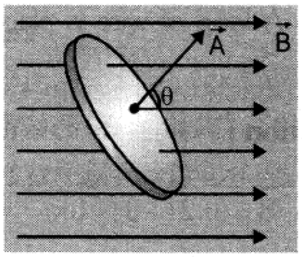
ELECTROMAGNETIC INDUCTION

INTRODUCTION

In year 1820 Orested discovered the magnetic effect of current. Faraday gives the thought that reverse of this phenomenon is also possible i.e. current is also produced by magnetic field. Faraday threw a magnet in a coil which is connected by a sensitive galvanometer when the magnet passes through the coil the galvanometer gives instantaneous deflection.

- **Magnetic Flux**

  The magnetic flux (ϕ) linked with a surface held in a magnetic field (B) is defined as the number of magnetic field lines crossing that area (A). If θ is the angle between the direction of the field and normal to the area, (area vector) then $\phi = \vec{B}.\vec{A} = BA\cos\theta$



- **Flux Linkage**

  If a coil has more than one turn, then the flux through the whole coil is the sum of the fluxes through the individual turn.

  If the magnetic field is uniform, the flux through one turn is $\phi = BA\cos\theta$

  If the coil has N turns, the total flux linkage $\phi = NBA\cos\theta$

- Magnetic field lines are imaginary, magnetic flux is a real scalar physical quantity with dimensions

  $[\phi] = B \times \text{area} = \left[\frac{F}{IL}\right][L^2]$ $\quad [\because F = B\,I\,L\sin\theta] \quad \therefore [\phi] = \left[\frac{MLT^{-2}}{AL}\right][L^2] = [ML^2T^{-2}A^{-1}]$

- **SI unit of magnetic flux f**

  $\because [M L^2 T^{-2}]$ corresponds to energy

  $\frac{\text{joule}}{\text{ampere}} = \frac{\text{joule}\times\text{sec}}{\text{coulomb}}$ = weber (WB) or T–m^2 (as tesla = WB/m^2)

  CGS UNIT of Magnetic flux ϕ : maxwell (Mx), 1Wb = 10^8 Mx

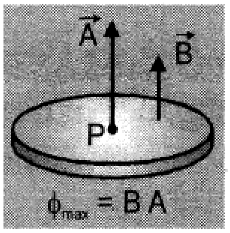
GOLDEN KEY POINTS

- For a given area flux will be maximum :

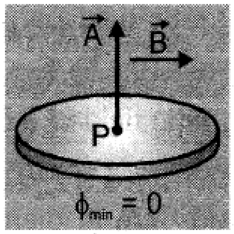
  when magnetic field $\vec{B}$ is normal to the area (transverse field)

  $\theta = 0° \Rightarrow$ $\cos\theta$ = maximum = 1

  $\phi_{max} = BA$



- For a given area flux will be minimum :
  when magnetic field $\vec{B}$ is parallel to the area (longitudinal field)
  $\theta = 90°$ $\Rightarrow$ $\cos\theta = 0$
  $\phi_{min} = 0$



ILLUSTRATIONS

Illustration 1

A rectangular loop of area 0.06 m^2 is placed in a magnetic field 12. T with its plane inclined 30° to the field direction. Find the flux linked with plane of loop.

Solution :

Area of loop A = 0.06 m^2, B = 1.2 T and $\theta = 90° - 30° = 60°$

So, the flux linked with the loop is $\phi = BA\cos\theta = 1.2 \times 0.06 \times \cos 60° = 1.2 \times 0.06 \times 1/2 = 0.036$ Wb

Illustration 2

A loop of wire is placed in a magnetic field $\vec{B} = 0.3\hat{j}T$. Find the flux through the loop if area vector is $\vec{A} = (2\hat{i} + 5\hat{j} - 3\hat{k})m^2$

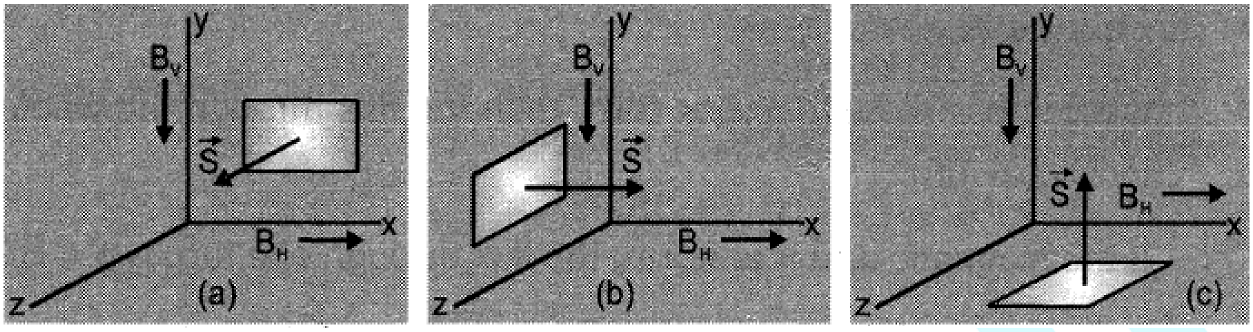
Solution :

$\vec{B} = (0.2\hat{i} + 0\hat{j} + 0\hat{k})$ Flux linked with the surface $\phi = \vec{B}.\vec{A} = (0.3\hat{j}).(2\hat{i} + 5\hat{j} - 3\hat{k})m^2$

= 1.5 Wb

Illustration 3.

At a given plane, horizontal and vertical components of earth's magnetic field B_H and B_V are along x and y axes respectively as shown in figure. What is the total flux of earth's magnetic field associated with an area S_1 if the area S is in (a) x–y plane (b) y–z plane and (c) z–x plane.



Solution :

$\vec{B} = \hat{i}B_H - \hat{j}B_V$ = constant, so $\phi = \vec{B}.\vec{S}$ [$\vec{B}$ = constant]

(a) For area in x–y plane $\vec{S} = S\hat{k}$ $\phi_{xy} = (\hat{i}B_H - \hat{j}B_V).(\hat{k}S) = 0$

(b) For area S in y–z plane $\vec{S} = S\hat{i}$ $\phi_{yz} = (\hat{i}B_H - \hat{j}B_V).(\hat{i}S) = B_HS$

(c) For area S in z–x plane $\vec{S} = S\hat{j}$ $\phi_{zx} = (\hat{i}B_H - \hat{j}B_V).(\hat{j}S) = -B_VS$

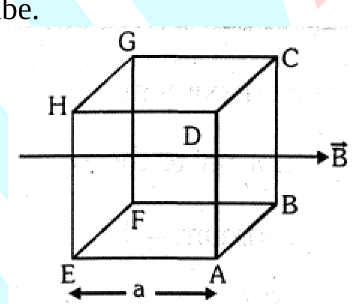
Negative sign implies that flux is directed vertically downwards.

BEGINNER'S BOX–1

1. A coil of 100 turns, 5cm^2 area is placed in external magnetic field of 0.2 Tesla (S.I.) in such a way that it makes an angle 30° with the field direction. Calculate magnetic flux through the coil (in weber).

2. A coil of N turns, A area is placed in uniform transverse magnetic field B. If it is turn through 180° about its one of the diameter in 2 seconds. Find rate of change of magnetic flux through the coil.

3. A square cube of side 'a' is placed in uniform magnetic field 'B'. Find magnetic flux through each face of the cube.



4. The magnetic field perpendicular to the plane of a loop of area 0.1 m^2 is 0.2 T. Calculate the magnetic flux through the loop.

5. The magnetic field in a certain region is given by $\vec{B} = (4\hat{i} - \hat{k})$ tesla. How much magnetic flux passes through the loop of area 0.1m^2 in this region if the loop lies flat in xy plane ?

6. A solenoid 4cm in diameter and 20cm, in length has 250 turns and carries a current of 15A. Calculate the flux through the surface of a disc of 10cm radius that is positioned perpendicular to and centered on the axis of the solenoid.

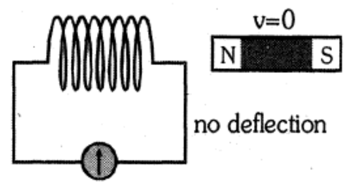
ELECTROMAGNETIC INDUCTION

Michael Faraday demonstrated the reverse effect of Oersted experiment. He explained the possibility of producing emf across the ends of a conductor when the magnetic flux linked with the conductor changes. This was termed as electromagnetic induction. The discovery of this phenomenon brought about a revolution in the field of electric power generation.

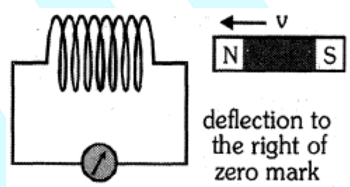
FARADAY'S EXPERIMENT

Faraday performed various experiments to discover and understand the phenomenon of electromagnetic induction. Some of them are :

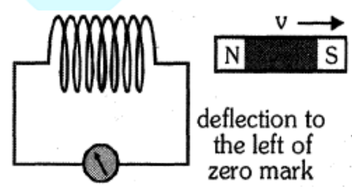
- When the magnetic is held stationary anywhere near or inside the coil, the galvanometer does not show any deflection.



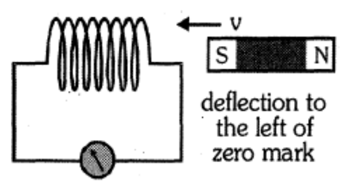
- When the N–pole of a strong bar magnet is moved towards the coil, the galvanometer shows a deflection right to the zero mark.



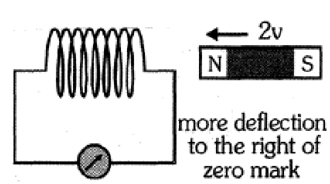
- When the N–pole of a strong bar magnet is moved away from the coil, the galvanometer shows a deflection left to the zero mark.



- If the above experiments are repeated by bringing the S–pole of the, magnet towards or away from the coil, the direction of current in the coil is opposite to that obtained in the case of N–pole.



- The deflection in galvanometer is more when the magnet moves faster and less when the magnet moves slower.



CONCLUSIONS

Whenever there is a relative motion between the source of magnetic field (magnet) and the coil, an emf is induced in the coil. When the magnetic and coil move towards each other then the flux linked with the coil increases and emf is induced. When the magnet and coil move away from each other the magnetic flux linked with the coil decreases, again an emf is induced. This emf lasts so long as the flux is changing.

Due to this emf an electric current start to flow and the galvanometer shows deflection.

the deflection in galvanometer last as long the relative motion between the magnet and coil continues.

Whenever relative motion between coil and magnet takes place an induced emf produced in coil. If coil is in closed circuit then current and charge is also induced in the circuit. This phenomenon is called electro magnetic induction.

FARADAY'S LAWS OF ELECTROMAGNETIC INDUCTION

Based on his experimental studies on the phenomenon of electromagnetic induction, Faraday proposed the following two laws.

- **First law**

  Whenever magnetic flux linked with a closed circular changes, an emf is induced in the circuit. The induced emf lasts so long as the change in magnetic flux continues.

- **Second law**

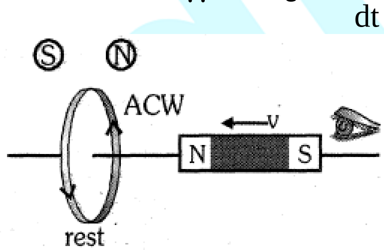
  The magnitude of emf induced in a closed circuit is directly proportional to rate of change of magnetic flux linked with the circuit. If the change in magnetic flux in a time dt is $d\phi$ then

$$e \propto \frac{d\phi}{dt}$$

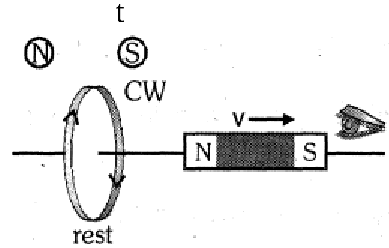
LENZ'S LAW

The Russian scientist H.F. Lenz in 1835 discovered a simple law giving the direction of the induced current produced in a circuit. Lenz's law states that the induced current produced in a circuit always flow in such a direction that it opposes the change or cause that produced it. If the coil has N number of turns and ϕ is the magnetic flux linked with each turn of the coil then, the total magnetic flux linked with the coil at any time = $N\phi$

$$\therefore \quad e = -\frac{d}{dt}(N\phi) = -N\frac{d\phi}{dt} = -\frac{N(\phi_2 - \phi_1)}{t}$$



(Coil face behaves as North pole to oppose the motion of magnet.)

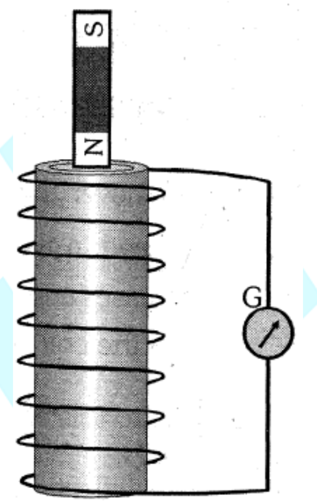


(Coil face behaves as South pole to oppose the motion of magnet.)

$e = (-)\frac{d\phi}{dt}$, here negative sign indicates the concept of Lenz law.

LENZ'S LAW – A CONSEQUENCE OF CONSERVATION OF ENERGY

Copper coils are wound on a cylindrical cardboard and the two ends of the coil are connected to a sensitive galvanometer. When a bar magnet is move towards the coil (fig.) The upper face of the coil near the magnet acquired north polarity.

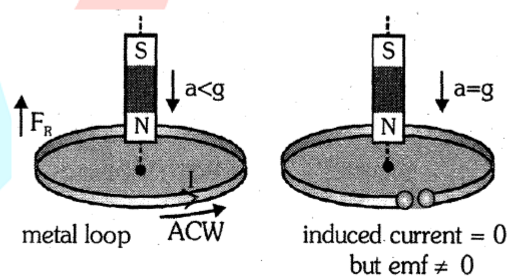


Consequently work has to be done to move the magnetic further against the force of repulsion. When we withdraw the magnet away from the coil, its nearer face acquires south polarity. Now the work done is against the face of attraction. When the magnet is moved, the number of magnetic lines of force linking the coil changes, which causes an induced current of flow through the coil. The direction of the induced current, according to Lenz's law is always to oppose the motion of the magnet.

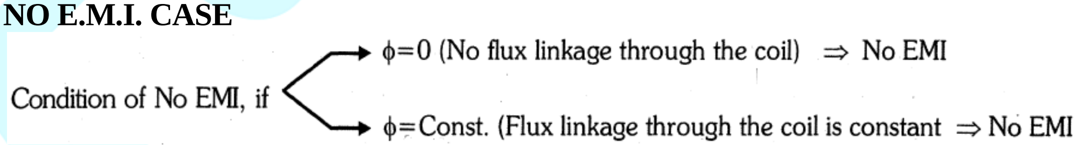
The work done in moving the magnet is converted into electrical energy. This energy is dissipated as heat energy in the coil. Therefore, the induced current always flows in such a direction to oppose the cause. Thus it is proved that Lenz's law is the consequence of conservation of energy.

GOLDEN KEY POINTS

- Induced emf does not depends on nature of the coil and its resistance.
- Magnitude of induced emf is directly proportional to the relative speed of coil magnet system, ($e \propto v$).
- Induced current also depends on resistance of coil (or circuit).
- Induced emf does not depends on resistance of circuit, It exist in open circuit also.
- In all E.M.I. phenomenon, induced emf is non–zero induced parameter.

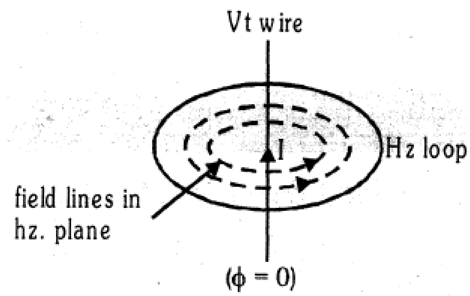


- Induced change in any coil (or circuit) does not depends on time in which change in flux occurs i.e. it is independent from rate of change of flux or relative speed of coil–magnet system.
- Induced change depends on change in flux through the coil and nature of the coil (or circuit) i.e. resistance.
- **NO E.M.I. CASE**

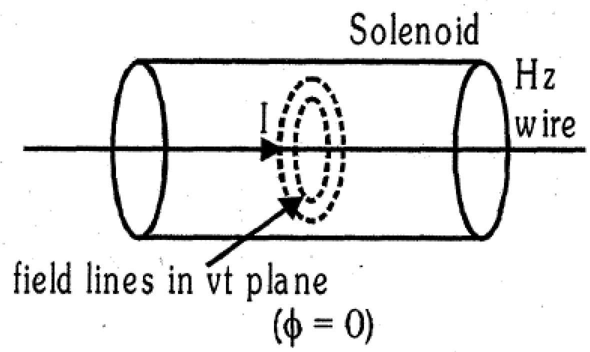


Cases

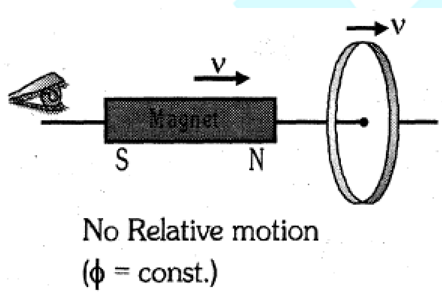
(i) If current I increases with respect to time, no emf induced in loop because no flux associated with it, as plane of circular field lines of straight wire is parallel to the plane of loop.



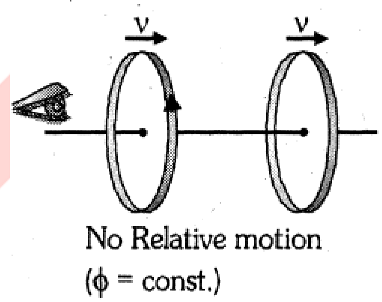
(ii) If current I increases with respe3ct to time no emf induced in solenoid because no flux associated with solenoid

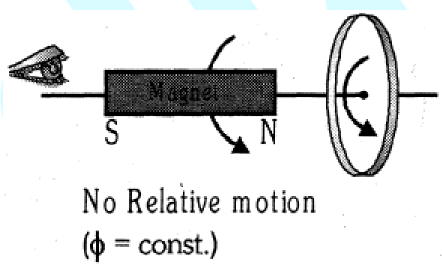


(iii) (a)

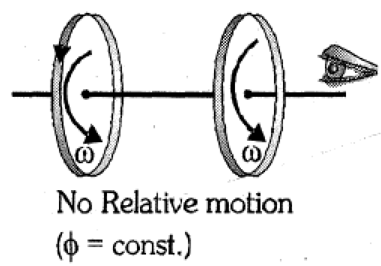


(b)



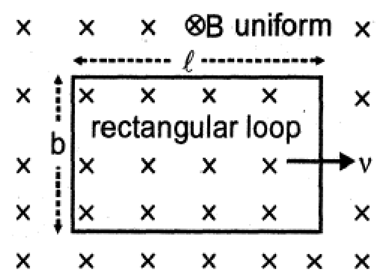


(c)

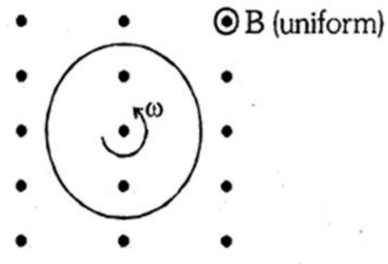


(d)

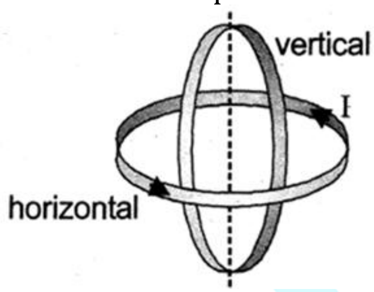
(iv) Any rectangular coil or loop translates within the uniform transverse magnetic field, no emf induced in it because its flux remains constant.



(v) Any coil or loop rotates about its geometrical axis in uniform transverse magnetic field, no emf induced in it because its flux remains constant.



(vi) If current of one coil (or loop) either increase or decrease, no emf induced in another coil (or loop) because no flux associated for the coils (or loops) which are placed mutually perpendicular.



ILLUSTRATIONS

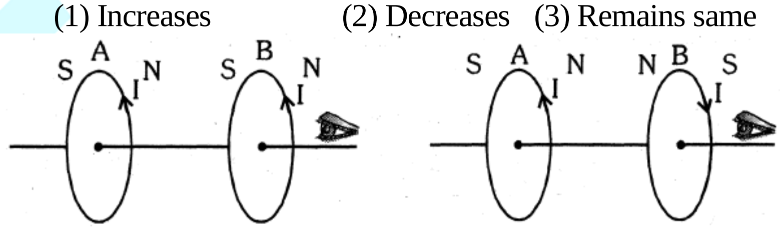
Illustrations 4.

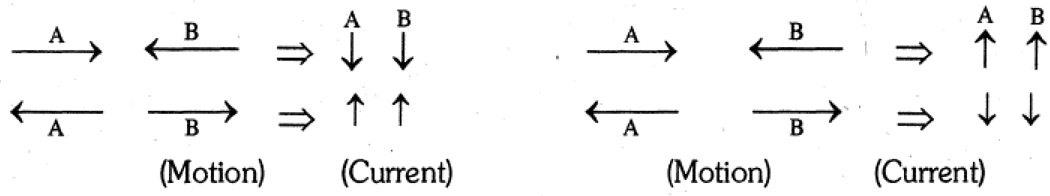
Two identical co–axial circular coils carries equal currents :–

(a) In same direction (b) In opposite direction.

If both the coils moves towards each other and away from each other respectively then current in both coils :–

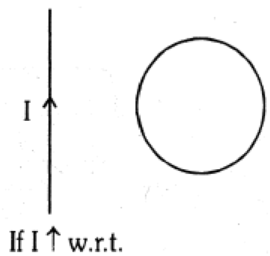
(1) Increases (2) Decreases (3) Remains same (4) None

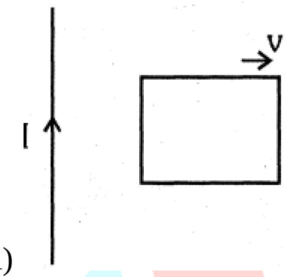


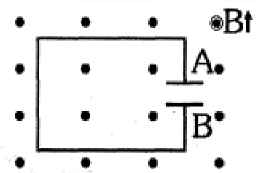


BEGINNER'S BOX–2

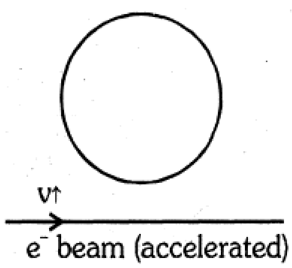
1. Find direction of induced current for the given cases :–
(Where w.r.t. = with respect to time, ob = observer =)

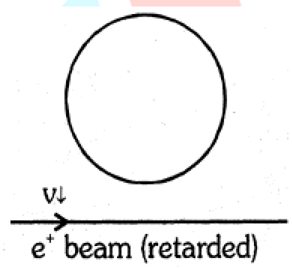
(i)


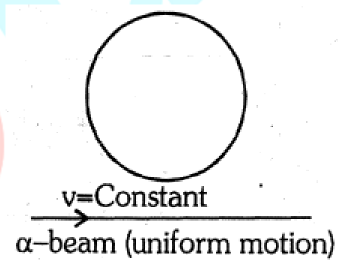
(ii)


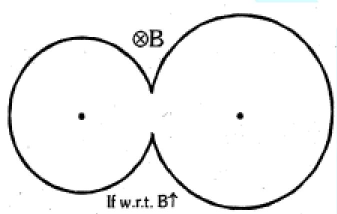
(iii)


What is the nature of charge on plates of capacitor ?

(iv)


(v)


(vi)


(vii) (a)


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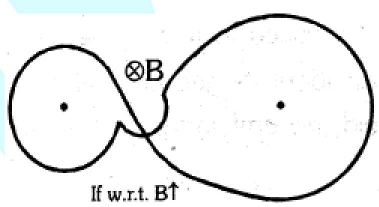

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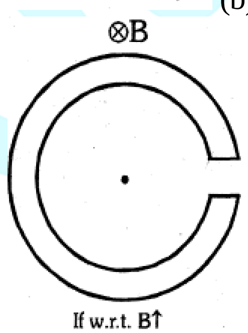

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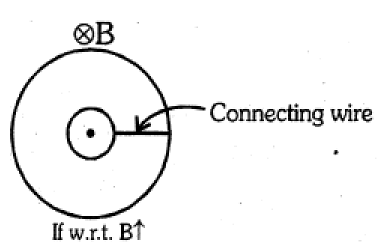
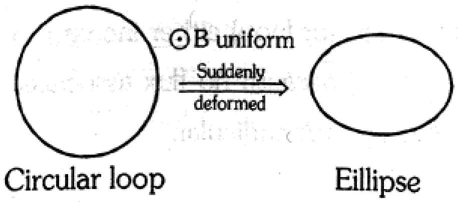
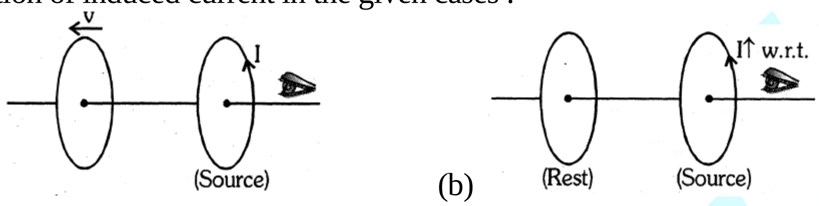




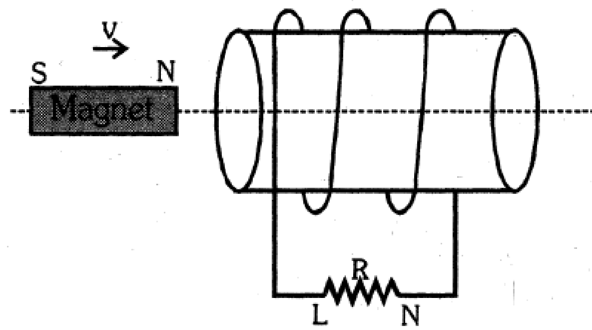
(viii)

2. Find direction of induced current in the given cases :–

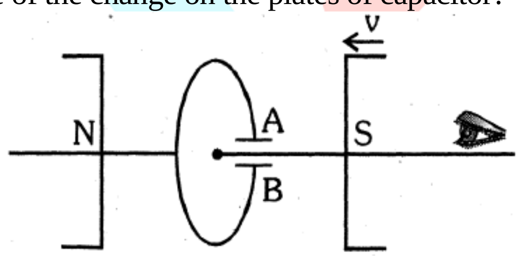
(i) (a) (b)



(ii) What is the direction of induced current in resistance 'R' ?



(iii) What is the nature of the change on the plates of capacitor?



INDUCED PARAMETERS

(i) Induced emf (e) (ii) Induced current (I) (iii) Induced charge (q)

(iv) Induced heat (H) (v) Induced electric field (E_m)

Let for a coil its mag. flux changes by $\Delta\phi$ in time interval Δt and total resistance of coil–circuit is R.

Now rate of change of flux = $\frac{\Delta\phi}{\Delta t}$ Average induced emf $e_{av} = \frac{-\Delta\phi}{\Delta t}$

(i) Instantaneous **induced emf e** = $\lim_{\Delta t \to 0}\left(\frac{-\Delta\phi}{\Delta t}\right)$

$$\boxed{e = -\frac{d\phi}{dt}}$$

(ii) Induced current flow at this instant $I = \frac{e}{R}$

$$\boxed{I = \frac{-1}{R}\left(\frac{d\phi}{dt}\right)}$$

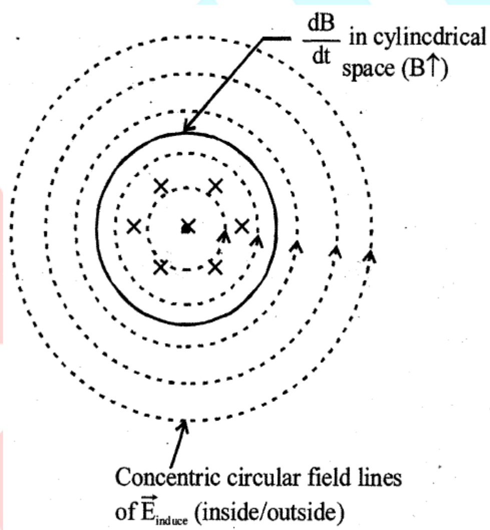
(iii) In time interval dt, induced charge dq = Idt

$$\boxed{dq = -\frac{d\phi}{R}}$$

(iv) **Induced heat** :– $\boxed{H = \int_0^t I^2 R dt = \int_0^t \frac{e^2}{R} dt}$

(v) **Induced electric field and its properties** :

When magnetic field changes with time in region then an electric field induces within and outside the region.



- This field is different from the conservative electrostatic field produced by stationary charges.
- Its field lines are always in closed curves.0
- Relation $\vec{F} = q\vec{E}$ is valid for this field
- For induced electric field $\oint \vec{E}_{ind} \cdot d\vec{l} \neq 0$

  (But for electrostatic field $\oint \vec{E} \cdot d\vec{l} = 0$; always)
- From faraday law of emf e = $-\frac{d\phi}{dt}$ or

  $$\boxed{\oint \vec{E}_{ind} \cdot d\vec{l} = -\frac{d\phi}{dt}}$$
- Direction of induced electric field is the same as direction of induced current.

E_{ind} due to $\frac{dB}{dt}$:–

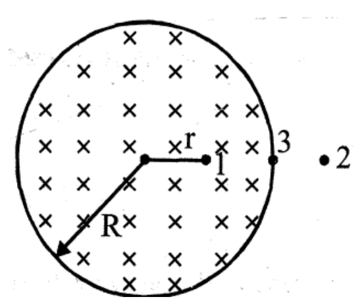
Use $\oint \vec{E}_{ind} \cdot d\vec{l} = -\frac{d\phi}{dt}$ and for

symmetrical situation Eλ = $\left|\frac{d\phi}{dt}\right| = \frac{AB}{dt}$

λ = length of closed loop

A = Area in which magnetic field is changing

Case I (r < R) (inside) :



E(2πr) = $\frac{AdB}{dt}$

E(2πr) = $\pi r^2 \left(\frac{dB}{dt}\right)$

$\boxed{E = \frac{r}{2}\frac{dB}{dt}}$ ($E_{inside} \propto r$)

Case I (r < R) (inside) :

$$E(2\pi r) = A\left(\frac{dB}{dt}\right)$$

$$E(2\pi r) = \pi R^2\left(\frac{dB}{dt}\right) \Rightarrow \boxed{E = \frac{R^2}{2r}\frac{dB}{dt}} \quad (E_{out} \propto \frac{1}{r})$$

Case III (r = R) surface :

$$E(2\pi R) = \pi R^2\left(\frac{dB}{dt}\right) \Rightarrow \boxed{E = \frac{R}{2}\frac{dB}{dt}}$$

BEGINNER'S BOX–3

1. Flux linked through following coils changes with respect to time then for which coil an e.m.f. is not induced :–

(1) Copper coils (2) Wood coil (3) Iron coil (4) None

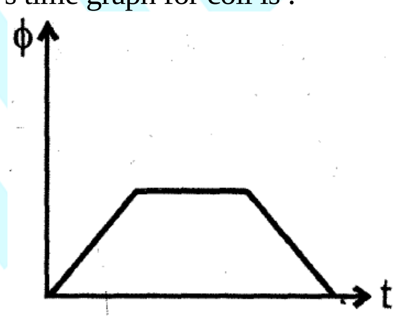
2. A coil and a magnet moves with their constant speeds 5 m/sec. and 3 m/sec. respectively, towards each other, then induced emf in coil is 16 mV. If both are moves in same direction, then induced emf in coil:–

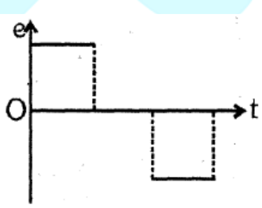
(1) 15 mV (2) 4 mV (3) 64 mV (4) Zero

3. Magnetic flux ϕ (in Weber) linked with a closed circuit of resistance 10 ohm varies with time t (in seconds) as $\phi = 5t^2 - 4t + 1$. The induced emf in the circuit at t = 0.2 sec. is :–

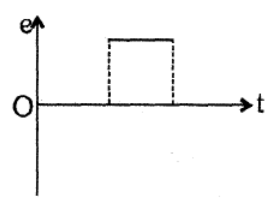
(1) 0.4 V (2) – 0.4 V (3) – 2.0 V (4) 2.0 V

4. Magnetic flux linked through the coil changes with respect to time according to following graph, then induced emf v/s time graph for coil is :–

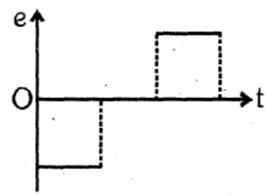


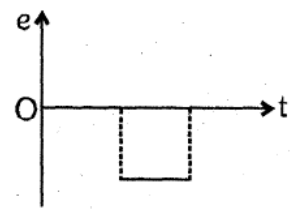


(1)



(2)

(3) 

(4) 

5. The radius of a circular coil having 50 turns is 2 cm. Its plane is normal to the magnetic field. The magnetic field changes from 2T to 4T in 3.14 sec. The induced emf in coil will be :–

(1) 0.4V (2) 0.0V (3) 4 mV (4) 0.12 V

6. Magnetic field changes at the rate of 0.4 T/sec. in a square coil of side 4 cm. kept perpendicular to the field. If the resistance of the coil is $2 \times 10^{-3}\ \Omega$, then induced current in coil is :–

(1) 0.16 A (2) 0.32 A (3) 3.2 A (4) 1.6 A

7. A short bar magnet allowed to fall along the axis of horizontal metallic ring. Starting from rest, the distance fallen by the magnet in one second may be :–

(1) 4.0 m. (2) 5.0 m. (3) 6.0 m. (4) 7.0 m.

8. In a circuit a coil of resistance 2Ω, then magnetic flux changes from 2.0Wb to 10.0Wb in 0.2 sec. The charge flow in the coil during this time is :

(1) 5.0 C (2) 4.0 C (3) 1.0 C (4) 0.8 C

9. A circular loop of radius 2 cm, is placed in a time varing magnetic field with rate of 2T/sec. Then induced electric field in this loop will be :–

(1) 0 (2) 0.02 V/m (3) 0.01 V/m (4) 2 V/m

TYPES OF E.M.I

For a loop flux, ($\phi = BA\cos\theta$) changes w.r.t. time in following three manner and according to it electro magnetic induction is classified in three ways :–

(i) If $(A, \theta) \rightarrow$ const & $\frac{dB}{dt} \rightarrow \frac{d\phi}{dt} \Rightarrow$ **Static EMI** → (1) Self Induction (In this case EMI occurs for rest coil) / (2) Mutual Induction

(ii) If $(B, \theta) \rightarrow$ const & $\frac{dA}{dt} \rightarrow \frac{d\phi}{dt} \Rightarrow$ Dynamic EMI (In this case EMI occurs for a moving straight wire)

(iii) If $(A, B) \rightarrow$ const & $\frac{d\theta}{dt} \rightarrow \frac{d\phi}{dt} \Rightarrow$ Periodic E.M.I. (In this case E.M.I. occurs for a rotating coil)

STATIC E.M.I. $\Rightarrow \frac{dI}{dt} \rightarrow \frac{dB}{dt} \rightarrow \frac{d\phi}{dt} \Rightarrow$ Static EMI

SELF INDUCTION

When current through the coil changes, with respect to time then magnetic flux linked with the coil also changes with respect to time. Due to this an emf and a current induced in the coil. According to Lenz law induced current opposes the changes in magnetic flux. This phenomenon is called self induction of coil. Considering this coil circuit in two cases :

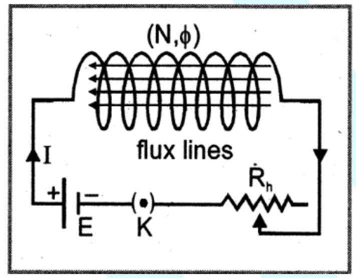
Case I : Current through the coil is constant :–

If $I \rightarrow B \rightarrow \phi \rightarrow$ Const. $\Rightarrow$ No EMI

total flux of coil $(N\phi) \propto$ current through the coil

$N\phi \propto I$

$N\phi = LI$



$\boxed{L = \frac{N\phi}{I} = \frac{NBA}{I} = \frac{\phi_{Total}}{I}}$, Where L : self inductance of coil

S.I. unit of L $\rightarrow 1\,\frac{\text{weber}}{A} = 1$ henry $= 1\frac{N-m}{A^2} = 1\frac{J}{A^2}$ **Dimensions :** $[M^1L^2T^{-2}A^{-2}]$

Sp. Note :– L is constant of coil it does not depends on current through the coil.

Case : Current through the coil changes w.r.t.

If $\frac{dI}{dt} \rightarrow \frac{dB}{dt} \rightarrow \frac{d\phi}{dt} \Rightarrow$ Static EMI

$N\phi = LI$

$-N\frac{d\phi}{dt} = -L\frac{dI}{dt}$, where $-N\frac{d\phi}{dt}$ called self induced emf of coil 'e_S'

$\boxed{e_S = -L\frac{dI}{dt}}$ **S.I. unit of L** $\rightarrow \frac{V-\text{sec}}{A}$

- **Self–inductance of a solenoid**

  Let cross–sectional area of solenoid =A, Current flowing through it = I

  Length of the solenoid = λ, then $\phi = NBA = N\frac{\mu_0 NI}{l}A = \frac{\mu_0 N^2 A}{l}I$

  But $\phi = LI \therefore L = \frac{\mu_0 N^2 A}{l}$ or $L_m = \frac{\mu_0 \mu_r N^2 A}{l}$

  If no iron or similar material is nearby, then the value of self–inductance depends only on the geometrical factors (length, cross–sectional area, number of turns).

ILLUSTRATIONS

Illustrations 6.

The current in a solenoid of 240 turns, having a length of 12 cm and a radius of 2 cm, changes at the rate of 0.8 As^{-1}. Find the emf induced in it.

Solution

$$|\varepsilon| L\frac{dI}{dt} = \frac{\mu_0 N^2 A}{l}\cdot\frac{dI}{dt} = \frac{4\pi\times10^{-7}\times(240)^2\times\pi\times(0.02)^2}{0.12}\times 0.8 = 6\times10^{-4}\ \text{V}$$

Illustration 7.

A soft iron core is introduced in an inductor. What is the effect on the self–inductance of the inductor?

Solution

Since soft iron has a large relative permeability therefore the magnetic flux and consequently the self–inductance is considerably increased.

BEGINNER'S BOX–4

1. The value of self inductance of a coil is 5H. The value of current changes from 1A to 2A in 5 sec., then value of induced emf in it :

(1) 10V (2) 0.1V (3) 1.0V (4) 100V

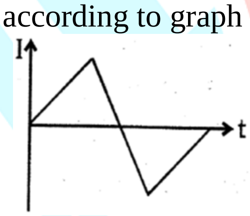
2. A coil of self inductance 2H carries a 2A current. If direction of current is reversed in 1 sec., then induced emf in it :–

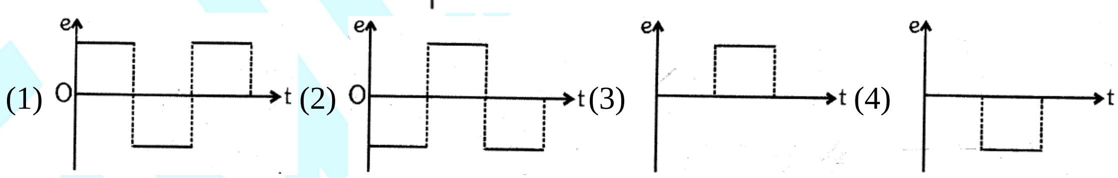
(1) – 8V (2) 8V (3) – 4V (4) Zero

3. For a coil having L = 2mH, current flow through it is $I = t^2e^{-t}$ then the time at which emf becomes zero

(1) 2 sec. (2) 1 sec. (3) 4 sec. (4) 3 sec.

4. Current through the coil varies according to graph then induced emf v/s time graph is





5. A solenoid have the self inductance 2H. If length of the solenoid is doubled having turn density and area constant then new self inductance is :–

(1) 4H (2) 1H (3) 8H (4) 0.5 H

6. A solenoid wound over a rectangular frame. If all the linear dimensions of the frame are increased by a factor 3 and the number of turns per unit length remains the same, the self inductance increased by a factor of :–

(1) 3 (2) 9 (3) 27 (4) 63

7. A coil of inductance 2H has a current of 5.8 A. The flux in weber through the coil is :

(1) 0.29 (2) 2.9 (3) 3.12 (4) 11.6

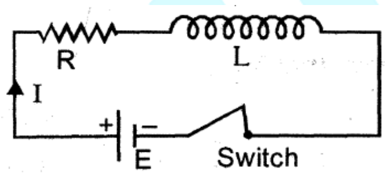
RL D.C Circuit

Case I : Current Growth :

Consider an inductance L and a resistance R (including the resistance of the coil L) connected in series to a battery of emf E. When the switch S is closed, the current in the circuit begins to grow. After the key is closed the current changes from zero to some value. The current rises gradually rather than instantly. It takes some time before the current reaches its steady value $I_0 = E/R$. The effect of the inductance in a dc circuit is to increase the time taken by the current to reach its limiting value I_0.

At any instant, Kirchoff's voltage law for the loop gives

$$E - L = \frac{dI}{dt} = RI$$



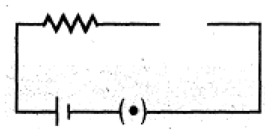
On rearranging the equation, we get

$$\frac{dI}{\frac{E}{R} - I} = \frac{R}{L} dt$$

On integrating both the sides we get

$$\boxed{I = I_0(1 - e^{-t/\lambda})} \text{ where } I_0 = \frac{E}{R} \text{ and } \lambda = \frac{L}{R}$$

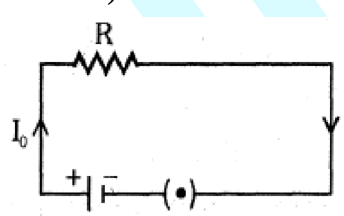
(i) Just after the closing of the key inductance behaves like open circuit and current in circuit is zero.



(Open circuit, t = 0, I = 0)

(Inductor provided infinite resistance)

(ii) Some time after closing of the key inductance behaves like simple connecting wire (short circuit) and circuit is constant.



(Shot circuit, $t \to \infty$, $I \to I_0$)

(Inductor provide zero resistance)

$$\boxed{I_0 = \frac{E}{R}}$$ (Final, steady, maximum or peak value of current)

Sp. Note : Peak value of current in circuit does not depends on self inductance of coil.

(iii) Time constant of circuit (λ) : $\boxed{\lambda = \frac{L}{R}}$ Its SI unit is second(s)

It is a time in which current increases up to 63% or 0.63 times of peak current value.

(iv) Half life (T) – It is a time in which current increases upto 50% or 0.50 times of peak current value.

$$I = I_0\,(1 - e^{-t/\lambda})$$

$t = T,\ I = I_0/2\}$ $$\frac{I_0}{2} = I_0\,(1 - e^{-T/\lambda}) \Rightarrow e^{-T/\lambda} = \frac{1}{2} \Rightarrow e^{T/\lambda} = 2$$

$$\frac{T}{\lambda}\log_e e = \log_e 2$$

$$\boxed{\begin{array}{l} T = 0.693\lambda \\ T = 0.693\dfrac{L}{R}\,\text{sec} \end{array}}$$

(v) Rate of growth of current at any instant :–

$$\boxed{\left(\frac{dI}{dt}\right) = \frac{E}{L}\left(e^{-t/\lambda}\right)}$$

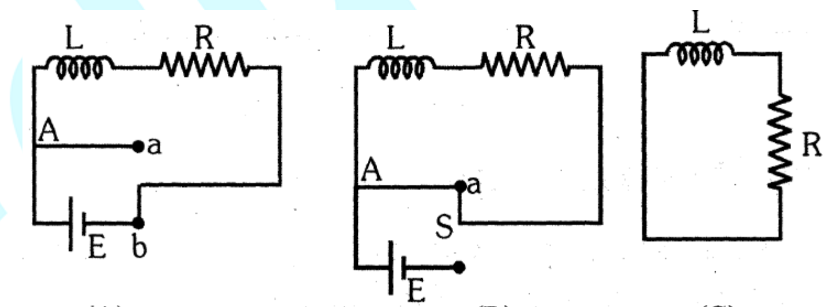
$$t = 0 \Rightarrow \left(\frac{dI}{dt}\right)_{max} = \frac{E}{L}$$

$$t \to \infty \Rightarrow \left(\frac{dI}{dt}\right)_{min} \to 0$$

Sp. Note : Maximum or initial value of rate of growth of current does not depends upon resistance of coil.

Case–II : Current Decay :

Consider the arrangement shown in figure (A). The sliding switch S can be slided up and down. Let the switch S connected the point b. The circuit is complete and a steady current $i = I_0$ is maintained through the circuit. Suddenly at t = 0, the switch S is moved to connect the point a. This completes the circuit through the wire Aa and disconnects the battery from the circuit [Figure (B)]. The special arrangement of the switch ensures that the circuit through the wire Aa is completed before the battery is disconnected. (Such a switch is called make before break switch). The equivalent circuit is redrawn in figure (C).



(A) (B) (C)

As the battery is disconnected, the current decreased in the circuit. This induced as emf is the inductor.

As that is only emf in the circuit, we have

$$-L\frac{dI}{dt} = RI \qquad \text{or} \qquad \frac{dI}{I} = -\frac{R}{L}dt$$

on integrating both the sides, we get

$$\boxed{I = I_0 e^{\frac{-Rt}{L}} = I_0 e^{-t/\lambda}}$$

where λ = L/R is the time constant of the circuit.

(Just after opening of key) $t = 0 \Rightarrow I = I_0 = \dfrac{E}{R}$

(Some time after opening of key) $t \to \infty \Rightarrow I \to 0$

(i) **Time constant (λ) :** – It is a time in which current decreases up to 37% or 0.37 times of peak current value.

$$\boxed{\lambda = \frac{L}{R}\,\text{sec}}$$

(ii) **Half life (T) :–** It is a time in which current decreases upto 50% or 0.50 times of peak current value.

$$\boxed{T = (0.693)\lambda\,\text{sec}}$$

(iii) **Rate of decay of current at any instant :–**

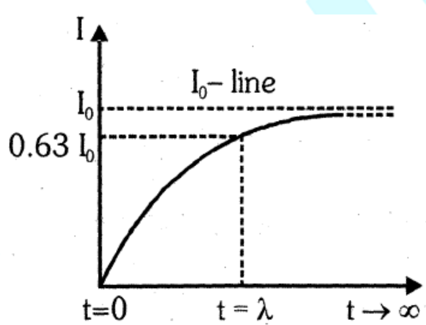
$$\boxed{\left(-\frac{dI}{dt}\right) = \left(\frac{E}{L}\right)e^{-t/\lambda}}$$

$\rightarrow$ $t = 0 \Rightarrow \left(-\dfrac{dI}{dt}\right)_{max.} = \dfrac{E}{L}$

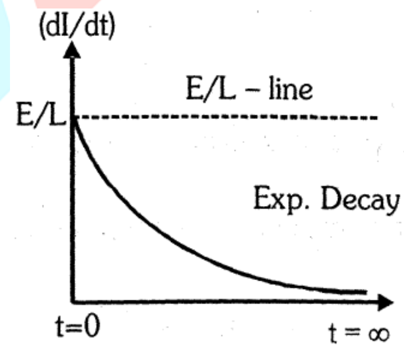
$\rightarrow$ $t \to \infty \Rightarrow \left(-\dfrac{dI}{dt}\right)_{min} \to 0$

Special graph for R–L circuit :–

- **Current Growth :–**

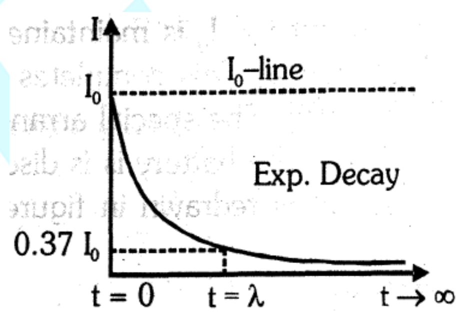


(a)

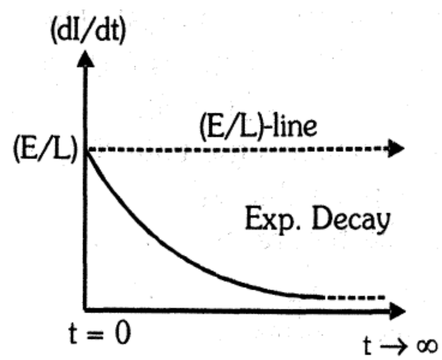


(b)

- **Current decay : –**



(a)

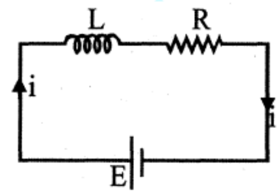


(b)

ENERGY STORED IN AN INDUCTOR

The battery that estabilishes the current in an inductor has to do work against opposing induced emf. The energy supplied by the battery is stored in the inductor.

let I be the instantaneous current in the LR circuit. Applying Kirchoff's voltage law, we get



$$E = iR + L\frac{di}{dt}$$

The instantaneous power supplied by the battery is given by

$$\boxed{P = Ei = i^2R + Li\frac{di}{dt}}$$

where i^2R is the power dissipated in the resistor and the last term represent the rate average energy U is being supplied to the inductor. That is,

$$\frac{dU}{dt} = Li\frac{di}{dt} \quad \text{or} \quad dU = Li\,di$$

The total energy stored when the current increases from 0 to I is found by integration,

$$U = L\int_0^1 idi = \frac{1}{2}LI^2 \Rightarrow \boxed{U = \frac{1}{2}LI^2}$$

Energy Density of Inductor

For a solenoid the inductance is given as

$L = \mu_0 n^2 IA$

Since $B = \mu_0 nI$, therefore, $L = \dfrac{B^2 A l}{\mu_0 I^2}$

Thus, the energy stored in the solenoid is

$$U = \frac{1}{2}LI^2 = \frac{B^2}{2\mu_0}(A\,l)$$

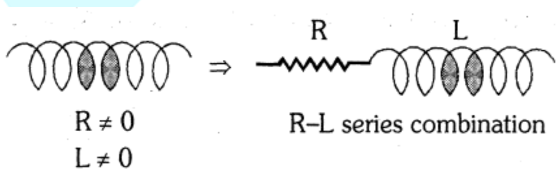
Since Al is the volume of the solenoid therefore energy stored per unit volume = $\dfrac{U}{Al} = \dfrac{B^2}{2\mu_0}$

Although we derived this result for a special case i.e. the solenoid but it is true in general.

Thus the energy density of a magnetic field in free space = $\dfrac{B^2}{2\mu_0}$.

GOLDEN KEY POINTS

- Thin wire ___________ $R \neq 0$ & $L = 0 \Rightarrow$  Role of R $\rightarrow$ to opposes flow of current, now this wire moulded in form of coil.



Role of L $\rightarrow$ to opposes changes in current, if current becomes constant, then no role of 'L'

Sp. Note **:** Resistance is possible without inductance but inductance is not possible without resistance.

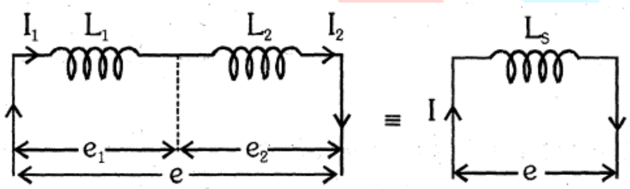
- If w.r.t. $I \uparrow \Rightarrow \frac{dI}{dt}(+ve) \Rightarrow e_S$ (–Ve) opposite emf $\Rightarrow \boxed{E_{net} = E - e_S}$
- If w.r.t $I \downarrow \Rightarrow \frac{dI}{dt}(-ve) \Rightarrow e_S$ (+Ve) same directed emf $\Rightarrow \boxed{E_{net} = E + e_S}$
- **Current variation with key :–**

  (a) Just closing of key $\Rightarrow I \uparrow = dI\ (+ve) \Rightarrow e_S\ (-ve)$

  (b) At the time of sudden opening of key, due to high inductance of circuit a high momentarily emf (surge) induces and sparking occurs at key position. To avoid sparking a capacitor is connected parallel to the key.
- Self inductance always opposes the change of current in electric circuit so it is also caled inertia of electric circuit.

Combination of Inductance :

(a) Series fashion :



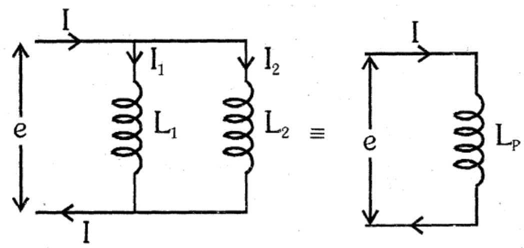
Potential devides, $e = e_1 + e_2$

$$L_S \frac{dI}{dt} = L_1 \frac{dI_1}{dt} + L_2 \frac{dI_2}{dt} \qquad \left(\text{as } e = -L\frac{dI}{dt}\right)$$

Current remains same $I = I_1 = I_2$ i.e. $\frac{dI}{dt} = \frac{dI_1}{dt} = \frac{dI_2}{dt}$; $\boxed{L_S = L_1 + L_2}$

(b) Parallel fashion :

Current devides, $I = I_1 + I_2$



$$\frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt}$$

$$\frac{e}{L_P} = \frac{e_1}{L_1} + \frac{e_2}{L_2} \left[\text{as } e = -L\frac{dI}{dt} \text{ i.e. } \frac{dI}{dt} = -\frac{e}{L}\right]$$

Potential remains same, $e = e_1 = e_2$

$$\frac{1}{L_P} = \frac{1}{L_1} + \frac{1}{L_2} \Rightarrow L_P = \boxed{\frac{L_1 L_2}{L_1 + L_2}}$$

ILLUSTRATIONS

Illustration 8.

An electromagnet has stored 648 J of magnetic energy, when a current of 9A exists in its coils. What average emf is induced if the current is reduced to zero in 0.45 s ?

Solution :

Magnetic energy $U = \frac{1}{2}LI^2 \Rightarrow L = \frac{2U}{I^2} = \frac{2 \times 648}{9 \times 9} = 16H$

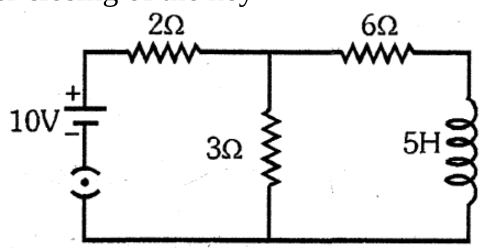
Induced emf $e = L\left(\frac{\Delta I}{\Delta t}\right) = (16)\left(\frac{9}{0.45}\right) = 320V$

Illustration 9.

Calculate current, which given by battery for the following circuit.

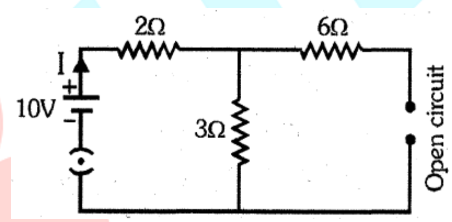
(a) Just after closing of the key.

(b) Some time after closing of the key



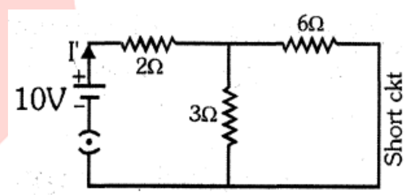
Solution :

(a) Just after closing of the key :–



$$\text{Current } I = \frac{E}{r_{net}} = \frac{10}{2+3} = 2A$$

(b) Some time after Closing of the key :–



$$\text{Current } I' = \frac{E}{r_{net}} = \frac{10}{4} = 2.5A,$$

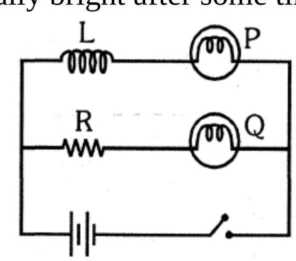
$$\text{Where } r_{net} = 2 + \frac{3\times 6}{3+6}$$

Illustration 10.

Figure shows an inductor L, a resistor R connected in parallel to a battery through a switch. The resistance of resistance R is same as that of the coil that makes L. Two identical bulb are put in each arm of the circuit.

(a) Which of two bulbs lights up earlier when S is closed?

(b) Will the bulbs be equally bright after some time?



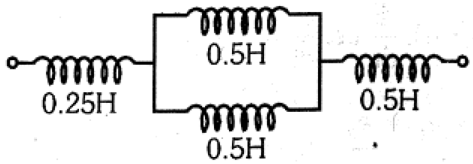
Solution :

(i) When switch is closed induced e.m.f. in inductor i.e. back e.m.f. delays the glowing of bulb P so bulb Q light up earlier.

(ii) Yea. At steady state inductance effect becomes meaningless so both identical bulbs become equally bright after some time.

Illustration 11.

Three inductances are connected as shown in figure. Find the equivalent inductance of circuit.



Solution :

$$\text{Lequivalent} = 0.25 + \left(\frac{0.5 \times 0.5}{0.5 + 0.5}\right) + 0.5 \ = 1\text{H}$$

BEGINNER'S BOX–5

1. L, C and R respectively indicate inductance, capacitance and resistance. Select the combination, which does not have dimensions of frequency :–

(1) 1/RC (2) R/L (3) $1/\sqrt{LC}$ (4) C/L

2. A coil of 10 H inductance and 5 Ω resistance is connected to 5 volt battery in series. The current in ampere in circuit 2 seconds after switched is on :–

(1) e^{-1} (2) $(1-e^{-1})$ (3) (1–e) (4) e

3. An L–R circuit consists of an inductance of 8mH and a resistance of 4Ω. The time constant of the circuit is :–

(1) 2ms (2) 12ms (3) 32ms (4) 500 s

4. In an L–R circuit, time constant is that time in which current grows from zero to the value (Where I_0 is the steady current) :

(1) 0.63 I_0 (2) 0.50 I_0 (3) 0.37 I_0 (4) I_0

5. An inductor of 20 H and a resistance of 10 Ω, are connected to a battery of 5 volt in series, then initial rate of change of current is :–

(1) 0.5 amp/s (2) 2.0 amp/s (3) 2.5 amp/s (4) 0.25 amp/s

6. A coil of L = 5 × 10^{-3} H and R = 18 Ω is abruptly supplied a potential of 5 volts. What will be the rate of change of current in 0.001 second ? ($e^{-3.6}$ = 0.0273)

(1) 27.3 amp/sec. (2) 27.8 amp/sec. (3) 2.73 amp/sec. (4) 2.78 amp/sec.

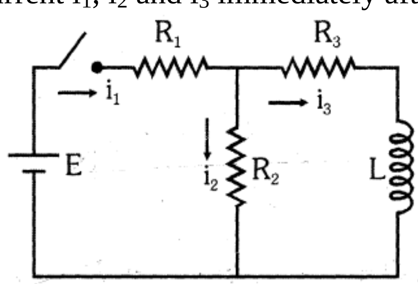
7. A coil of inductance 8.4 mH and resistance 6Ω is connected to a 12V battery in series. The current in the coil is 1.0A at approximately the time :–

(1) 500s (2) 20s (3) 35ms (4) 1ms

8. The dimensions of combination $\frac{L}{CVR}$ are same as dimensions of :–

(1) Change (2) Current (3) Charge^{-1} (4) Current^{-1}

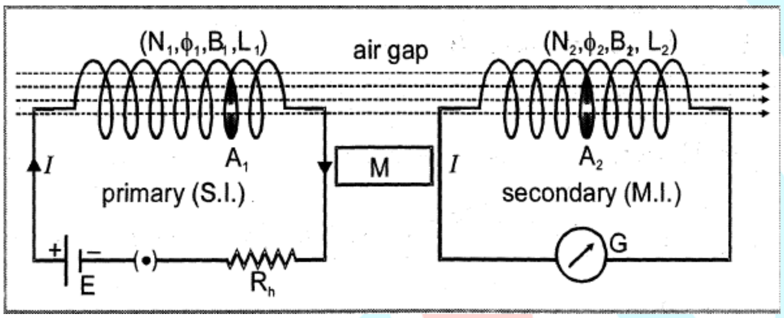
9. In the circuit shown in adjoining fig E = 10V, R_1 = 1Ω, R_2 = 2Ω, R_3 = 3Ω and L = 2H. Calculate the value of current i_1, i_2 and i_3 immediately after key S is closed :–



(1) 3.3 amp, 3.3 amp, 3.3 amp (2) 3.3 amp, 3.3 amp, 0 amp
(3) 3.3 amp, 0 amp, 0 amp (4) 3.3 amp, 3.3 amp 1.1

Mutual Induction :

Basic Concept : Whenever current passing through primary coil or circuit change with respect to time then magnetic flux in neighbouring secondary coil or circuit will also changes with respect to time. According to Lenz Law for opposition of flux change an emf and a current induced in the neighbouring coil or circuit. This phenomenon called as 'Mutual induction'.



Due to Air gap, $\phi_2 < \phi_1$ always and $\phi_2 = B_1A_2$ ($\theta = 0°$)

Case–I : When current through primary is constant :

Total flux of secondary is directly proportional to current flow through the primary coil

$N_2\,\phi_2 \propto I_1$

$N_2\,\phi_2 = MI_1$

$$\boxed{M = \frac{N_2\phi_2}{I_1} = \frac{N_2B_1A_2}{I_1} = \frac{(\phi_T)_s}{I_p}}$$, Where M : mutual inductance of circuits.

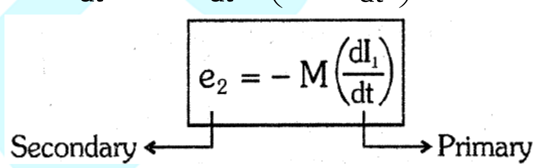
- The units and dimension of M are same as 'L'.
- The mutual inductance does not depends upon current through the primary and it is constant for both circuits.

Case–II : When current through primary changes w.r.t. :

If $\frac{dI_1}{dt} \rightarrow \frac{dB_1}{dt} \rightarrow \frac{d\phi_1}{dt} \rightarrow \frac{d\phi_2}{dt} \Rightarrow$ Static EMI

$N_2f_2 = MI_1$

$-N_2\frac{d\phi_2}{dt} = -M\frac{dI_1}{dt}$, $\left(-N_2\frac{d\phi_2}{dt}\right)$ called total mutual induced emf of secondary coil e_2.

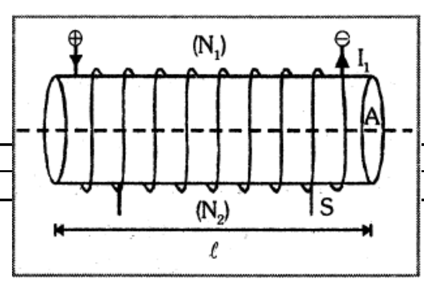


Different mutual inductances :–

(a) In terms of their number of turns (b) In terms of their self inductances

(a) In terms of their number of turns (N_1, N_2) :–

(1) Two co–axial solenoids $(M_{S_1S_2})$ **:–**

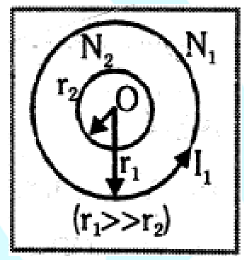


$$M_{S_1S_2} = \frac{N_2B_1A}{I_1}$$

$$= \frac{N_2}{I_1}\left(\frac{\mu_0 N_1 I_1}{l}\right)A\text{, where } B_1 = \frac{\mu_0 N_1 I_1}{l}$$

$$\Rightarrow \boxed{M_{S_1S_2} = \left(\frac{\mu_0 N_1 N_2 A}{l}\right)}$$

(2) Two concentric and coplanar coils $(M_{C_1C_2})$ **:–**



$$M_{C_1C_2} = \frac{N_2B_1A_2}{I_1}\text{, where } B_1 = \frac{\mu_0 N_1 I_1}{2r_1} \ \& \ A_2 = \pi r_2^2$$

$$M_{C_1C_2} = \frac{N_2}{I_1}\left(\frac{\mu_0 N_1 I_1}{2r_1}\right)(\pi r_2^2) \Rightarrow \boxed{M_{C_1C_2} = \frac{\mu_0 N_1 N_2 \pi r_2^2}{2r_1}}$$

(b) In terms of their self inductances (L_1, L_2) :–

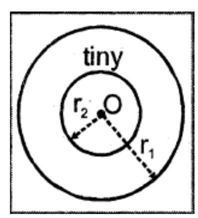
For two magnetically coupled coils :–

$\boxed{M = K\sqrt{L_1L_2}}$, where 'K' is coupling factor between two coils and its range $0 \le K \le 1$

- For ideal coupling $K_{max} = 1$ $\Rightarrow$ $\boxed{M_{max} = \sqrt{L_1L_2}}$ (Where M is geometrical mean of L_1 & L_2)
- For real coupling $(0 < K < 1) \Rightarrow \boxed{M = K\sqrt{L_1L_2}}$
- Value of coupling factor 'K' decides from fashion of coupling.

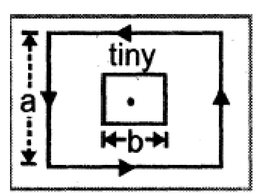
GOLDEN KEY POINTS

- Two concentric and coplanar loop :–



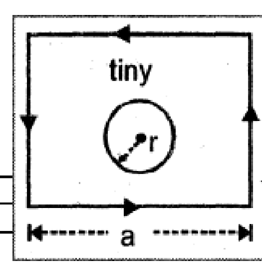
$\boxed{M \propto \frac{r_2^2}{r_1}}$ $(r_1 >> r_2)$

- Two concentric and coplanar square loops :–



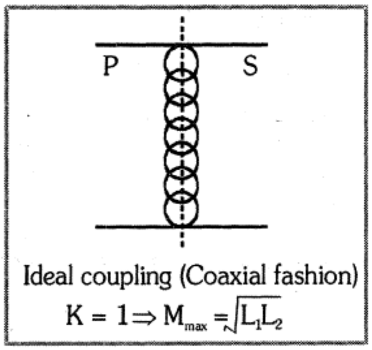
$\boxed{M \propto \frac{b^2}{a}}$

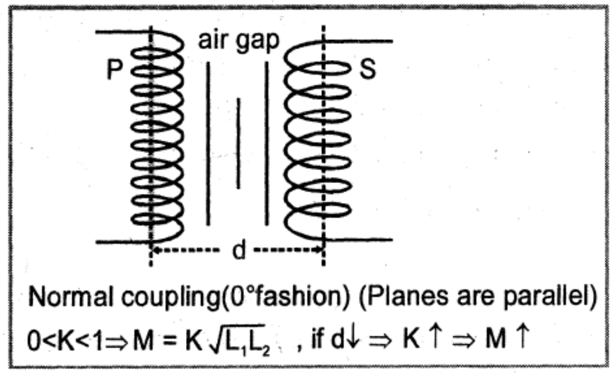
- A square and a circular concentric and coplanar loop :–

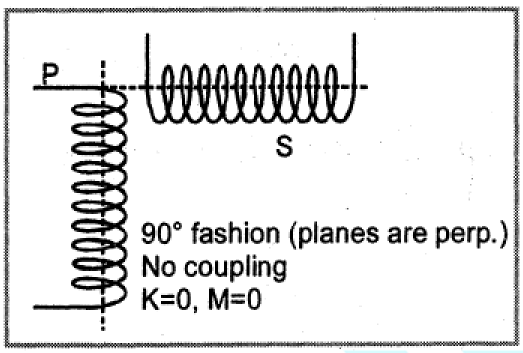


$$\boxed{M \propto \frac{r^2}{a}}$$

- **Different fashions of coupling :–**







- 'K' also defined as $k = \frac{\phi_s}{\phi_p} = \frac{\text{mag. flux linked with Secondary}}{\text{mag. flux linked with Primary}}$

'M' depends on :–
Number of turns (N_1, N_2)
Self inductances (L_1, L_2)
Area of cross section
Magnetic permeability of medium (μ_r)
Distance between two coils (As d↓ = M↑)
Orientation between two coils
Coupling factor 'K' between two coils.

ILLUSTRATIONS

Illustration 12.

A solenoid has 2000 turns wound over a length of 0.3 m. The area of cross–section is 1.2×10^{-3} m^2. Around its central section a coil of 300 turns is closely wound. If an initial current of 2A is reversed in 0.25 s, find the emf induced in the coil.

Solution :

$$M = \frac{\mu_0 N_1 N_2 A}{l} = \frac{4\pi \times 10^{-7} \times 2000 \times 300 \times 1.2 \times 10^{-3}}{0.3} = 3 \times 10^{-3} \text{ H}$$

$$\varepsilon = -M\frac{\Delta I}{\Delta t} = -3\times10^{-3}\left[\frac{-2-2}{0.25}\right] = 48 \times 10^{-3}\ V = 48\ mV$$

Illustration 13.

On a cylindrical rod two coils are wound one above the other. What is the coefficient of mutual induction if the inductance of each coil is 0.1 H?

Solution :

One coil is wound over the other and coupling is tight, so K = 1,

$$M = \sqrt{L_1L_2} = \sqrt{0.1\times0.1} = 0.1H$$

Illustration 14.

How does the mutual inductance of a pair of coils change when :

(i) the distance between the coils is increased?

(ii) the number of turns in each coil is decreased ?

(iii) a thin iron rod is placed between the two coils, other factors remaining the same? Justify your answer in each case.

Solution

(i) The mutual inductance of two coil, decreases when the distance between them is increased. This is because the flux passing from one coil to another decreases.

(ii) Mutual inductance $M = \frac{\mu_0 N_1 N_2 A}{l}$ i.e., $M \propto N_1 N_2$

Clearly, when the number of turns N_1 and N_2 in the two coils is decreased, the mutual inductance decrease.

(iii) When an iron rod is placed between the two coils the mutual inductance increases, because $M \propto$ permeability (μ)

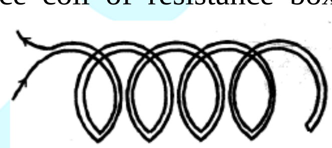
Illustration 15.

A coil is wound on an iron core and looped back on itself so that the core has tow sets of closely would wires in series carrying current in the opposite sense. What do you expect about its self–inductance? Will it be larger or small?

Solution :

As the two sets of wire carry in opposite directions, their induced emf's also act in opposite direction. These induced emf's tend to cancel each other, making the self–inductance of the coil very small.

Note : Resistance coil of resistance box, wound in two layer in opposite manner. The self inductance of coil becomes negligible.



$R \neq 0$

$L \approx 0$ (Non inductive resistance)

BEGINNER'S BOX–6

1. The mutual inductance between a primary and secondary circuits is 0.5H. The resistance of the primary and the secondary circuits are 20Ω and 5Ω respectively. To generate a current of 0.4A in the secondary, current in the primary must be changed at the rate of :–

(1) 4.0 A/s (2) 16.0 A/s (3) 1.6 A/s (4) 8.0 A/s

2. Two coils A and B having turns 300 and 600 respectively are placed near each other, on passing a current of 3.0 ampere in A, the flux linked with A is 1.2×10^{-4} weber and with B it is 9.0×10^{-5} weber. The mutual inductance of the system is :–

(1) 2×10^{-5} H (2) 3×10^{-5} H (3) 4×10^{-5} H (4) 6×10^{-5} H

3. If the current in a primary circuits is $I = I_0 \sin \omega t$ and the mutual inductance is M, then the value of induced voltage in secondary circuit will be :–

(1) $e = MI_0\omega \cos \omega t$ (2) $e = -MI_0\omega \cos \omega t$ (3) $e = [M\omega \cos \omega t]/I_0$ (4) $e = -(M\omega \cos \omega t)/I_0$

4. An a.c. of 50 Hz and 1 A peak value flows in primary coil transformer whose mutual inductance is 1.5 H. Then peak value of induced emf in secondary is :–

(1) 150 V (2) 150 πV (3) 300 V (4) 200 V

5. The number of turn of primary and secondary coil of a transformer is 5 and 10 respectively and the mutual inductance is 25 H. If the number of turns of the primary and secondary is made 10 and 5, then the mutual inductance of the coils will be :–

(1) 6.25 H (2) 12.5 H (3) 25 H (4) 50 H

6. The length of a solenoid is 0.3m and the number of turns is 2000. The area of cross–section of the solenoid is 1.2×10^{-3} m^2. Another coil of 300 turns is wrapped over the solenoid. A current of 2A is passed through the solenoid and its direction is changed in 0.25 sec. then the induced emf in coil :–

(1) 4.8×10^{-2} V (2) 4.8×10^{-3} V (3) 3.2×10^{-4} V (4) 3.2×10^{-2} V

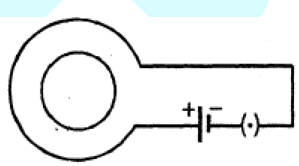
7. Two conducting loops of radii R_1 and R_2 are concentric and are placed in the same plane. If $R_1 > R_2$, the mutual inductance M between them will be directly proportional to :–

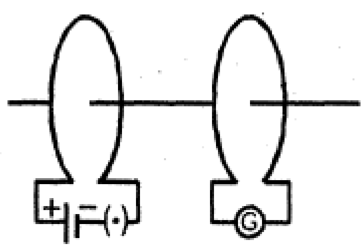
(1) R_1/R_2 (2) R_2/R_1 (3) R_1^2/R_2^2 (4) R_2^2/R^1

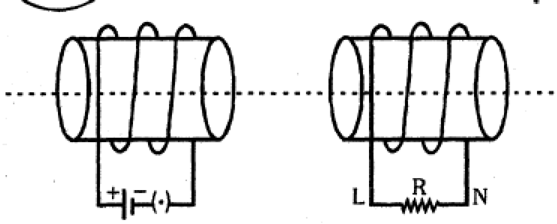
8. Find direction of induced current in secondary circuit for the following changes in primary circuit :–

(a) Key is just closed (b) Some time after the closing of key

(c) Key is just opened

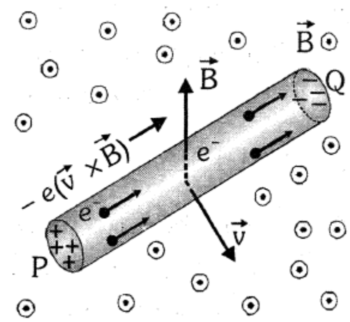
(i) 

(ii) 

(iii) 

DYNAMIC E.M.I. $\left[\frac{dA}{dt} \rightarrow \frac{d\phi}{dt}\right]$

Motional emf from Lorentz force



A conductor PQ is placed in a uniform magnetic field B, directed normal to the plane of paper outwards. PQ is moved with a velocity v, the free electrons of PQ also move with the same velocity. The electron experience a magnetic Lorentz force, $F_m = -e(\vec{v}\times\vec{B})$. According to Fleming's left hand rule, this force acts in the direction PQ and hence the free electrons will move towards Q. A negative charge accumulates at Q and a positive charge at P. An electric field E is setup in the conductor from P to Q. Force exerted by electric field on the free electrons is, $\vec{F}_e = -e\vec{E}$

The accumulation of charge at the two ends continues till these two forces balance each other.

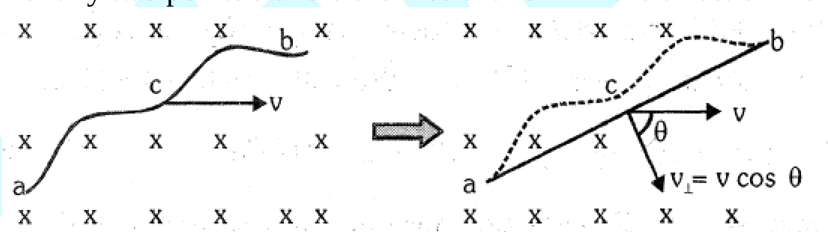
So $\vec{F}_m = -\vec{F}_e \Rightarrow e(\vec{v}\times\vec{B}) = -e\vec{E} \Rightarrow \vec{E} = -(\vec{v}\times\vec{B})$

The potential difference between the ends P and Q is $V = -\vec{E}.\vec{l} = (\vec{v}\times\vec{B}).\vec{l}$. It is the magnetic force on the moving free electrons that maintains the potentials difference and produces the emf $\varepsilon = Blv$ (for $\vec{B}\perp\vec{l}$)

As this emf is produced due to the motion of a conductor, so it is called a motional emf.

The concept of motional emf for a conductor can be generalized for any shape moving in any magnetic field uniform or not. For an element $d\vec{l}$ of conductor the contribution de to the emf is the magnitude dl multiplied by the component of $\vec{v}\times\vec{B}$ parallel to $d\vec{l}$, that is $de = (\vec{v}\times\vec{B}).d\vec{l}$

For any two points a and b the motional emf in the direction from b to a is,

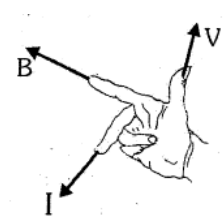


$$e = \int_b^a (\vec{v}\times\vec{B}).d\vec{\ell}$$

Motion emf in wire acb in a uniform magnetic field is the motional emf in an imaginary wire ab. Thus, $e_{acb} = e_{ab}$ = (length of ab) ($v_\perp$) (B), $v_\perp$ = the component of velocity perpendicular to both $\vec{B}$ and ab. From right hand rule : b is at higher potential and a at lower potential. Hence, $V_{ba} = V_b - V_a$ = (ab) (v cos θ) (B)

- **Direction of induced current or HP end of the rod find out with the help (HP → Higher potential) of**

(i) Fleming right hand rule



Fore finger → In eternal field $\vec{B}$ direction.

Thumb → In the direction of motion $(\vec{v})$ of conductor.

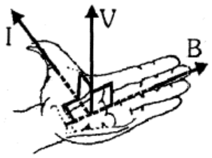
Middle finger → It indicates HP end of conductor/direction of induced current.

(ii) Left hand palm rule

Fingers → In external field $(\vec{B})$ direction.

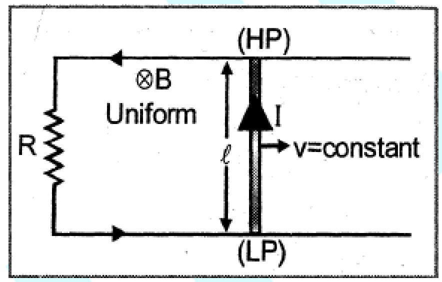
Palm → In direction of motion $(\vec{v})$ of conductor.

Thumb → It indicates HP end of conductor/direction of induced current in conductor.



- **Motion of straight conductor in horizontal plane**

For the given circuit, If metal rod moves with uniform velocity 'v' by an external agent.



- Induced emf in circuit $\boxed{e = Bvl}$
- Current flows through circuit $\boxed{I = \frac{e}{R} = \frac{Bvl}{R}}$
- Retarding opposing force exerted on metal rod by action of induced current

$$\vec{F}_m = I(\vec{l} \times \vec{B}) \quad \Rightarrow \quad F_m = BIl, \quad \text{where } \theta = 90° \quad \boxed{F_m = \frac{B^2 l^2 v}{R}}$$

- External mechanical force required for uniform velocity of metal rod.
For constant velocity resultant force on metal rod must zero and for that $F_{ext} = F_m$

$$\boxed{F_{ext.} = F_m = \frac{B^2 l^2 v}{R}} \Rightarrow \text{If } (B, l, R) \to \text{const. } \boxed{F_{ext.} \propto v}$$

- For uniform motion of metal rod, The rate of doing mechanical work by external agent or mechanical power delivered by external source given as :–

$$P_{mech} = P_{ext} = \vec{F}_{ext}.\vec{v} = F_{ext} v$$

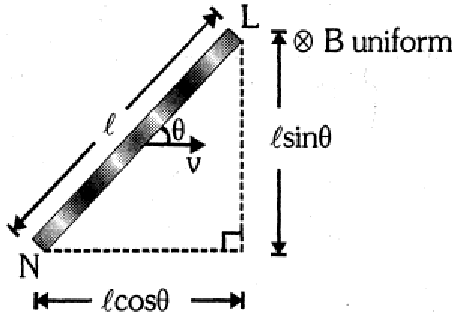
$$\boxed{p_{ext.} = p_m = \frac{B^2 l^2 v^2}{R}} \Rightarrow \text{If } (B.l, R) \to \text{const.} \Rightarrow \boxed{p_{mech.} \propto v^2}$$

- Rate of heat dissipation across resistance or thermal power developed across resistance is :–

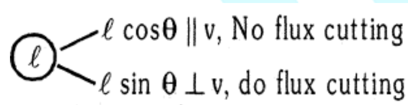
$$P_{th} = I^2 R = \frac{1}{R}\left(\frac{Bvl}{R}\right)^2 \Rightarrow \boxed{p_{th} = \frac{B^2 l^2 v^2}{R}}$$

It is clear that $p_{th} = p_{mech}$ which is consistent with the principle of conservation of energy.

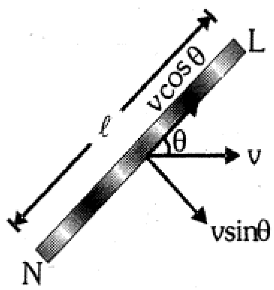
• $\left\{ \begin{array}{l} \vec{l}\ \theta\ \vec{v} \\ \vec{l} \perp \vec{B} \\ \vec{v} \perp \vec{B} \end{array} \right\}$



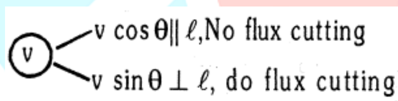
$\boxed{e_d = Bv(l\sin\theta)}$



OR



$\boxed{e_d = B(v\sin\theta)l}$

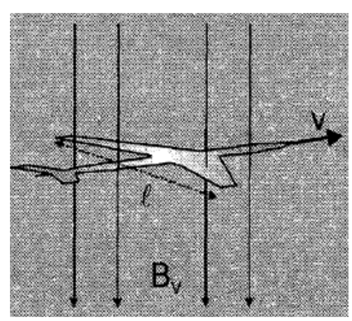


ILLUSTRATIONS

Illustration 16.

An aircraft with a wing span of 40 m flies with a speed of 1080 kmh^{-1} in the eastward direction at a constant altitude in the northern hemisphere, where the vertical component of earth's magnetic field is 1.75×10^{-5} T. Find the emf that develops between the tips of the wings.

Solution



The metallic part between the wing–tips can be treated as a single conductor cutting flux–lines due to vertical component of earth's magnetic field. so emf is induced between the tips of its wings.

Here I = 40 m, $B_V = 1.75 \times 10^{-5}$ T

$$v = 1080\ kmh^{-1} = \frac{1080 \times 1000}{3600} ms^{-1} = 300\ ms^{-1}$$

Illustration 17.

Figure shows a rectangular conducting loop of resistance R, width L, and length b being pulled at constant speed v through a region of width d in which a uniform magnetic field B is set up by an electromagnet. Let L = 40 mm, b = 10 cm, d = 15 cm, R = 1.6 Ω, B = 2.0 T and v = 1.0 m/s

(i) Plot the flux ϕ through the loop as a function of the position x of the right side of the loop.

(ii) Plot the induced emf as a function of the position of the loop.

Solution

(i) When the loop is not in the field :

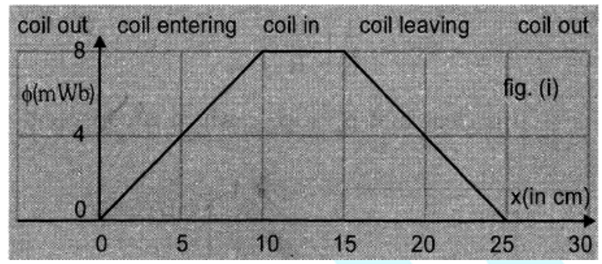
The flux linked with the loop $\phi = 0$

When the loop is entirely in the field :

Magnetic flux linked with the loop

$\phi = BLb = 2 \times 40 \times 10^{-3} \times 10 \times 10^{-2}$

$= 8$ m Wb



When the loop is entering the field :

The flux linked with the loop $\phi = B\,L\,x$

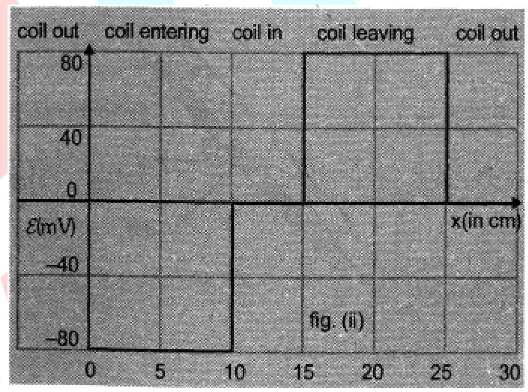
When the loop is leaving the field :

The flux $\phi = B\,L\,[b - (x - d)]$

(ii) Induced emf is $e = -\dfrac{d\phi}{dt} = -\dfrac{d\phi}{dx}\dfrac{dx}{dt} = -\dfrac{d\phi}{dx}v = -$ slope of the curve of figure (i) × v

The emf for 0 to 10 cm :

$$e = -\frac{(8-0)\times 10^{-3}}{(10-0)\times 10^{-2}}\times 1 = -80 \text{ mV}$$



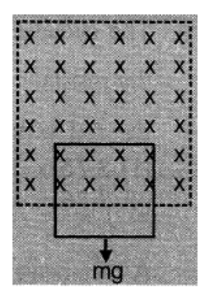
The emf for 10 to 15 cm : $e = 0 \times 1 = 0$

The emf for 15 to 25 cm :

$$e = -\frac{(0-8)\times 10^{-3}}{(25-15)\times 10^{-2}}\times 1 = +80 \text{mV}$$

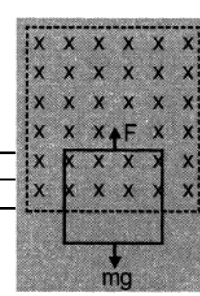
Illustration 18.

A horizontal magnetic field B is produced across a narrow gap between square iron pole–pieces as shown. A closed square wire loop of side l, mass ma and resistance R is allowed of fall with the top of the loop in the field. Show that the loop attains a terminal velocity given by $v = \dfrac{Rmg}{B^2 l^2}$ while it is between the poles of the magnet.



Solution :

As the loop falls under gravity, the flux passing through it decreases and so an induced emf is set up in it. Then a force F which opposes its fall. When this force becomes equal to the gravity force mg, the loop attains a terminal velocity v.



The induced emf e = B v l, and the induced current is $i = \frac{e}{R} = \frac{Bvl}{R}$

The force experienced by the loop due to this current is F = B l i = $\frac{B^2vl^2}{R}$

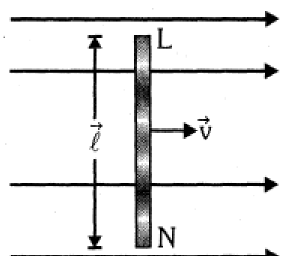
When v is the terminal (constant) velocity F = mg

or $\frac{B^2vl^2}{R} = mg$ or $v = \frac{Rmg}{B^2l^2}$

BEGINNER'S BOX–7

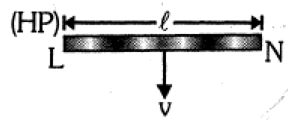
1. Find the Given Parameter when straight conductor moves in external magnetic field :–

(i)



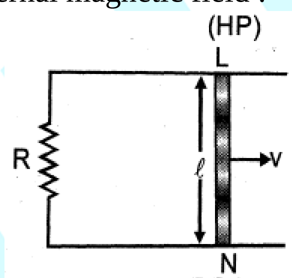
HP end = ?

(ii)



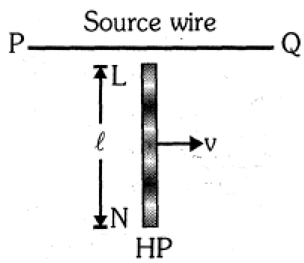
Direction of ext. B = ?

(iii)



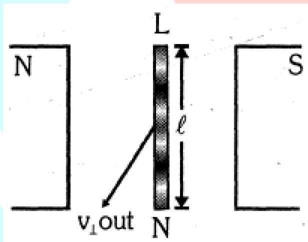
Direction ext. B = ?

(iv)



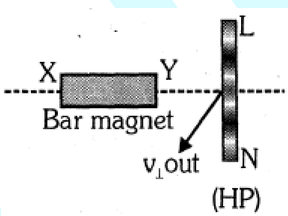
Direction of current in source wire PQ = ?

(v)



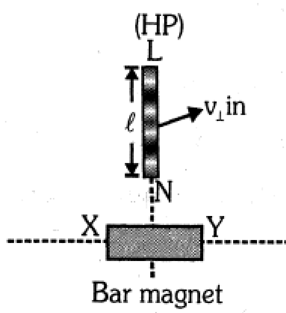
LP end = ?

(vi)



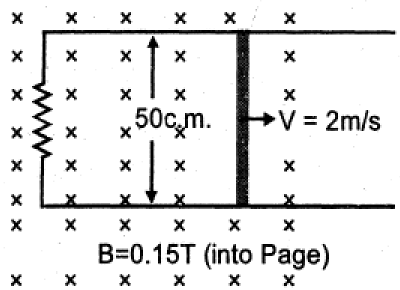
Poles of magnet=?

(vii)



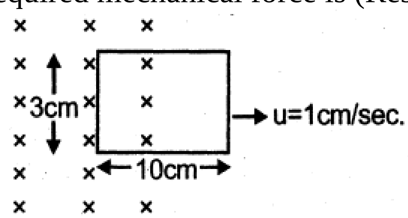
Poles of magnet = ?

2. A metallic rod completes its circuit as shown in the figure. The circuit is normal to a magnetic field of B= 0.15 T. If the resistance of the circuit is 3Ω the force required to move the rod with a constant velocity of 2m/sec. is :



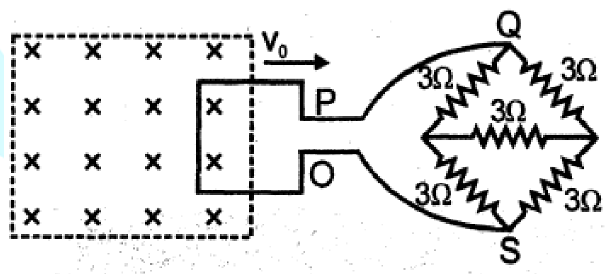
(1) 3.75×10^{-3} N (2) 3.75×10^{-2} N (3) 3.75×10^{2} N (4) 3.75×10^{-4} N

3. A rectangular loop sides 10 cm and 3cm moving out of a region of uniform magnetic field of 0.5 T directed normal to the loop. If we want to move loop with a constant velocity 1 cm/sec. then required mechanical force is (Resistance of loop = 1mΩ) :–



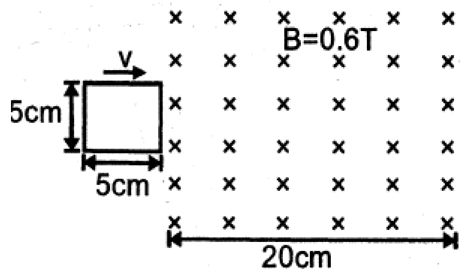
(1) 2.25×10^{-3} N (2) 4.5×10^{-3} N (3) 9×10^{-3} N (4) 1.25×10^{-3} N

4. A metallic square wire loop of side 10 cm and resistance 1 Ω is moved with a constant velocity v_0 in a uniform magnetic field of induction B = 2 T as shown in the figure. The magnetic field perpendicular to the plane of the loop. The loop is connected to a network of resistors each of value 3 ohm. the resistance of the lead wires OS and PQ are negligible. What should be the speed of the loop so as to have a steady current of 1 mA in it? Give the direction of current in the loop.



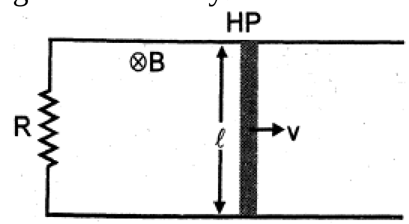
(1) 2×10^{-2} m/sec. , anticlockwise direction
(2) 4×10^{-2} m/sec. , anticlockwise direction
(3) 2×10^{-2} m/sec. , clockwise direction
(4) 4×10^{-2} m/sec. , clockwise direction

5. Figure shows a square loop of side 5 cm being moved towards right at a constant speed of 1cm/sec. The front edge just enters the 20 cm wide magnetic field at t = 0. Find the induced emf in the loop at t = 2s and t = 10s.



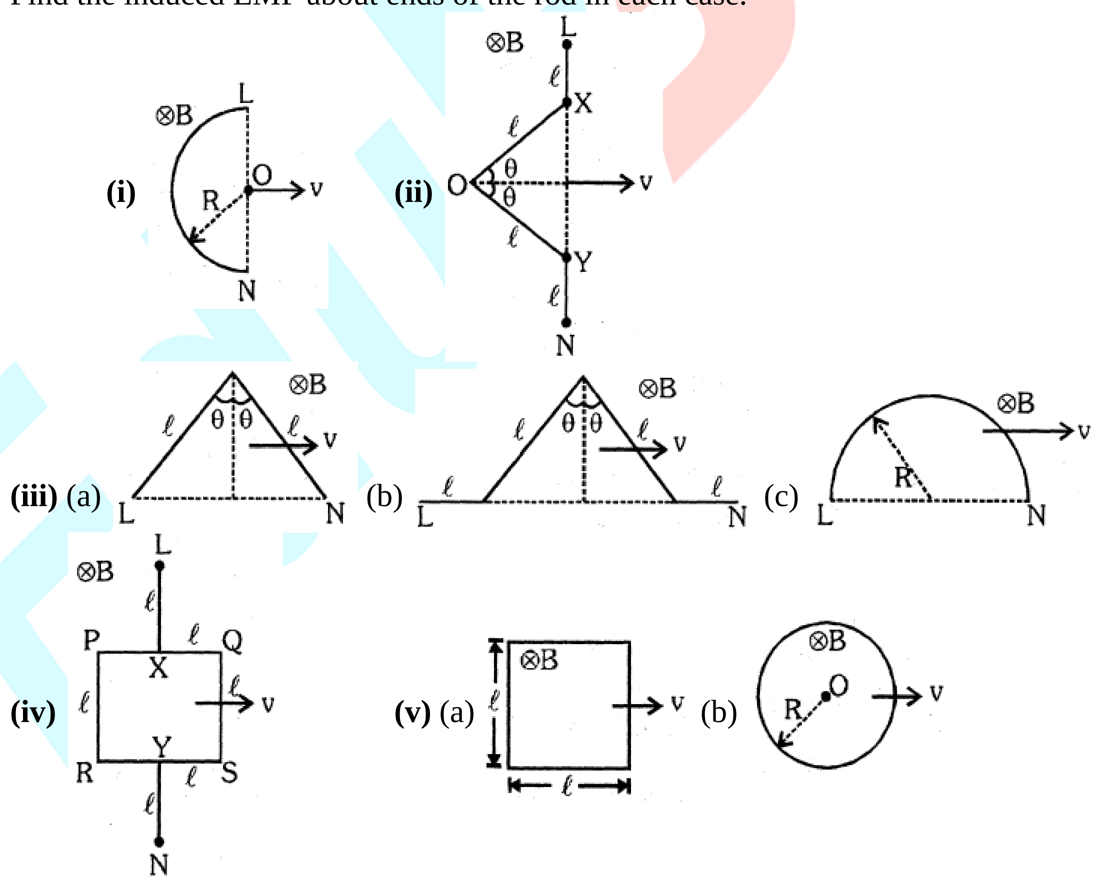
(1) 3×10^{-2}, zero (2) 3×10^{-2}, 3×10^{-4} (3) 3×10^{-4}, 3×10^{-4} (4) 3×10^{-4}, zero

6. A conducting rod moves towards right with constant velocity v in uniform transverse magnetic field. Graph between force applied by the external agent v/s velocity and power supplied by the external agent v/s velocity.

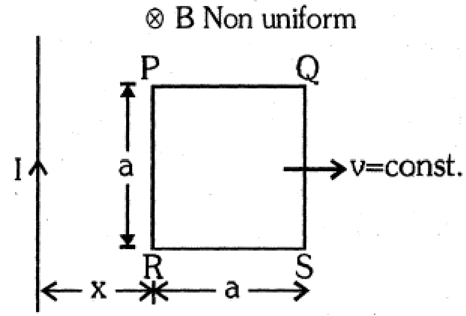


(1) St. line, parabola (2) Parabola, st. line (3) St. line, St. line (4) Parabola, Parabola

7. Find the induced EMF about ends of the rod in each case.



8. Find the EMF induced in metal loop when it moves in non–uniform magnetic field



- **MOVING CONDUCTING ROD IN EARTH'S MAGNETIC FIELD**

(Assume angle of declination is zero)

Case–I Placed Horizontally and moves in horizontal plane.

If its ends in
- E – W direction ⇒ B_v cuts
- N – S direction ⇒ B_v cuts

} Dynamic emf :– $e_d = B_v vl$

Case–II Hold vertically and moves in horizontal plane :–

In it moves on
- E – W line ⇒ B_H cuts } Dynamic emf :– $e_d = B_H v\ell$
- N – S line ⇒ No flux cutting ⇒ No Dy. EMI

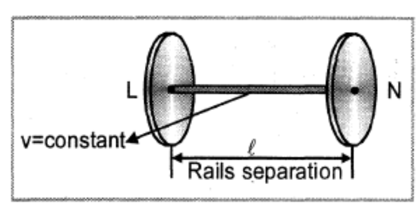
Case–III Placed horizontally and allow to fall under gravity in vertical plane :–

If its ends in
- E – W direction ⇒ B_H cuts } Dynamic emf :– $e_d = B_H v\ell$
- N – S direction ⇒ No flux cutting ⇒ No Dyn. EMI

- **Applications (Hz → Horizontal, V_t → vertical)**

(i) Moving Train (Hz – Hz) :– Induced emf across of moving train is :–

$e_{LN} = B_v v\ell$
- *At equator ⇒ $e_{LN} = 0$ ($B_v = 0$)
- At poles $e_{LN} \to$ max. ($B_v \to$ max)

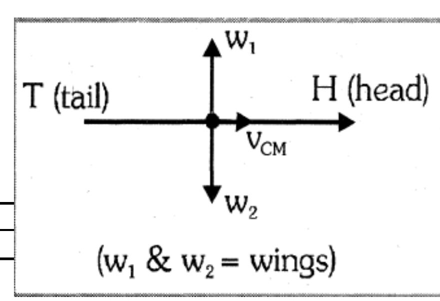


where $B_v = B\sin\theta$, θ angle of dip at that place

v → m/sec.

(ii) Moving Aeroplane :

Motion of aeroplane can be deal as motion of two metal rods (H–T) and (w_1–w_2) which are perpendicular to each other. For (H–T) conductor



$\vec{l} \parallel \vec{v}_{cm}$, so (H–T) conductor never do flux cutting hence no induced emf across (H–T) of aeroplane of its any sort of motion, only ($w_1 - w_2$) conductor can do flux cutting.

(a) When aeroplane flying at a certain height ie parallel to earth surface (Hz – Hz) :

If wings ($w_1 - w_2$) in
- E – W direction $\Rightarrow B_V$ cuts
- N – S direction $\Rightarrow B_V$ cuts

Induced emf across wings of aeroplane given as (both cases) :–

$\boxed{e_{w_1w_2} = B_V l_{w_1w_2} v}$, where $B_V = B\sin\theta$ [θ angle of dip.]

(b) When aeroplane dives vertically (Hz – Vt) :–

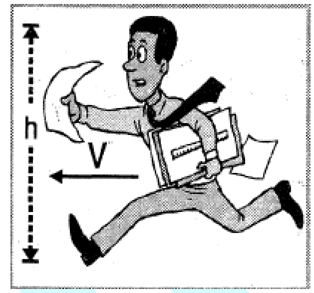
If wings ($w_1 - w_2$) in
- E – W direction $\Rightarrow B_H$ cuts
- N – S direction $\Rightarrow$ No flux cutting $\Rightarrow$ No Dy. EMI

Induced emf across wings of aeroplane given as (only in one case)

$\boxed{e_{w_1w_2} = B_H l_{w_1w_2} v}$, where $B_H = B\cos\theta$ [θ angle of dip.]

(iii) Human body (Vt – Hz) :

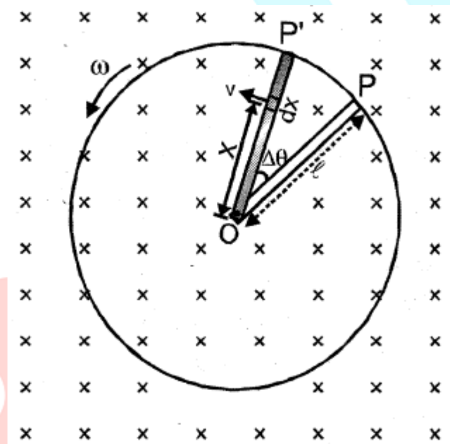
A human body of height 'h' moves with constant velocity v then induced emf between his head and feet, if it moves along :



→ E – W line ⇒ B_H cuts } Dynamic emf ⇒ $\boxed{e_d = B_H vh}$

→ *N – S line ⇒ No flux cutting ⇒ No Dyn. EMI

INDUCED E.M.F. DUE TO ROTATION OF A CONDUCTOR ROD IN A UNIFORM MAGNETIC FIELD



Let a conducting rod is rotating in a magnetic field around an axis passing through its one end, normal to its plane.

Consider an small element dx at a distance x from axis of rotation.

Suppose velocity of this small element = v

So, according to Lorent's formula induced e.m.f. across this small element

$d\varepsilon = Bv.\, dx$

ϴ This small element dx is at distance x from O (axis of rotation)

∴ Linear velocity of this element dx is $v = \omega x$

substitute of value of v in equation (i) $d\varepsilon = B\,\omega\, x\, dx$

Every element of conducting rod is normal to magnetic field and moving in perpendicular direction to the field

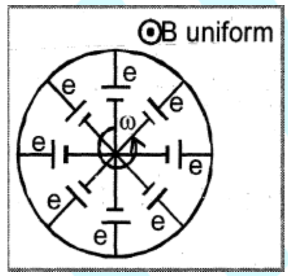
So, net induced e.m.f. across conducting rod $\varepsilon = \int d\varepsilon = \int_0^l B\omega\, x\, dx = \omega B\left(\frac{x^2}{2}\right)_0^l$

or $\varepsilon = \frac{1}{2}B\omega l^2$ $\quad \varepsilon = \frac{1}{2}B \times 2\pi f \times l^2$ [f = frequency of rotation]

$= Bf(\pi l^2)$ area transversed by the rod $A = \pi l^2$ or $\varepsilon = BAf$

GOLDEN KEY POINTS

- During rotational motion of disc it cuts the magnetic flux.
- A metal disc can be assumed to made of uncountable radial conductors. When metal disc rotates in uniform transverse magnetic field these radial conductors cuts the magnetic flux and because of this flux cutting all becomes identical cells each of emf 'e', where $e = \frac{1}{2}B\omega R^2$, as shown in following figure and periphery of disc becomes equipotential.



- All identical cells connected in parallel fashion so net emf

  $e_{net} = e$ (emf of single cell)

  $$e_{net} = \frac{1}{2}B\omega R^2$$
  $$\omega = 2\pi f$$
  , where R is radius of disc.

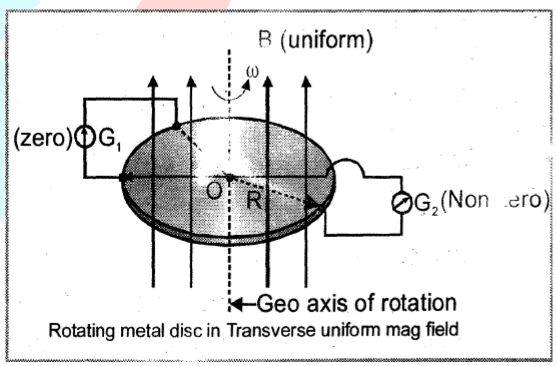
- Net induced emf between centre and rim of disc is $\frac{1}{2}B\omega R^2$.
- Reading of Galvanometers :–

  G–1 : – Its reading is zero if it is connected between any two peripheral points or diametrical opposite ends.

  G–2 :– Its reading is non zero if it is connected between centre and peripheral point.

- Faraday Copper disc generator (Based on Dynamic EMI) :–

  When disk rotates about its axis perpendicular to magnetic field then emf induces about its centre and peripheral points.

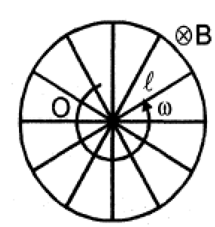


Rotating metal disc in Transverse uniform mag field

ILLUSTRATIONS

Illustration 20.

A conducting cycle wheel with each spoke of length *l*, is rotating about its geometrical axis with uniform angular velocity ω in uniform magnetic field as shown in figure. Find induced emf between its centre and rim.



Solution.

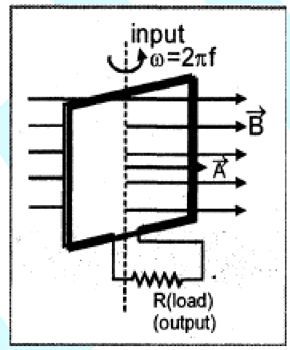
Due to flux cutting each metal spoke becomes identical cell of emf e (say), all such identical cells connected in parallel fashion e_{net} = e (emf of single cell)

$$\boxed{\begin{array}{l} e_{net} = \frac{1}{2}B\omega l^2 \\ \omega = 2\pi f \end{array}}$$

Sp. Note : This emf does not depends on number of spokes ('N') in wheel.

PERIODIC E.M.I. $\left[\frac{d\theta}{dt} \to \frac{d\phi}{dt}\right]$

When a coil, which is placed in uniform magnetic field, rotates with constant angular frequency about shown axis then magnetic flux through the coil changes periodically with respect to time so an emf of periodic nature induced in coil. This phenomenon known ass periodic emi.



- **Magnetic flux through the rotating coil at any instant 't' :–**

  ϕ = NBA cosθ = NGBA cos ωt (as $\theta = \omega t$)

  $\boxed{\phi = \phi_0 \cos\omega t}$ where $\boxed{\phi_0 = NBA}$ $\Rightarrow$ flux amplitude or max. flux

  Sp. Note :– Magnetic flux changes periodically with respect to time.

- Induced emf in rotating coil at any instant 't' :–

  $$e = -\frac{d\phi}{dt} = NBA\omega\sin\omega t$$

  $\boxed{e = e_0 \sin\omega t}$, where $\boxed{e_0 = NBA\omega = \phi_0\omega}$ $\Rightarrow$ emf amplitude or max. emf

- Induced current in load circuit at any instant 't' :–

  $$I = e/R = \frac{e_0}{R}\sin\omega t$$

  $\boxed{I = I_0 \sin\omega t}$, where $\boxed{I_0 = \frac{e_0}{r} = \frac{NBA\omega}{R} = \frac{\phi_0\omega}{R}}$ $\Rightarrow$ Current Amplitude or max. current

- Induced emf also changes in periodic manner thats why this phenomenon called periodic EMI.
- Phase difference between magnetic flux through the coil and induced emf is 90°.

  (a) When plane of coil perpendicular to $\vec{B}$ $\Rightarrow$ f_{max} and e_{min} = 0

  (b) When plane of coil parallel to $\vec{B}$ $\Rightarrow$ 0 and e_{max}
- Induced emf and current acquires their max and min values simultaneously ie phase difference between both induced parameters is zero.
- Frequency of induced parameter = Rotational frequency of coil = f.

- Induced emf and current changes their value with respect to time according to sine function, hence they called as sinusoidal induced quantities.

MAIN APPLICATIONS OF E.M.I. :

(A) Generator (G) → Based on periodic EMI

(B) DC Motor

(C) Transformer (T) → Based on Mutual induction (Static EMI)

Sp. points :–

(a) There is no mechanical losses for transformer, because it has no moving element and hence efficiency of transformer is higher than Generator and motor.

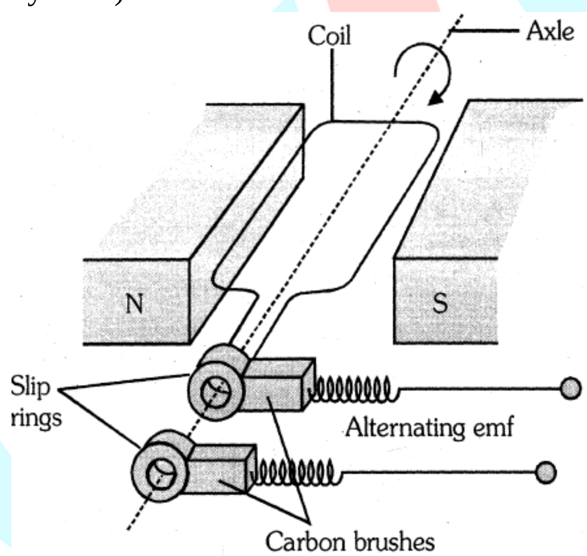
(b) Common losses for Generator and transformer

(i) Joule heating losses or Cu losses

(ii) Iron losses : (a) Eddy currents losses (b) Hysteresis losses

(iii) Flux leakage losses

(A) Generator (or Dynamo) :–

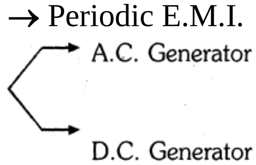


AC generator

(i) Work :– It converts mechanical energy into electrical energy.

(ii) Working principle → Periodic E.M.I.

(iii) Types of Generator



(Acc. to output)

(iv) Generator has basic three sections

(a) Armature circuit (Internal circuit)

(b) Conveyor system (Connector of two circuit)

(c) Load circuit (External circuit)

(v) Basic difference between A.C. G. and D.C. G. in conveyor system.

For A.C. ⇒ In conveyor system → **Slip Rings**, Electric Brushes

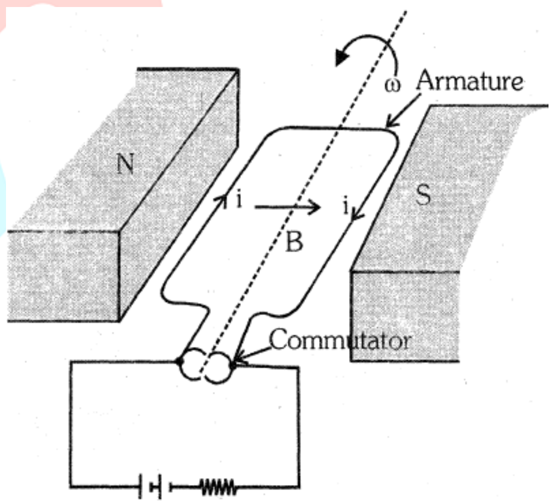
For D.C. ⇒ In conveyor system → **Split Rings (Commutator)**, Electric Brushes

GOLDEN KEY POINTS

- Special chart for Rotating coil

Phy. parameter	Equation	Max value	Frequency ($f = \omega/2\pi$)
(a) Magnetic flux	$\phi = \phi_0 \cos\omega t$	$\phi_0 = NBA$	f
(b) Induced emf	$e = e_0 \sin\omega t$	$e_0 = NBA\omega$	f
(c) Induced current	$I = I_0 \sin\omega t$	$I_0 = \dfrac{NBA\omega}{R}$	f

(B) DC Motor :



- It's a device, that converts electrical energy into mechanical energy (rotational energy).
- Principle : When current carrying coil is placed in magnetic field it experiences a torque.
- Working : When DC motor is connected to a dc source, a current flow in coil, which reverses it direction in regular intervals so that magnetic torque act on coil in same direction. The direction of current is reversed by commutator. Due to rotation of coil in magnetic field, magnetic flux linked with coil changes with time hence E.M.F. is induced, which opposes the current in the coil. This e.m.f. is known as back emf.

If coil of N turns, each of area A rotates with constant angular velocity. Then peak value of back e.m.f. is given by :

$e_0 = NBA\omega$

i.e. $e_0 \propto \omega$

- Equation of motor (D.C.) :

$E - e = iR$

At $t = 0$, when motor is switched ON, $\omega = 0$

$\Rightarrow$ e = 0 and $i = \frac{E}{R}$ = Maximum

as t↑, ω↑ ⇒ e↑ but i↓

and when motor rotate with maximum angular speed, current is minimum.

$$i_{min} = \frac{E - e_{maximum}}{R}$$

- Efficiency of motor :

$$\eta = \frac{P_{out}}{P_{in}} \times 100 = \frac{ei}{Ei} \times 100 \qquad \boxed{\eta = \frac{e}{E} \times 100}$$

Note : When $e = \frac{E}{2}$, then efficiency of motor is maximum.

$\eta_{maximum}$ = 50%

ILLUSTRATIONS

Illustration 21.

A circular coil of radius 8.0 cm and 20 turns rotates about its vertical diameter with an angular speed of 50 s^{-1} in a uniform horizontal magnetic field of magnitude 3×10^{-2} T. Obtain the maximum and average induced emf in the coil. If the coil forms and closed loop of resistance 10Ω, how much power is dissipated as heat? What is the source of this power?

Solution

Induced emf in coil :–

e = NBAωsinωt

$e_{max} = NBA\omega = NB(\pi r^2)\omega = 20 \times 3.0 \times 10^{-2} \times \pi \times 64 \times 10^{-4} \times 50 = 0.603$ V

e_{avg} is zero over a one cycle

$$I_{max} = \frac{e_{max}}{R} = \frac{0.603}{10} = 0.0603 \text{ A}$$

$$P_{avg} = \frac{I^2_{maxR}}{2} = 0.018\text{W}$$

The induced current causes a torque opposing the rotation of the coil. An external agent (rotor) must supply torque (and do work) to counter this torque in order to keep the coil rotating uniformly. Thus, the source of the power dissipated as heat in the coil is the external rotor.

Illustration 22.

As a.c. generator consists of a coil of 50 turns and area 2.5 m^2 rotating at an angular speed of 60 rad sec^{-1} in a uniform magnetic field B = 0.30 T between two fixed pole pieces. The resistance of the circuit including that of the coil is 500Ω.

(a) Calculate the maximum current drawn from the generator.

(b) What will be the orientation of the coil w.r.t. the magnetic field to have (i) maximum magnetic flux (ii) zero magnetic flux.

(c) Would the generator work if the coil were stationary and instead the poles were rotated with same speed as above.

Solution

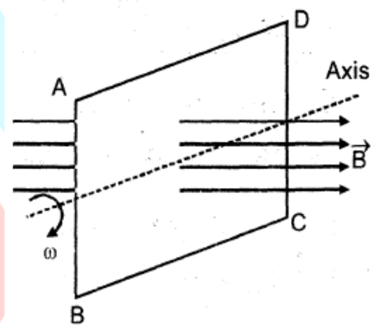
(a) Maximum current, $I_{max} = \frac{e_{max}}{R} = \frac{NBA\omega}{R} = \frac{50\times 0.3\times 2.5\times 60}{500} = 4.5$ A

(b) Flux is maximum, when plane of coil is perpendicular to the magnetic field. Flux is zero when plane of coil is parallel to the magnetic field.

(c) Yes, it will work.

BEGINNER'S BOX–8

1. A rectangular coil ABCD is rotated in uniform magnetic field with constant angular velocity about its one of the diameter as shown in figure. The induced emf will be maximum, when the plane of the coil is :–



(1) Perpendicular to the magnetic field
(2) Making an angle of 30° with the magnetic field.
(3) making an angle of 45° with the magnetic field.
(4) Parallel to the magnetic field.

2. A rectangular coil has 60 turns and its length and width is 20 cm and 10 cm respectively. The coil rotates at a speed of 1800 rotation per minute in a uniform magnetic field of 0.5 T about its one of the diameter. The maximum induced emf will be :

(1) 98 V (2) 110 V (3) 113 V (4) 118 V

(C) TRANSFORMER

(i) **Working principle :–** Mutual induction

(ii) **Transformer has basic two section :–**

(a) Shell :– It consist of primary and secondary coils of copper. The effective resistance between primary and secondary coil is infinite because electric circuit between two is open ($R_{ps} = \infty$)

(b) Core :– Both Cu coils are tightly wound over a bulk metal piece of high magnetic permeability (eg. soft iron) called core. Both coils are electrically insulated to core but core part magnetically coupled to both the coils.

(iii) **Work :–** It regulates A.C. voltage and transfers the electrical power without change in frequency of input supply. (The alternating current changes itself)

(iv) Special Points :–

- It can't work with D.C. supply. If a battery is connected to its primary then output is across secondary is always zero i.e. No working of transformer.
- It can't called 'Amplifier' as it has no power gain like **transistor**.
- It has no moving part hence there are no mechanical losses in transformer, so it efficiency is higher than generator and motor.

(v) Types (According to voltage regulation) :–

Type
- Step up transformer (S.U.T), $N_s > N_p$
- Step down transformer (S.D.T) $N_s < N_p$

(vi) S.U.T. $\Rightarrow$ converts **low voltage, high current** in to **high voltage, low current**

S.D.T. $\Rightarrow$ converts **high voltage, low current** in to **low voltage, high current**

(vii) Power transmission is carried out always at "High voltage, low current" so that voltage drop and power losses are minimum in transmission line.

$\boxed{\text{Voltage drop} = I_L R_L}$, I_L : Line current, R_L : total line resistance,

$$I_L = \frac{\text{Power to be transmitted}}{\text{Line voltage}}$$

$\boxed{\text{Power losses} = I_L^2 R_L}$

(viii) Sending power always at high voltage & low current (By S.U.T.) and Receiving power always at low voltage & high current (BY S.D.T.)

(ix) High voltage coil having more number of turns and always **made of thin wire** and high current coil having less number of turns and always **made of thick wires**.

(x) Ideal Transformer : (η = 100%)

(a) No flux leakage :–

$$\phi_s = \phi_p \Rightarrow \frac{-d\phi_s}{dt} = \frac{-d\phi_p}{dt}$$

$e_s = e_p = e$ induced emf per turn of each coil is also same.

total induced emf for secondary $E_s = N_s e$

total induced emf for primary $E_p = N_p e$

$$\frac{E_s}{E_p} = \frac{N_s}{N_p} = n \text{ or } p \;.....(1)$$

where
- n : turn ratio
- p : transformation ratio

(b) No load condition :–

$V_P = E_P$ & $E_S = V_S$

$$\boxed{\frac{V_S}{V_P} = \frac{N_S}{N_P}} \quad \ldots.(2)$$

$$\boxed{\frac{V_S}{V_P} = \frac{N_S}{N_P} = n \text{ or } p} \quad \ldots.(3) \quad [\text{from } (1) \,\&\, (2)]$$

(c) No power loss :–

$P_{out} = P_{in}$

$V_S I_S = V_P I_P$

$$\boxed{\frac{V_S}{V_P} = \frac{I_P}{I_S}} \quad \ldots(4)$$

from equation (3) & (4)

$$\boxed{\frac{V_S}{V_P} = \frac{I_P}{I_S} = \frac{N_S}{N_P} = n \text{ or } p}$$

Sp. Note : Generally transformers deals in ideal condition i.e. $P_{in} = P_{out}$, if other information are not given.

(xi) Real transformer ($\eta \neq 100\%$) :– Some power is always lost due to flux leakage, hysteresis, eddy currents, and heating of coils. hence $P_{out} < P_{in}$ always.

Efficiency of transformer $\boxed{\eta = \frac{P_{out}}{P_{in}} = \frac{V_S I_S}{V_P I_P} \times 100}$

Losses in Transformer :–

(i) Copper or joule heating losses :–

Where : These losses occurs in both coils of shell part.

Reason : Due to heating effect of current ($H = I^2Rt$).

Remedy : To minimise these losses, high current coil always made up with thick wire and for removal of produced heat, circulation of mineral oil should be used.

(ii) Flux leakage losses :–

Where : These losses occurs in between both the coil of shell part.

Cause : Due to air gap between both the coils.

Remedy : To minimise these losses both coils are tightly wound over a common soft iron core (high magnetic permeability) so a closed path of magnetic field lines formed itself within the core and tires to makes coupling factor $K \to 1$

(iii) Iron losses :–

Where : These losses occurs in core part.

Types :
- (i) Hysteresis losses
- (ii) Eddy currents losses

(a) Hysteresis losses :–

Cause : Transformer core always present in the effect of alternating magnetic field ($B = B_0 \sin\omega t$) so it will magnetised & demagnetised with very high frequency (f = 50 Hz).

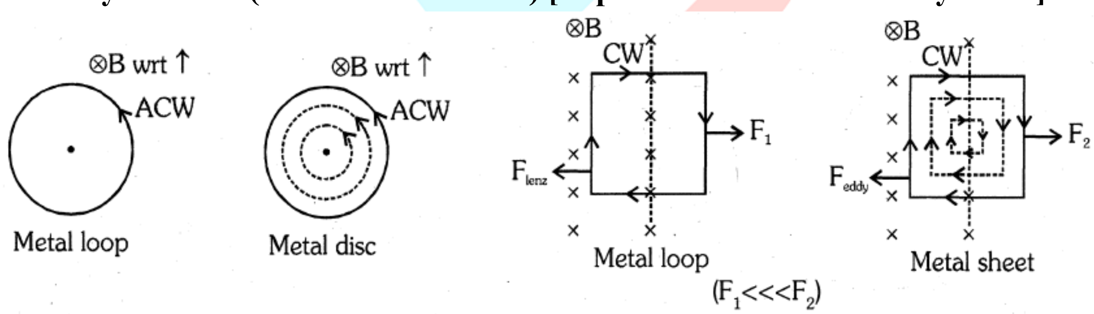
During its demagnetization a part of magnetic energy left inside core part in form of residual magnetic field. Finally this residual energy waste as heat.

Remedy : To minimise these losses material of transformer core should be such that it can be easily magnetised & demagnetised. For this purpose magnetic soft materials should be used.

Ex. : soft Iron { Low retentivity, Low coercivity

(b) Eddy current losses :

*** Eddy currents (or Focalt's currents) [Experimental verification by focalt]**



Def. : It is a group of induced currents which are produced, when metal bodies placed in time varying magnetic field or they moves in external magnetic field in such a way that flux through them changes with respect to time.

GOLDEN KEY POINTS

(i) These currents are produced only in closed path within the entire volume and on the surface of metal body. Therefore their measurement is imossible.

(ii) Circulation plane of these currents is always perpendicular to the external magnetic field direction.

(iii) Generally resistance of metal bodies is low so magnitude of these currents is very high.

(iv) These currents can heat up the metal body and some time body will melt out (Application : Induction furnace)

(v) Due to these induced currents a strong eddy force (or torque) acts on metal body which always opposes the translatory (or rotatory) motion of metal body, according to Lenz law.

(vi) Transformer :

Cause : Transformer core is always present in the effect of alternating magnetic field ($B = B_0 \sin\omega t$). Due to this eddy currents are produced in its volume, so a part of magnetic energy of core is wasted as heat.

Remedy: To minimise these losses transformer core should be laminated with the help of lamination process, circulation path of eddy current is greatly reduced & net resistance of system is greatly increased. So these currents become feeble.

Application of eddy currents :–

(i) Induction furnace
(ii) Dead beat galvanometer
(iii) Electric Brakes
(iv) Induction motor
(v) Car speedometer
(vi) Energy meter

ILLUSTRATIONS

Illustration 23.

A power transmission line feeds input power at 2300 V to a step down transformer having 4000 turns in its primary. What should be the number of turns in the secondary to get output power at 230 V?

Solution

$V_p = 2300$ V; $N_P = 4000$, $V_S = 230$ V

$$\boxed{\frac{V_S}{V_P} = \frac{N_S}{N_P}} \qquad \therefore \qquad N_S = N_P \times \frac{V_S}{V_P} = 4000 \times \frac{230}{2300} = 400$$

Illustration 24.

The output voltage of an ideal transformer, connected to a 240 V a.c. mains is 24 V. When this transformer is used to light a bulb with rating 24V, 24W calculate the current in the primary coil of the circuit.

Solution

$V_P = 240$ V $\qquad V_S = 24$ V, $\qquad V_S I_S = 24$ W

Current in primary coil $I_P = \dfrac{V_S I_S}{V_P} = \dfrac{24}{240} = 0.1$A

BEGINNER'S BOX-9

1. Why the core of transformer is laminated?

2. A step down transformer is used to reduce the main supply of 220 V to 11 V. If the primary coil draws a current of 5A and the current in secondary coil 90A, What is the efficiency of the transformer?

3. Why can't transformer be used to step up d.c. voltage?

4. Write two applications of eddy currents.

ANSWERS

Beginner's Box–1

1. 5 × 10–3 Weber **2.** –NBA

3. Face ABCD ⇒ $+Ba^2$, Face EFGH ⇒ $-Ba^2$, Remaining faces ⇒ Zero

4. 0.02 Wb **5.** –0.1 Wb **6.** 29.6×10^{-6} Wb

Beginner's Box–2

1. **(i)** Anticlockwise **(ii)** Clockwise
(iii) A – Positive charge, B – Negative charge **(iv)** Anticlockwise
(v) Anticlockwise **(vi)** No induced current
(vii) (a) Anticlockwise **(b)** Anti clockwise in bigger loop & clockwise in smaller loop
(c) Anti clockwise in bigger loop & clockwise in smaller loop
(d) Anticlockwise in both loop & through connecting wire zero current
(viii) Anticlockwise

2. **(i) (a)** Anticlockwise (ACW), **(b)** Clockwise (CW)
(ii) N to L
(iii) Plate A – Positive charge, Plate B- Negative charge

BEGINNER'S BOX–3

1. (4) **2.** (2) **3.** (4) **4.** (3) **5.** (2) **6.** (2) **7.** (1)
8. (2) **9.** (2)

BEGINNER'S BOX–4

1. (3) **2.** (2) **3.** (1) **4.** (2) **5.** (1) **6.** (4) **7.** (4)

BEGINNER'S BOX–5

1. (4) **2.** (2) **3.** (1) **4.** (1) **5.** (4) **6.** (1) **7.** (4)
8. (4) **9.** (2)

BEGINNER'S BOX–6

1. (1) **2.** (2) **3.** (2) **4.** (2) **5.** (3) **6.** (1) **7.** (4)
8. (i) (a) ACW, (b) Zero, (c) CW
(ii) (a) ACW, (b) Zero, (c) CW
(iii) (a) L to N, (b) Zero, (c) N to L

BEGINNER'S BOX–7

1. (i) No induced EMF (ii) B outwards
(iii) B inwards (iv) Direction of current Q to P

(v) Low potential is N (vi) Y south & X north
(vii) X south; Y north

2. (1) **3.** (1) **4.** (3) **5.** (4) **6.** (1)

7. (i) 2 BvR, (ii) 2 Bvl (1 + sin θ), (iii) (a) Zero, (b) Zero, (c) Zero (iv) 3 Bvl
(v) (a) Zero; (b) Zero

8. $e_{net} = \dfrac{\mu_0 Iva^2}{2\pi x(x+a)}$

BEGINNER'S BOX–8

1. (4) **2.** (3)

BEGINNER'S BOX–9

1. To reduce eddy current
2. 90%
3. Working of transformer is based on mutual induction
4. Application of eddy current
(i) Induction furnace,
(ii) Electric Brakes