COMPOUND INTEREST

INTEREST

Interest is the fixed amount paid on borrowed money.

The sum lent is called the **Principal.**

The sum of the principal and interest is called the **Amount.**

Interest is of two kinds:

(i) Simple interest (ii) Compound interest

Compound interest: Money is said to be lent at compound interest when at the end of a year or other fixed period, the interest that has become due is not paid to the lender, but is added to the sum lent, and the amount thus obtained becomes the principal in the next year or period. The process is repeated until the amount for the last period has been found. Hence, When the interest charged after a certain specified time period is added to form new principal for the next time period, the interest is said to be compounded and the total interest accurse is compounded and the total interest accrued is compound interest.

❖ C.I. = $p\left[\left(1+\frac{r}{100}\right)^{n}-1\right]$;

❖ Amount(A) = $P\left(1+\frac{r}{100}\right)^{n}$

Where n is number of time period.

❖ If rate of compound interest differs from year to year, then

Amount = $P\left(1+\frac{r_1}{100}\right)\left(1+\frac{r_2}{100}\right)\left(1+\frac{r_3}{100}\right)$.....

Example 1:

If `60000 amounts to `68694 in 2 years then find the rate of interest.

Solution:

Given: A = `68694

P = `60000

n = 2 years

r=?

∴ $A = P\left(1+\frac{r}{100}\right)^{n}$

∴ $68694 = 60000\left(1+\frac{r}{100}\right)^{2}$

⇨ $\frac{68694}{60000} = \left(1+\frac{r}{100}\right)^{2}$

⇨ $\frac{11449}{10000} = \left(1+\frac{r}{100}\right)^{2}$

⇨ $1+\frac{r}{100} = \sqrt{\frac{11449}{10000}} = \sqrt{1.1449}$

⇨ $1+\frac{r}{100} = 1.07$

⇨ $\frac{r}{100} = 1.07 - 1 = 0.07$

∴ $r = 0.07 \times 100 = 7\%$

Example 2:

In how many years, the sum of □10000 will become `10920.25 if the rate of compound interest is 4.5% per annum?

Solution:

A = `10920.25

P =`10000

Rate of interest = 4.5%

Time (n) = ?

∴ $A = P\left(1+\frac{r}{100}\right)^{n}$

∴ $10920.25 = 10000\left(1+\frac{4.5}{100}\right)^{n}$

$$\frac{10920.25}{10000} = \left(1+\frac{0.9}{20}\right)^{n} = \left(\frac{20.9}{20}\right)^{n} = 436.81/400$$

$$\left(\frac{20.9}{20}\right)^{n} = \left(\frac{20.9}{20}\right)^{n} = \left(\frac{20.9}{20}\right)^{n}$$

Hence `10000 will become `10920.25 in 2 years at 4.5%.

❖ **Compound interest – when interest is completed annually but time is in fraction**

If time = $t\frac{p}{q}$ years, then

$$\boxed{A=P\left(1+\frac{r}{100}\right)^{t}\left(1+\frac{\frac{p}{q}r}{100}\right)}$$

Example 3:

Find the compound interest on □8000 at 15% per annum for 2 year 4 months, compound annually.

Solution:

Time = 2 years 4 months = $2\frac{4}{12}$ years = $2\frac{1}{3}$years

Amount = `$\left[8000\left\{\left(1+\frac{15}{100}\right)^{2}\left(1+\frac{\frac{1}{3}\times 15}{100}\right)\right\}\right]$

=`$\left(8000\times\frac{23}{20}\times\frac{23}{20}\times\frac{21}{20}\right)$ = `11109

∴ C.I. = ` (11109 - 8000) = `3109

⇨ **Compound interest – when interest is calculated half-yearly**

Since r is calculated half-yearly therefore the rate per cent will become half and the time will become twice, i.e.,

Rate per cent when interest is paid half-yearly = $\frac{r}{2}$%

and time = 2×time given in years

Hence,

$$\boxed{A=P\left(1+\frac{r}{2\times 100}\right)^{2n}}$$

Example 4:

What will be the compound interest on `4000 in 4 years at 8 per cent annum. If the interest is calculated half-yearly.

Solution:

Given: P = Rs.4000, r = 8%, n = 4years

Since interest is calculated half-yearly, therefore ,

r = $\frac{8}{2}$% = 4% and n = 4×2 = 8 half years

∴ A=4000$\left(1+\frac{4}{100}\right)^{8}$ =4000× $\left(\frac{26}{25}\right)^{8}$

= 4000× 1.3685 = 5474.2762

Amount = `5474.28

∴ Interest = Amount - Principal

= `5474.28 - `4000 = `1474.28

❖ **Compound Interest-when interest is calculated quarterly**

Since 1 year has 4 quarters, therefore rate of interest will become $\frac{1}{4}$th of the rate of interest per annum, and the time period will be 4 times the time given in years

Hence, for quaterly interest

$$\boxed{A=P\left(1+\frac{r/4}{100}\right)^{4\times n} = P\left(1+\frac{r}{400}\right)^{4n}}$$

Example 5:

Find the compound interest on `25625 for 12 months at 16% per annum, compound quaterly.

Solution:

Principal(P)= `25625

Rate(r) = 16% =$\frac{16}{4}$% = 4%

Time = 12 months = 4 quaters

A = 25625$\left(1+\frac{4}{100}\right)^{4}$ = 25625 $\left(\frac{26}{25}\right)^{4}$

25625×$\frac{26}{25}\times\frac{26}{25}\times\frac{26}{25}\times\frac{26}{25}$=□29977.62

C.I. =A-P = 29977.62-25625 = `4352.62

❖ **Difference between compound Interest and Simple Interest**

When T=2

(i) C.I.-S.I. = P$\left(\frac{R}{100}\right)^{2}$

(ii) C.I.-S.I. = $\frac{R\times S.I.}{2\times 100}$

When T=3

(i) C.I.-S.I. = $\frac{PR^{2}}{10^{4}}\left(\frac{300\times R}{100}\right)$

(ii) C.I.-S.I. = $\frac{S.I.}{3}\left[\left(\frac{R}{100}\right)^{2}+3\left(\frac{R}{100}\right)\right]$

When C.I. is compound annually, the ratio of S.I. to C.I. at the same rate per annum and for the same period is given by $\frac{\text{S.I.}}{\text{C.I.}} = \frac{r1}{100\left[\left(1+\frac{r}{100}\right)^{i}-1\right]}$

Example 6:

The difference between compound interest and simple interest on a certain amount of money at 5% per annum for 2 years is `15. Find the sum:

(a) `4500 (b) `7500

(c) `5000 (d) `6000

Solution:

(d) Let the sum be `100.

Therefore, SI=$\frac{100\times5\times2}{100}$ = `10

and CI=100$\left(1+\frac{5}{100}\right)^2$ - 100

= $100\times\frac{21\times21}{20\times20} - 100$ = `$\frac{41}{4}$

Difference of CI and SI = $\frac{41}{4} - 10 = \frac{1}{4}$

If the difference is $\frac{1}{4}$, the sum = 100

⇨ If the difference is `15, the sum

= 400× 15 =Rs.6000

❖ **POPULATION FORMULA**

The original population of a town is P and the annual increase is R%, then the population in years is $p\left(\frac{R}{100}\right)^{n}$ and if the annual decrease is P $\left(1+\frac{R}{100}\right)^{n}$ R%, then the population in a year is given by a change of sign in the formula i.e P$\left(1-\frac{R}{100}\right)^{n}$

Example 7:

If the annual increase in the population of a town is 4% and the present population is 15625 what will be the population in 3 years.

Solution:

$15625\left(1+\frac{4}{100}\right)^{3}$

Required population: $15625(1.04)^3$=17576

NOTE:

❖ A certain sum is lent out on a certain rate of interest for a certain period. Again the same sum is out on x% higher rate of interest for y% higher period. Then the % increase in S.I is given by$\left(x+y+\frac{xy}{100}\right)$%

❖ P is lent out at the rate of R_1% and P_2 is lent out at the rate of R_2%. Then over all rate of interest will be

R=$\frac{P_1R_1+P_2P_2}{P_1+P_2}$

❖ $\frac{1}{x_1}$ part of the principal is lent out on R_1% rate of interest,

❖ $\frac{1}{x_2}$ part of the principal is lent out on R_2% rate of interest,.....,

❖ $\frac{1}{x_n}$ part on R_n% rate of interest. The over all rate of interest on whole sum is equal to

$$\frac{1}{x_1}\times R_1+\frac{1}{x_2}\times R_2+\cdots+\frac{1}{x_n}\times R_n\Big)$$

EFFECTIVE RATE

If `1 is deposited at 4% compounded quaterly, a calculator can be used to find that at the end of one year, the compound amount is ` 1.0406, an increase of 4.06% over the original `1. The actual in the money is somewhat higher than the stated increase of 4%. To differentiate between these two numbers, 4% is called the nominal or stated rate of interest, while 4.06% is called the effective rate. To avoid confusion between stated rates and effective rates, we shall continue to use r for the stated rate and we will use r_e for the effective rate.

Example 8:

Find the effective rate corresponding to a stated rate of 6% compound semiannually.

Solution:

A calculator shows that `100 at 6% compounded semiannually will grow to

A=$100\left(1+\frac{.06}{2}\right)^{2}$= $100(1.03)^2$=$ 106.09

Thus, the actual amount of compound interest is

`106.09 - `100=`6.09. Now if you earn `6.09 interest on

`100 in 1 year with annual compounding, your rate is 6.09/100=.0609=6.09%

Thus, the effective rate is r_e = 6.09%

NOTE:

In the preceding example we found the effective rate by dividing compound interest for 1 year by the original principal. The same thing can be done with any principal P and rate r compounded m times per year.

Effective rate =$\frac{\text{compound interest}}{\text{principal}}$

$r_e=\frac{\text{compound amont} - \text{principal}}{\text{principal}}$

$$=\frac{P\left(1+\frac{r}{m}\right)^{m}-P}{P}=\frac{P\left[\left(1+\frac{r}{m}\right)^{m}-1\right]}{P}$$

$$=r_e=\left(1+\frac{r}{m}\right)^{m}-1$$

Example 9:

A bank pays interest of 4.9% compounded monthly. Find the effective rate.

Solution:

Use the formula given above with r=.049 and m= 12.

The effective rate is $r_e=\left(1+\frac{.049}{12}\right)^{12}-1$

=1.050115575-1=.0501 or 5.01%

- Present worth of `p due n years hence

  Present worth=$\frac{p}{\left(1+\frac{r}{100}\right)^{n}}$

- Equal annual instalement to pay the borrowed amount

  Let the value of each instalement = `x

  Rate = r% and time = n years

  Then, Borrowed Amount

  $=\frac{x}{\left(1+\frac{r}{100}\right)}+\frac{x}{\left(1+\frac{r}{100}\right)^{2}}+\ldots\ldots+\frac{x}{\left(1+\frac{r}{100}\right)^{n}}$