PIPES AND CISTERNS

1. Pipes and cisterns

Pipes and cisterns is just another application of the concept of time and work. While we see only +ve work being done in normal cases of time and work, in case of pipes and cisterns, −ve work is also possible.

Given that pipes A and B can fill a tank in 20 min and 25 min working individually ⇒ this statement is similar to "A can do a work in 20 min and B can do the same work in 25 min."

Again, given that pipe C can empty a tank in 40 min we can say this statement is similar to "C can demolish a wall in 40 min (assuming that the work is building or demolishing the wall)

Let us understand this with the help of an example.

Example 14 A and B are two taps which can fill a tank individually in 10 min and 20 min respectively. However, there is a leakage at the bottom, which can empty a filled tank in 40 min. If the tank is empty initially, how much time will both the taps take to fill the tank (leakage is still there)?

Solution Let us assume the units of work = LCM of (10, 20, 40) = 40 units

Work done by Tap A/min = 4 units/min (Positive work)

Work done by Tap B/min = 2 units/min (Positive work)

Work done by leakage/min = 1 unit/min (Negative work)

Net work done/min = 5 units/min

Hence time taken = 8 min

Example 15 Pipe A can fill a tank in 3 hour. But there is a leakage also, due to which it takes 3.5 hour for the tank to be filled. How much time will the leakage take in emptying the tank if the tank is filled initially?

Solution Assume the total units of work = 10.5 units

Work done by Tap A/h = 3.5 units/h (Positive work)

Work done by leakage/h = 3 units/h (Negative work)

Net work done/h = 0.5 units/h

$$\text{So, the time taken} = \frac{10.5}{0.5} = 21 \text{ hour}$$

Alternatively, due to the leakage, the pipe is required to work for an extra half an hour. So, the quantity filled by pipe in half an hour is being emptied by the leakage in 3.5 hour. Hence, the quantity filled by pipe in 3 hour will be emptied by the leakage in 21 hour.

2. Variable work

The concept of variable work comes from the possibility

$\Rightarrow$ that the rate of working can be different or

$\Rightarrow$ can be dependent upon some external agent.

In these cases, the rate of work will be proportional to some external factor.

Understand this with the help of a simple statement: The rate of the flow of water from a pipe is directly proportional to the area of the cross section of the pipe.

Example 16 There are three inlet taps whose diameters are 1 cm, 2 cm and 3 cm respectively. The rate of flow of the water is directly proportional to the square of the diameter. It takes 9 min for the smallest pipe to fill an empty tank. Find the time taken to fill an empty tank when all the three taps are opened.

Solution The rate of flow of a diameter^2, or, rate of flow = K $\times$ diameter^2 (where K is a constant)

For 1st tap, rate of flow = K $\times$ 1

For 2nd tap, rate of flow = K $\times$ 4

For 3rd tap, rate of flow = K $\times$ 9

We know, the quantity filled will be equal to the product of the rate of flow and time.

So, the quantity filled by the smallest pipe = K $\times$ 1 $\times$ 9 = 9K = Capacity of tank

Quantity of water filled by all the taps together in 1 min = 9K + 4K + 1K = 14K

Assume that all the taps working together take 't' min.

So, $14\ K \times t = 9\ K$

So, the time taken $t = \frac{9K}{14K} = \frac{9}{14}$ min