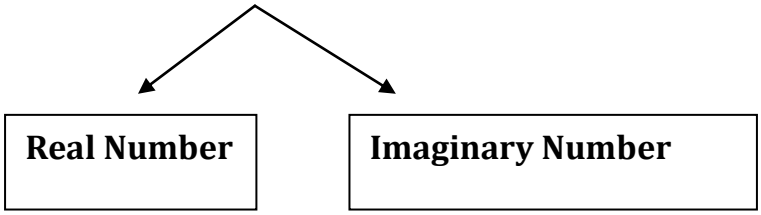
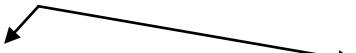
NUMBER SYSTEM

- Decimal System – 10 symbols, that's why it is called decimal system i.e. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
- Complex Number –



- Real Number :-



Rational Number

Irrational Number $\rightarrow$ non terminating non repeating number

eg. (i) $\sqrt{2}, \sqrt{3}$

(ii) π, e

$$0.\bar{3} = 0.33333\ldots\ldots = \frac{3}{9}$$

$$0.2\bar{3} = 0.23333\ldots\ldots = \frac{23-2}{90} = \frac{21}{90}$$

$$0.1\overline{26} = 0.1262626 = \frac{126-1}{990} = \frac{125}{90}$$

Ques :– $4\frac{3}{5} + 0.262626\ldots\ldots\ldots\ldots\ldots + 7\frac{1}{9} + 5.2323\ldots\ldots\ldots$

Sol :

$$= 4\frac{3}{5} + \frac{26}{99} + 7\frac{1}{9} + 5\frac{23}{99}$$

$$= 16\left(\frac{3}{5} + \frac{26}{99} + \frac{1}{9} + \frac{23}{99}\right)$$

$$= 16\left(\frac{297+300}{495}\right)$$

$$= 16\frac{597}{495}$$

$$= 17\frac{102}{495}\ Ans$$

Rational Number

Integers

- **Negative numbers**
- **0 (Zero)**
  Zero is origin of number it is neither negative nor positive.
- **Positive numbers.**

Eg. ... -3, -2, -1, 0, 1, 2, 3 ,...

Fraction Number

- **A numerical quantity that is not a whole number**

e.g. $\frac{1}{9}$ **, 0.5**

- Natural Numbers – Positive integers are called natural numbers
  Eg. - 0, 1, 2, 3, 4, 5, 6 ……
- Whole Numbers – There is no fractional, no decimal and no negatives.
  Eg. – 0, 1, 2, 3, 4, 5, 6 ……
- Prime Number – Number which has exactly two divisor is called prime number.
  - 1 is not prime number
  - 2 is only even prime number
  - There are 25 prime numbers between 1 to 100
  - Average of 1st ten prime number is 12.9
  - There are 15 prime numbers between 1 to 50
  - Test of Prime Number
  - When any prime number divided by 6 it gives 1 and 5 as remainder but vice – versa is not true i.e. it is not necessary that every number whose remainder is 1 or 5 when divided by 6 is a prime number.
  - We will search a nearby perfect square and divide that number with small digit of perfect square number if it is divisible then it is not prime number if it is not then it is prime number.

137	$(12)^2$	137 is not divisible by 2, 3, 5, 7
	2, 3, 5, 7	Hence it is prime number

Ques : - $48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3^1$; **$N = p^a \times q^b \times r^c$**

i) How may different prime number involved in 48. = **p, q, r**

ii) Find total prime numbers in 48 = **a + b + c** = 5 (4 + 1)

iii) Find total factors of 48 = **(a + 1) (b+1) (c+1)** = (4+1) (1 + 1)

$= 5 \times 2 = 10$

- Composite Number –

  Number which has more than two divisor is called composite number.
  - 4 is smallest composite number
  - Average of 1st 10 composite number is

Ques :- Find average of all prime number between 1 to 50

Sol. :- $A = \frac{2+3+5+7+11+13+17+19+23+29+31+37+41+43+47}{15}$

$A = \frac{328}{15} = 21.86$ Ans.

Ques :- N be the prime number. What is the remainder when it is divided by 6.

Sol. :- 1 and 5

- Perfect Number
- Number for which sum of all factors excluding itself is equal to number is called perfect number.

  Ex - **6** = 1 + 2 + 3 +.....

  28 = 1 + 2 + 4 + 7 + 14 +
- Product of three consecutive number is divisible by 6.

  Ex. – (i) $2 \times 3 \times 4$ (ii) $13 \times 14 \times 15$ (iii) $1001 \times 1002 \times 1003$

  Ques : - There is a positive number $(n^3 - n)$ must be divisible by ?

  Sum :- **6** i.e. $(n^3 - n) = (n - 1)(n)(n+1)$

  Similar to $6 \times 7 \times 8$

  Ques :- Prime factors of 72

  $72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$ **p^a, q^b, r^c**

  (i) Different prime number are involved in 72 **p, q, r**

= 2 (2,3)

(ii) Total prime number involved in 72 **a + b + c**

= 5 (2, 2, 2, 3, 3)

(iii) Total factors of 72 **(a +1) × (b + 1)**

= (3 + 1) × (2 + 1)

= 4×3 = 12

Odd Factors $= 2^0$

(0 + 1) × (2 + 1)

= 1×3

= 3

- Odd में even की power को zero कर देंगे

Even Factors

= Total – odd

= 12 - 3

= 9

(iv) How many different ways number (72) can be represented as product of two numbers.

Sol. : - $\frac{\text{Number of factors}}{2} = \frac{12}{2} = 6$; $\frac{(a + 1)(b + 1)(c + 1)}{2}$

(v) How many different ways number (72) can be represented as product of 2 different numbers.

Sol. : - $225 = 3^2 \times 5^2$

= (2 + 1) (2 + 1) = 9 ; $\frac{\text{⊕} - 1}{2}$

= $\frac{9-1}{2}$

= 4

Note : - When total number of factors is an odd number, number must be a perfect square.

(vi) How many different ways 225 can be written as product of 2 numbers.

Sol. :- $\frac{(2+1)(2+1)+1}{2} = \frac{9+1}{2} = \frac{10}{2} = 5$; $\frac{\text{⊕} - 1}{2}$

(vii) Sum of all factors of 72

Sol. : - = $\frac{2^{3+1}-1}{2-1}\times\frac{3^{2+1}}{3-1}$; $\frac{p^{a+1}-1}{p-1}\times\frac{q^{b+1}-1}{q-1}\times\frac{r^{c+1}-1}{r-1}$

= 15 ×13 = **195** Ans.

(viii) Find product of all factors of 72

Sol. :- $(72)^{\frac{12}{2}} = (72)^6$; N $\frac{(a+1)(b+1)(c+1)}{2}$

- **Zero – Number of zeroes**

10 = 2×5

100 = 2×2× 5× 5

1000 = 2 ×2×2×5×5×5

$2^{17}\times 5^{17}$ = 17 zeroes

$2^{22}\times 5^{15}$ = 15 zeroes

$$\begin{array}{r|l} 2 & 10 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

Ques :- Find number of zeroes in 555×101×216

Sol. 555 ×101× 216 = 5×111×101×216

= **1 zero** Ans.

Ques :- Find number of zeroes in right hand of 625×202×155

Sol. : - 625×101×2×155

So, Number of zero will be **1** Ans. ∴ Pair of (2, 5) is one

- Factorial – It is product of 'n' natural number

  i.e. n! = n×(n – 1)×(n – 2)×(n – 3)×…… 3×2×1

Ques : - Find number of zero in right hand of 25 !

Sol. :- 25 ! = 25×24×23…….. 3×2×1

Count five

So Number of pair (2, 5) will be six, So Number of zero will be **6**

Method I | Method II

5	25
5	5
	1

number of 5 = 5 + 1
in 25 ! = 6

$25 \times 2 = 5.0$

$5 \times 2 = 1.0$

So 5 + 1 = **6** Ans.

2	25
2	12
2	6
2	3
	1

number of 2 in 25! .....
= 22

Number of pairs = **6** Ans.

Ques :- Find number of zero in right end of 68 !

Sol. :-

5	68
5	13
	2

number of 5 = 13 + 2
= 15

2	68
2	34
2	17
2	8
2	4
2	2
	1

no. of 2 = 34+17+8+4+2+1
= 66

So, number of zero = **15** Ans.

Method 2 No of 5 is 68!

$68 \times 2 = 13.6 = 13$

$13 \times 2 = 2.6 = 2$

15

Number of zero = **15** Ans.

Ques : - Find number of zero in right end of 317 !

Sol. : -

Number of five

$317 \times 2 = 63.4 = 63$

$63 \times 2 = 12.6 = 12$

$12 \times 2 = 2.4 = 2$

77

Number of 2

= 311

2	317
2	158
2	79
2	39
2	19
2	9
2	4
2	2
	1

Number of zero = **77** Ans.

Ques :- Find number of 3 in 178 !

Sol. :-

3	178
3	59
3	19
3	6
	2

Number of 3 in 178 ! = 59 + 19 + 6 + 2

= **86** Ans.

Rule

Number of 5 in 100 ! = 25 – 1 = 24; $\frac{100}{4}=25$

Number of 5 in 200 ! = 50 – 1= 49 ; $\frac{200}{4}=50$

Number of 5 in 300 ! = 75 – 1 = 74 ; $\frac{300}{4}=75$

Number of 5 in 400 ! = 100 – 1 = 99 ; $\frac{400}{4}=100$

Number of 5 in 1000 ! = 250 – 1 = 249 ; $\frac{1000}{4}=250$

Ques : - Find number of zero in product of 1st 125 multiple of 3

Sol. :- $3\times6\times9\times12\times$ …….$\times$ 375

i.e. $3^{125}\times125$!

Number of 5 in 125 = 25 + 5 + 1 = 31

So number of zero is **31** Ans

Ques :- Find number of zero in $10\times20\times30\times$ ……$\times$ 1000

Sol. $10^{100}\times100$!

Number of zero = 100 + 24

= **124** Ans.

- **Division** Division) Dividend (Quotient

Remainder

Dividend = Divisor × Quotient + Remainder

Ques : - N is a number which is when divided by 77 leaves remainder 30 and quotient is 11. Find value of N.

Sol. :- $N = 77 \times 11 + 30$

$= 847 + 30$

$=$ **N = 877** Ans.

Ques. : - $\frac{771}{9}$; Find remainder

Sol. : - $\frac{771}{9} = \frac{500+200+50+21}{9} = 5+2+5+3 = \frac{15}{9}$; Remainder $= 6$ Ans

$2N = 2 \times 57 \times Q + 2 \times 45$

- Fermat's Theorem
  अगर Dividend को Divisor से divide किया जाए और वह Divisor उस Divisor का co – prime ना हो तथा Divisor prime हो तो प्रत्येक Prime से 1 कम पर Remainder 1 होता है अर्थात् Divisor 17 हो तो प्रत्येक 16वें term पर remainder 1 होगा।
- When any number N is divided by P which are coprime and P is a prime number, N^{P-1} must leave remainder 1

Ques. :- $\frac{500^{265}}{131}$, Find remainder

Sol. :- For every 130, remainder 1 will be repeated.

$$\text{So, } \frac{500^{5}}{131} = \frac{\overbrace{500 \times 500 \times 500}^{107} \times \overbrace{500 \times 500}^{52} \times \overbrace{500}^{52} \text{ - } 24}{131}$$

$$= \frac{52 \times 52 \times -24}{131}$$

So remainder $= 80$ Ans.

Ques :- $\frac{700^{60}}{59}$, Find remainder

Sol. :- 5 (-8 and -8) So, $\frac{700 \times 700}{59}$ So remainder $= 5$ Ans.

- According to Wilson Theorem when any number P, which is a prime number divided (P – 1) It leaves -1 remainder.
- Wilson Theorem

i) $\frac{(n-1)!}{n}$ so remainder $= n-1; -1$

ii) $\frac{(n-1)!+1}{n}$, so remainder $= 0$

Note :-

- If any number's digit is repeated in 3 or multiples of 3 then the digit is divisible by | 3 and 37 |

  Ex – 888, 888888, 222222, 111111111, 444, etc.

Ques. :- What should be added to the number 222221 that it is divisible by 37.

Sol. :- | Add 1 | Ans.

- Number 123123 is always divisible by 1001
- A number written in abcabc format, is always divisible by 1001

Ques :- Find remainder of $\frac{32^{32^{32}}}{3}$

Sol. :- remainder = 1

- **Euler's Theorem**

  When any number $N^{f(p)}$ when divided by P, must leave remainder 1, where f (P) number of positive co – prime integers less than P.

  $\frac{N^{f(P)}}{P}$ then $R \to 1$ Where N is any number

  When any number λ is repeated n times and divided by P where P is a prime number (P -1) times must be divisible by P

  Eg. $\frac{88888888 \ldots\ldots\ldots 2 \ 2 \text{ times}}{7}$

  $\text{Remainder} = 1$

- **Unit digit finding**

  Find unit digit

  $72 \times 18 = 6$

  $79 + 28 = 7$

  $78 - 19 = 9$

  $72 \times 19 \times 84 \times 57 = 4$

  $72 \times 19 \times 84 \times 57 = 2$

- Unit digit of 2^x will be 2, 4, 8, 6. It is in | Pair of 4 |. It will repeat after it.

$2^1 = \underline{2}$ $2^5 = 3\underline{2}$

$2^2 = \underline{4}$ $2^6 = 6\underline{4}$

$2^3 = \underline{8}$ $2^7 = 12\underline{8}$

$2^4 = 1\underline{6}$ $2^8 = 25\underline{6}$

Ques :- Find unit digit of 2500

Sol. :- $(2^{500}) = (2^4)^{125}$

So | Unit digit = 6 |

Ques :- 2^{123}, Find unit digit

Sol. :- $(2^{123}) = 2^3 =$ | Unit digit = 8 |

- Unit digit of 3x

$3^1 = 3$ $3^5 = 3$ | Repeated every 4 |

$3^2 = 9$ $3^6 = 9$

$3^3 = 27$ $3^7 = 7$

$3^4 = 81$ $3^8 = 1$

Ques = Find unit digit $(89375283)^{95793205}$

Sol. :- 3^{05} | Unit digit = 3 |

Ques :- $(793572)^{793572} \times (57293)^{57293}$ Find unit digit.

Sol. :- 272×393

$= 6 \times 3$ So | Unit digit = 8 | Ans.

$= 8$

-

Number	7^1	7^2	7^3	7^4	7^5
Unit Digit	7	9	3	1	7

On Every 4th position it is repeated

Ques :- $(5793827)^{5308497} + (59283)^{94007} \times (3842)^{9873}$

Sol. :- $= 7^{97} + 3^7 \times 2^{73}$

$= 7^1 + 3^3 \times 2^1$ So | Unit digit = 1 | Ans.

$= 7 + 7 \times 2$

$= 21$

-

Number	8^1	8^2	8^3	8^4	8^5
Unit Digit	8	4	2	6	8

Ques. :- $8^{25^{125^{625}}}$, Find unit digit.

Sol. :- $8^{25} = 8^1 = 8$

So | Unit digit = 8 | Ans.

➢

Number	4^1	4^2	4^3
Unit Digit	4	6	4

= Repeated on every 2nd position

➢

Number	9^1	9^2	9^3
Unit Digit	9	1	9

= Repeated on every 2nd position

Ques :- $(597)^{93850} + (2803)^{5790}\ (675)^{5938} + (694)^{589}$, Find unit digit.

Sol. :- $= (7)^{50} + (3)^{90} \times 5 + 4$

$= 9 + 9 \times 5 + 4$

$= 9 + 45 + 4$

$=$ 8 Ans.

Ques :- Find unit digit of 1 ! + 2 ! + 3 ! + ... + 500 !

Sol. :- 1 ! + 2 ! + 3 ! + 4 ! + 5 ! + ... + 500 !

After 5 ! unit digit become zero.

So we will find unit digit of = 1 ! + 2 ! + 3 ! + 4 !

$= 1 + 2 + 6 + 24$

$= 33 =$ 3 Ans.

- Divisibility Rule

➢ 4 - Last two digit is divisible by 4 then number will be divisible

Ex – 1139420, 620, 724, etc .

Ques :- Given number 895379*6 is divisible by 4, then find sum of possible values of ✱

Sol.:- Sum of all value of * = 1 + 3 + 5 + 7 + 9

= 25 Ans.

Ques :- Given number 8973056✱#2 is completely divisible by 8 then what is the maximum multiple of ✱ and #

Sol. :- Maximum of ✱ × # = 9 × 9

= 81 Ans.

✱	#
0	3
0	7
1	1
1	5

9	9

- 2

$2^1 = 2$ →Last 1 digit is divisible by 2

$2^2 = 4$ →Last 2 digit is divisible by 4

$2^3 = 8$ →Last 3 digit is divisible by 8

$2^4 = 16$ →Last 4 digit is divisible by 16

.

.

.

2^x → Last x digits are divisible by 2x

2

Ques :- 379358✱9357 is divisible by 3, then find the possible values of ✱

Sol. :-

~~3~~ ~~7~~ ~~9~~ ~~3~~ ~~5~~ ~~8~~ ✱ ~~9~~ ~~3~~ ~~5~~ ~~7~~

1

4

7

So possible values are 3 Ans.

Ques :- 9358079352921 ✱is divisible by 3 then find the multiple of minimum and maximum value of

Sol. :- 9 3 5 8 0 7 9 3 5 2 9 2 1 ✱

Minimum 0

Maximum 9

So minimum × maximum = 0 × 9

= 0 Ans.

- Divisibility Rule of 7

  $10 \times N \pm 1$

  $21 = 10 \times 2 + 1$ So, N = 2

- Multiples

Ques : From 1 to 100, how many multiples of 7 ?

Sol. :- $\frac{\cancel{100}}{\cancel{7}}$ 14 Ans.

Ques :- From 1 to 100, multiples of 5.

Sol. :- $\frac{\cancel{100}}{\cancel{5}}$ 20 Ans.

HCF and LCM

LCM – Lowest Common Multiple

ii) Divisor Method

2	12	18	20
2	6	9	10
3	3	9	5
3	1	3	5
5	1	1	5
	1	1	1

LCM = 2× 2× 3× 3× 5

LCM = 180

i) Prime Factorization Method

$12 = 2\times 2\times 3 = 2^2\times 3$

$18 = 2\times 3\times 3 = 2\times 3^2$

$20 = 2\times 2\times 5 = 2^2\times 5$

Highest power

$=2^2\times 3^2\times 5$

LCM = 180

Ex. – (i) 25, 20, 18

= 25×4×9

= 900 LCM

(ii) 21, 15, 10

= 21×5×2

= 210 LCM

(iii) 35, 20, 15

LCM = 35×4×3

= 420

(iv) 27, 21, 15

LCM = 27×7×5

= 945

(v) 45, 30, 21

LCM = 45×2×7

LCM = 630

Tpye – 1= Least number which when divided by a, b, c leave remainder 'k'

Ques :- Find least number which is when divided by 21, 24, 30 and leaves remainder 7 in each case.

Sol. :- 21, 24, 30

LCM = 30×4×7

LCM = $\boxed{840}$

So required Number = 840 + 7

= $\boxed{847}$ Ans

Ques :- Find largest number of 5 digit which is when divided by 24, 15 and 12 leaves remainder 9 in each case.

Sol. :- LCM = 24, 15, 12

= 24×5

= 120

$\boxed{120\ K + 9}$

$$
\begin{array}{r|l|l}
120 & 99999 & 833 \\
 & 960 & \\
\hline
 & 399 & \\
 & 360 & \\
\hline
 & 399 & \\
 & 360 & \\
\hline
 & 39 &
\end{array}
$$

Required. Number = 99960 + 9

= $\boxed{99969}$ Ans.

= 99999 – 39

= 99960

Tpye = 2

= Least number which is when divided by a, b , c leaves remainder x, y, z respectively. But difference between divisor and remainder is constant. (k)

$$= \boxed{\text{LCM (a, b, c)} - K}$$

$$\frac{(a, b, c)}{k, k, k}$$

Ques :- Find smallest number which is when divided by 16 leaves remainder 10 ; 21 leaves remainder 15 ; 15 leaves remainder 9

Sol. :-

	16,	21,	15
Rem.	10,	15	9
Const (k)	6	6	6

LCM = 16, 21, 15

= 21 × 16 × 5

LCM = 1680

Required Number = 1680 – 6

= $\boxed{1674}$ Ans.

Ques:- Find 4 digit smallest number which is when divided by 20, 16 and 12 leaves remainder 16, 12, 8 respectively.

Sol. :- LCM = 20, 16, 12

= 20 × 4 × 3

= 240 K – 4

Required Number = 1200 – 4

= $\boxed{1196}$ Ans.

240) 1000 (4

960

40

= 1000 + 200

= $\boxed{1200}$

Tpye –3 Neither remainder nor difference is same

Ques 1 :- Find least number which is when divided by 17 leaves remainder 1 and when divided by 23 leaves remainder 11

Sol. :- $\boxed{17x + 1 = 23y + 11}$ in ssc $\boxed{x = y}$ easy concept

$$17x = 23y + 10$$

∴ x and y is integer $\boxed{x = \dfrac{23y + 10}{17}}$

X	$\frac{33}{17}$	$\frac{56}{17}$	$\frac{79}{17}$	6	
Y	1	2	3	4	

Required Number = 234 + 11

= $\boxed{103}$ Ans.

Ques :- Find smallest number which is when divided by 19 leaves remainder 2 and when divided by 29 leaves remainder 19

Sol. :- $19x + 2 = 29y + 19$

$$19x = 29y + 17$$

Y	4	
X	7	

$$x = \frac{29y + 17}{19}$$

$\boxed{y = 4}$ $\boxed{x = 7}$

Required Number = 29 × 4 + 19

= 116 + 19

= $\boxed{135}$ Ans.

Ques :- In a class when 4 students sit on each bench 6 students could not find their position whereas when 5 students could not find their position whereas when 5 students sit on each bench one bench one seat remain vacant. Find number of students in the class also find number of bench.

Sol. :- $4x + 6 = 5x - 6$

Number of bench = $\boxed{x = 12}$ Ans.

Number of students = 124 + 6

= $\boxed{54}$ Ans.

Application of LCM

Ques :- There are 4 bells ring at an interval of 36 sec, 40 sec, 24 sec, 42 sec after how many minute all bell ring together.

Sol. :- 36, 40, 24, 42

LCM = 42 × 6 × 10 sec

= $\boxed{42 \text{ minute}}$ Ans.

Ques :- There are 3 guns fire bullets at an interval of 12 min, 15 min and 21 min if all gun fire on 10 am then find how many times gun fires bullet upto night 12.

Sol. :- 12, 15, 21

LCM = 21 × 5 × 4

= $\boxed{7 \text{ hours}}$ Ans.

1st firing @ 10 am

2nd firing @ 5 pm

3rd firing @ 12 last night

So $\boxed{3 \text{ times}}$ Ans. or $\boxed{\frac{14}{7}}$ = 2 + 1

$\boxed{= 3 \text{ times}}$

$$\text{Lcm of Fraction} = \left(\frac{P}{q}, \frac{r}{s} \right)$$

$$\text{LCM} = \frac{\text{LCM of Numerator}}{\text{HCF of Denominator}} = \frac{\text{LCM of P, r}}{\text{HCF of q, s}}$$

- HCF Highest Common Factor
  Highest Common Divisor

Eg. 48 = 1, 2, 3, 4, 5, 6, 8, 12, 16, 24, 48
60 = 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

HCF

HCF of 48 and 60 = 12

Type – 1

Ques :- Find highest number by which when 135, 200, 270 are divided and leaves remainder 5, 5, 10.

Sol. :-

Number	135	200	270
Remainder	5	5	10
	130	195	260

HCF of 130, 195, 260 = 65 Ans.

Ques :- Find highest number by which when 590 and 1250 are divided leaves remainder 6 and 10.

Sol. :-

Number	590	1250
Remainder	6	10
	584	1240

H.C.F. = 73 Ans.

$N^1 \times N^2 = HCF \times LCM$

→ HCF should be factor of LCM

- **Application of HCF**

Ques :- There are three vessel containing milk 250 lit, 175 lit, and 100 lit. find minimum number of times in which all can be measured through a far.

Sol. $\text{HCF of } \frac{(250}{25}, \frac{175}{25}, \frac{100)}{25} = 25$

$\text{Number of times} = 10 + 7 + 4 = 21 \text{Ans.}$

Ques :- There are three rods whose lengths are 216m, 144m, 180m. Find minimum number of pieces with equal length without wastage.

Sol. :- $\text{HCF of } \frac{(\cancel{216}, \cancel{144}, \cancel{180})}{\cancel{36} \quad \cancel{36} \quad \cancel{36}} = 36$

Number of pieces = 6 + 4 + 5 = 15 Ans.