **$\cos^n\theta + \sec^n\theta = 2$**

Ex.24 If cosθ+secθ = 2, find the value of $\cos^{10}\theta + \sec^{10}\theta$ = ?

Sol. cosθ+secθ = 2

cosθ = secθ = 1

Then, $\cos^{10}\theta + \sec^{10}\theta$

$= (1)^{10} + (1)^{10} = 1+1 = 2$

(c) **If tanθ + cotθ = 2**

so tanθ = cotθ =1

$\tan^n\theta + \cot^n\theta = 2$

Ex.25 If tanθ + cotθ = 2 find the value of $\tan^{50}\theta + \cot^{60}\theta$

Sol. tanθ + cotθ = 2

tanθ = cotθ =1

$\tan^{50}\theta + \cot^{60}\theta = (1)^{50}+(1)^{60}=1+1=2$

(d) **If sinA + cosB = 2**

Then A = 90º

B = 0º

TYPE - VIII

Series Base ⟶

Ex. 26 The value of cos1º, cos2º,cos3º...... $\cos 179^0$ is:-

(a) 1 (b) -1 (c) 2 (d) 0

Sol. Q $\cos 90^0 = 0$

$\therefore \cos 1^0 . \cos 2^0 .... \cos 179^0 = 0$

Ex. 27 The value of tan1° tan2°tan3° ....tan 89° is :

(a) 1 (b) 0 (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

Sol. tan 1°.tan 2°. tan3°......tan 45° .....tan88°tan89°

= (tan 1°.tan 89°) (tan2°. tan88°) .... tan 45°

= (tan 1°.cot 1°). (tan 2°. cot2°) .... tan 45° = 1

[Θ tan (90° – θ) = cot θ,tan θ.cotθ = 1]

Ex.28 The value of : cos20º + cos40º + cosº + ... + cos160º + cos180º is:-

(a) 1 (b) -1

(c) 0 (d) $\frac{1}{2}$

Sol. $\cos(180^0 - \theta) = -\cos\theta$

$\therefore \cos 160^0 = \cos(180^0 - 20^0) = -\cos 20^0$

similarly

⇒ cos140° = – cos40°,

cos120° = – cos60°

cos100° = – cos80°

Now,

⇒ $(\cos 20^0 + \cos 160^0) +$

$(\cos 40^0 + \cos 140^0)$

$+(\cos 60^0 + \cos 120^0)$

$+(\cos 80^0 + \cos 100^0) + \cos 180^0$

$= (\cos 20^0 - \cos 20^0) + (\cos 40^0 - \cos 40^0) +$

$(\cos 60^0 - \cos 60^0) + (\cos 80^0 - \cos 80^0)$

$+ \cos 180^0$

⇒ $\cos 180^0 = -1$

Ex. 29 sin²5° + sin²6° + ...sin²84° + sin²85° = ?

(a) $39\frac{1}{2}$ (b) $40\frac{1}{2}$

(c) 40 (d) $39\frac{1}{\sqrt{2}}$

Sol. Let the number of terms be n, then By $t_n = a + (n – 1)d$

Here,

⇒ a = 5, d = 1

⇒ 85 = 5 + (n – 1) 1

⇒ n – 1 = 85 – 5 = 80

⇒ n = 81

∴ sin²5°+ sin²6° + ... + sin²45° + ... + sin²84° + sin² 85°

= (sin²5° + sin²85°) + (sin²6° + sin² 84°) + ...+ to 40 terms + sin² 45°

= (sin²5° + cos² 5°) + (sin²6° + cos²6)+.... + to 40 terms + sin²45°

$= 40 + \frac{1}{2} = 40\frac{1}{2}$

ê sin(90° – θ) = cosθ ú
ê sin²θ + cos²θ= 1 ú

Ex.30 The value of sin10º + sin20º + ... + sin340º + sin350º

Sol. sin10º+sin20º+...+ sin340º+sin350º

Þ sin(360º – 350º) + sin(360º–340º) +...sin180º ....sin340º + sin350º

Þ – sin350º–sin340º ....+ sin180º + ... sin340º+ sin350º = 0

[sin{360º–θ}= – sinθ,sin180º = 0])

TYPE- IX

(A)

$\sin^2\theta + \cos^2\theta = 1$
or
$\sin^2\theta = 1 - \cos^2\theta$
or
$\cos^2\theta = 1 - \sin^2\theta$

Ex. 31 What is the value of sin²1000° + cos² 1000°?

(a) 1000 (b) 100 (c) 10 (d) 1

Sol.(d) sin²1000° + cos²1000° = 1

for every value of q in sin² q + cos² q will be 1

Ex. 32 If sin²60° + cos²(3x – 9°) = 1 Then value of x is

Sol. sin²60° + cos²(3x – 9)° = 1

This is similar to sin² q + cos² q

Þ So,

60° = 3x – 9

69° = 3x

x = 23°

Ex.33 If 3sin²α + 7 cos²α = 4, then the value of tan α is (where 0 < α <90°) :

(a) $\sqrt{2}$ (b) $\sqrt{5}$

(c) $\sqrt{3}$ (d) $\sqrt{6}$

Sol.(a) 3 sin²α + 7(1– sin²α) = 4

⇒ 3 sin²α + 7 – 7 sin²α = 4

⇒ 7– 4 sin²α = 4

⇒ 4 sin²α = 3 ⇒ sinα = $\frac{\sqrt{3}}{2}$

So, α = 60°

tanα = tan60° = $\sqrt{3}$

Ex.34 If $\cos^4\theta - \sin^4\theta = \frac{2}{3}$, then the value of 2 cos²θ – 1 is :

(a) 0 (b) 1 (c) $\frac{2}{3}$ (d) $\frac{3}{2}$

Sol.(c) $\cos^4\theta - \sin^4\theta = \frac{2}{3}$

$\Rightarrow (\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta) = \frac{2}{3}$

$\Rightarrow \cos^2\theta - \sin^2\theta = \frac{2}{3}$

$\Rightarrow \cos^2\theta - (1 - \cos^2\theta) = \frac{2}{3}$

$\Rightarrow 2\cos^2\theta - 1 = \frac{2}{3}$

Ex.35 $\sin\theta + \sin^2\theta = 1$
Find the value of $\cos^2\theta + \cos^4\theta$

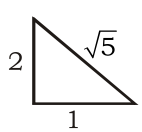
Sol. $\sin\theta + \sin^2\theta = 1$
$\sin\theta = 1 - \sin^2\theta$
$\sin\theta = \cos^2\theta$
Now, $\cos^2\theta + \cos^4\theta$
Put the value $\cos^2\theta$
$\cos^2\theta + (\cos^2\theta)^2$
$\sin\theta + \sin^2\theta = 1$ (given)

TYPE - X

(A) $\mathbf{1 + \tan^2\theta = \sec^2\theta}$
$\mathbf{\sec^2\theta - 1 = \tan^2\theta}$
$\mathbf{\sec^2\theta - \tan^2\theta = 1}$

Ex.36 If $\sec^2\theta + \tan^2\theta = 9$ find the value of $\sin\theta$ $(0° < \theta < 90°)$

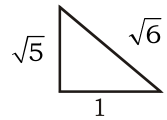
Sol. $\sec^2\theta + \tan^2\theta = 9$
$1 + \tan^2\theta + \tan^2\theta = 9$
$2\tan^2\theta = 8$
$\tan^2\theta = 4$
$\tan\theta = 2, = \frac{p}{b}$

Now,


$\sin\theta = \frac{p}{h} = \frac{2}{\sqrt{5}}$

Ex.37 If $\sec^2\theta + \tan^2\theta = 11$, find the value of $\operatorname{cosec}\theta$ $(\because 0° < \theta < 90°)$

Sol. $\sec^2\theta + \tan^2\theta = 11$
$1 + \tan^2\theta + \tan^2\theta = 11$
$2\tan^2\theta = 10$
$\tan^2\theta = 5$
$\tan\theta = \sqrt{5}$

Now,


$\operatorname{cosec}\theta = \frac{\sqrt{6}}{\sqrt{5}}$

Ex.38 If $\sec^2\theta + \tan^2\theta = \frac{5}{3}$ find the value of θ. while $(0° \le \theta \le 90°)$

Sol. $\sec^2\theta + \tan^2\theta = \frac{5}{3}$

$1 + \tan^2\theta + \tan^2\theta = \frac{5}{3}$

$2\tan^2\theta = \frac{5}{3} - 1$

$2\tan^2\theta = \frac{2}{3}$

$\tan\theta = \frac{1}{\sqrt{3}}$

so, $\theta = 30°$

Ex.39 If $\tan^2\alpha = 1 + 2\tan^2\beta$, find the value of $\sqrt{2}\cos\alpha - \cos\beta = ?$

Sol. $\tan^2\alpha = 1 + 2\tan^2\beta$ (Using identity)

$\Rightarrow \sec^2\alpha - 1 = 1 + 2(\sec^2\beta - 1)$

$\Rightarrow \sec^2\alpha - 1 = 1 + 2\sec^2\beta - 2$

$\Rightarrow \sec^2\alpha - 1 = 2\sec^2\beta - 1$

$\Rightarrow \sec^2\alpha = 2\sec^2\beta$

$\Rightarrow \sec\alpha = \sqrt{2}\sec\beta$

$\Rightarrow \frac{1}{\cos\alpha} = \sqrt{2}\left(\frac{1}{\cos\beta}\right)$

$\Rightarrow \cos\beta = \sqrt{2}\cos\alpha$

$\Rightarrow \sqrt{2}\cos\alpha - \cos\beta = 0$

Alternative:-

$\alpha = 45°$ & $\beta = 0°$ satisfies

$\tan^2\alpha = 1 + 2\tan^2\beta$

put $\alpha = 45°$ & $\beta = 0°$ in $\sqrt{2}\cos\alpha - \cos\beta$

$= \sqrt{2}\cos 45° - \cos 0° = 1 - 1 = 0$

(B). $\mathbf{(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1}$

$\mathbf{\sec\theta - \tan\theta = \frac{1}{\sec\theta + \tan\theta}}$

$\mathbf{\sec\theta + \tan\theta = \frac{1}{\sec\theta - \tan\theta}}$

if $\mathbf{\sec\theta + \tan\theta = \mathit{x}}$

then, $\mathbf{\sec\theta - \tan\theta = \frac{1}{x}}$

Ex.40 If $\sec\theta + \tan\theta = 3$, find the value of $\cos\theta$

Sol. $\sec\theta + \tan\theta = 3$ ....(i)

then $\sec\theta - \tan\theta = \frac{1}{3}$ ....(ii)

adding (i) + (ii)

$2\sec\theta = 3 + \frac{1}{3}$

$2\sec\theta = \frac{10}{3}$

$\sec\theta = \frac{5}{3}$

Hence, $\cos\theta = \frac{3}{5}$

TYPE- XI

(A) $\mathbf{1 + \cot^2\theta = \operatorname{cosec}^2\theta}$
$\mathbf{\operatorname{cosec}^2\theta - 1 = \cot^2\theta}$
$\mathbf{\operatorname{cosec}^2\theta - \cot^2\theta = 1}$

Ex.41 $\operatorname{cosec}^2\theta + 2\cot^2\theta = 10$, then find the value of $\sin\theta + \cos\theta$ when $0° < \theta < 90°$

Sol. $\operatorname{cosec}^2\theta + 2\cot^2\theta = 10$
$1 + \cot^2\theta + 2\cot^2\theta = 10$
$3\cot^2\theta = 9$
$\cot\theta = \sqrt{3}$
So, $\theta = 30°$
Now, $\sin\theta + \cos\theta$
$= \sin 30° + \cos 30°$
$= \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}$

Ex.42 If $\operatorname{cosec}^2\theta + \cot^2\theta = 3$, find the value of $\cos\theta$. when $(0° < \theta < 90°)$

Sol. $\operatorname{cosec}^2\theta + \cot^2\theta = 3$
$1 + \cot^2\theta + \cot^2\theta = 3$
$2\cot^2\theta = 2$
$\cot^2\theta = 1$
$\cot\theta = 1$

So, $\theta = 45^\circ$

Now, $\cos\theta$

$= \cos 45^\circ = \frac{1}{\sqrt{2}}$

(B) $\mathbf{cosec^2\theta - cot^2\theta = 1}$

$\mathbf{(cosec\theta - cot\theta)(cosec\theta + cot\theta) = 1}$

$\mathbf{cosec\theta - cot\theta}$

$\mathbf{= \frac{1}{cosec\theta + cot\theta}}$

If $\mathbf{cosec\theta - cot\theta = x}$,

then , $\mathbf{cosec\theta + cot\theta = \frac{1}{x}}$

Ex.43 If $\text{cosec}\theta - \cot\theta = 4$, find the value of $\cos\theta$

Sol. $\text{cosec}\theta - \cot\theta = 4$ .....(i)

then, $\text{cosec}\theta + \cot\theta = \frac{1}{4}$

.....(ii)

(i) and (ii)

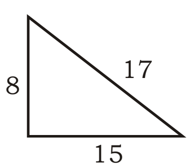
$2\,\text{cosec}\theta = 4 + \frac{1}{4}$

$2\,\text{cosec}\theta = \frac{17}{4}$

$\text{cosec}\theta = \frac{17}{8} = \frac{h}{p}$

$b = \sqrt{(17)^2 - (8)^2} = 15$

Now,



$\cos\theta = \frac{b}{h} = \frac{15}{17}$

Ex.44 If $\text{cosec}\theta + \cot\theta = \sqrt{5} + 2$, then find the value of $\sin\theta$.

Sol. $\text{cosec}\theta + \cot\theta = \sqrt{5} + 2$

.....(i)

Then, $\text{cosec}\,\theta - \cot\theta = \frac{1}{\sqrt{5}+2} = \sqrt{5} - 2$

.....(ii)

(i) + (ii)

$2\,\text{cosec}\theta = 2\sqrt{5}$

$\text{cosec}\theta = \sqrt{5}$

So, $\sin\theta = \frac{1}{\sqrt{5}}$

(c) $\mathbf{a\,cosec\theta - b\,cot\theta = c}$

$\mathbf{b\,cosec\theta - a\,cot\theta = d}$

or

$\mathbf{a\,cot\theta - b\,cosec\theta = d}$

$\mathbf{(i)^2 - (ii)^2}$

Then, $\mathbf{(a^2 - b^2 = c^2 - d^2)}$

or

$\mathbf{a\,cosec\theta + b\,cot\theta = c}$

.... (i)

$\mathbf{b\,cosec\theta + a\,cot\theta = d}$

.......(ii)

$\mathbf{(i)^2 - (ii)^2}$

$\mathbf{(a^2 - b^2 = c^2 - d^2)}$

Ex.45 If $4\,\text{cosec}\theta + 5\cot\theta = 7$, then find the value of $5\,\text{cosec}\theta + 4\cot\theta = ?$

Sol. $4\,\text{cosec}\theta + 5\cot\theta = 7$ (given)

$5\,\text{cosec}\theta + 4\cot\theta = m$ (let)

Using identity

$(4)^2 - (5)^2 = (7)^2 - (m)^2$

$16 - 25 = 49 - m^2$

$m^2 = 49 + 9$

$m = \pm\sqrt{58}$

TYPE – XII

(A) **If A+B = 45° or 225° then,**

(i) **(1+tanA) (1+tanB) = 2 and**

(ii) **(1–cotA) (1– cotB) = 2**

Proof

(i) A+B = 45°

Both side take tan.

tan (A+B)= tan45°

$\frac{\tan A + \tan B}{1 - \tan A \tan B} = 1$

$\Rightarrow$ tanA + tanB = 1– tanA tanB

$\Rightarrow$ tanA + tanB + tanA tanB = 1

Adding 1 both side.

$\Rightarrow$ 1+ tanA + tanB + tanA tanB = 1+1

$\Rightarrow$ 1(1+tanA) + tanB (1+tanA) = 2

Hence, (1+tanB) (1+tanA) = 2

(ii) A+B = 45

Both side take cot.

cot (A+B)= cot45°

Þ $\frac{\cot A \cdot \cot B - 1}{\cot A + \cot B} = \frac{1}{1}$

Þ cotA cotB –1 = cotA + cot B

Þ cotA [cotB –1] –1– cotB +1 – 1=0

Þ cotA [cotB –1] –1 [cotB –1] = 2

Þ (cotA–1)(cotB –1) = **2**

Ex.46 Find the value of (1+tan5°) (1+tan40°)

Sol. A+B = 5° + 40° = 45°

then, (1+tan5°) (1+tan40°) = 2

Ex.47 Find the value of (1+tan1°) (1+tan2°) (1+tan3°) (1+tan44°) (1+tan43°) (1+tan42°)

Sol. (1+tan1°) (1+tan44°)

(1+tan2°) (1+tan43°)

(1+tan3°) (1+tan42°)

1°+44° = 2°+43°

= 3°+42° = 45°

so, 3 pair of such term

= 2×2×2 = 8

TYPE - XIII

Morri's law

If $\mathbf{4\theta < 60^\circ}$

(i) $\mathbf{\sin\theta . \sin 2\theta . \sin 4\theta = \frac{1}{4}\sin 3\theta}$

(ii) $\mathbf{\cos\theta . \cos 2\theta . \cos 4\theta = \frac{1}{4}\cos 3\theta}$

(iii) $\mathbf{\tan\theta . \tan 2\theta . \tan 4\theta = \tan 3\theta}$

For all value of θ

(i) $\mathbf{\sin(60-\theta)\,\sin\theta . \sin(60+\theta)}$

$\mathbf{= \frac{1}{4}\sin 3\theta}$

(ii) $\mathbf{\cos(60-\theta)\,\cos\theta . \cos(60+\theta)}$

$\mathbf{= \frac{1}{4}\cos 3\theta}$

(iii) $\mathbf{\tan(60-\theta)\,\tan\theta . \tan(60+\theta) = \tan 3\theta}$

Ex.48 The value of tan10° tan20° tan40° =?

Sol. Here $\theta = 10^\circ$

\

tan(10°)tan(2×10°)tan(4×10°) = tan(3×10°)

Þ $\tan 30^\circ = \frac{1}{\sqrt{3}}$

Ex.49 The value of sin20°.sin40°.sin80° =?

Sol. sin20°.sin40°.sin80°

– – –

θ $60-\theta$ $60+\theta$

Here, $\theta = 20^\circ$

$= \frac{1}{4}\sin 3\theta$

$= \frac{1}{4}\sin(3\times 20^\circ)$

$= \frac{1}{4}\sin 60^\circ$

$\frac{1}{4}\times\frac{\sqrt{3}}{2}=\frac{\sqrt{3}}{8}$

Ex.50 The value of

$\sin\frac{\pi}{9}\cdot\sin\frac{5\pi}{9}\cdot\sin\frac{7\pi}{9}\cdot\sin\frac{3\pi}{9}\cdot$ is equal to

Sol. $\text{in}\frac{\pi}{9}\cdot\sin\frac{5\pi}{9}\cdot\sin\frac{7\pi}{9}\cdot\sin\frac{3\pi}{9}\cdot$

Put value of π = 180°

$\Rightarrow$ sin20°. sin100°. sin140°. sin60°

$\Rightarrow$ sin20°. sin(180°–80°). sin(180°–40°). $\frac{\sqrt{3}}{2}$

$\Rightarrow$ sin20°. sin40°. sin80°. $\frac{\sqrt{3}}{2}$

[$\because$ sin(180° – θ)= sinθ]

$\Rightarrow$ $\frac{1}{4}$ sin60° $\times\frac{\sqrt{3}}{2}$

$\Rightarrow$ $\frac{1}{4}\times\frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}\Rightarrow\frac{3}{16}$

TYPE - XIV

T-radius of Multiple Angles :-

(i) $\sin 2A = 2\sin A.\cos A = \frac{2\tan A}{1+\tan^2 A}$

(ii) $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1$

$= 1 - 2\sin^2 A = \frac{1-\tan^2 A}{1+\tan^2 A}$

(iii) $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

(iv) $\sin 3A = 3\sin A - 4\sin^3 A$

(v) $\cos 3A = 4\cos^3 A - 3\cos A$

(vi) $\tan 3A = \frac{3\tan A - \tan^3 A}{1-3\tan^2 A}$

(vii) sin C + sin D = 2 $\sin\left(\frac{C+D}{2}\right).\cos\left(\frac{C-D}{2}\right)$

(viii) sin C – sin D

= 2 $\cos\left(\frac{C+D}{2}\right).\sin\left(\frac{C-D}{2}\right)$

(ix) cos C + cos D =

2 $\cos\left(\frac{C+D}{2}\right).\cos\left(\frac{C-D}{2}\right)$

(x) cos C – cos D =

2 $\sin\left(\frac{C+D}{2}\right).\sin\left(\frac{D-C}{2}\right)$

Ex.51. If $\sin 2x = \frac{1}{5}$, the value of $(\sin x + \cos x)$ is :-

(a) $\sqrt{\frac{7}{5}}$ (b) $\sqrt{\frac{4}{5}}$

(c) $\sqrt{\frac{6}{5}}$ (d) $\sqrt{\frac{2}{5}}$

Sol. $\sin 2x = \frac{1}{5}$

add 1 both side

$1 + \sin 2x = 1 + \frac{1}{5} = \frac{6}{5}$

$\therefore \sin^2 x + \cos^2 x + 2\sin x.\cos x = \frac{6}{5}$

$[\because \sin^2 x + \cos^2 x = 1$ and $\sin 2x = 2\sin x.\cos x]$

$\Rightarrow (\sin x + \cos x)^2 = \frac{6}{5}$

$\Rightarrow \sin x + \cos x = \sqrt{\frac{6}{5}}$

Ex.52 The value of

cos15°. $\cos 7\frac{1}{2}^0 . \sin 7\frac{1}{2}^0$. =?

Sol. cos15°. $\cos 7\frac{1}{2}^0 . \sin 7\frac{1}{2}^0$

Multiply and divide by 2

$\Rightarrow$ cos15°. $\frac{1}{2}\left[2\cos 7\frac{1}{2}^0 . \sin 7\frac{1}{2}^0\right]$

$\Rightarrow$ $\frac{1}{2}$ cos15°×sin2. $\frac{15}{2}$

$\Rightarrow$ $\frac{1}{2}$ cos15°.sin15°

Again multiply and divide by 2

$\Rightarrow$ $\frac{1}{2\times 2}$ ×2 sin15.cos15°

$\Rightarrow$ $\frac{1}{4}$.sin30° $\Rightarrow \frac{1}{4}\times\frac{1}{2} = \frac{1}{8}$

TYPE- XV

Trigonometry expression is independent of angle so we can put any value of θ except result should not indeterminate

$\left(\infty, \frac{\infty}{\infty}, \frac{0}{0}, \frac{1}{0}\right)$

Note:- $\frac{1}{\infty}$ =0, $\frac{1}{\infty+1} = \frac{1}{\infty}$ =0

$\Rightarrow$ It is better to put θ = 0° if expression does not contain cosecθ or cotθ otherwise θ = 45°

(i) If sinθ, cosθ in equation, Try to put θ=0° or 90°

(ii) If sinθ, cosθ, tanθ, secθ, cosecθ, cotθ try to put θ = 45°

Ex.53 If tanθ+sinθ=m and tanθ-sinθ= n, then find the value of m² –n² =?

(a) $4\sqrt{mn}$ (b) mn

(c) m²n² (d) m³n³

Sol. m²-n²=(tanθ-sinθ)²-(tanθ-sinθ)²

= 4 tan θ . sin θ

$[\because (a+b)^2 - (a-b)^2 = 4ab]$

= 4 tanθ .sinθ

= $4\sqrt{\tan^2\theta.\sin^2\theta}$

= $4\sqrt{\tan^2\theta(1-\cos^2\theta)}$

= $4\sqrt{\tan^2\theta - \tan^2\theta.\cos^2\theta}$

= $4\sqrt{\tan^2\theta - \frac{\sin^2\theta}{\cos^2\theta}.\cos^2\theta}$

= $4\sqrt{\tan^2\theta - \sin^2\theta}$

Now, mn = (tanθ + sinθ) (tanθ – sinθ)

mn = tan²θ – sin²θ ....(i)

From equation (i)

= $4\sqrt{mn}$

Alternative:-

Let θ = 45°

m = tanθ + sinθ = 1 + $\frac{1}{\sqrt{2}}$

n = tanθ – sinθ = 1 – $\frac{1}{\sqrt{2}}$

Now, m² – n² = (m + n) (m –n)

= (2) × $\left(2\times\frac{1}{\sqrt{2}}\right)$ = 4 × $\frac{1}{\sqrt{2}}$

take option (a) $4\sqrt{mn}$

= mn = $\left(1+\frac{1}{\sqrt{2}}\right)\left(1-\frac{1}{\sqrt{2}}\right)$

= (1)² – $\left(\frac{1}{\sqrt{2}}\right)^2$ = 1– $\frac{1}{2}$ = $\frac{1}{2}$

So, $4\sqrt{mn}$ = 4× $\sqrt{\frac{1}{2}}$

So option (a) is correct.