Straight Line

1. Equation of Straight Line

A relation between x and y which is satisfied by co-ordinates of every point lying on a line is called the equation of Straight Line. Every linear equation in two variable x and y always represents a straight line.

eg. $3x + 4y = 5$, $-4x + 9y = 3$ etc.

General form of straight line is given by

$ax + by + c = 0$.

2. Equation of Straight line Parallel to Axes

(i) Equation of x axis $\Rightarrow y = 0$.

Equation a line parallel to x axis (or perpendicular to y-axis) at a distance 'a' from it $\Rightarrow y = a$.

(ii) Equation of y axis $\Rightarrow x = 0$.

Equation of a line parallel to y-axis (or perpendicular to x axis) at a distance 'a' from it $\Rightarrow x = a$.

eg. Equation of a line which is parallel to x-axis and at a distance of 4 units in the negative direction is $y = -4$.

3. Slope of a Line

If θ is the angle made by a line with the positive direction of x axis in anticlockwise sense, then the value of $\tan\theta$ is called the Slope (also called gradient) of the line and is denoted by m or slope $\Rightarrow m = \tan\theta$

eg. A line which is making an angle of 45º with the x-axis then its slope is $m = \tan 45^\circ = 1$.

Note :

(i) Slope of x axis or a line parallel to x-axis is $\tan 0^\circ = 0$.

(ii) Slope of y axis or a line parallel to y-axis is $\tan 90^\circ = \infty$.

(iii) The slope of a line joining two points (x_1, y_1) and (x_2, y_2) is given by $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

eg. Slope of a line joining two points (3, 5) and (7, 9) is $= \dfrac{9-5}{7-3} = \dfrac{4}{4} = 1$.

4. Different forms of the Equation of Straight line

4.1 Slope - Intercept Form :

The equation of a line with slope m and making an intercept c on y–axis is $y = mx + c$. If the line passes through the origin, then $c = 0$. Thus the equation of a line with slope m and passing through the origin $y = mx$.

4.2 Slope Point Form :

The equation of a line with slope m and passing through a point (x_1, y_1) is

$$y - y_1 = m(x - x_1)$$

4.3 Two Point Form :

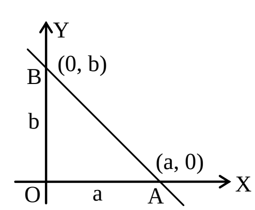
The equation of a line passing through two given points (x_1, y_1) and (x_2, y_2) is -

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

4.4 Intercept Form :

The equation of a line which makes intercept a and b on the x-axis and y-axis respectively is $\dfrac{x}{a} + \dfrac{y}{b} = 1$.

Here, the length of intercept between the co-ordinates axis $= \sqrt{a^2 + b^2}$



Area of $\Delta OAB = \dfrac{1}{2}$ OA. OB $= \dfrac{1}{2}$ a.b.

4.5 Normal (Perpendicular) Form of a Line :

If p is the length of perpendicular on a line from the origin and α is the inclination of perpendicular with x- axis then equation on this line is

$$x\cos\alpha + y\sin\alpha = p$$

4.6 Parametric Form (Distance Form) :

If θ be the angle made by a straight line with x-axis which is passing through the point (x_1, y_1) and r be the distance of any point (x, y) on the line from the point (x_1, y_1) then its equation.

$$\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r$$

5. Reduction of general form of Equations into Standard forms

General Form of equation $ax + by + c = 0$ then its-

(i) Slope Intercept Form is

$$y = -\frac{a}{b}x - \frac{c}{b}$$, here slope $m = -\frac{a}{b}$, Intercept $C = \frac{c}{b}$

(ii) Intercept Form is

$$\frac{x}{-c/a} + \frac{y}{-c/b} = 1$$, here x intercept is $= -c/a$, y intercept is $= -c/b$

(iii) Normal Form is to change the general form of a line into normal form, first take c to right hand side and make it positive, then divide the whole equation by $\sqrt{a^2+b^2}$ like

$$-\frac{ax}{\sqrt{a^2+b^2}} - \frac{by}{\sqrt{a^2+b^2}} = \frac{c}{\sqrt{a^2+b^2}},$$

here $\cos\alpha = \frac{a}{\sqrt{a^2+b^2}}$, $\sin\alpha = \frac{b}{\sqrt{a^2+b^2}}$ and

$$p = \frac{c}{\sqrt{a^2+b^2}}$$

6. Position of a point relative to a line

(i) The point (x_1, y_1) lies on the line $ax + by + c = 0$ if, $ax_1 + by_1 + c = 0$

(ii) If $P(x_1, y_1)$ and $Q(x_2, y_2)$ do not lie on the line $ax + by + c = 0$ then they are on the same side of the line, if $ax_1+by_1+ c$ and $ax_2 + by_2 +c$ are of the same sign and they lie on the opposite sides of line if $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ are of the opposite sign.

(iii) (x_1, y_1) is on origin or non origin sides of the line $ax + by + c = 0$ if $ax_1 + by_1 + c = 0$ and c are of the same or opposite signs.

7. Angle between two Straight lines

The angle between two straight lines $y = m_1x + c_1$ and $y = m_2x + c_2$ is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$$

Note :

(i) If any one line is parallel to y axis then the angle between two straight line is given by

$$\tan\theta = \pm\frac{1}{m}$$

Where m is the slope of other straight line

(ii) If the equation of lines are $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ then above formula would be

$$\tan\theta = \left|\frac{a_1b_2 - b_1a_2}{a_1a_2 + b_1b_2}\right|$$

(iii) Here two angles between two lines, but generally we consider the acute angle as the angle between them, so in all the above formula we take only positive value of $\tan\theta$.

7.1 Parallel Lines :

Two lines are parallel, then angle between them is 0

$$\Rightarrow \frac{m_1 - m_2}{1 + m_1m_2} = \tan 0^o = 0$$

$$\Rightarrow m_1 = m_2$$

Note : Lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are parallel $\Leftrightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2}$

7.2 Perpendicular Lines :

Two lines are perpendicular, then angle between them is 90°

$$\Rightarrow \frac{m_1 - m_2}{1 + m_1 m_2} = \tan 90^\circ = \infty$$

$$\Rightarrow m_1 m_2 = -1$$

Note : Lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are perpendicular then $a_1a_2 + b_1b_2 = 0$

7.3 Coincident Lines :

Two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are coincident only and only if $\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

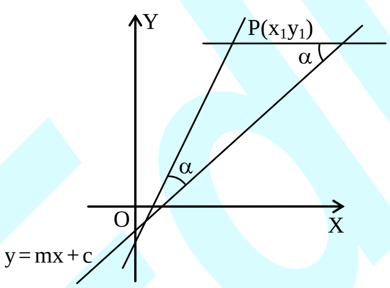
8. Equation of Parallel & Perpendicular lines

(i) Equation of a line which is parallel to $ax + by + c = 0$ is $ax + by + k = 0$

(ii) Equation of a line which is perpendicular to $ax + by + c = 0$ is $bx - ay + k = 0$

The value of k in both cases is obtained with the help of additional information given in the problem.

9. Equation of Straight lines through (X_1, Y_1) making an angle a with = mx + c



$$y - y_1 = \frac{m \mu \tan\alpha}{1 \pm m \tan\alpha}(x - x_1)$$

10. Length of Perpendicular

The length P of the perpendicular from the point (x_1, y_1) on the line $ax + by + c = 0$ is given by

$$P = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

Note :

(i) Length of perpendicular from origin on the line $ax + by + c = 0$ is $c / \sqrt{a^2 + b^2}$

(ii) Length of perpendicular from the point (x_1, y_1) on the line $x \cos\alpha + y \sin\alpha = p$ is -

$x_1 \cos\alpha + y_1 \sin\alpha = p$

10.1 Distance between Two Parallel Lines :

The distance between two parallel lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is

$$\frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$$

Note :

(i) Distance between two parallel lines $ax + by + c_1 = 0$ and $kax + kby + c_2 = 0$ is

$$\frac{\left|c_1 - \frac{c_2}{k}\right|}{\sqrt{a^2 + b^2}}$$

(ii) Distance between two non parallel lines is always zero.

11. Condition of Concurrency

Three lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ and $a_3x + b_3y + c_3 = 0$ are said to be concurrent, if they passes through a same point. The condition for their concurrency is

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

Again, to test the concurrency of three lines, first find out the point of intersection of any two of the three lines. If this point lies on the remaining lines then the three lines are concurrent.

Note : If P = 0, Q = 0, R = 0 the equation of any three line and P + Q + R = 0 the line are concurrent. But its converse is not true i.e. if the line are concurrent then it is not necessary that P + Q + R = 0

12. Bisector of Angle between two Straight line

(i) Equation of the bisector of angles between the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

(ii) To discriminate between the acute angle bisector and the obtuse angle bisector : If θ be the angle between one of the lines and one of the bisector, find $\tan\theta$. If $|\tan\theta| < 1$ then $2\theta < 90°$ so that this bisector is the acute angle bisector, If $|\tan\theta| > 1$, then we get the bisector to be the obtuse angle bisector.

(iii) First write the equation of the lines so that the constant terms are positive. Then

(a) If $a_1a_2 + b_1b_2 > 0$ then on taking positive sign in the above bisectors equation we shall get the obtuse angle bisector and on taking negative sign we shall get the acute angle bisector.

(b) If $a_1a_2 + b_1b_2 < 0$, the positive sign give the acute angle and negative sign gives the obtuse angle bisector.

(c) On taking positive sign we shall get equation of the bisector of the angle which contains the origin and negative sign gives the equation of the bisector which does not contain origin.

Note : This is also the bisector of the angle in which origin lies (since c_1, c_2 are positive and it has been obtained by taking positive sign)

13. Lines passing through the point of intersection of two lines

If equation of two lines $P = a_1x + b_1y + c_1 = 0$ and $Q = a_2x + b_2y + c_2 = 0$, then the equation of the lines passing through the point of intersection of these lines is $P + \lambda Q = 0$ or $(a_1x + b_1y + c = 0) + \lambda(a_2x + b_2y + c_2 = 0) = 0$; Value of λ is obtained with the help of the additional information given in the problem.