Wave Optics

NATURE OF LIGHT

Newton's corpuscular theory of light

This theory was given by Newton.

- **Characteristics of the theory**
  - (i) Extremely minute, very light and elastic particles are being constantly emitted by all luminous bodies (light sources} in all directions which are known as corpuscles.
  - (ii) These corpuscles travel with the speed of light.
  - (iii) When these corpuscles strike the retina of our eye then they produce the sensation of vision.
  - (iv) The velocity of these corpuscles in vacuum is 3 × 108 m/s.
  - (v) The different colours of light are due to different size of these corpuscles.
  - (vi) The rest mass of these corpuscles is zero.
  - (vii) The velocity of these corpuscles in an isotropic medium is same in all directions but it changes with the change of medium.
  - (viii) These corpuscles travel in straight lines.
  - (ix) These corpuscles are invisible.

- **The phenomena explained by this theory**
  - (i) Reflection and refraction of light.
  - (ii) Rectilinear propagation of light.
  - (iii) Existence of energy in light.

- **The phenomena not explained by this theory**
  - (i) Interference, diffraction, polarisation, double refraction and total internal reflection.
  - (ii) Velocity of light being greater in rarer medium than that in a denser medium.
  - (iii) Photoelectric effect and Compton effect.

Huygen's Wave theory of light

This theory was enunciated by Huygen in a hypothetical medium known as luminiferrous ether.

Ether is that imaginary medium ·which prevail~ in all space and is isotropic, perfectly elastic and massless.

The velocity of light in a medium is constant but changes with change of medium.

This theory is valid for all types of waves.

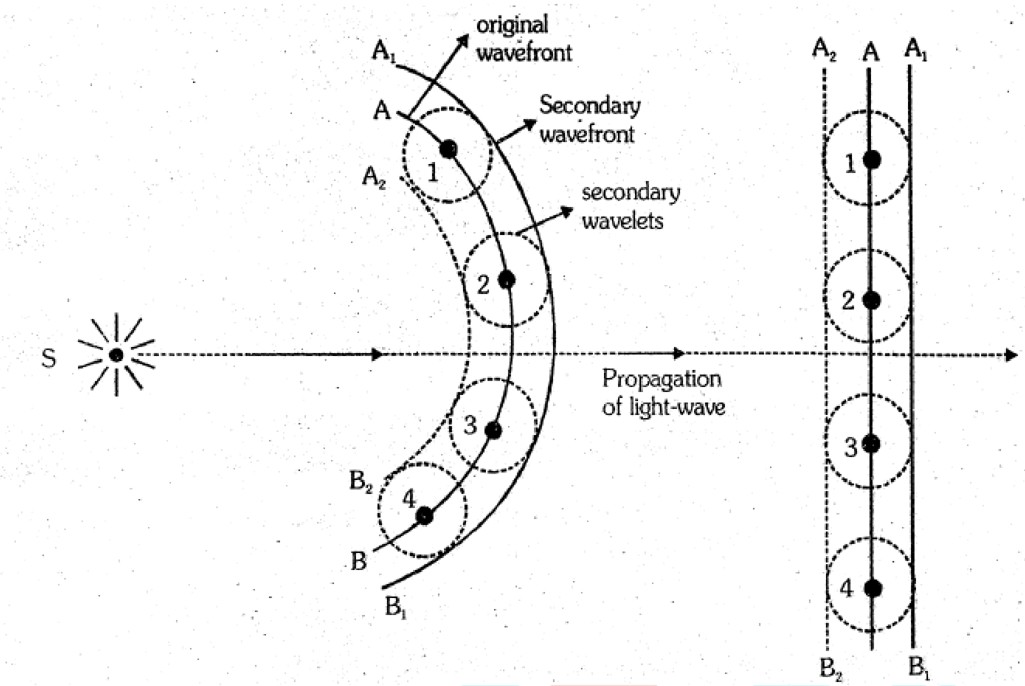
(i) The locus of all ether particles vibrating in same phase is known as wavefront.

(ii) Light travels in the medium in the form of wavefront.

(iii) When light travels in a medium then the particles of medium start vibrating and consequently a disturbance is created in the medium.·

(iv) Every point on the wavefront becomes the source of secondary wavelets. It emits secondary wavelets in all directions which travel with the speed of light.

(v) The tangent plane to these secondary wavelets represents the new position of wave front.



The phenomena explained by this theory

(i) Reflection, refraction, interference, diffraction.

(ii) Rectilinear propagation of light

(iii) Velocity of light in rarer medium being greater than that in denser medium.

Phenomena not explained by this theory

(i) Photoelectric effect and Raman effect.

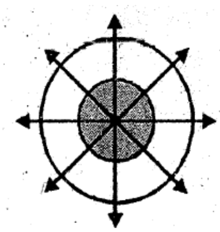
WAVEFRONT, VARIOUS TYPES OF WAVEFRONT

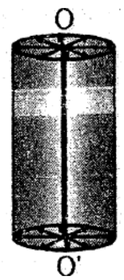
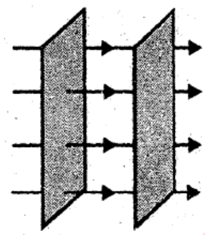
- **Wavefront**

  The locus of all the particles vibrating in the same phase in known as wavefront.

- **Types of wavefront**

  The shape of wavefront depends upon the shape of the light source from, the wavefront originates. On this basis there are three types of wavefronts

S.No.	Wavefront	Shape of light source	Diagram of shape of wavefront	Variation of amplitude (A) with distance	Variation of Intensity (I) with distance
1.	Spherical	Point source		$A \propto \frac{1}{d}$ or $A \propto \frac{1}{r}$	$I \propto \frac{1}{r^2}$

2.	Cylindrical	Linear source or slit		$A \propto \frac{1}{\sqrt{d}}$ or $A \propto \frac{1}{\sqrt{r}}$	$I \propto \frac{1}{r}$
3.	Plane	Extended large source or point source situated at very large distance		A = constant	I = constant

CHARACTERISTIC OF WAVEFRONT

(a) The phase difference between various particles on the wavefront is zero.
(b) These wavefronts travel with the speed of light in all directions in an isotropic medium.
(c) A point source of light always gives rise to a spherical wavefront in an isotropic medium.
(d) In anisotropic medium it travels with different velocities in different directions.
(e) Normal to the wavefront represents a ray of light.
(f) It always travels in the forward direction in the medium.

INTERFERENCE OF LIGHT

When two light waves of same frequency with constant phase difference superimpose over each other, then the resultant amplitude (or intensity) in the region of superimposition is different from the amplitude (or intensity) of individual waves.
This modification in intensity in the region of superposition is called interference.

(a) Constructive interference

When resultant intensity is greater than the sum of two individual wave intensities $[I > (I_1 + I_2)]$, then the interference is said to be constructive.

(b) Destructive interference

When the resultant intensity is less than the sum of two individual wave intensities $[I < (I_1 + I_2)]$, then the interference is said to destructive.
There is no violation of the law of conservation of energy in interference. Here, the energy from the points of minimum energy is shifted to the points of maximum energy.

TYPES OF SOURCES

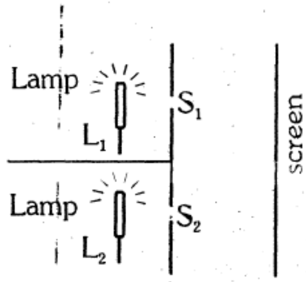
- **Coherent sources**

  Two sources are said to be coherent if they emit light waves of the same frequency and start with same phase or have a constant phase difference. They are obtained from a single source.

  Note : Laser is a source of monochromatic light waves of high degree of coherence.

- **Incoherent sources**

  Two independent monochromatic sources, emit waves of same frequency. But the waves are not in phase. So they are incoherent. This is because, atoms cannot emit light waves in same phase and these sources are said to be incoherent sources. By using two independent laser beams it has been possible to record the interference pattern.



Independent Source
(non-coherent source)

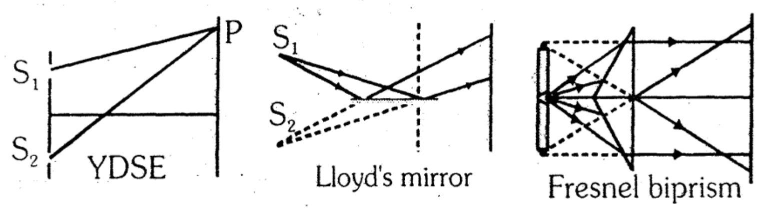
METHODS OF OBTAINING COHERENT SOURCE

- **Division of wave front**

  In this method, the wavefront is divided into two or more parts by reflection or refraction using mirrors, lenses or prisms.

Illustration :

Young's double slit experiment. Fresnel's Biprism and Lloyd's single mirror method.

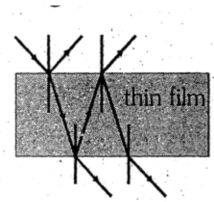


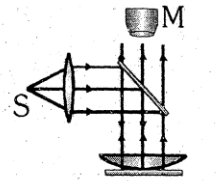
- **Division of amplitude**

  The amplitude of incoming beam is divided into two or more parts by partial reflection or refraction. These divided parts travel different paths and are finally brought together to produce interference.

Illustration :

The brilliant colour seen in a thin film of transparent material like soap film, oil film, Michelson's Interferro Meter, Newtons' ring etc.





Newton's rings

Condition for sustained interference

To obtain the stationary interference pattern, the following conditions must be fulfilled:

(a) The two sources should be coherent, i.e., they should vibrate in the same phase or there should be a constant phase difference between them. .

(b) The two sources must emit continuous waves of same wavelength and frequency.

(c) The separation between two coherent sources should be small.

(d) The distance of the screen from the two sources should be large.

(e) For good contrast between maxima and minima; the amplitude of two interfering waves should be as nearly equal as possible and the background should be dark.

(f) For a large number of fringes in the field of View, the sources should be narrow and monochromatic.

ANALYSIS OF INTERFERENCE OF LIGHT

When two light waves having same frequency and equal or nearly equal amplitude are moving in the same direction superimpose then different points have different light intensities. At some point the intensity of light is maximum and at some paint it is minimum this phenomenon is known as interference of light.

Let two waves having amplitude a_1 and a_2 and same frequency, and constant phase difference ϕ superpose. Let their displacement are:

$y_1 = a_1 \sin \omega t$ and $y_2 = a_2 \sin(\omega t + \phi)$.

$y = y_1 + y_2 = A \sin(\omega t + \theta)$.

Where A = Amplitude of resultant wave

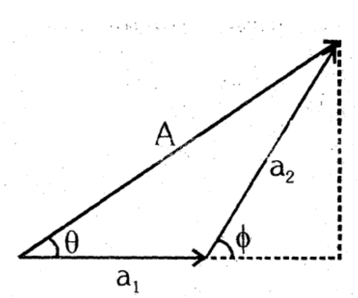
ϕ = New initial phase angle

Phasor diagram

By right angle triangle :

$$A^2 = (a_1 + a^2 \cos \phi)^2 + (a^2 \sin \phi)^2$$

Resultant amplitude $A' = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos\phi}$



Phase angle $\phi = \tan^{-1}\left(\dfrac{a_2 \sin\phi}{a_1 + a_2 \cos\phi}\right)$

Intensity $\propto (\text{Amplitude})^2 \Rightarrow \propto A^2 \Rightarrow I = KA^2$

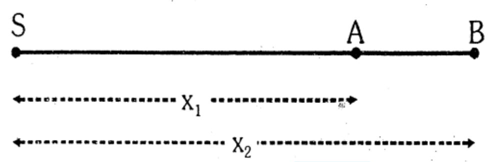
So, $I_1 = Ka_1^2$ and $I_2 = Ka_2^2$

$\therefore \quad I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi$

Here, $2\sqrt{I_1}\sqrt{I_2}\cos\phi$ is known as interference factor

If the distance of a source from two points A and B is x_1 and x_2 then Path difference $\delta = x_2 - x_1$

Phase difference $\phi = \frac{2\pi}{\lambda}(x_2 - x_1)$



$\Rightarrow \phi = \frac{2\pi}{\lambda}\delta$

Time difference $\Delta t = \frac{\phi}{2\pi} t$

$$\frac{\text{Phase difference}}{2\pi} = \frac{\text{Path difference}}{\lambda} = \frac{\text{Time difference}}{T} \Rightarrow \frac{\phi}{2\pi} = \frac{\delta}{\lambda} = \frac{\Delta t}{T}$$

TYPES OF INTERFERENCE

Constructive Interference

When both waves are in same phase then phase difference is an even multiple of $\pi \Rightarrow \phi = 2n\pi$; $n = 0, 1, 2, \ldots\ldots\ldots$

- Path difference is an even multiple of $\frac{\lambda}{2}$

Q $\quad \frac{\phi}{2\pi} = \frac{\delta}{\lambda}$

$\Rightarrow \quad \frac{2n\pi}{2\pi} = \frac{\delta}{\lambda} \quad \Rightarrow \quad 2n\left(\frac{\lambda}{2}\right)$

$\Rightarrow \quad \delta = n\lambda$ (where m= 0, 1, 2,…..

- When time difference is an even multiple of $\frac{T}{2}$ $\quad \therefore \quad \Delta t = 2n\left(\frac{T}{2}\right)$
- In this condition the resultant amplitude and intensity will be maximum.

$$A_{max} = (a_1 + a_2) \Rightarrow \quad I_{max} = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2} = (\sqrt{I_1} + \sqrt{I_2})^2$$

Destructive Interference

When both the waves are in opposite phase. So phase difference is an odd multiple of π.

$$\phi = (2n - 1)\pi \; ; \; n = 1, 2, \ldots..$$

- When path difference is an odd multiple of $\frac{\lambda}{2}$, $\delta = (2n-1)\frac{\lambda}{2}$, $n = 1, 2, \ldots\ldots.$
- When time difference is an odd multiple of $\frac{T}{2}$, $\Delta t = (2n-1)\frac{T}{2}$, $(n = 1, 2, \ldots\ldots)$

In this condition the resultant amplitude and intensity of wave be minimum

$$A_{min} = (a_1 - a_2) \qquad \Rightarrow \quad I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$$

GOLDEN KEY POINTS

- Interference follows law of conservation of energy.
- Average Intensity $I_{av} = \frac{I_{max} + I_{min}}{2} = I_1 + I_2 = a_1^2 + a_2^2$
- Intensity $\propto$ width of slit $\propto$ (amplitude)$^2 \Rightarrow I \propto w \propto a^2 \Rightarrow \frac{I_1}{I_2} = \frac{w_1}{w_2} = \frac{a_1^2}{a_2^2}$
- $\frac{I_{max}}{I_{min}} = \left[\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right]^2 = \left[\frac{a_1 + a_2}{a_1 - a_2}\right]^2 = \left[\frac{a_{max}}{a_{min}}\right]^2$
- Fringe visibility $V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \times 100\%$ when $I_{min} = 0$ then fringe visibility is maximum

  i.e. when both slits are of equal width the fringe visibility is the best and equal to 100%

ILLUSTRATIONS

Illustrations 1

If two waves represented by y_1 = 4 sin ωt and y_2 = 3sin $\left(\omega t + \frac{\pi}{3}\right)$ interfere at a point. Find out the amplitude of the resulting wave.

Solution

Resultant amplitude $A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi} = \sqrt{(4)^2 + (3)^2 + 2.(4)(3)\cos\frac{\pi}{3}} \Rightarrow A ; 6$

Illustration 2

Two beams of light having intensities I and 4I interferer to produce a fringe pattern on a screen. The phase difference between the beam is $\frac{\pi}{2}$ at point A and 2π at point B. Then find out the difference between the resultant intensities at A and B.

Solution

Resultant intensity $I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi$

Resultant intensity at point A is $I_A = I + 4I + 2\sqrt{I}\sqrt{4I}\cos\frac{\pi}{2} = 5I$

Resultant intensity at point B is $I_B = I + 4I + 2\sqrt{I}\sqrt{4I}\cos 2\pi = 9I$ (Q $\cos 2\pi = 9$)

$\therefore \quad I_B - I_A = 9I - 5I = 4I$

Illustration 3

In an interference pattern, the slit widths are in the ratio 1 : 9. Then find out the ratio of minimum and maximum intensity.

Solution

Slit width ratio

$$\frac{w_1}{w_2} = \frac{1}{9}$$

$$\because \quad \frac{I_1}{I_2} = \frac{w_1}{w_2} = \frac{a_1^2}{a_2^2} = \frac{1}{9} \quad \Rightarrow \quad \frac{a_1}{a_2} = \frac{1}{3}$$

$$\therefore \quad \frac{I_{min}}{I_{max}} = \frac{(a_1 - a_2)^2}{(a_1 + a_2)^2} = \frac{(a_1 - 3a_1)^2}{(a_1 + a_1)^2} = \frac{4}{16} = 1:4$$

Illustration 4

The intensity variation in the interference pattern obtained with the help of two coherent sources is 5% of the average intensity. Find out the ratio of intensities of two sources.

Solution

Let I_{avg} = 100 units

$$\frac{I_{max}}{I_{min}} = \frac{105}{95} = \frac{21}{19}$$

$$\Rightarrow \quad \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{21}{19} \quad \Rightarrow \quad \frac{a_1 + a_2}{a_1 - a_2} = \sqrt{\frac{21}{19}} = 1.05 \quad \Rightarrow \quad a_1 + a_2 = 1.05\, a_1 - 1.05 a_2$$

$$\Rightarrow \quad 0.05 a_1 = 2.05\, a_2 \quad \Rightarrow \quad \frac{a_1}{a_2} = \frac{2.05}{0.05} = \frac{41}{1} \quad \therefore \quad \frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = \frac{1680}{1}$$

Illustration 5

Can two different bulbs, similar in all respect act as coherent sources? Give reasons for your answer.

Solution

No, because the light waves emitted by two independent bulbs will not have stable constant phase difference

<u>BEGINNER'S BOX-1</u>

1. Consider interference between two sources of intensities I and 41. Obtain intensity at a point where phase difference is $\frac{\pi}{2}$

2. Two coherent sources whose intensity ratio is 81 : 1 produce interference fringes. Calculate the ratio of intensity of maxima and minima in the fringe system.

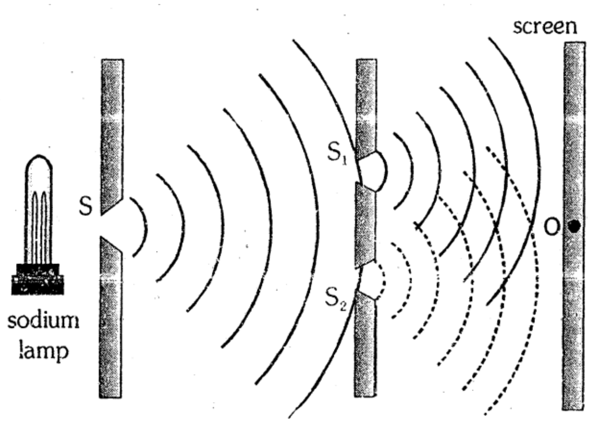
3. Consider interference between two sources of intensity I and 41. Find the resultant intensity. where phase difference is

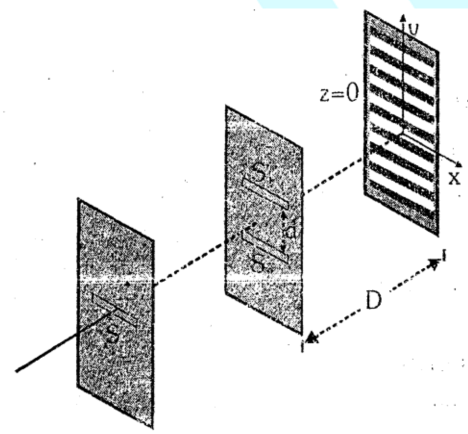
(a) $\frac{\pi}{4}$ (2) π (3) 4π

YOUNG'S DOUBLE SLIT EXPERIMENT (YDSE)

According to Huygen, light is a wave. It is proved experimentally by YDSE.

S is a narrow slit illuminated by a monochromatic source of light sends wave fronts in all directions. Slits S_1 and S_2 become the source of secondary wavelets which are in phase and of same frequency. These waves are superimposed on each other which give rise to interference. Alternate dark and bright bands are obtained on a screen (called interference fringes) placed certain distance from the plane of slit S_1 and S_2 . Central fringe is always bright (due to path length S_1O and S_2O to centre of screen are equal) and is called central maxima.





- In YDSE division of wavefront takes place.
- If one of the two slit is closed, the interference pattern disappears. It shows that two coherent sources are required to produce interference pattern.
- If white light is used as parent source, then the fringes will be coloured and of unequal width.
  - (i) Central fringe will be white.
  - (ii) The fringe closest on either side of the central white fringe is red and the farthest will appear blue. After a few fringes, no clear fringe pattern is seen.

CONDITION FOR BRIGHT AND DARK FRINGES

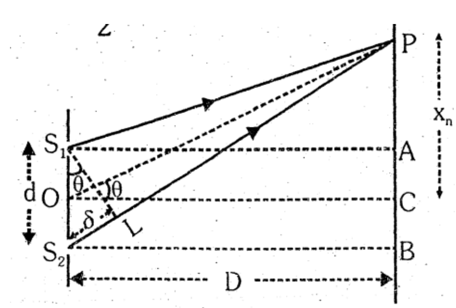
Bright Fringe

D = distance between slit and screen, d = distance between slit S_1 and S_2

Bright fringe occurs due to constructive interference.

Q For constructive interference path difference should be even multiple of $\frac{\lambda}{2}$

$\therefore$ Path difference $\delta = PS_2 - PS_1 = S_2L = (2n)\frac{\lambda}{2}$



In ΔPCO $\tan\theta = \frac{x_n}{D}$; In ΔS_1S_2L $\sin\theta = \frac{\delta}{d}$

$\delta = n\lambda$ for bright fringes

If θ is small then $\tan\theta$; $\sin\theta$ $\Rightarrow$ $\frac{x_n}{D} = \frac{\delta}{d}$

The distance of n^{th} bright fringe from the central bright fringe $x_n = n\dfrac{D\lambda}{d}$

Dark Fringe

Dark fringe occurs due to destructive interference.

Q For destructive interference path difference should be odd multiple of $\dfrac{\lambda}{2}$.

$\therefore$ Path difference $\delta = (2m-1)\dfrac{\lambda}{2}$

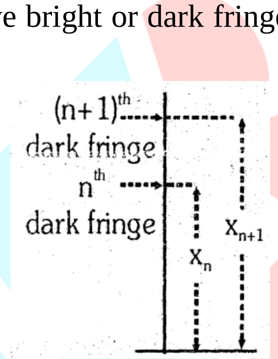
The distance of the m^{th} dark fringe from the central bright fringe $x_m = \dfrac{(2m-1)D\lambda}{2d}$

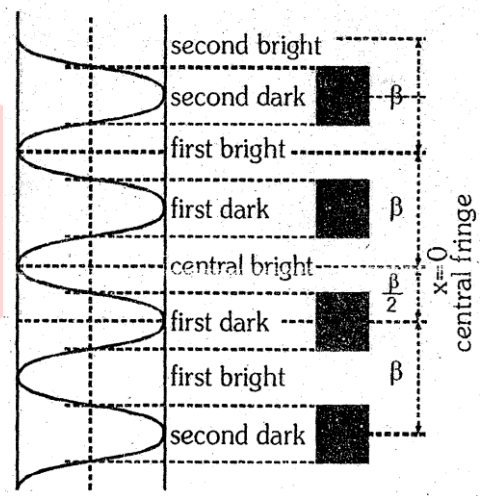
FRINGE WIDTH

The distance between two successive bright or dark fringe is known as fringe width.

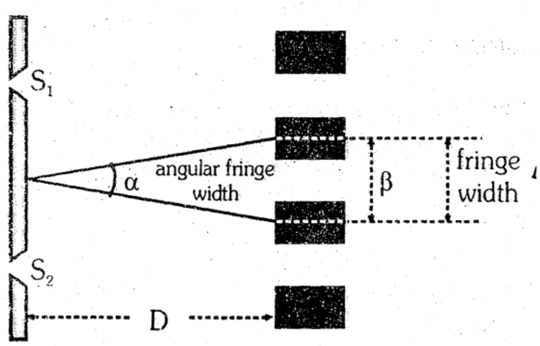
$$\beta = x_{n+1} - x_n = \frac{(n+1)D\lambda}{d} - \frac{nD\lambda}{d}$$

Fringe Width $\beta = \dfrac{D\lambda}{d}$





ANGULAR FRINGE WIDTH



Angular Fringe width, $\alpha = \dfrac{\beta}{D}$, $\alpha = \dfrac{\lambda}{d}\ \left[\because \dfrac{\beta}{D} = \dfrac{\lambda}{d}\right]$

- The distance of nth bright fringe from the central bright fringe $x_n = \frac{n\lambda D}{d} = n\beta$
- The distance between n_1 and n_2 bright fringe $x_{n_2} - x_{n_1} = n_2\frac{\lambda D}{d} - n_1\frac{\lambda D}{d} = (n_2 - n_1)\beta$
- The distance between of m^{th} dark fringe from central fringe $x_m = \frac{(2m-1)D\lambda}{2d} = \frac{(2m-1)\beta}{2}$
- The distance of nth bright fringe from m^{th} dark fringe

$$x_n - x_m = n\frac{D\lambda}{d} - \frac{(2m-1)D\lambda}{2d} = n\beta - \frac{(2m-1)\beta}{2}$$

$$x_n - x_m = \left[n - \frac{(2m-1)}{2}\right]\beta$$

GOLDEN KEY POINTS

- If the whole apparatus is immersed in a liquid of refractive index μ, then wavelength of light $\lambda' = \frac{\lambda}{\mu}$ since $\mu = 1$ so $\lambda' < \lambda \Rightarrow$ wavelength will decrease. Hence fringe width $(\beta \propto \lambda)$ will decrease $\Rightarrow$ fringe width in liquid $\beta' = \frac{\beta}{\mu}$. Angular width will also decrease.
- With increase in distance between slit and screen D, angular width of maxima does not change, fringe width β increases linearly with D but the intensity of fringes decreases.
- If an additional phase difference of π is created in one of the wave then the central fringe becomes dark.
- When wavelength λ_1 is used to obtain a fringe n_1. At the same point wavelength λ_2 is required to obtain a fringe n_2 then $n_1\lambda_1 = n_2\lambda_2$
- When waves from two coherent sources S_1 and S_2 interfere in space the shape of the fringe is hyperbolic with foci at S_1 and S_2.

ILLUSTRATIONS

Illustration 6

Laser light of wavelength 630 nm incident on a pair of slits produces an interference pattern in which the fringes are separated by 8.1 mm. A second laser light produces an interference pattern in which the fringes are separated by 7.2 mm. Calculate the wavelength of the second light.

Solution

Fringe separation is given by $\beta = \frac{\lambda D}{d}$ i.e. $\frac{\beta_2}{\beta_1} = \frac{\lambda_2}{\lambda_1} \Rightarrow \lambda_2 = \frac{\beta_2}{\beta_1} \times \lambda_1 = \frac{7.2}{8.1} \times 630 = 560$ nm

Illustration 7

A double slit is illuminated by light of wavelength 6000 Å. The slits are 0.1 cm apart and the screen is placed one metre away. Calculate

(i) The angular position of the 10^{th} maximum is radians and

(ii) separation between the two adjacent minima

Solution

(i) λ = 6000 Å = 6×10^{-7} m, d = 0.1 cm = 1×10^{-3} m, D = 1m , n = 10

Angular position $\theta_n = \frac{n\lambda}{d} = \frac{10 \times 6 \times 10^{-7}}{10^{-3}} = 6 \times 10^{-3}$ rad

(ii) Separation between two adjacent minima = fringe width β

$$\beta = \frac{\lambda D}{d} = \frac{6 \times 10^{-7} \times 1}{1 \times 10^{-3}} = 6 \times 10^{-4}\,\text{m} = 0.6\,\text{mm}$$

Illustration 8

In Young's double slit experiment the fringes are formed at a distance of 1 m from double slits of separation 0.12 mm. Calculate

(i) The distance of 3rd dark band from the centre of the screen.

(ii) the distance of 3rd bright band from the centre of the screen, given λ = 6000 Å

Solution

(i) From m^{th} dark fringe $x_m = (2m-1)\frac{D\lambda}{2d}$,

Given D = 1 m = 100 cm , d = 0.12 mm = 0.012 cm

$$x_m = \frac{(2 \times 3 - 1) \times 100 \times 6 \times 10^{-5}}{2 \times 0.012} = 1.25\,\text{cm} \qquad [\because\ m = 3 \text{ and } \lambda = 6 \times 10^{-5}\text{ cm}]$$

(ii) For nth bright fringe $x_n = \frac{nD\lambda}{d} \Rightarrow x_n = \frac{3 \times 100 \times 6 \times 10^{-5}}{0.012} = 1.5\,\text{cm} \qquad [\because\ n = 3]$

Illustration 9

In Young's double slit experiment the two slits are illuminated by light of wavelength 5890Å and the distance between the fringes obtained on the screen is 0.2°. The whole apparatus in immersed in water, then find out angular fringe widh, (refractive index of water = $\frac{4}{3}$)

Solution

$$\alpha_{air} = \frac{\lambda}{d} \Rightarrow \alpha_{air} = 0.2° \Rightarrow \alpha \propto \lambda \Rightarrow \frac{\alpha_w}{\alpha_{air}} = \frac{\lambda_w}{\lambda_{air}} \Rightarrow \lambda_w = \frac{\lambda_{air}}{\mu} \Rightarrow \alpha_w = \frac{\alpha_{air}\lambda}{\mu\lambda} \Rightarrow \frac{0.2 \times 3}{4} = 0.15°$$

Illustration 10

The path difference between two interfering waves at a point on screen is 171.5 times the wavelength. If the path difference is 0.21029 cm. Find the wavelength.

Solution

Path difference = 1715 $\lambda = \frac{343}{2}\lambda$ = odd multiple of half wavelength. It means dark finge is observed.

According to question $0.01029 = \frac{343}{2}\lambda \Rightarrow \lambda = \frac{0.01029 \times 2}{343} = 6 \times 10^{-5} \text{cm} \Rightarrow \lambda = 6000 \text{ Å}$

Illustration 11

In young's double slit interference experiment, the distance between two sources is 0.1 /π mm. The distance of the screen from the source is 25 cm. Wavelength of light used is 5000Å. Then what is the angular position of the first dark fringe?

Solution

The angular position $\theta = \frac{\beta}{D} = \frac{\lambda}{d} (\because \beta = \frac{\lambda D}{d})$ The first dark fringe will be at half the fringe width from the midpoint of central maximum. Thus the angular position of first dark fringe will be

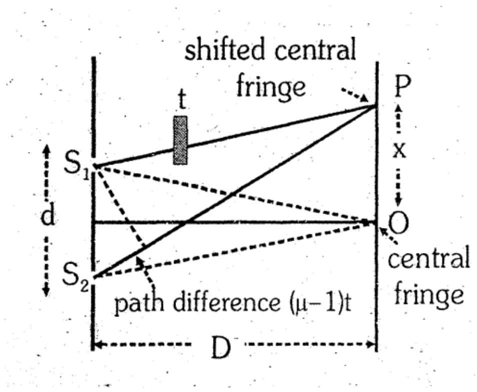
$$\alpha = \frac{\theta}{2} = \frac{1}{2}\left[\frac{\lambda}{d}\right] = \frac{1}{2}\left[\frac{5000 \times \pi}{1 \times 10^{-3}} \times 10^{-10}\right]\frac{180}{\pi} = 0.45°$$

BEGINNER'S BOX-2

1. In Young's the slits are 2mm apart and are illuminated with a mixture of two wavelength, λ = 7500 Å and λ = 9000 Å. At what minimum distance from the common central bright fringe on screen 2 m. from the slits will a bright fringe from one interference pattern coincide with a bright fringe from the other ?
2. In a Young's double slit experiment the angular width of fringe formed on a distant screen is 0.1 radian. Find the distance between the two slits if wavelength of light used is 6000 Å.
3. Two slits' in Young experiment are 0.02 cm apart. The interference fringes for light of wavelength 6000Å are form on a screen 80 cm away. Calculate the distance of the fifth bright fringe.
4. In Young's double slit experiment, two slits are separated by 3 mm distance and illuminated by light of wavelength 480 nm. The screen is at 2m from the plane of the slits. Calculate the separation between the 8^{th} bright fringe and the 3^{rd} dark fringe obtained with respect to central Bright fringe.
5. In a double slit experiment with monochromatic light, fringes are obtained on a screen placed at some distance moved by 5×10^{-2} m towards the slits, the change in fringe width is 3×10^{-5} m. If the distance between slits is 10^{-3} m. Calculate the wavelength of light used.
6. In young's experiment for interference of light the slits 0.2 cm apart are illuminated by yellow light (λ = 5896 Å). What would be the fringe width· on a screen placed 1m from the plane of slits. What will be the fringe width if the system is immersed in water.
7. The distance between the coherent source is 0.3 mm and the screen is 90 cm from the sources. The second dark band is 0.3 cm away from central bright fringe. Find the wavelength and the distance of the fourth bright fourth bright fringe from central bright fringe.
8. State two conditions to obtain sustained interference of light. In Young's double slit experiment, using light of wavelength 400 nm, interference fringes of width 'X' are obtained. The wavelength of light is increased to 600 nm and the separation between the slits is halved. If one wants the observed fringe width on the screen to be the same in the two cases, find the ratio of the distance between the screen and the plane of the slits in the two arrangements.
9. In a Young's double slit experiment, light has a frequency of 6×10^{14} Hz. The distance between the centres of adjacent bright fringes is 0.75 mm. If the screen is 1.5 m away then find the distance between the slits.
10. In a Young's experiment, the width of the fringes obtained with light of wavelength 6000 Å is 2.0 mm. What will the fringe width, if the entire apparatus is immersed in a liquid of refractive index 1.33 ?

EFFECT OF THIN FILMS

When a glass plate of thickness t and refractive index μ is placed in front of one of the slits in YDSE then the central fringe shifts towards that side in which glass plate is placed because extra path difference is introduced by the glass plate. In the path S_1P distance travelled by wave in air (S_1P – t)



Distance travelled by wave in the sheet = t

Time taken by light to reach up to point P will be same from S_1 and S_2.

$$\frac{S_2P}{c} = \frac{S_1P - t}{c} + \frac{t}{c/\mu} \Rightarrow \frac{S_2P}{c} = \frac{S_1P + (\mu - 1)t}{c} \Rightarrow S_2P = S_1P + (\mu - 1)t \Rightarrow S_2P - S_1P\ (\mu - 1)t$$

Path difference = $(\mu - 1)t \Rightarrow$ Phase difference $\phi = \frac{2\pi}{\lambda}(\mu - 1)t$;

Number of fringes displaced = $\frac{(\mu - 1)t}{\lambda}$

Distance of shifted fringes from central fringe $x = \frac{D(\mu - 1)t}{d}$ $\left[\because \frac{xd}{D} = (\mu - 1)t\right]$

$\therefore \quad x = \frac{\beta(\mu - 1)t}{\lambda}$ and $\beta = \frac{D\lambda}{d}$

COLOURS IN THIN FILMS

When white light is incident on a this film (like oil film on the surface of water or a soap bubble) then interference takes place between the waves reflected from its two surface and waves refracted through it. Then intensity becomes maximum and minimum as a result of interference and colours are seen.

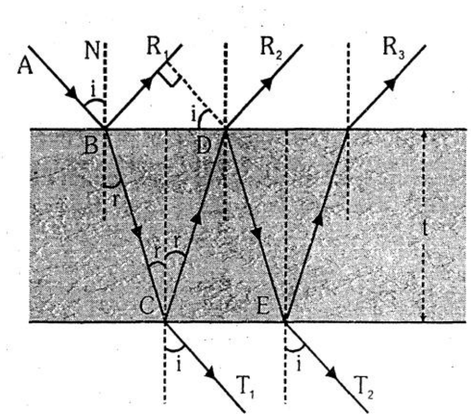
(i) The source of light must be an extended source.

(ii) The colours obtained in reflected and transmitted light are mutually complementary.

(iii) The colours obtained in thin films are due to interference whereas those obtained in prism are due to dispersion.

INTERFERENCE DUE TO THIN FILMS

Consider a thin transparent film of thickness t and refractive index μ. Let a ray of light AB be incident on the film at B. At B, a part of light is reflected along BR_1 and a part of light refracted along BC. At C a part of light is reflected along CD and a part of light transmitted along CT_1. At D, a part of light is refracted along DR_2 and a part of light is reflected along DE. Thus interference in this film takes place due to reflected light in between BR_1 and DR_2 also in transmitted light in between CT_1 and ET_2. Here coherent sources are obtained by division of amplitude.



- **Reflected System**

  The path difference between BR_1 and DR_2 is $x = 2\mu t \cos r$. Reflection from the surface of denser medium

  involves an additional phase difference of π or path difference $\lambda/2$. Therefore the effective path difference

  between BR_1 and DR_2 is $\Rightarrow x' = 2\mu t \cos r - \lambda/2$

  Maximum or constructive Interference occurs when path difference between the light waves is $n\lambda$.

  $2\mu t \cos r - \lambda/2 = n\lambda \Rightarrow 2\mu t \cos r = n\lambda + A/2$

  So the film will appear bright if $2\mu t \cos r = (2n + 1)\lambda/2$ $(n = 0, 1, 2, 3 \ldots)$

- **For minima or destructive interference** :

  When path difference is odd multiple of $\frac{\lambda}{2} \Rightarrow 2\mu t \cos \mu - \frac{\lambda}{2} = (2n-1)\frac{\lambda}{2}$

  So the film will appear dark if $2\mu t \cos r = n\lambda$

- **For transmitted system**

  Since no additional path difference between transmitted rays CT_1 and ET_2.

  So the net path difference between them is $x = 2\mu t \cos r$

  For maxima $2\mu t \cos r = n\lambda$, $n = 0, 1, 2 \ldots\ldots\ldots\ldots$.

  For minima $2\mu t \cos r = (2n+1)\frac{\lambda}{2}$, $n = 0, 1, 2 \ldots\ldots\ldots\ldots$.

USES OF INTERFERENCE EFFECT

Thin layer of oil on water and soap bubbles show different colours due to interference of waves reflected from two surfaces of their films. Here we get two coherent beams by division of amplitude making use of partial reflection and partial refraction .

Use

- Used to determine the wavelength of light precisely.
- Used to determine refractive index or thickness of transparent sheet.
- Used in holography to produce 3-D images.

GOLDEN KEY POINTS

- If a glass plate of refractive index μ_1 and μ_2 having same thickness t is placed in the path of rays coming from S_1 and S_2 then path difference $x = \frac{D}{d}(\mu_1 - \mu_2)t$.
- Distance of displaced fringe from central fringe $x = \frac{\beta(\mu_1 - \mu_2)t}{\lambda}$ $\quad \because \frac{\beta}{\lambda} = \frac{D}{d}$

ILLUSTRATIONS

Illustration 12

Light of wavelength 6000 Å is incident on a thin glass plate of refractive index 1.5 such that angle of refraction into the plate is 60°. Calculate the smallest thickness of plate which will make it appear dark by reflection.

Solution

$2\mu t \cos r = n\lambda$

$$t = \frac{n\lambda}{2\mu\cos r} = \frac{1\times 6\times 10^{-7}}{2\times 1.5\times \cos 60} = \frac{6\times 10^{-7}}{1.5} = 4\times 10^{-7}\,m$$

Illustration 13

Light is incident on a gloss plate (μ = 1.5) such that angle of refraction is 60°. Dark band is observed corresponding to the wavelength of 6000Å . If the thickness of glass plate is 1.2×10^{-3} mm. calculate the order of the interference band for reflected system

Solution

$\mu = 1.5$, r = 60°, λ = 6000 Å = 6×10^{-7} m

t = 1.2×10^{-3} mm = 1.2×10^{-6} m

For dark band in the reflected light $2\mu t \cos r = n\lambda$

$$n = \frac{2\mu t\cos r}{\lambda} = \frac{2\times 1.5\times 1.2\times 10^{-6}\times \cos 6°}{6\times 10^{-7}} = \frac{2\times 1.5\times 1.2\times 10^{-6}\times \frac{1}{2}}{6\times 10^{-7}} = 3$$

Thus third dark band is observed.

BEGINNER'S BOX-3

1. On placing a thin sheet of mica of thickness 12×10^{-7} m in the path of one of interfering beams in a Young's experiment, it is found that the central bright band shifts a distance equal to the width of a bright fringe. If the wavelength of light used is 6×10^{-7} m, then find refractive index of mica.

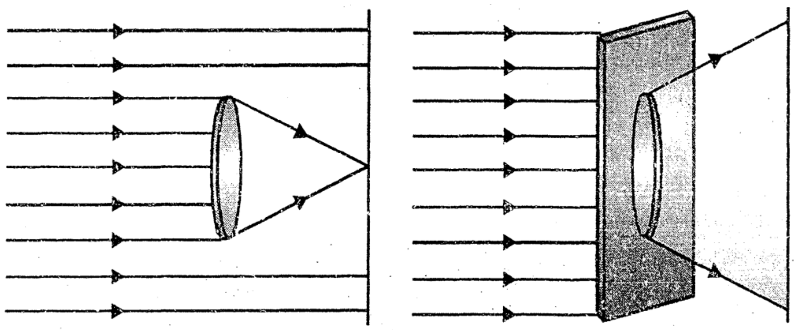
2. A central fringe of the interference produced by light of wavelength 6000Å is shifted to the position of 5^{th} bright fringes by introducing a thin glass plate of refractive index 1.5. Calculate the thickness of the plate.

3. White light is incident on a soap film of refractive index $\frac{4}{3}$ at an angle of refraction 30°. The reflected light is observed to have a dark band for wavelength 6×10^{-5} cm. Calculate the minimum thickness of the film.

4. A soap solution film of $\mu = \frac{4}{3}$ is illuminated by white light incident with angle of refraction is 65°. In reflected light, dark band was found corresponding to wavelength 5500Å. Calculate the minimum thickness of the film.

DIFFRACTION OF LIGHT

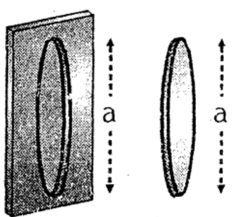
Bending of light rays from sharp edges of an opaque obstacle or aperture and its spreading in the geometrical shadow region is defined as diffraction of light or deviation of light from its rectilinear propagation tendency is defined as direction of light.



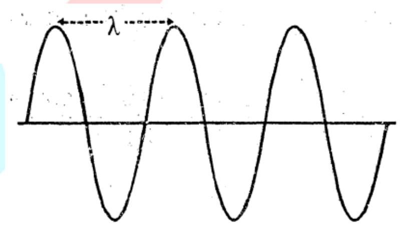
diffraction from obstacle   diffraction from aperture

- Diffraction was discovered by Grimaldi.
- Theoretically explained by Fresnel.
- Diffraction is possible in all type of waves means mechanical or electromagnetic waves shows diffraction depends on two factors :

(i) Size of obstacles or aperture   (ii) Wavelength of the wave



aperture obstacle



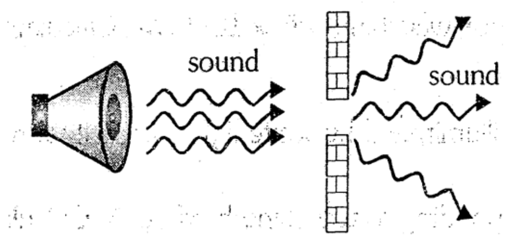
Condition of diffraction : Size of obstacle or aperture should be nearly equal to the wavelength of light.

i.e. $\lambda ; a ;$ $\frac{a}{\lambda} ; 1$

If size of obstacle is much greater than wavelength of light, then rectilinear motion of light is observed.

- It is practically observed when size of aperture or obstacle is greater than 50 λ. The obstacle or aperature does not shows diffraction.
- Wavelength of light is of the order 10^{-7} m. In general obstacle of this wavelength is not present so light rays does not show diffraction and it appears to travel in straight line Sound wave shows more diffraction as compared to light rays because wavelength of sound is large (16 mm to 16 m). So it is generally diffracted by the objects of our daily life.
- Diffraction of ultrasonic waves is also not observed easily because their wavelength is of the order of about 1 cm. Diffraction of radio wave is very conveniently observed because of its very large

wavelength (2.5 m to 250 m). X-ray can be diffracted easily by crystals. Its was discovered by Lave.



diffraction of sound from a window

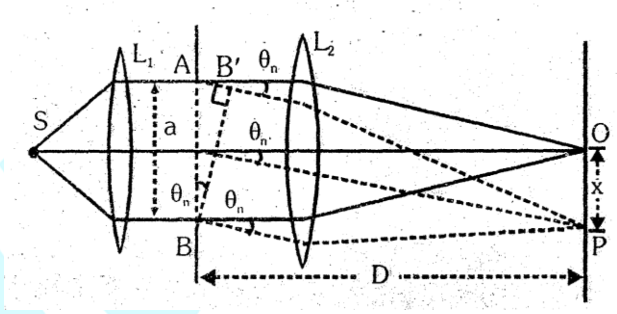
FRAUNHOFER DIFFRACTION

In fraunhofer diffraction both source and screen are effectively at infinite distance from the diffracting device and pattern is the image of source modified by diffraction effects.

Diffraction at single slit, double slit and diffraction grating are the example of Fraunhofer diffraction.

FRAUNHOFER DIFFRACTION DUE TO SINGLE SLIT

AB is single slit of width a, Plane wavefront is incident on a slit AB. Secondary wavelets coming from every part of AB each the axial point O in same phase forming the central maxima. The intensity of central maxima is maximum in this diffraction, where θ_n represents direction of n^{th} minima, Path difference $AB' = a\sin\theta_n$



For n^{th} minima a sin $\theta_n = n\lambda$

$\therefore \quad \sin\theta_n \approx \theta_n = \frac{n\lambda}{a} \qquad$ (if θ_n is small)

- When path difference between the secondary wavelets coming from A and B is $n\lambda$ or $2n\left[\frac{\lambda}{2}\right]$ or even multiple of $\frac{\lambda}{2}$ then minima occurs.

For minima $a\sin\theta_n\left[\frac{\lambda}{2}\right]$ where n = 1, 2, 3, …..

- When path difference between the secondary wavelets coming from A and B is $(2n+1)\frac{\lambda}{2}$ or odd multiple of $\frac{\lambda}{2}$ then maxima occurs.

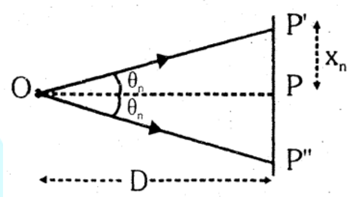
  For maxima $\quad a\sin\theta_n = (2n+1)\frac{\lambda}{2}$ $\quad$ where n = 1, 2, 3, …….

  n = 1 → first maxima and n = 2 seconds → maxima
- Alternate ordered minima and maxima occurs on both sides of cental maxima.

  For n^th^ minima

  If distance of n^{th} minima from central maxima = x_n

  Distance of slit from screen = D, with of slit = a



  Path difference $\delta = a\sin\theta_n = \frac{2n\lambda}{2}$ $\quad\Rightarrow\quad$ $\sin\theta_n = \frac{n\lambda}{a}$

  In $\Delta POP'$ $\tan\theta_n = \frac{x_n}{D}$

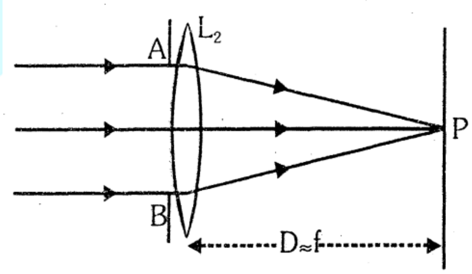
  If θ_n is small $\Rightarrow$ $\sin\theta_n \approx \tan\theta_n \approx \theta_n$

  $x_n = \frac{n\lambda D}{a}$ $\quad\Rightarrow\quad$ $\theta_n = \frac{x_n}{D} = \frac{n\lambda}{a}$ First minima occurs on both sides of central maxima.

  For first minima $x = \frac{D\lambda}{a}$ and $\theta = \frac{x}{D} = \frac{\lambda}{a}$
- Linear width of central maxima $\quad w_x = 2x \quad\Rightarrow\quad w_x = \frac{2D\lambda}{a}$
- Angular width of central maxima $\quad w_\theta = 2\theta = \frac{2\lambda}{a}$

SPECIAL CASE

Lens L_2 is shifted very near to slit AB. In this case distance between slit and screen will be nearly equal to the focal length of lens L_2. (i.e. D ≈ f) $\theta_n = \frac{x_n}{f} = \frac{n\lambda}{a}$ $\Rightarrow$ $x_n = \frac{n\lambda f}{a}$

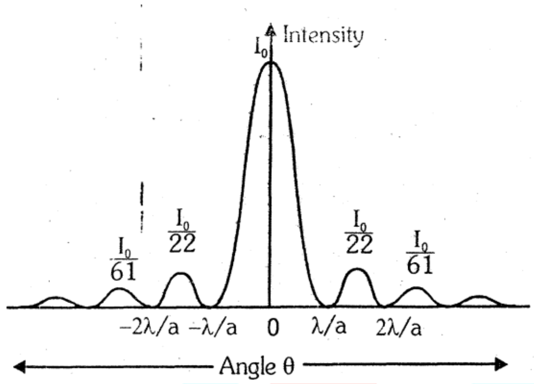


$w_x = \frac{2\lambda f}{a}$ and angular width of central maxima $w_\theta = \frac{2x}{f} = \frac{2\lambda}{a}$

Fringe width : Distance between two consecutive maxima (bright fringe) or minima (dark fringe) is known as fringe width. Fringe width of central maxima is doubled then the width of other maxima i.e.

$$\beta = x_{n+1} - x_n (n+1)\frac{\lambda D}{a} - \frac{n\lambda D}{a} = \frac{\lambda D}{a} \quad \text{(other than central maxima)}$$

Intensity curve of Fraunhofer's diffraction



DIFFERENCE BETWEEN INTERFERENCE AND DIFFRACTION (FOR FRAUNHOFER SINGLE SLIT) :

	Interference		Fraunhofer diffraction
(1)	It is the phenomenon of superposition of two waves coming from two different coherent sources.	**(1)**	It is the phenomenon of superposition of two waves coming from two different parts of the same wave front.
(2)	In interference pattern, all bright lines are equally bright and equally spaced.	**(2)**	All bright lines are not equally bright and equally wide. Brightness and width goes on decreasing with the angle of diffraction.
(3)	All dark lines are totally dark.	**(3)**	Dark lines are perfectly dark. Their contrast with bright lines and width goes on decreasing with angle of diffraction.
(4)	In interference bands are large in number.	**(4)**	In diffraction bands are few in number.

GOLDEN KEY POINTS

- The width of central maxima $\propto \lambda$, that is, more for red colour and less for blue.
  i.e. $w_x \propto \lambda$ as $\lambda_{blue} < \lambda_{red} \Rightarrow w_{blue} < w_{red}$
- For obtaining the fraunhofer diffraction, focal length of second lens (L_2) is used.
  $w_x \propto \lambda \propto f \propto \frac{1}{a}$ width will be more for narrow slit.
- By decreasing linear width of slit, the width of central maxima increase.
- The angular width of a beam of light of wavelength λ on account of diffraction at a slit of width a is given by $\theta = \frac{\lambda}{a}$
  If this beam is allowed to travel a distance Z,

linear width acquired by the beam = $Z\theta = \frac{Z\lambda}{a}$

Let Z_F be that value of Z for which the width of the beam becomes equal to a_1 i.e.

$\frac{Z_F\lambda}{a}a$ or $Z_F = \frac{a^2}{\lambda}$

Z_F is called the Freshnel distance.

ILLUSTRATIONS

Illustration 14

Light of wavelength 6000 Å is incident normally on a slit of width 24 × 10^{-5} cm. Find out the angular position of second minimum from central maximum?

Solution

a sin θ = 2λ given λ = 6 × 10^{-7}m, a = 24 × 10^{-5} × 10^{-2} m

$$\sin\theta = \frac{2\lambda}{a} = \frac{2\times 6\times 10^{-7}}{24\times 10^{-7}} = \frac{1}{2}$$

$\therefore \quad \theta = 30°$

Illustration 15

Light of wavelength 6328 Å is incident normally on a slit of width 0.2 mm. Calculate the angular width of central maximum on a screen distance 9 m ?

Solution

Given, λ = 6.328 × 10^{-7} m, a = 0.2 × 10^{-3} m

$$w_\theta = \frac{2\lambda}{a} = \frac{2\times 6.328\times 10^{-7}}{2\times 10^{-4}}\text{ radian} = \frac{6.328\times 10^{-3}\times 180}{3.14} = 0.36°$$

Illustration 16

Light of wavelength 5000Å is incident on a slit of width 0.1 mm. Find out the width of the central bright line on a screen distance 2m from the slit?

Solution

$$w_x = \frac{2f\lambda}{a} = \frac{2\times 2\times 5\times 10^{-7}}{10^{-4}} = 20\text{ mm}$$

Illustration 17

The Fraunhofer diffraction pattern of a single slit is formed at the focal plane of a lens of focal length 1m. The width of the slit is 0.3 mm. If the third minimum is formed at a distance of 5 mm from the central maximum then calculate the wavelength of light.

Solution

$$x_n = \frac{nf\lambda}{a} \quad \Rightarrow \quad \lambda = \frac{ax_n}{fn} = \frac{3\times 10^{-4}\times 5\times 10^{-3}}{3\times 1} = 5000\text{ Å} \qquad [\because n = 3]$$

Illustration 18

Find the half angular width of the central bright maximum in the Fraunhofer diffraction pattern of a slit of width 12 × 10^{-5} cm when the slit is illuminated by monochromatic light of wavelength 6000 Å.

Solution

Q $\sin\theta = \frac{\lambda}{a}$ θ = half angular width of the central maximum

a = 12 × 10^{-5} cm, λ = 6000 Å = 6 × 10^{-5} cm

∴ $\sin\theta = \frac{\lambda}{a} = \frac{6\times10^{-5}}{12\times10^{-5}} = 0.50$

⇒ $\theta = 30°$

Illustration 19

Light of wavelength 6000 Å is incident on a slit of width 0.30 mm. The screen is placed 2 m from the slit. Find (a) the position of the first dark fringe and (b) the width of the central bright fringe.

Solution

The first fringe is on either side of the central bright fringe.

Here n = ±1, D = 2 m, λ = 6000Å = 6 × 10^{-7} m

∴ $\sin\theta = \frac{x}{D}$ ⇒ a = 0.30 mm = 3 × 10^{-4} m ⇒ a sin θ = nλ ⇒ $\frac{ax}{D} = n\lambda$

(a) $x = \frac{n\lambda D}{a}$ ⇒ $x = \pm\left[\frac{1\times6\times10^{-7}\times2}{3\times10^{-4}}\right] = \pm4\times10^{-3}\text{m}$

The position and negative signs corresponds to the dark fringes on either side of the central bright fringe.

(b) The width of the central bright fringe y = 2x = 2 × 4× 10^{-3} = 8 × 10^{-3} m = 8 mm

Illustration 20

A Slit of width a is illuminated by monochromatic light of wavelength 650nm at normal incidence. Calculate the value of a when -

(a) the first minimum falls· at an angle of diffraction of 30°

(b) the first maximum falls at an angle of diffraction of 30°.

Solution

(a) For first minimum $\sin\theta_1 = \frac{\lambda}{a}$

∴ $a = \frac{\lambda}{\sin\theta_1} = \frac{650\times10^{-9}}{\sin 30°} = \frac{650\times10^{-9}}{0.5} = 1.3\times10^{-6}\text{m}$

(b) For first minimum $\sin\theta_1 = \frac{3\lambda}{2a}$

$$\therefore \qquad a = \frac{3\lambda}{2\sin\theta} = \frac{3\times 650\times 10^{-9}}{2\times 0.5} = 1.95\times 10^{-6}\,m$$

Illustration 21

Red light of wavelength 6500Å from a distant source falls on a slit 0.50 mm wide. What is the distance between the first two dark bands on each side of the central bright of the diffraction pattern observed on a screen placed 1.8 m from the slit.

Solution

Given λ = 6500 Å = 65 × 10^{-8} m, a = 0.5 mm = 0.5 × 10^{-3} m. D = 1.8 m

Required distance between first two dark banks will be equal to width of central maxima.

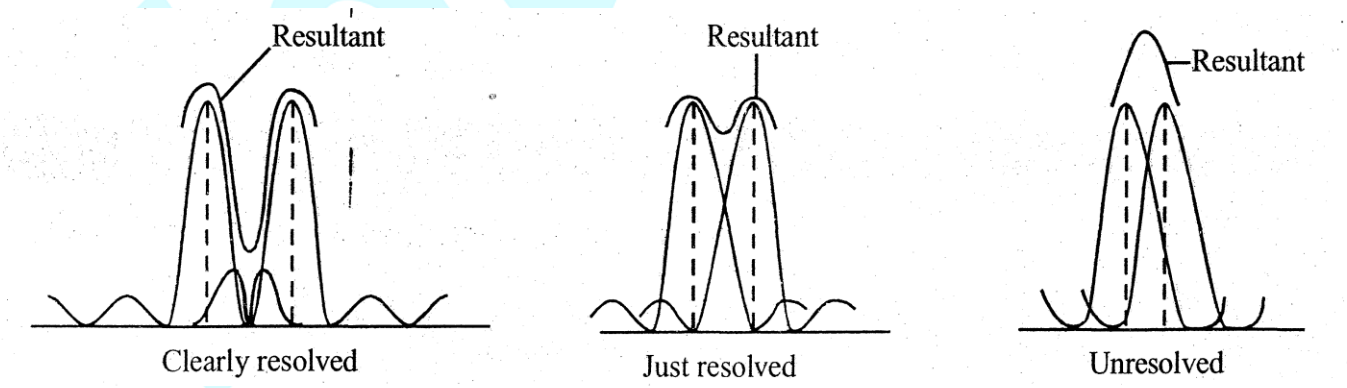
$$W_x = \frac{2\lambda D}{a} = \frac{2\times 6500\times 10^{-10}\times 1.8}{0.5\times 10^{-3}} = 468\times 10^{-5}\,m = 4.68\,mm$$

BEGINNER'S BOX-4

1. A slit of width 0.15 cm is illuminated by light of wavelength 5 × 10^{-5} cm and a diffraction pattern is obtained on a screen 2.1m away. Calculate the width of central maxima.
2. The light of wavelength 600nm is incident normally on a slit of width 3mm. Calculate the angular width of central maximum on a screen kept 3m away from the slit.
3. Red light of wavelength 6500 Å from a distant source falls on a slit 0.50 mm wide. What is the distance between the first two dark bands on each side of the central bright of the diffraction pattern observed on a screen placed 1.8 m from the slit.
4. In a single slit diffraction experiment first minimum for λ_1 = 660 nm coincides with first maxima for wavelength λ_2. Calculate λ_2.

RAYLEIGHT'S CRITERION FOR RESOLUTION

When a point source of light is imaged by an optical system with a circular aperture, the image is an Airy disc. If two points are very close, their Airy discs will overlap and we may not be able to resolve them, i.e. distinguish separate images.



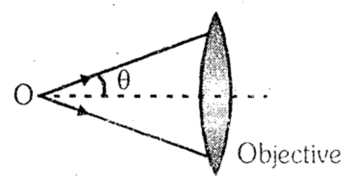
Two points are just resolved by an optical system when the central maximum of the diffraction pattern due to one falls on the first minimum of the diffraction pattern of the other.

RESOLVING POWER (R.P.)

A large number of images are formed as a consequence of light diffraction from a source. If two sources are separated such that their central maxima do not overlap, their images can be distinguished and are said to be resolved R.P. of an optical instrument is its ability to distinguish two neighbouring point.

(1) **Microscope :** In reference to a microscope, the minimum distance between two lines at which they are just distinct is called Resolving limit (RL) and it's reciprocal is called Resolving power (RP).

$$\text{R.L.} = \frac{1.22\,\lambda}{2\mu\sin\theta} \text{ and R.P.} = \frac{2\mu\sin\theta}{1.22\,\lambda} \Rightarrow \text{R.P.} \propto \frac{1}{\lambda}$$



λ = Wavelength of light used to illuminate the object

μ = Refractive index of the medium between object and objective.

θ = Half angle of the cone of light from the point object, $\mu \sin\theta$ = Numerical aperture.

(2) **Telescope :** Smallest angular separations ($d\theta$) between two distant object, whose images are separated in the telescope is called resolving limit. So resolving limit $d\theta = \frac{1.22\,\lambda}{a}$ and resolving power.

$$(\text{RP}) = \frac{1}{d\theta} = \frac{a}{1.22\,\lambda} \quad \Rightarrow \quad \text{R.P.} \propto \frac{1}{\lambda}$$ where a = aperture of objective.

ILLUSTRATIONS

Illustrations 22

The Hale telescope of Mount Palomar has a diameter of 200 inch. What is its limiting angle of resolution for 600 nm light?

Solution

Here, $a = 200 \text{ in} = 200 \times 2.54 \text{ cm} = 508 \text{ cm} = 5.08 \text{ m}$

$\lambda = 600 \text{ nm} = 600 \times 10^{-9} \text{ m} = 6.00 \times 10^{-7} \text{ m}$

$$\Delta\theta = 1.22\left(\frac{\lambda}{a}\right) = 1.22\left(\frac{6.00\times10^{-7}\,\text{m}}{5.08\,\text{m}}\right) = 1.44\times10^{-7}\,\text{rad}$$

BEGINNER'S BOX-5

1. Calculate the resolving power of a telescope, assuming the diameter of the objective lens to be 6 cm and the wavelength of light used to be 540 nm.
2. Calculate the limit of resolution of a microscope if an object of numerical aperture 0.12 is viewed by using light of wavelength 6×10^{-7} m.

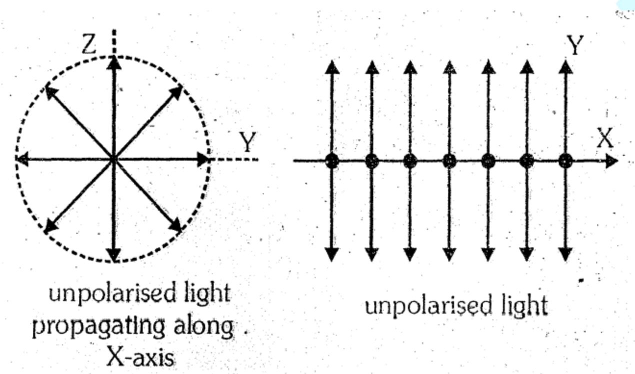
POLARISATION

Experiments on interference and diffraction have shown that light is a form of wave motion. These effects do not tell us about the type of wave motion i.e., whether the light waves are longitudinal or transverse.

The phenomenon of polarization has helped to establish beyond doubt that light waves are transverse waves.

UNPOLARISED LIGHT

An ordinary beam of light consists of a large number of waves emitted by the atoms of the light source. Each atom produces a wave with its own orientation of electric vector $\vec{E}$ so all direction of $\vec{E}$ are equally probable.



The resultant electromagnetic: wave is a superposition of waves produced by the individual atomic sources and it is called unpolatised light. In ordinary or unpolarised light, the vibrations of the electric vector occur symmetrically in all possible directions in a plane perpendicular to the direction of propagation of light.

POLARISATION

The phenomenon of restricting the vibration of light (electric vector) in a particular direction perpendicular to the direction of propagation of wave is called polarization of light. In polarized light, the vibration of the electric vector occur in a plane perpendicular to the direction of propagation light and are confined to a single direction in the plane (do not occur symmetrically in all possible directions). After polarisalion the vibrations become symmetrical about the direction of propagation of light.

POLARISER

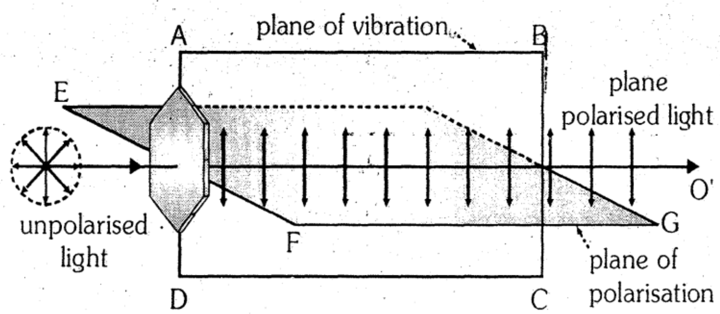
Tourmaline crystal : When light is passed through a tourmaline crystal cut parallel to its optic axis, the vibrations of the light carrying out of the tourmaline crystal are confined only to one direction in a plane perpendicular to the direction of propagation of light. The emergent light from the crystal is said to be plane polarized light.

Nicol Prism : A nicol prism is an optical device which can be used for the production and detection of plane polarized light. It was invented by William Nicol in 1828.

Polaroid : A polariod is a thin commercial sheet in the form of circular disc which makes use of the property of selective absorption to produce an intense beam of plane polarized light.

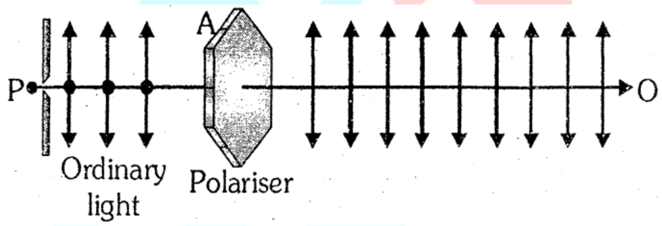
PLANE OF POLARIZATION AND PLANE OF VIBRATION

The plane in which vibrations of light vector and the direction of propagation lie is known as plane of vibration A plane normal to the plane of vibration and in which no vibration takes place is known as plane of polarization.



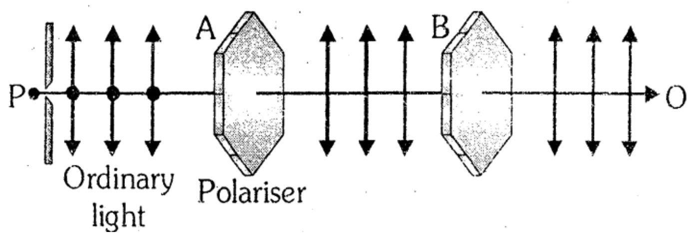
EXPERIMENTAL DEMONSTRATION OF POLARISATION OF LIGHT

Take two tourmaline crystals cut parallel to their crystallographic axis (optic axis).

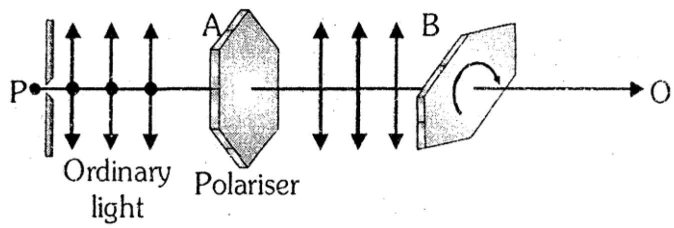


Find hold the crystal A normally to the path of a beam of colour light. The emergent beam will be slightly coloured. Rotate the crystal A about PO. No change in the intensity or the colour of the emergent beam of light.

Take another crystal B and hold it in the path of the emergent beam of so that its axis is parallel to the axis of the crystal A. The beam of light passes through both the crystals and outcoming light appears coloured.



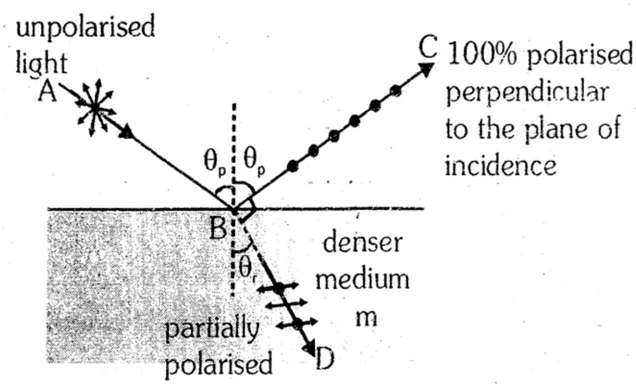
Now, rotate the crystal B about the axis PO. It will be seen that the intensity of the emergent beam decreases and when the axes of both the crystals are at right angles to each other no light comes out of the crystal B.



If the crystal B is further rotated light reappears and intensity becomes maximum again when their axis are parallel. This occurs after a further rotation of B through 90°. This experiment confirms that the light waves are transverse in nature. The vibrations in light waves are perpendicular to the direction of propagation of the wave. First crystal A polarizes the light so it is called polarizer. Second crystal B, analyses the light whether it is polarized or not, so it is called analyser.

METHODS OF OBTAINING PLANE POLARISED LIGHT

- **Polarisation by reflection :** The simplest method to produce plane polarized light is by reflection. This method was discovered by Malus in 1808. When a beam of ordinary light is reflected from a surface, the reflected light is partially polarized. (The degree of polarization of the polarized light in the reflected beam is greatest when it is incident at an angle called polarizing angle or Brewster's angle)



- **Polarising angle :** Polarising angle is that angle of incidence at which the reflected light is completely plane polarization.
- **Brewster's law :** When unpolarised light strikes at polarising angle θ_P on an interface separating a rare medium from a denser medium of refractive index μ, such that $\mu = \tan\theta_P$ then the reflected light (light in rare medium) is completely polarised. Also reflected and refracted rays are normal. to each other. This relation is known as Brewster's law. The law state that the tan function of the polarizing angle of incidence of a transparent medium is equal to its refractive index

$$\mu = \tan\theta_P$$

In case of polarisation by reflection :

(i) For $i = \theta_P$ refracted light is partially polarised.

(ii) For $i = \theta_P$ reflected and refracted rays are perpendicular to each other.

(iii) For $i < \theta_P$ or $i > \theta_P$ both reflected and refracted light become partially polarised.

According to Snell's law

$$\mu = \frac{\sin\theta_p}{\sin\theta_r} \qquad \text{.....(i)}$$

But according to Brewster's law

$$\mu = \tan\theta_p = \frac{\sin\theta_p}{\cos\theta_p} \qquad \text{.....(ii)}$$

From equation (i) and (ii)

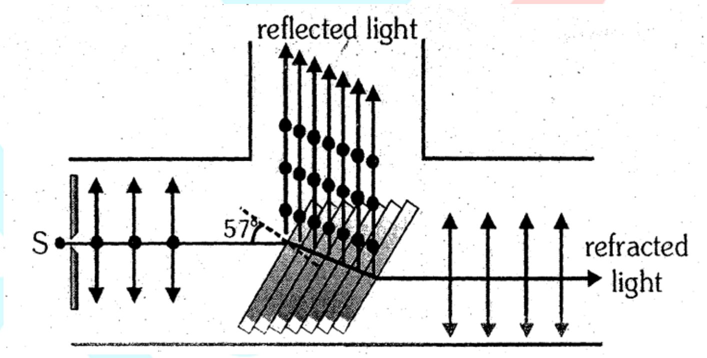
$$\frac{\sin\theta_p}{\sin\theta_r} = \frac{\sin\theta_p}{\cos\theta_p} \quad \Rightarrow \quad \sin\theta_r = \cos\theta_p$$

$$\therefore \quad \sin\theta_r = \sin(90° - \theta_p) \Rightarrow \quad \theta_r = 90° - \theta_p \quad \text{or} \quad \theta_p + \theta_r = 90°$$

Thus reflected and refracted rays are mutually perpendicular.

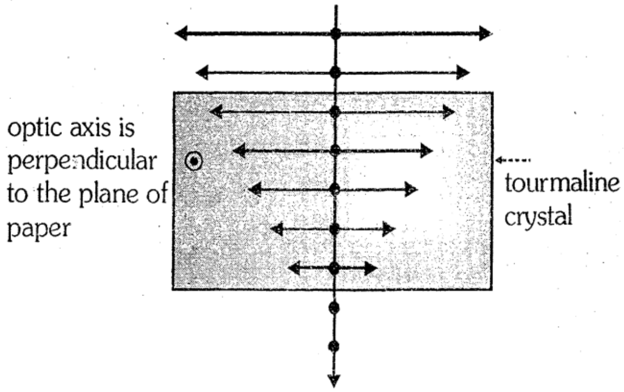
By Refraction

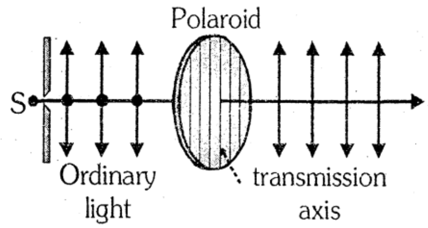
In this method, a pile of glass plates is formed by taking 20 to 30 microscope slides and light is made to be incident at polarizing angle 57°. According Brewster's law, the reflected light will be plane polarized with vibrations perpendicular to the plane of incidence and the transmitted light will be partially polarized. Since in one reflection about 15% of the light with vibration perpendicular to plane of paper is reflected therefore after passing through a number of plates emerging light will become plane polarized with vibrations in the plane of paper.



By Dichroism

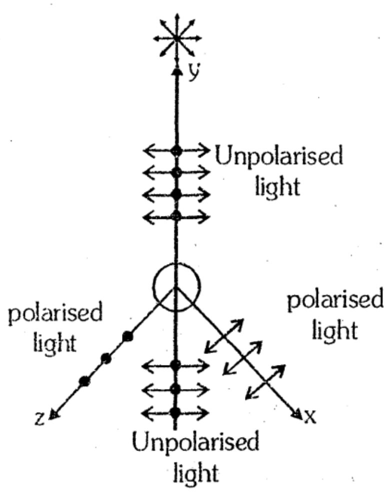
Some crystals such as tourmaline and sheets of iodosulphates of quinine have the property of strongly absorbing the light with vibrations perpendicular of a specific direction (called transmission axis) and transmitting the light with vibration parallel to it. This selective absorption of light is called dichroism. So if unpolarised light passes through proper thickness of these crystals, the transmitted light will plane polarized with vibrations parallel to transmission axis. Polaroids work on this principle.





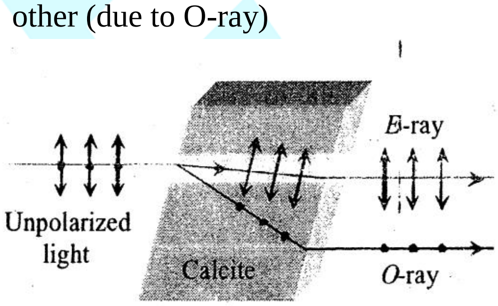
By scattering

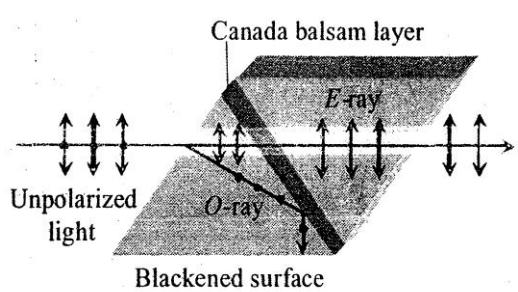
When light is incident on small particles of dust, air molecules etc. (having smaller size as compared to the wavelength of light), it is absorbed by the electrons and is re-radiated in all directions. The phenomenon is called as scattering. Light scattered in a direction at right angle to the incident light is always plane-polarised.



By Double refraction

It was found that in certain crystals such as calcite, quartz and tourmaline, etc. incident unpolarised light splits in into two light beams of equal intensities with perpendicular polarizations. One of the rays behaves as ordinary light and is called O-ray (ordinary ray) while the other does not obey laws of refraction and is called E-ray (extra ordinary ray). This is why when an object is seen through these crystal is rotated, one image (due to E-ray) rotates around the other (due to O-ray)

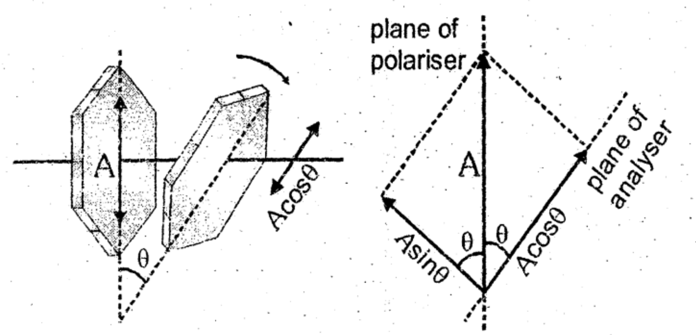




By using the phenomenon of double refraction .and isolating one ray from the other we can obtain plane polarised light which actually happens in a Nicol-prism. Nicol-prism is made up of calcite crystal and in it E-ray is isolated from O-ray through total internal reflection of O-ray at Canada balsam layer and then absorbing it at the blackened surface as shown in figure.

Laws of Malus

- When a completely plane polarised light beam is incident on analyser, then intensity of emergent light varies as the square of cosine of the angle between the planes of transmission axis of the analyser and the polarizered.
  $I \propto \cos^2\theta \Rightarrow I = I_0 \cos^2\theta$



  (i) If $\theta = 0°$ then $I = I_0$ maximum value (Parallel arrangement)
  (ii) If $\theta = 90°$ then $I = 0$ minimum value (Crossed arrangement)

- If plane polarised light of intensity $I_0 (= KN^2)$ is incident on a polaroid and its vibrations of amplitude A make angle θ with transmission axis, then the component of vibrations parallel to transmission axis will be $A \cos \theta$ while perpendicular to it will be $A \sin \theta$.
  Polaroid will pass only those vibrations which are parallel to transmission axis i.e. $A \cos \theta$,
  Q $I_0 \propto A^2$
  So the intensity of emergent light $I = K(A \cos \theta)^2 = KA^2 \cos^2 \theta$
- If an unpolarised light is converted into plane pdarised light' its intensity becomes half.
- If light of intensity I_1, emerging from one polaroid called polariser is incident on a second polaroid (called analyser) the intensity of light emerging from the second polaroid is
  $I_2 = I_1\cos^2\theta$ θ = angle between the transmission axis of the two polaroids.

GOLDEN KEY POINTS

- Our eyes are nearly insensitive lo polarisation of light, but this is not universally true for animals.
- If the angle of incidence is 0° or 90° the reflected beam is unpolarised.
- A Nicol prism is made by cutting a calcite crystal in a certain way.

ILLUSTRATIONS

Illustration 23

For a given median, the polarizing angle is 60°. What will be the critical angle for this medium?

Solution

Here $i_p = 60°$

Thus, $\mu = \tan i_p = \tan 60° = \sqrt{3}$

If i_c is the critical angle for the medium

$\mu = \frac{1}{\sin i_c}$ or $\sin i_c = \frac{1}{\mu} = \frac{1}{\sqrt{3}}$ or $\sin i_c = 0.5774$ or $i_c = 35°16'$

Illustration 24

A ray of light is incident on the surface of glass plate of refracive index 1.5 at polarising angle. What is the angle of refraction?

Solution

As $\mu = \tan i_p$, $\tan i_p = 1.5$ or $i_p = \tan^{-1}(1.5)$ or $i_p = 56°19'$

As $r + i_p = 90°$, $r = 90° - i_p = 33°41'$

BEGINNER'S BOX-6

1. Refractive index of water is 1.33. Calculate the angle of polarisation for light reflected from the surface of a lake. ($\tan^{-1} 1.33 = 53°4'$)

2. A ray of light strikes a glass plate at an angle of 60°. If the reflected and the refracted rays are perpendicular to each other, find the index of refraction of glass.

3. A parallel beam of light is incident at an angle of 60° on a plane glass surface and the reflected beam is completely polarised.

(a) What is the angle of refraction in glass?

(b) What is the refraction index of glass?

4. Light reflected from the surface of a glass plate of refractive index 1.732 is linearly polarised. Calculate the angle of refraction in glass.

5. Critical angle for a certain wavelength of light in case of glass is 40°. Find the polarising angle and angle of refraction in glass corresponding to this $\left(\sin 40° = \frac{2}{3}\right)$.

ANSWERS

BEGINNER'S BOX-1

1.	5I	**2.**	$\frac{25}{16}$	**3.**	**(a)** 7.8I **(b)** I **(c)** 9I

BEGINNER'S BOX-2

1.	4.5 mm	**2.**	6 μm	**3.**	1.2 cm	**4.**	1.76×10^{-3} m
5.	6×10^{-7} cm	**6.**	0.2948 mm, 0.2211 mm				
7.	8×10^{-3} m, $\lambda = 0.66 \times 10^{-6}$ m			**8.**	$\frac{3}{1}$	**9.**	10^{-3} m
10.	1.5 mm						

BEGINNER'S BOX-3

1.	1.5	**2.**	6×10^{-6} m	**3.**	2.59×10^{-5} cm	**4.**	4125 Å

BEGINNER'S BOX-4

1.	1.4 mm	**2.**	0.023°	**3.**	4.68 mm	**4.**	440 nm

BEGINNER'S BOX-5

1.	9.1×10^{4}	**2.**	3.05×10^{-6} m

BEGINNER'S BOX-6

1.	53°4′	**2.**	1.732	**3.**	**(a)** 30° **(b)** 1.732	**4.**	30°
5.	57.3°, 32.7°						