MIXTURE AND ALLIGATIONS

Simple Mixture: When two different ingredients are mixed together, it is known as a simple mixture.

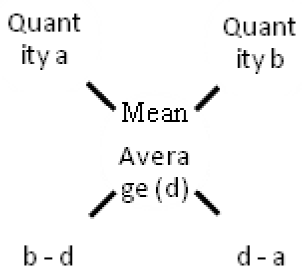
Compound Mixture: When two or more simple mixtures are mixed together to form another mixture, it is known as a compound mixture,

Alligation: Alligation is nothing but a faster technique of solving problems based on the weighted average situation as applied to the case of two groups being mixed together.
The word ‘Alligation’ literally means ‘linking’.

Alligation rule: It states that when different quantities of the same or different ingredients of different costs are mixed together to produce a mixture of a mean cost, the ratio of their quantities is inversely proportional to the difference in their cost from the mean cost.

$$\frac{\text{Quantity of Cheaper}}{\text{Quantity of Dearer}} = \frac{\text{Price of Dearer} - \text{Mean Price}}{\text{Mean Price} - \text{Price of Cheaper}}$$

Graphical representation of Alligation Rule:



$$\frac{Quantiy\ of\ a}{Quantity\ of\ b} = \frac{b-d}{d-a}$$

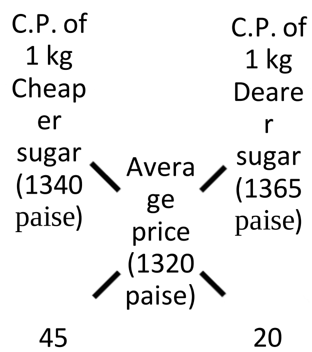
Applications of Alligation Rule:

(i) To find the mean value of a mixture when the prices of two or more ingredients, which are mixed together and the proportion in which they are mixed are given.
(ii) To find the proportion in which the ingredients at given prices must be mixed to produce a mixture at a given price.

Example l:

In what proportion must sugar at`13.40 per kg be mixed with sugar at ` 13.65 per kg, so that the mixture be worth `13.20 a kg?

Solution:



$$\frac{\text{Quantity of cheaper sugar}}{\text{Quantity of dearer sugar}} = \frac{45}{20} = \frac{9}{4}$$

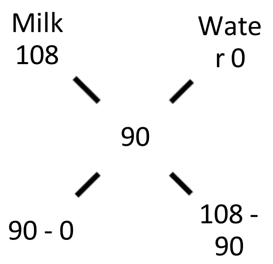
∴They must be mixed in the ratio 9:4.

Example 2:

A mixture of a certain quantity of milk with 16 litres of water is worth 90 P per litre. If pure milk be worth `1.08 per litre, how much milk is there in the mixture?

Solution:

The mean value is 90P and the price of water is 0 P.



By the Alligation Rule, milk and water are in the ratio of 5:1.
Quantity of milk in the mixture = 5 × 16 = 80 litres.

Price of the Mixture:

When quantities Q_i of ingredients M_i's with the cost C_i's are mixed then cost of the mixture C_m is given by

$$C_m = \frac{\sum C_i Q_i}{\sum Q_i}$$

Example 3:

5 kg of rice of `6 per kg is mixed with 4 kg of rice to get a mixture costing `7 per kg. Find the price of the costlier rice.

Solution:

Let the price of the costlier rice be □x.
By direct formula,

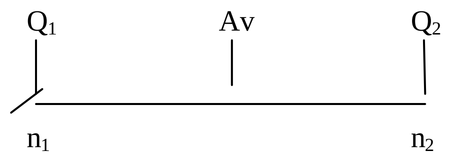
$$7 = \frac{6 \times 5 + 4 \times x}{9}$$

$$\Rightarrow 63 - 30 = 4x \Rightarrow 4x = 33$$

$$\Rightarrow x = \frac{33}{4} = 8.25$$

Straight line approach of Alligation

Let Q_1 and Q_2 be the two quantities, and n_1 and n_2 are the number of elements present in the two quantities respectively,



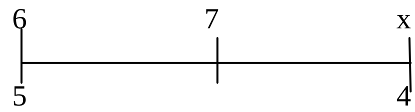
where Av is the average of the new group formed then n_1 corresponds to Q_2 – Av, n_2 corresponds to Av – Q_1 and (n_1 + n_2) corresponds to $Q_2 - Q_1$.
Let us consider the previous example.

Example 4:

5 kg of rice at `6 per kg is mixed with 4 kg of rice to get a mixture costing `7 per kg. Find the price of the costlier rice.

Solution:

Using straight line method,



4 corresponds to 7 – 6 and 5 corresponds to x – 7.
i.e. 4 → 1
5 → 1.25
Hence, x – 7 =1.2
⇒ x= 8.25

Example 5:

A jar contains a mixture of two liquids P and Q in the ratio 4:l.When 15 litres of the mixture is taken out and 15 litres of liquid Q is poured into the jar, the ratio becomes 2:3. How many litres of liquid P was contained in the jar.

Solution:

Fraction of Q in original mixture

$$= \frac{1}{1+4} = \frac{1}{5}$$

Fraction of Q in resulting mixture

$$= \frac{3}{2+3} = \frac{3}{5}$$

Q in mixure 1/5 　　　　Pure Q 1

　　　　　　3/5

2/5 　　　　　　　　2/5

Thus, the original mixture and liquid Q are mixed in the same ratio.

∴ If 15 litres of liquid Q is added, then after taking out 15 litres of mixture from the jar, there should have 15 litres of mixture left.

So, the quantity of mixture in the jar

= 15 + 15 = 30 litres

and quantity of P in the jar

$\frac{30}{5} \times 4 = 24$ litres.

Alligation Rule for Compound Mixture:

Remember that in compound mixture, same mixtures i.e. mixtures of same ingredients are mixed together in different proportion to make a new mixture.

Let Mixture 1 has ingredients A and B in ratio a:b and Mixture 2 has ingredients A and B in ratio x: y.

Now, M unit of mixture l and N unit of mixture 2 are mixed to form compound mixture. Then, in the resultant mixture, the ratio of A and B is

(i) $$\frac{\text{Quantity of ingredient A}}{\text{Quantity of ingredient B}} = \frac{q_A}{q_B} = \frac{M\left(\frac{a}{a+b}\right)+N\left(\frac{x}{x+y}\right)}{M\left(\frac{b}{a+b}\right)+N\left(\frac{y}{x+y}\right)}$$

And,

Quantity of A in resultant mixture

$$= \frac{q_A}{q_A + q_B} \times (M + N)$$

Quantity of B in resultant mixture

$$= \frac{q_B}{q_A + q_B} \times (M + N)$$

(ii) When qA and qB are known and M and N have to be found out

$$\frac{\text{Quantity of mixture 1}}{\text{Quantity of mixture 2}} = \frac{Q_1}{Q_2}$$

$$= \frac{\frac{x}{x+y} - \frac{q_A}{q_A+q_B}}{\frac{q_A}{q_A+q_B} - \frac{a}{a+b}}$$

And,

Quantity of mixture 1

$= \frac{Q_1}{Q_1+Q_2} \times$Quantity of resultant mixture

Quantity of mixture 2

$= \frac{Q_2}{Q_1+Q_2} \times$Quantity of resultant mixture

REMOVAL AND REPLACEMENT

(i) Let a vessel contains Q unit of mixture of ingredients A and B. From this, R unit of mixture is taken out and replaced by an equal amount of ingredient B only.

If this process is repeated n times, then after n operations

$$\frac{Quantity\ of\ A\ left}{Quantity\ of\ A\ originally\ present} = \left(1 - \frac{R}{Q}\right)^n$$

and Quantity of B left = Q - Quantity of A Left

(ii) Let a vessel contains Q unit of ingredient A only. From this R unit of ingredient A is taken out and replaced by an equal amount of ingredient B.

If this process is repeated n times, then after n operations,

Quantity of A left $= Q\left(1 - \frac{R}{Q}\right)^n$

Quantity of B = 1 - Quantity of A left

Example 6:

A container contains 40 litres of milk. From this container, 4 litres of milk was taken out and replaced by water. This process was repeated further two times. How much milk is now contained by die container?

Solution:

Milk	Water
To start with 40 litres	4litres

After 1st operation 36 litres After 2nd operation$36-\frac{4}{40}\times$ 36 =32.4 litres After 3rd operation $32.4-\frac{4}{40}\times 32.4$ $=32.4-3.24=7.6-0.$	$4-\frac{4}{40}\times 4+4$ =4-0.4+4 = 7.6 litres $7.6-\frac{4}{40}\times 7.6+4$ $76+4$

∴The quantity of milk in the container is 29.16 litres.

Short-cut Method:

Quantity of milk in container:

$40\left(1-\frac{4}{40}\right.^{3}=29.16$ litres

Examples 7:

A dishonest hair dresser uses a mixture having 5 parts pure After shave lotion and 3 parts of pure water. After taking out some portion of the mixture, he adds equal amount of pure water to the remaining portion of the mixture such that the amount of Aftershave lotion and water become equal. The part of the mixture taken out is

Solution:

Let quantity of pure After shave lotion = 5kg
and quantity of pure water = 3 kg
∴ Total quantity of the mixture = 8 kg
Again let x kg of mixture is taken out of 8kg of mixture.
Now, the amount of Aftershave lotion left $=\left(5-\frac{5x}{8}\right)kg$
and the amount of water left $=\left(3-\frac{3x}{8}\right)$kg
∴ The amount of water after adding x kg of water becomes
According to question,

$$5-\frac{5x}{8}=3+\frac{5x}{8}$$

$$\Rightarrow \frac{10x}{8}=2\Rightarrow x=\frac{8}{5}$$

$\Rightarrow \frac{1}{5}$of the 8 kg mixture is taken out.

If in x litres mixture of A and B, the ratio of A and B is a: b, the quantity of B to be added in order to make the ratio c : d is $\frac{x(ad-bc}{c(a+b)}$

If x glasses of equal size are filled with a mixture of milk and water. The ratio of milk and water in each glass are as follows:

$a_1:b_1, a_2:b_2, a_3:b_3 ... a_x:b_x$

If the content of all the x glasses are emptied into a single large vessel, then proportion of milk and water in it is given by

$$\frac{a_1}{a_1+b_1}+\frac{a_2}{a_2+b_2}+\cdots+\frac{a_x}{a_x+b_x}\Big):\ \frac{b_1}{a_1+b_1}+\frac{b_2}{a_2+b_2}+\cdots+\frac{b_x}{a_x+b_x}$$

Example 8:

In four vessels each of 20 litres capacity mixture of milk and water is filled. The ratio of milk and water are 2:1, 3:l, 3:2 and 1:1 in the four respective vessels. If alt the four vessels are emptied into a single large vessel, find the proportion of milk and water in the mixture.

Solution:

$$\frac{2}{3}+\frac{3}{4}+\frac{3}{5}+\frac{1}{2}\Big):\Big(\frac{1}{3}+\frac{1}{4}+\frac{2}{5}+\frac{1}{2}\ =\frac{151}{60}:\frac{89}{60}$$

$$=151:89$$

Example 9:

The ratio of water and milk in a 30 litres mixture is 7:3. Find the quantity of water to be added to the mixture in order to make this ratio 6: 1.

Solution:

In this example the ratio of water: milk is given and water is further added. But in the above formula ratio of A:B is given and quantity B is added. So the formula in this changed scenario becomes:

Quantity of B added $= \frac{x(bc-ad}{d(a+b)}$

∴ Required quantity

$$= \frac{30(3 \times 6 - 7 \times 1)}{1(7+3)} = \frac{30(18-7)}{1 \times 10}$$

$= \frac{30 \times 11}{10} = 33$ litres.

A mixture contains A and B in the ratio a:b. If x litres of B is added to the mixture, A and B become in the ratio a: c. Then the quantity of A in the mixture is given by and that of B is given by $\frac{ax}{c-b}$ and that of B is given by $\frac{bx}{c-b}$

Example 10:

A mixture contains beer and soda in the ratio of 8:3. On adding 3 litres of soda, the ratio of beer to soda becomes 2:1 (i.e., 8:4). Find the quantity of beer and soda in the is mixture.

Solution:

Quantity of beer in the mixture

$= \frac{8 \times 3}{4-3} = 24$litres

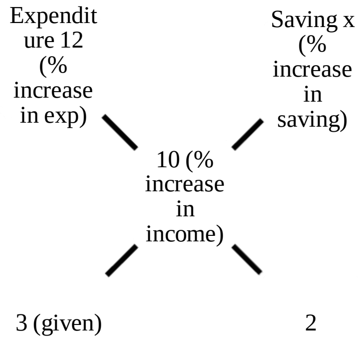
and the quantity of soda in the mixture

$= \frac{3 \times 3}{4-3} = 9$litres.

Example 11:

Mira's expenditure and savings in the ratio 3:2.Her income increase by 10%. Her expenditure also increases by 12%. By how many %does her saving increase?

Solution:



We get two valuesofx, 7 and 13.But to get a viable answer, we must keep in mind the central value (10) must lie between x and 12. Thus the value of x should be 7 and not 13.

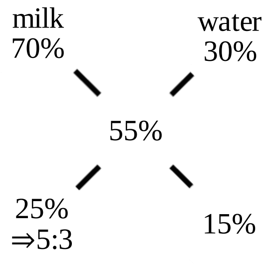
∴required % increase = 7%

Example 12:

A vessel of 80 litre is filled with milk and water, and 30% of water is taken out of the vessel. It is the vessel is vacated by 55%. Find the initial milk and water.

Solution:

Here the % values of milk and water that is taken from the vessel should be taken into consideration.



Ratio of milk to water= 5:3

∴ quantity of milk $= \frac{80}{5+3} \times 50$ litres

and quantity of water $= \frac{80}{5+3} \times 3 = 30$ litres

Nine litres are from drawn from a case full of water and it is then filled with milk. Nine litres of mixture are drawn and the cask is again filled with milk. The quantity of water now left in the cask is to that of the milk in it as 16:9, How much does the cask hold?

Solution:

Let there be x litres in the cask.From the above formula we have, after n operations:

$$\frac{\text{Water left in vessel after n operations}}{\text{Whole quantity of milk in vessel}} = \frac{x-y}{x}^{\,n}$$

Thus in this case, $\frac{x-9}{x}^{\,2} = \frac{16}{16+9} = \frac{16}{25}$

∴ x = 45 litres

Example 14:

`1500 in invested in two such part that if one invested at 16%, and the other at 5% the total interest in one year from both investments is `85. How much invested at 5%?

Solution:

If the whole money is invested at 6% the annual income is 6% of `1,500 = `90. If the whole money is invested at 5%, the annual income is 5% of `1,500 =`75. But real income = `85.

∴Applying the alligation rule, we have

6% $90 \ \ \ \ \ \ \ \ 5% $75

\ /

$85

/ \

$10 2 \ \ \ \ \ \ \ \ $5 1

Money invested at 5% = $\frac{1}{3}$×`1500=`500

Example 15:

Three vessels containing mixtures of milk and water are of capacities which are in the ratio 1:2:3. The ratios of milk and water in the three vessels are 4:1, 3:2 and 2:3 respectively. If one-fourth the contents of the first vessel, one-third of that of the second vessel and half of that of the third vessel are mixed; what is the ratio of milk and water in the new mixture?

Solution:

Part of milk in the resultant solution

$$= \frac{1}{4}\times\frac{1}{6}\times\frac{4}{5}+\frac{1}{3}\times\frac{2}{6}\times\frac{3}{5}+\frac{1}{2}\times\frac{3}{6}\times\frac{2}{5} = \frac{1}{5}$$

Part of water in the resultant solution

$$= \frac{1}{4}\times\frac{1}{6}\times\frac{1}{5}+\frac{1}{3}\times\frac{2}{6}\times\frac{2}{5}+\frac{1}{2}\times\frac{3}{6}\times\frac{3}{5} = \frac{73}{360}$$

Ratio of milk-to water $= \frac{1}{5} : \frac{73}{360} = 72 : 73$

Example 16:

Sea water contains 5 % salt by weight. How many kg of fresh water must be added to 60 kg if sea water for the content of salt in solution to be made 3%.

Solution:

Let x kg of fresh water is added to sea water

$$\frac{q_{salt}}{(q_{salt} + q_{water})} = \frac{5\%\ of\ 60}{60+x} = \frac{3}{100}$$

(given 3% salt in solution)

$$\frac{3}{(60+x)} = \frac{3}{100} = x = 40kg$$

∴40 kg of fresh water must be added to sea water.