TIME AND WORK

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There is a definite relationship between the amount of time taken, the number of persons doing the work and the unit of work done. Under this topic we study the phenomenon of accomplishment of a work with relation to the time. There are certain formulae that demonstrate the above-mentioned relationship and exhibit how each of these factors influence the other two.

TIME–WORK EQUIVALENCE

The essence of time-work equivalence lies in the fact that it exhibits the most fundamental relationship between the three factors as mentioned above viz., work, time and the agent which is completing the work. That is,

Work done = Number of days × Number of men

$W = M \times D$

This gives us an important concept of man-days.

Suppose there are 20 persons working for 10 days to complete a job, then the total work done is equal to 200 man-days. Now, if we change the number of days in which the work is to be completed, then the other factor i.e., the number of persons will change accordingly, so that, the product of the factors becomes equal to 200 man-days. Product-stability-ratio (Chapter 3) is a very effective tool to calculate this.

Example 1 Seven persons can clean 7 floors by 7 mops in 7 days. In how many days can 5 persons clean 5 floors by 5 mops?

Solution This problem can be solved through several methods.

Method 1 To clean 7 floors, we need to have 7 × 7 = 49 man-days.

So, to clean 5 floors, we need to have 35 man-days.

So, 35 = D × 5. So, D = 7 days

Method 2 Using ratio proportion, less work and less men are involved here.

So, the number of days = $7 \times \frac{5}{7} \times \frac{7}{5} = 7$ days

Method 3 Let us try to have a mental image of this situation: There is a building with seven floors namely $F_1, F_2, \ldots, F_7$ and seven persons $P_1, P_2, \ldots P_7$ are cleaning this building in such a way that one floor is being cleaned by each one of them. Since it takes 7 days to complete the whole work, it can be inferred that everybody is taking 7 days to clean his respective floor. So, if there are just floors and five persons are cleaning these five floors, then it will take them seven days (assuming that the top two floors have been demolished.)

Now, depending upon different situations, three conditions are possible, in the relationship W = MD (where W = quantity of work, M = number of persons, D = number of days)

Condition 1: W is constant

$M \times D = \text{Constant}$

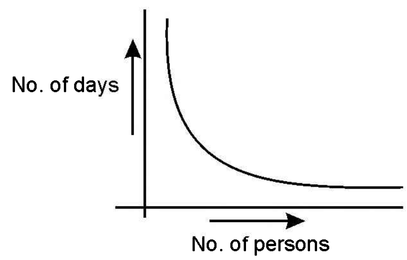
$M \propto 1/D$

It can be observed that if the work done is constant, then the number of persons is inversely proportional to the number of days, which means that the multipliers of M and D will be reciprocal. Extending this situation, if 10 persons can do a work in 20 days, then 5 persons can do the same work in 40 days, or, 20 persons will do the same work in 10 days. Further it can be summarized as

W	=	M	×	D
200	=	10	×	20
200	=	5(10 × 1/2)	×	40(20 × 2)
200	=	20(10 × 2)	×	10(20 × 1/2)

Thus, it can be said that the multiplier of M and multiplier of D are reciprocal to each other.

It can be seen with the help of the graph given below



Example 2 Yadavjee contractor undertakes to get a work done in 50 days by 50 labourers. After 40 days, he realizes that only 50% of the work is done. How many more men should be employed so that the work is complete on time?

Solution

W	=	M	×	D	
50%		50		40	...(i)
Rest 50%		(50 + M)		10	...(ii)

Since work is constant in both the cases, so, the number of men and the number of days will be reciprocal to each other. As the number of days left in (ii) is 1/4th of initial period (i), so the number of persons will become 4 times of the initial number of persons.

Hence, the number of persons = 50 × 4 = 200. So, M = 150 men

Example 3 In the above question, if the schedule can go behind by 10 days, then how many extra men are required to complete the work?

Solution So, now we will have to complete the work in 60 days.

W	=	M	×	D	
50%		50		40	.....................(i)
Rest 50%		(50 + M)		20	(ii)

Since work is constant in both the cases, so the number of men and the number of days will be reciprocal to each other. As, the number of days left in (ii) is 1/2 of initial period (i), so the number of persons will become 2 times of the initial number of persons.

Hence, the number of persons = 50 × 2 = 100. So, M = 50 men

Example 4 In question 2, he realizes after 40 days that work is only 20% complete. How many extra men should be employed now so that the work is completed on time?

Solution

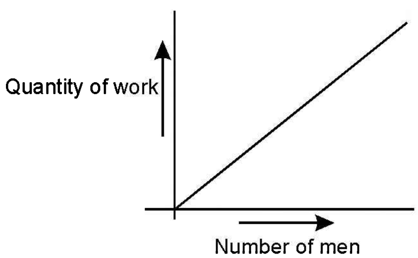
W =	M ×	D
20%	50	40 .... (i)
20%	200	10 .... (ii)
So, 80%	800	10 .... (iii)

Hence, 750 more persons are needed to complete the job on time.

Condition 2: D is constant

W α M

More work will be done if we employ more men and vice-versa. It means that multiplier of W and M will be same. It can be seen with the help of the graph given below



Condition 3: M is constant

W α D

In general, we can summarize that

$$\frac{M_1D_1}{W_1} = \frac{M_2D_2}{W_2}$$

Example 5 12 persons can cut 10 trees in 16 days. In how many days can 8 persons cut 12 trees?

Solution Here $W_1 = 10\ W_2 = 12$
$M_1 = 12\ M_2 = 8$
$D_1 = 16\ D_2 = ?$

Putting the values in the equation $W_1/W_2 = (M_1/M_2) \times (D_1/D_2)$

We get $\frac{10}{12} = \frac{12}{8} \times \frac{16}{D_2} \Rightarrow D_2 = 28.8$ days

INDIVIDUAL WORK AND INDIVIDUAL EFFICIENCY

Individual Work

If Amit can do a certain work in 10 days, then he will finish 1/10th of the work in one day.

Example 6 Amit can do some work in 12 days and Vinit can do the same work in 15 days. In how many days will both of them do the work when working together?

Solution Assume total work = 1 unit

Work done by Amit in one day = 1/12 unit

Work done by Vinit in one day = 1/15 unit

Work done by both of them in one day when working together = (1/12) + (1/15) = 9/60 unit

Hence, they will be doing the whole work in 60/9 days $= 6\frac{2}{3}$ days

LCM method of solving time and work questions

This can be understood in terms of the above written example in the following way:

Let us assume total work to be equal to the LCM of the days taken by Amit and Vinit (i.e., of 10 and 15)

Assume work = 60 units

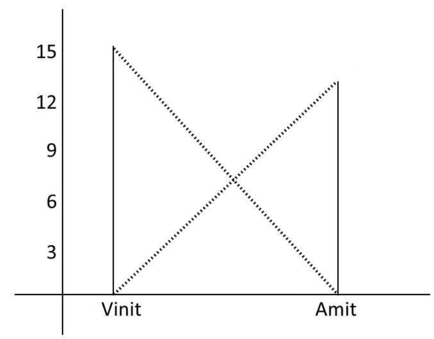
Work done by Amit in one day = 5 units

Work done by Vinit in one day = 4 units

Work done by both of them in one day when working together = 9 units

So, the number of days taken by both of them when working together = 60/9 = $6\frac{2}{3}$ days

Alternatively, we can do this problem with the help of a graph also.



We have created the pillars of the number of days of Amit and Vinit. Now the point of intersection of the top of the first pillar to the bottom of the second pillar and the top of second pillar and the bottom of the first pillar is the number of days taken by both of them when working together. In the above drawn graph, it can be clearly seen that the point of intersection of both the straight lines is a bit more than 6.

Despite graphical method appearing easier than the earlier two methods, the usage of this method should be avoided due to the complexity of denoting the points on the graph paper.

Example 7 A, B and C can do a piece of work individually in 8, 12 and 15 days respectively. A and B start working, but A quits after working for 2 days. After this, C joins B till the completion of work. In how many days will the work be complete?

Solution Let us assume that the Work = LCM (8, 12, 15) = 120 units

So, the work done by A in one day = 15 units

Work done by B in one day = 10 units

Work done by C in one day = 8 units

Work done by A and B in two days = 2 × 25 = 50 units
Remaining work = 70 units
Work done by C and B in one day = 18 units

Time taken to complete the remaining work by C and B = $\frac{70}{18} = 3\frac{16}{18}$ days

So, the total number of days = $5\frac{16}{18}$ days

Example 8 A and B together can do a work in 12 days, B and C together can do the same work in 10 days and A and C together can do the same work in 8 days. In how many days will the work be complete if A, B and C are working together?

Solution Let us assume work = LCM of (12, 10, 8) = 120 units

So, A and B are doing 10 units in one day, B and C are doing 12 units in a day and A and C are doing 15 units in a day.

Adding all these, 2 (A + B + C) are doing 37 units in a day.

⇒ (A + B + C) are doing $\frac{37}{2}$ = 18.5 units in a day

So, time taken to complete the work = $\frac{120}{18.5}$ days = 6.48 days

Individual Efficiency

Efficiency is also known as work-rate.

If A is taking less number of days with respect to B to complete the same work, we can say that the efficiency of A is more than the efficiency of B.

So, more the efficiency, less will be the number of days and less the efficiency, more will be the number of days to do a certain work. We have observed individual efficiency in case of percentage also (Product Stability ratio).

Now, assume A takes 20 days to complete a work and B takes 25 days to complete the same work. It means A is doing 5% (100%/20) work in one day and B is doing 4% (100%/25) work in a day. So, efficiency of A is 25% more than efficiency of B.

General expression correlating time taken and efficiency

If efficiency of A is x per cent more than the efficiency of B and B takes 'B' days to complete the work, then A will take $\left(\frac{B}{100 + x} \times 100\right)$ days to complete the same work.

If efficiency of A is x% less than the efficiency of B and B takes 'B' days to complete the work, then A will take $\left(\frac{B}{100 - x} \times 100\right)$ days to complete the same work.

So, if A is 20% more efficient than B and B takes 'B' days to complete the work, then A will take $\frac{B}{1.2}$ days to do the same work.

With this, it can also be observed that if work is constant then time taken is inversely proportional to efficiency.

Example 9 John is thrice as efficient as Abraham and hence completes a work in 60 days less than the number of days taken by Abraham. What will be the number of days taken by both of them when working together?

Solution Since John is thrice as efficient as Abraham, so the number of days taken by him will be 1/3rd the number of days taken by Abraham. If John is taking x days, then Abraham will take $3x$ days to complete the same work.

Now, $3x - x = 2x = 60$ days

So, $x = 30$ days and $3x = 90$ days

Let us assume that the total work = 90 units (LCM of 30 and 90)

So, the total work done by both of them in one day = 3 + 1 = 4 units of work.

So, the total number of days = $\frac{90}{4}$ days = 22.5 days

Example 10 A can do a work in 4 days. Efficiency of B is half the efficiency of A, efficiency of C is half the efficiency of B and efficiency of D is half the efficiency of C. After they have been grouped in two pairs it is found that the total number of days taken by one group is 2/3rd the time taken by the other group. Which of the following is a possible group? (CAT 2001)

(a) AB (b) BC (c) CD (d) AC

Solution Number of days taken by A = 4 days

Number of days taken by B = 8 days
Number of days taken by C = 16 days
Number of days taken by D = 32 days
Assume that the total work = 32 units
So, the work done by A = 8 units
Work done by B = 4 units
Work done by C = 2 units
Work done by D = 1 units

It can be observed that the work done by B and C together in one day is 2/3rd the work done by A and D. So the groups are—AD and BC.

Example 11 In a nuts and bolts factory, one machine produces only nuts at the rate of 100 nuts per minute and needs to be cleaned for 5 min after production of every 1,000 nuts. Another machine produces only bolts at the rate of 75 bolts per minute and needs to be cleaned for 10 min after production of every 1,500 bolts. If both the machines start their production at the same time, what is the minimum duration required for producing 9,000 pairs of nuts and bolts?

(a) 130 min (b) 135 min (c) 170 min (d) 180 min

Solution

Machine I

Number of nuts produced in one min = 100
Time required to produce 1,000 nuts = 10 min
Cleaning time for nuts = 5 min
Overall time to produce 1,000 nuts = 15 min.

Over all time to produce 9,000 = 138 min – 5 min = 133 min … (1)

Machine II

Time required to produce 75 bolts = 1 min
Time required to produce 1,500 bolts = 20 min
Cleaning time for bolts = 10 in
Effective time to produce 1,500 bolts = 30 min
Effective time to produce 9,000 bolts = 30 × 6 – 10 = 170 min … (2)
From (1) and (2)
Minimum time = 170 min

Example 12 A, B and C are assigned a piece of work which they can complete by working together in 15 days. Their efficiencies (measured in terms of rate of doing work) are in the ratio of 1:2:3. After $\frac{1}{3}$rd of the work is completed, one of them has to be withdrawn due to budget constraint. Their wages per day are in the ratio of 3:5:6. The number of days in which the remaining two persons can complete the work (at optimal cost) is

(a) 18 (b) 20 (c) 15 (d) 12

Solution A, B and C together in 15 days

= A alone in 90 days, B alone in 45 days, C alone in 30 days

Wages per day per unit work for A, B and C are $\frac{3}{1}:\frac{5}{2}:\frac{6}{3}$

Hence, A is the least efficient and hence, must be done away with.

For B and C, the whole work can be finished in 18 days and hence, remaining 2/3rd of the work can be finished in 12 days only.

COLLECTIVE WORK AND COLLECTIVE EFFICIENCY

When people of different efficiencies start working together, the method of time-work equivalence to find the time or the amount of work done cannot be used. In those cases, we will be required to relate time with the efficiency of a group.

For example, if 5 men and 8 women can do a piece of work in 10 days, from this information we can not find out that in how many days a man or a woman can do the work individually, since we are not aware of their individual efficiencies. However, if we get a similar equation like x men and y women can do the same work in ‘p’ days, then we can correlate these two equations to find the number of days taken by one man and one woman to do the work.

Example 13 10 men and 9 women can do a piece of work in 20 days. The same work can be done by 6 men and 12 women in 30 days. In how many days can the same work be done by 1 man and 1 woman?

Solution 10 men and 9 women can do a work in 20 days ⇒ 20 × (10 men and 9 women) can do the same work in 1 day.

Similarly, 6 men and 9 women can do the same work in 30 days ⇒ 30 × (6 men and 12 women) can do the same work in 1 day.

So, efficiency of 20 × (10 men and 9 women) = efficiency of 30 × (6 men and 12 women)
Or, 200 men + 180 women = 180 men + 360 women
Or, 20 men = 180 women ⇒ 1 man = 9 women
So, the total work = 20 × (10 men and 9 women) = 20 × (90 + 9) women = 99 × 20 = 1980 units
Total number of persons employed =
1 man + 1 women = 10 women

So, the time taken = $\frac{1980}{10}$ = 198 days

EXTENSION OF THE CONCEPT OF TIME AND WORK

1. Pipes and cisterns

Pipes and cisterns is just another application of the concept of time and work. While we see only +ve work being done in normal cases of time and work, in case of pipes and cisterns, –ve work is also possible.

Given that pipes A and B can fill a tank in 20 min and 25 min working individually ⇒ this statement is similar to “A can do a work in 20 min and B can do the same work in 25 min.”

Again, given that pipe C can empty a tank in 40 min we can say this statement is similar to “C can demolish a wall in 40 min (assuming that the work is building or demolishing the wall)

Let us understand this with the help of an example.

Example 14 A and B are two taps which can fill a tank individually in 10 min and 20 min respectively. However, there is a leakage at the bottom, which can empty a filled tank in 40 min. If the tank is empty initially, how much time will both the taps take to fill the tank (leakage is still there)?

Solution Let us assume the units of work = LCM of (10, 20, 40) = 40 units

Work done by Tap A/min = 4 units/min (Positive work)
Work done by Tap B/min = 2 units/min (Positive work)
Work done by leakage/min = 1 unit/min (Negative work)
Net work done/min = 5 units/min
Hence time taken = 8 min

Example 15 Pipe A can fill a tank in 3 hour. But there is a leakage also, due to which it takes 3.5 hour for the tank to be filled. How much time will the leakage take in emptying the tank if the tank is filled initially?

Solution Assume the total units of work = 10.5 units

Work done by Tap A/h = 3.5 units/h (Positive work)
Work done by leakage/h = 3 units/h (Negative work)
Net work done/h = 0.5 units/h

So, the time taken = $\frac{10.5}{0.5}$ = 21 hour

Alternatively, due to the leakage, the pipe is required to work for an extra half an hour. So, the quantity filled by pipe in half an hour is being emptied by the leakage in 3.5 hour. Hence, the quantity filled by pipe in 3 hour will be emptied by the leakage in 21 hour.

2. Variable work

The concept of variable work comes from the possibility

⇒ that the rate of working can be different or

⇒ can be dependent upon some external agent.

In these cases, the rate of work will be proportional to some external factor.

Understand this with the help of a simple statement: The rate of the flow of water from a pipe is directly proportional to the area of the cross section of the pipe.

Example 16 There are three inlet taps whose diameters are 1 cm, 2 cm and 3 cm respectively. The rate of flow of the water is directly proportional to the square of the diameter. It takes 9 min for the smallest pipe to fill an empty tank. Find the time taken to fill an empty tank when all the three taps are opened.

Solution The rate of flow of a diameter^2, or, rate of flow = K × diameter^2 (where K is a constant)

For 1st tap, rate of flow = $K \times 1$

For 2nd tap, rate of flow = $K \times 4$

For 3rd tap, rate of flow = $K \times 9$

We know, the quantity filled will be equal to the product of the rate of flow and time.

So, the quantity filled by the smallest pipe = $K \times 1 \times 9 = 9K$ = Capacity of tank

Quantity of water filled by all the taps together in 1 min = 9K + 4K + 1K = 14K

Assume that all the taps working together take 't' min.

So, $14\,K \times t = 9\,K$

So, the time taken $t = \frac{9K}{14K} = \frac{9}{14}$ min

3. Alternate work

The concept of alternate work is analogous to the concept of man-days. As we have seen in the concept of man-hour that if 20 men can do a work in 10 days, then this work is equivalent to 200 man-days. However, in the case of alternate work, two or more than two people of different efficiencies work alternately or in some particular pattern.

Example 17 Navneet can build a wall in 30 days and Rakesh can demolish the same wall in 40 days. If they work on alternate days with Navneet starting the job on the 1st days, then in how many days will the wall be built for the first time?

Solution Let us assume the total units of work = 120 units

So, the wall built by Navneet in one day = 4 units

And wall demolished by Rakesh in one day = 3 units

So, effectively in two days, total wall built = 1 unit

Now, they work on alternate days, so days taken to built 116 units = 116 days

On 117th day Navneet will add another 4 units and so completing the construction of wall in 117 days.

(This problem can be understood well with another very traditional problem—A frog climbs up a pole 4 inches in 1 hour and slips 3 inches next hour. If height of the pole is 120 inches, then what is the time taken by the frog to reach the top of the pole?)